MATH 6250 Homework 6

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Problem 2

```
AAA = {{EE, FF}, {FF, GG}};
BBB = {{lll, mmm}, {mmm, nnn}};
```

```
AaA = Inverse[AAA]
```

$$\left\{\left\{\frac{GG}{-FF^2+EE\,GG},\ -\frac{FF}{-FF^2+EE\,GG}\right\},\ \left\{-\frac{FF}{-FF^2+EE\,GG},\ \frac{EE}{-FF^2+EE\,GG}\right\}\right\}$$

```
CCC = AaA.BBB
```

$$\left\{\left\{\frac{GG\,lll}{-FF^2+EE\,GG}-\frac{FF\,mmm}{-FF^2+EE\,GG},\ \frac{GG\,mmm}{-FF^2+EE\,GG}-\frac{FF\,nnn}{-FF^2+EE\,GG}\right\},\right.$$
$$\left.\left\{-\frac{FF\,lll}{-FF^2+EE\,GG}+\frac{EE\,mmm}{-FF^2+EE\,GG},\ -\frac{FF\,mmm}{-FF^2+EE\,GG}+\frac{EE\,nnn}{-FF^2+EE\,GG}\right\}\right\}$$

```
FullSimplify[Det[CCC]]
```

$$\frac{mmm^2-lll\,nnn}{FF^2-EE\,GG}$$

Problem 3

A

```
x[u_, v_] := {a Cos[u], a Sin[u], v};
```

```
Surface[x, u, v]
```

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General::stop : Further output of Power::infy will be suppressed during this calculation. >>

$$\text{Ip} = \begin{pmatrix} a^2 & 0 \\ 0 & 1 \end{pmatrix}$$

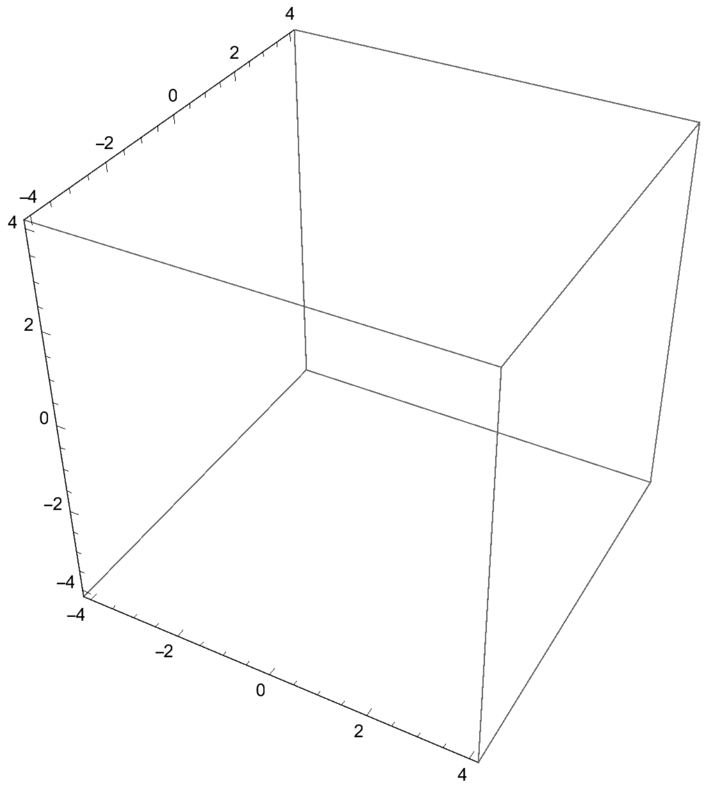
$$IIp = \begin{pmatrix} -\sqrt{a^2} & 0 \\ 0 & 0 \end{pmatrix}$$

$$Sp = \begin{pmatrix} -\frac{1}{\sqrt{a^2}} & 0 \\ 0 & 0 \end{pmatrix}$$

$$K = 0$$

$$H = -\frac{1}{2\sqrt{a^2}}$$

$$Vec = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



B

```
x[u_, v_] := {(a + b Cos[u]) Cos[v], (a + b Cos[u]) Sin[v], b Sin[u]};
Surface[x, u, v]
```

$$Ip = \begin{pmatrix} b^2 & 0 \\ 0 & (a + b\,\mathrm{Cos}[u])^2 \end{pmatrix}$$

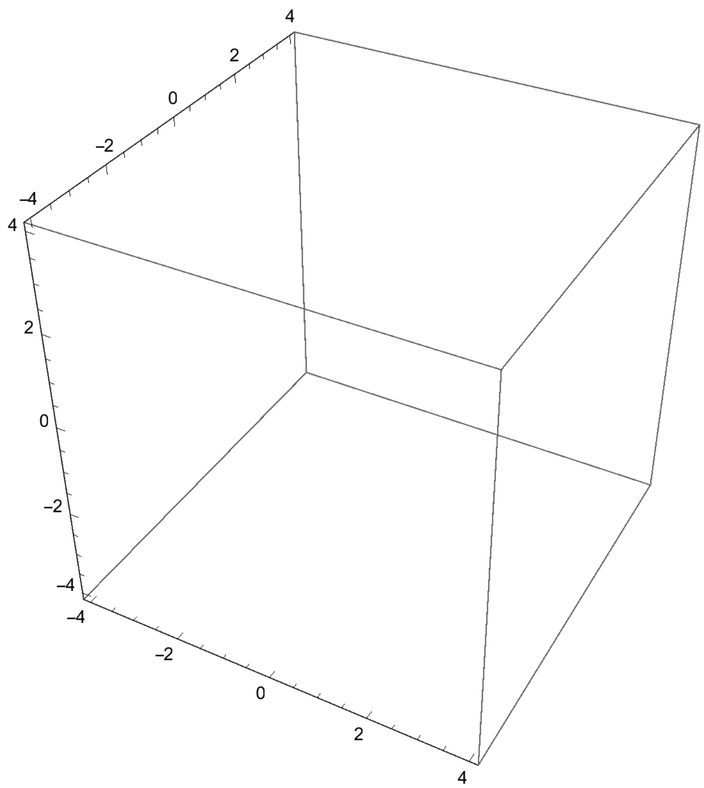
$$IIp = \begin{pmatrix} \frac{\sqrt{b^2\,(a+b\,\mathrm{Cos}[u])^2}}{a+b\,\mathrm{Cos}[u]} & 0 \\ 0 & \frac{\mathrm{Cos}[u]\,\sqrt{b^2\,(a+b\,\mathrm{Cos}[u])^2}}{b} \end{pmatrix}$$

$$Sp = \begin{pmatrix} \frac{a+b\,\mathrm{Cos}[u]}{\sqrt{b^2\,(a+b\,\mathrm{Cos}[u])^2}} & 0 \\ 0 & \frac{b\,\mathrm{Cos}[u]}{\sqrt{b^2\,(a+b\,\mathrm{Cos}[u])^2}} \end{pmatrix}$$

$$K = \frac{1}{b^2 + a\, b\, \mathrm{Sec}[u]}$$

$$H = \frac{a + 2\, b\, \mathrm{Cos}[u]}{2\sqrt{b^2\, (a + b\, \mathrm{Cos}[u])^2}}$$

$$\mathrm{Vec} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



C

```
x[u_, v_] := {u Cos[v], u Sin[v], b v};
```

```
Surface[x, u, v]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

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$$\mathrm{Ip} = \begin{pmatrix} 1 & 0 \\ 0 & b^2 + u^2 \end{pmatrix}$$

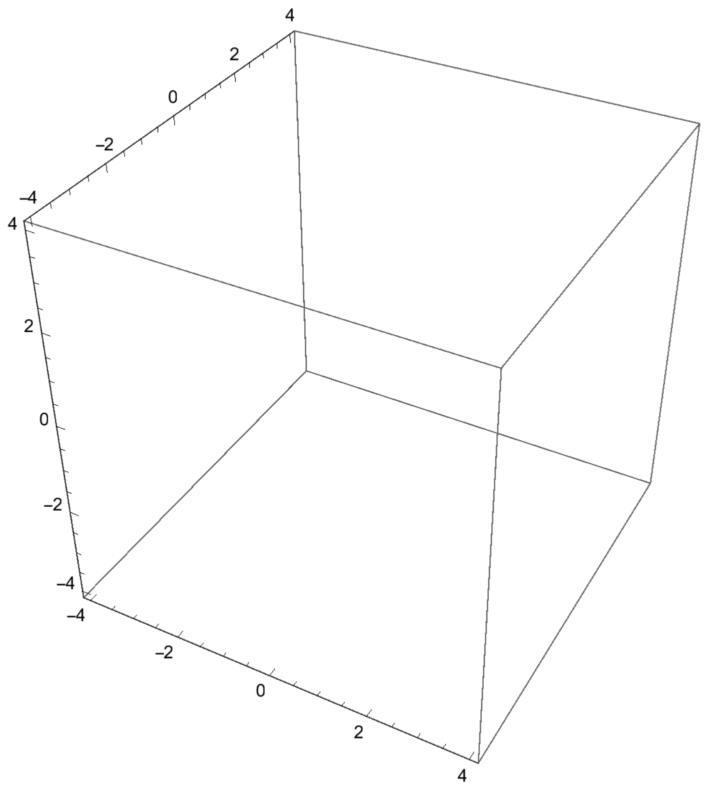
$$\mathrm{IIp} = \begin{pmatrix} 0 & -\frac{b}{\sqrt{b^2+u^2}} \\ -\frac{b}{\sqrt{b^2+u^2}} & 0 \end{pmatrix}$$

$$Sp = \begin{pmatrix} 0 & -\frac{b}{\sqrt{b^2+u^2}} \\ -\frac{b}{(b^2+u^2)^{3/2}} & 0 \end{pmatrix}$$

$$K = -\frac{b^2}{(b^2+u^2)^2}$$

$$H = 0$$

$$Vec = \begin{pmatrix} \sqrt{b^2+u^2} & 1 \\ -\sqrt{b^2+u^2} & 1 \end{pmatrix}$$



D

```
x[u_, v_] := a {Cosh[u] Cos[v], Cosh[u] Sin[v], u}
Surface[x, u, v]
```

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General::stop : Further output of Power::infy will be suppressed during this calculation. ≫

$$Ip = \begin{pmatrix} a^2\,\text{Cosh}[u]^2 & 0 \\ 0 & a^2\,\text{Cosh}[u]^2 \end{pmatrix}$$

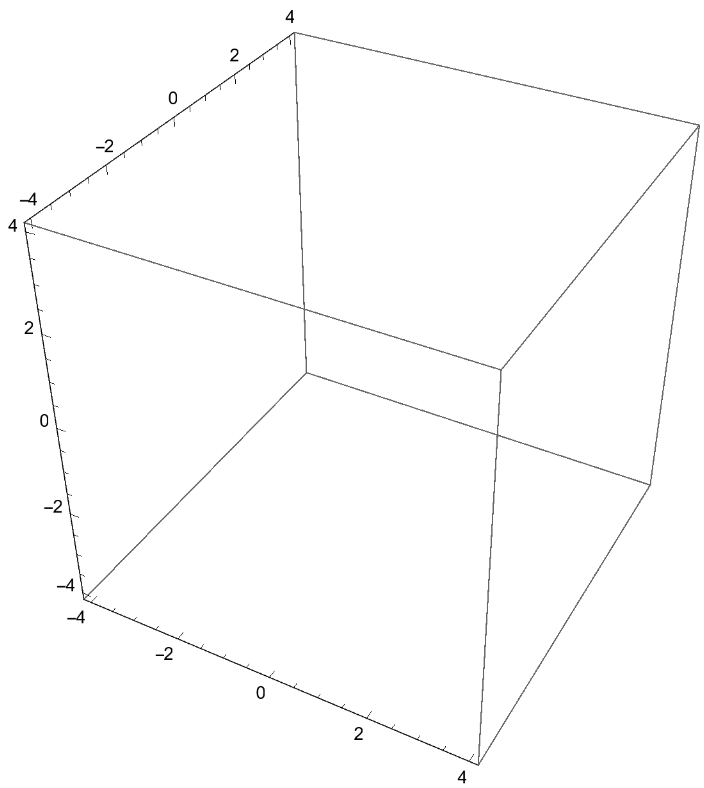
$$\text{IIp} = \begin{pmatrix} -\frac{a^3 \text{Cosh}[u]^2}{\sqrt{a^4 \text{Cosh}[u]^4}} & 0 \\ 0 & \frac{a^3 \text{Cosh}[u]^2}{\sqrt{a^4 \text{Cosh}[u]^4}} \end{pmatrix}$$

$$\text{Sp} = \begin{pmatrix} -\frac{a}{\sqrt{a^4 \text{Cosh}[u]^4}} & 0 \\ 0 & \frac{a}{\sqrt{a^4 \text{Cosh}[u]^4}} \end{pmatrix}$$

$$K = -\frac{\text{Sech}[u]^4}{a^2}$$

$$H = 0$$

$$\text{Vec} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



E

```
x[u_, v_] := {Sech[u] Cos[v], Sech[u] Sin[v], Tanh[u]}

Surface[x, u, v]
```

$$\text{Ip} = \begin{pmatrix} \text{Sech}[u]^2 & 0 \\ 0 & \text{Sech}[u]^2 \end{pmatrix}$$

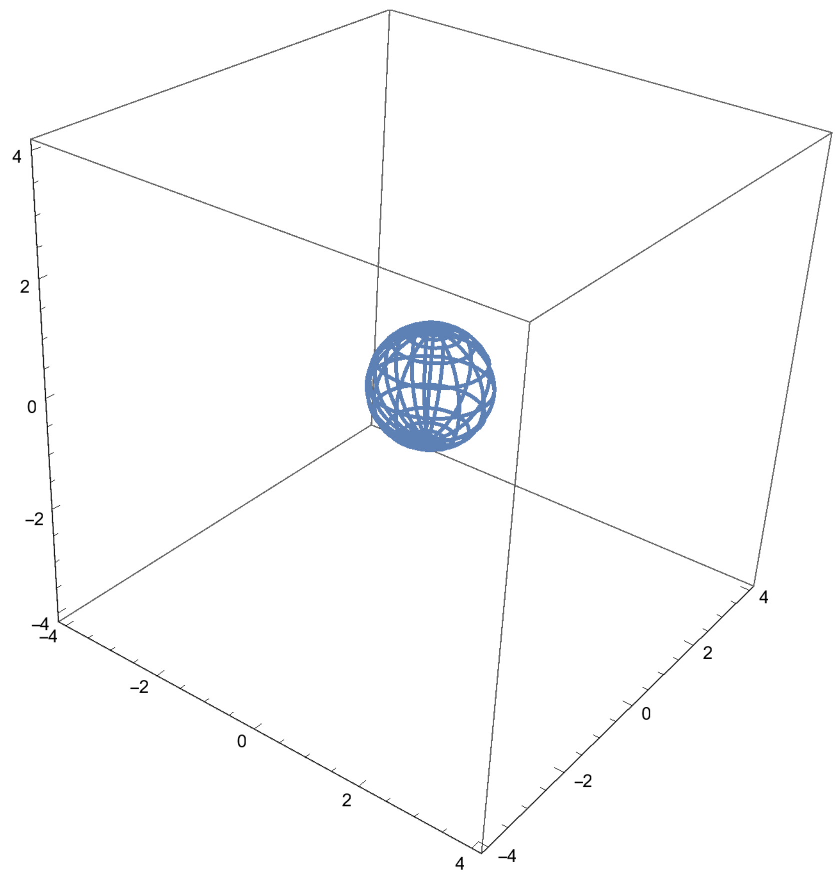
$$\text{IIp} = \begin{pmatrix} \sqrt{\text{Sech}[u]^4} & 0 \\ 0 & \sqrt{\text{Sech}[u]^4} \end{pmatrix}$$

$$Sp = \begin{pmatrix} \text{Cosh}[u]^2 \sqrt{\text{Sech}[u]^4} & 0 \\ 0 & \text{Cosh}[u]^2 \sqrt{\text{Sech}[u]^4} \end{pmatrix}$$

K = 1

$$H = \text{Cosh}[u]^2 \sqrt{\text{Sech}[u]^4}$$

$$Vec = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



F

```
x[u_, v_] := {u - u^3/3 + u v^2, v - v^3/3 + u^2 v, u^2 - v^2};
Surface[x, u, v]
```

$$Ip = \begin{pmatrix} (1 + u^2 + v^2)^2 & 0 \\ 0 & (1 + u^2 + v^2)^2 \end{pmatrix}$$

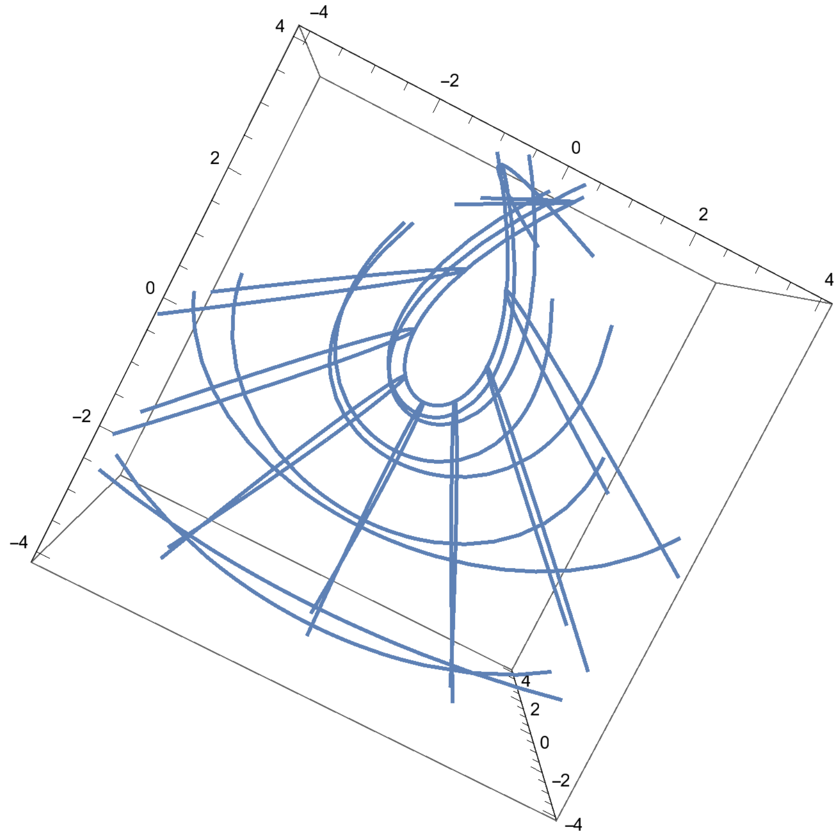
$$IIp = \begin{pmatrix} \frac{2 (1+u^2+v^2)^2}{\sqrt{(1+u^2+v^2)^4}} & 0 \\ 0 & -\frac{2 (1+u^2+v^2)^2}{\sqrt{(1+u^2+v^2)^4}} \end{pmatrix}$$

$$Sp = \begin{pmatrix} \frac{2}{\sqrt{(1+u^2+v^2)^4}} & 0 \\ 0 & -\frac{2}{\sqrt{(1+u^2+v^2)^4}} \end{pmatrix}$$

$$K = -\frac{4}{(1+u^2+v^2)^4}$$

$$H = 0$$

$$Vec = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



Random pseudosphere imaging

```
x[u_, v_] := {u - Tanh[u], Sech[u] Cos[v], Sech[u] Sin[v]};
Surface[x, u, v]
```

$$Ip = \begin{pmatrix} \mathrm{Tanh}[u]^2 & 0 \\ 0 & \mathrm{Sech}[u]^2 \end{pmatrix}$$

$$\text{IIp} = \begin{pmatrix} -\sqrt{\text{Sech}[u]^2\,\text{Tanh}[u]^2} & 0 \\ 0 & \sqrt{\text{Sech}[u]^2\,\text{Tanh}[u]^2} \end{pmatrix}$$

$$\text{Sp} = \begin{pmatrix} -\text{Coth}[u]^2\sqrt{\text{Sech}[u]^2\,\text{Tanh}[u]^2} & 0 \\ 0 & \text{Cosh}[u]^2\sqrt{\text{Sech}[u]^2\,\text{Tanh}[u]^2} \end{pmatrix}$$

$$K = -1$$

$$H = \frac{1}{4}\,(-3+\text{Cosh}[2\,u])\,\text{Coth}[u]^2\sqrt{\text{Sech}[u]^2\,\text{Tanh}[u]^2}$$

$$\text{Vec} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$