MATH 6250

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Functions

```
norm[j_] := FullSimplify[√(TrigReduce[j[[1]]^2 + j[[2]]^2 + j[[3]]^2])]
 (* Mathematica norm sucks *)
```

```
In[127]:= frenet[x_] := CellPrint[{
    Cell[
     TextData[{"T =", Cell[BoxData[ToBoxes[k = FullSimplify[D[x, s]]]]]}], "Text"],
    Cell[TextData[{"κ =", Cell[BoxData[ToBoxes[l = FullSimplify[norm[D[k, s]]]]]]}],
     "Text"],
    Cell[TextData[{"N =", Cell[BoxData[ToBoxes[m = FullSimplify[D[k, s] / l]]]]}],
     "Text"],
    Cell[TextData[{"B =", Cell[BoxData[
         ToBoxes[r = FullSimplify[Cross[D[x, s], m]]]]]}], "Text"],
    Cell[TextData[{"τ =", Cell[BoxData[ToBoxes[FullSimplify[D[m, s].r]]]]}],
     "Text"]
   }]
```

```
In[149]:= nonarcfrenet[x_] := CellPrint[{
    Cell[TextData[{"T =",
       Cell[BoxData[ToBoxes[k = FullSimplify[D[x, s] / norm[D[x, s]]]]]]}], "Text"],
    Cell[TextData[{"κ =", Cell[BoxData[ToBoxes[l = FullSimplify[norm[D[k, s]]]]]]}],
     "Text"],
    Cell[TextData[{"N =", Cell[BoxData[ToBoxes[m = FullSimplify[D[k, s] / l]]]]}],
     "Text"],
    Cell[TextData[{"B =", Cell[BoxData[
         ToBoxes[r = FullSimplify[Cross[D[x, s], m]]]]]}], "Text"],
    Cell[TextData[{"τ =", Cell[BoxData[ToBoxes[FullSimplify[D[m, s].r]]]]}],
     "Text"]
   }]
```

Test

Problem 1

A

In[131]:= `q[s_] := {1/Sqrt[2] Cos[s], 1/Sqrt[2] Cos[s], Sin[s]}`

$$q[s\_] := \left\{\frac{1}{\sqrt{2}} \text{Cos}[s], \frac{1}{\sqrt{2}} \text{Cos}[s], \text{Sin}[s]\right\}$$

In[132]:= `frenet[q[s]]`

$$T = \left\{-\frac{\text{Sin}[s]}{\sqrt{2}}, -\frac{\text{Sin}[s]}{\sqrt{2}}, \text{Cos}[s]\right\}$$

$$\kappa = 1$$

$$N = \left\{-\frac{\text{Cos}[s]}{\sqrt{2}}, -\frac{\text{Cos}[s]}{\sqrt{2}}, -\text{Sin}[s]\right\}$$

$$B = \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right\}$$

$$\tau = 0$$

B

In[133]:=

$$q[s\_] := \left\{\backslash[\text{RawEscape}]\ \sqrt{1+s^2}, \text{Log}\left[s+\sqrt{1+s\wedge 2}\right], 0\right\}$$

`frenet[q[s]]`

$$T = \left\{\frac{\backslash[\text{RawEscape}]}{(1+s^2)^{3/2}}, -\frac{s}{(1+s^2)^{3/2}}, 0\right\}$$

$$\kappa = \sqrt{\frac{\backslash[\text{RawEscape}]^2+s^2}{(1+s^2)^3}}$$

$$N = \left\{\frac{\backslash[\text{RawEscape}]}{(1+s^2)^{3/2}\sqrt{\frac{\backslash[\text{RawEscape}]^2+s^2}{(1+s^2)^3}}}, -\frac{s}{(1+s^2)^{3/2}\sqrt{\frac{\backslash[\text{RawEscape}]^2+s^2}{(1+s^2)^3}}}, 0\right\}$$

$$B = \left\{0, 0, -\frac{\backslash[\text{RawEscape}]}{(1+s^2)\sqrt{\frac{\backslash[\text{RawEscape}]^2+s^2}{(1+s^2)^3}}}\right\}$$

$$\tau = 0$$

C

In[137]:=

$$q[s\_] := \left\{\frac{1}{3}(1+s)^{3/2}, \frac{1}{3}(1-s)^{3/2}, \frac{1}{\sqrt{2}} s\right\}$$

In[138]:= `frenet[q[s]]`

$$T = \left\{\frac{\sqrt{1+s}}{2}, -\frac{\sqrt{1-s}}{2}, \frac{1}{\sqrt{2}}\right\}$$

$$\kappa = \frac{1}{2}\sqrt{\frac{1}{2-2 s^2}}$$

$$N = \left\{ \frac{1}{2\sqrt{1+s}\sqrt{\frac{1}{2-2s^2}}}, \frac{1}{\sqrt{2-2s}\sqrt{\frac{1}{1-s^2}}}, 0 \right\}$$

$$B = \left\{ -\frac{1}{2\sqrt{1-s}\sqrt{\frac{1}{1-s^2}}}, \frac{1}{2\sqrt{1+s}\sqrt{\frac{1}{1-s^2}}}, \sqrt{\frac{1}{2-2s^2}}\sqrt{1-s^2} \right\}$$

$$\tau = \frac{1}{2\sqrt{2-2s^2}}$$

Problem 3

A

In[139]:= $q[s\_] := \left\{ \frac{1}{3}(1+s)^{3/2}, \frac{1}{3}(1-s)^{3/2}, \frac{1}{\sqrt{2}} s \right\}$

In[140]:= `frenet[q[s]]`

$$T = \left\{ \frac{\sqrt{1+s}}{2}, -\frac{\sqrt{1-s}}{2}, \frac{1}{\sqrt{2}} \right\}$$

$$\kappa = \frac{1}{2}\sqrt{\frac{1}{2-2s^2}}$$

$$N = \left\{ \frac{1}{2\sqrt{1+s}\sqrt{\frac{1}{2-2s^2}}}, \frac{1}{\sqrt{2-2s}\sqrt{\frac{1}{1-s^2}}}, 0 \right\}$$

$$B = \left\{ -\frac{1}{2\sqrt{1-s}\sqrt{\frac{1}{1-s^2}}}, \frac{1}{2\sqrt{1+s}\sqrt{\frac{1}{1-s^2}}}, \sqrt{\frac{1}{2-2s^2}}\sqrt{1-s^2} \right\}$$

$$\tau = \frac{1}{2\sqrt{2-2s^2}}$$

B

In[146]:= $q[t\_] := \left\{ \frac{1}{2} e^t (\mathrm{Sin}[t] + \mathrm{Cos}[t]), \frac{1}{2} e^t (\mathrm{Sin}[t] - \mathrm{Cos}[t]), e^t \backslash .1d \right\}$

In[150]:= `nonarcfrenet[q[s]]`

$$T = \left\{ \frac{e^s \mathrm{Cos}[s]}{\sqrt{(1+\backslash.1d^2)\, e^{2s}}}, \frac{e^s \mathrm{Sin}[s]}{\sqrt{(1+\backslash.1d^2)\, e^{2s}}}, \frac{\backslash.1d\, e^s}{\sqrt{(1+\backslash.1d^2)\, e^{2s}}} \right\}$$

$$\kappa = \sqrt{\frac{1}{1+\backslash.1d^2}}$$

$$N = \left\{ -\frac{e^s \mathrm{Sin}[s]}{\sqrt{\frac{1}{1+\backslash.1d^2}}\sqrt{(1+\backslash.1d^2)\, e^{2s}}}, \frac{e^s \mathrm{Cos}[s]}{\sqrt{\frac{1}{1+\backslash.1d^2}}\sqrt{(1+\backslash.1d^2)\, e^{2s}}}, 0 \right\}$$

$$B=\left\{-\backslash .1d\sqrt{\frac{1}{1+\backslash .1d^2}}\sqrt{(1+\backslash .1d^2)\,e^{2s}}\,\text{Cos}[s],\; -\backslash .1d\sqrt{\frac{1}{1+\backslash .1d^2}}\sqrt{(1+\backslash .1d^2)\,e^{2s}}\,\text{Sin}[s],\; \sqrt{\frac{1}{1+\backslash .1d^2}}\sqrt{(1+\backslash .1d^2)\,e^{2s}}\right\}$$

$$\tau = \backslash .1d\, e^s$$

C

In[153]:= `q[t_] := {Sqrt[1 + t^2], t, Log[t + Sqrt[1 + t^2]]}`

$$q[t\_] := \left\{\sqrt{1+t^2},\, t,\, \text{Log}\left[t+\sqrt{1+t^2}\right]\right\}$$

In[154]:= `nonarcfrenet[q[s]]`

$$T=\left\{\frac{s}{\sqrt{2}\sqrt{1+s^2}},\, \frac{1}{\sqrt{2}},\, \frac{1}{\sqrt{2}\sqrt{1+s^2}}\right\}$$

$$\kappa=\frac{\sqrt{\frac{1}{(1+s^2)^2}}}{\sqrt{2}}$$

$$N=\left\{\sqrt{\frac{1}{(1+s^2)^2}}\sqrt{1+s^2},\, 0,\, -s\sqrt{\frac{1}{(1+s^2)^2}}\sqrt{1+s^2}\right\}$$

$$B=\left\{-s\sqrt{\frac{1}{(1+s^2)^2}}\sqrt{1+s^2},\, \sqrt{\frac{1}{(1+s^2)^2}}\,(1+s^2),\, -\sqrt{\frac{1}{(1+s^2)^2}}\sqrt{1+s^2}\right\}$$

$$\tau=\frac{1}{1+s^2}$$

D

In[156]:= $q[t\_] := \{e^t\,\text{Cos}[t],\, e^t\,\text{Sin}[t],\, e^t\}$

In[157]:= `nonarcfrenet[q[s]]`

$$T=\left\{\frac{e^s\,(\text{Cos}[s]-\text{Sin}[s])}{\sqrt{3}\sqrt{e^{2s}}},\, \frac{e^s\,(\text{Cos}[s]+\text{Sin}[s])}{\sqrt{3}\sqrt{e^{2s}}},\, \frac{e^s}{\sqrt{3}\sqrt{e^{2s}}}\right\}$$

$$\kappa=\sqrt{\frac{2}{3}}$$

$$N=\left\{-\frac{e^s\,(\text{Cos}[s]+\text{Sin}[s])}{\sqrt{2}\sqrt{e^{2s}}},\, \frac{e^s\,(\text{Cos}[s]-\text{Sin}[s])}{\sqrt{2}\sqrt{e^{2s}}},\, 0\right\}$$

$$B=\left\{\frac{\sqrt{e^{2s}}\,(-\text{Cos}[s]+\text{Sin}[s])}{\sqrt{2}},\, -\frac{\sqrt{e^{2s}}\,(\text{Cos}[s]+\text{Sin}[s])}{\sqrt{2}},\, \sqrt{2}\sqrt{e^{2s}}\right\}$$

$$\tau=e^s$$

E

In[159]:= `q[t_] := {Cosh[t], Sinh[t], t}`

In[160]:= `nonarcfrenet[q[s]]`

$$T = \left\{\frac{\text{Sinh}[s]}{\sqrt{1+\text{Cosh}[2\,s]}}, \frac{\text{Cosh}[s]}{\sqrt{1+\text{Cosh}[2\,s]}}, \frac{1}{\sqrt{1+\text{Cosh}[2\,s]}}\right\}$$

$$\kappa = \sqrt{\frac{1}{1+\text{Cosh}[2\,s]}}$$

$$N = \left\{\text{Cosh}[s]\sqrt{\text{Cosh}[s]^2}\left(\text{Sech}[s]^2\right)^{3/2}, 0, -\sqrt{\text{Cosh}[s]^2}\sqrt{\text{Sech}[s]^2}\,\text{Tanh}[s]\right\}$$

$$B = \left\{-\sqrt{\text{Cosh}[s]^2}\sqrt{\text{Sech}[s]^2}\,\text{Sinh}[s], \left(\text{Cosh}[s]^2\right)^{3/2}\text{Sech}[s]\sqrt{\text{Sech}[s]^2}, -\sqrt{\text{Cosh}[s]^2}\sqrt{\text{Sech}[s]^2}\right\}$$

$$\tau = 1$$

F

In[162]:=
```
q[t_] := {t, t^2/2, t Sqrt[1 + t^2] + Log[t + Sqrt[1 + t^2]]}
```

$$q[t\_] := \left\{t, \frac{t^2}{2}, t\sqrt{1+t^2} + \text{Log}\left[t + \sqrt{1+t^2}\right]\right\}$$

In[163]:=
```
nonarcfrenet[q[s]]
```

$$T = \left\{\frac{1}{\sqrt{5}\sqrt{1+s^2}}, \frac{s}{\sqrt{5}\sqrt{1+s^2}}, \frac{2}{\sqrt{5}}\right\}$$

$$\kappa = \frac{\sqrt{\frac{1}{(1+s^2)^2}}}{\sqrt{5}}$$

$$N = \left\{-s\sqrt{\frac{1}{(1+s^2)^2}}\sqrt{1+s^2}, \sqrt{\frac{1}{(1+s^2)^2}}\sqrt{1+s^2}, 0\right\}$$

$$B = \left\{-2\sqrt{\frac{1}{(1+s^2)^2}}(1+s^2), -2\,s\sqrt{\frac{1}{(1+s^2)^2}}(1+s^2), \sqrt{\frac{1}{(1+s^2)^2}}(1+s^2)^{3/2}\right\}$$

$$\tau = \frac{2}{\sqrt{1+s^2}}$$

G

In[164]:=
```
q[t_] := {1 - Sin[t] * Cos[t], (Sin[t])^2, Cos[t]}
```

In[165]:=
```
nonarcfrenet[q[s]]
```

$$T = \left\{-\frac{\sqrt{2}\,\text{Cos}[2\,s]}{\sqrt{3-\text{Cos}[2\,s]}}, \frac{\sqrt{2}\,\text{Sin}[2\,s]}{\sqrt{3-\text{Cos}[2\,s]}}, -\frac{\sqrt{2}\,\text{Sin}[s]}{\sqrt{3-\text{Cos}[2\,s]}}\right\}$$

$$\kappa = \sqrt{\frac{26-6\,\text{Cos}[2\,s]}{(-3+\text{Cos}[2\,s])^2}}$$

$$N = \left\{-\frac{(-6+\text{Cos}[2\,s])\,\text{Sin}[2\,s]}{(3-\text{Cos}[2\,s])^{3/2}\sqrt{\frac{13-3\,\text{Cos}[2\,s]}{(-3+\text{Cos}[2\,s])^2}}}, -\frac{3-12\,\text{Cos}[2\,s]+\text{Cos}[4\,s]}{2\,(3-\text{Cos}[2\,s])^{3/2}\sqrt{\frac{13-3\,\text{Cos}[2\,s]}{(-3+\text{Cos}[2\,s])^2}}}, -\frac{2\,\text{Cos}[s]}{(3-\text{Cos}[2\,s])^{3/2}\sqrt{\frac{13-3\,\text{Cos}[2\,s]}{(-3+\text{Cos}[2\,s])^2}}}\right\}$$

$$B = \left\{-\frac{2\,\text{Sin}[s]^3}{\sqrt{3-\text{Cos}[2\,s]}\sqrt{\frac{13-3\,\text{Cos}[2\,s]}{(-3+\text{Cos}[2\,s])^2}}}, \frac{-3\,\text{Cos}[s]+\text{Cos}[3\,s]}{2\sqrt{3-\text{Cos}[2\,s]}\sqrt{\frac{13-3\,\text{Cos}[2\,s]}{(-3+\text{Cos}[2\,s])^2}}}, -\frac{2}{\sqrt{3-\text{Cos}[2\,s]}\sqrt{\frac{13-3\,\text{Cos}[2\,s]}{(-3+\text{Cos}[2\,s])^2}}}\right\}$$

$$\tau = 2\left(1+\frac{4}{-13+3\,\mathrm{Cos}[2\,s]}\right)\mathrm{Sin}[s]$$