Review on Superconducting qubits

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Abstract

This paper is a brief review on the working of superconducting qubits, more specifically the transmon qubits, and how we can physically realize them. Entanglement between qubits is a fundamental aspect that is used in quantum algorithms. We see how we can engineer the interaction Hamiltonian which couples the qubits and also how we can use the familiar harmonic oscillator to model a qubit.

Any physical system once realised has errors and "noise" is one such form of errors. We first build on a theory to study the noises and decoherence which will be encountered in such a system. Then we try and model an actual noise in a real life system.

Acknowledgement

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1 Introduction

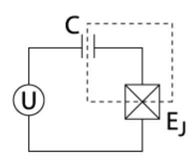
We have discussed the working of superconducting qubits from a physical perspective. Superconducting qubits are currently the most popular technology used to create qubits and are also used by companies like IBM and Google to name a few. Before diving into the workings of a superconducting qubit, we start off with some concepts related to superconductors and all components necessary to construct a superconducting qubit. These include Cooper pairs and the Josephson Junction which we talk about briefly.

We then understand the nature of the qubits and then we see how it can be engineered using circuits. The Hamiltonian is constructed by using a classical analogue and then trying to incorporate the same using quantum mechanics. We see how we can physically realize the interacting Hamiltonian and the quantum mechanical representations of the same.

Study of noise is a critical part for a system such as qubits since it will eventually cause errors during measurements. We start with a general study of noise, by studying its types. We then move towards modeling noise in actual systems by first introducing the Bloch sphere representation. We breifely study the meaning of decoherence and what it looks like for such a system like a superconducting qubit. Finally, the Bloch-Redfield model of decoherence is introduced and we try and measure a few physical quantities such as the longitudinal relaxation time.

2 Charge qubits

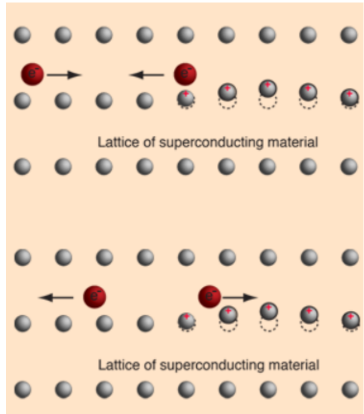
A charge qubit circuit typically has a gate voltage U, a gate capacitance C and a junction capacitance E_j, as shown in the figure attached. Charge states are states which represent the presence/absence of excess Cooper pairs in the island(the dotted line in the image consists of a typical "island"). A qubit whose basis states are these charge states are said to be Charge Qubits. In this section we first discuss about the Cooper pairs and Josephson effect which are fundamental to understanding the charge qubits. Then we move to discuss the typical Hamiltonian of a charge qubit.



2.1 Cooper pairs

The American physicist, Leon Cooper in 1956 for the first time described the term Cooper pair in context of Condensed Matter Physics. They are usually a pair or electrons, but could be any other fermions as well, which are bounded at low temperatures in a certain manner. This manner was described first by Leon Cooper. The BCS theory developed by John Bardeen, Leon Cooper, and John Schrieffer got them the Nobel Prize in 1972, and it discussed how these Cooper pairs were responsible for superconductivity. When a metal transitions into a superconducting state, the electrons sort of condense into a state leaving an energy band gap above them. These pair of electrons were observed to have a small attraction, which caused them to have an energy lower than the Fermi energy. This implied that the pair of electrons were in a bound state. It was found that an electron-phonon interaction was responsible for this attraction. The isotope effect on the superconducting transition temperature provided an experimental evidence for this sort of interaction with the lattice.

The best model to describe this phenomenon is the visual model of Cooper pair attraction. A visual model of the Cooper pair attraction has a passing electron which attracts the lattice, causing a slight ripple toward its path. There will be some other electron moving in the opposite direction which is attracted towards this ripple of lattice. This attraction can be seen as a coupling between the



two electrons. The isotope effect and the condensation to a boson-like state at critical temperature in superconductivity, both support this model of interaction experimentally.

2.2 Josephson Effect

When two superconductors are placed in close proximity separated using a barrier, there is a current produced across the barrier without application of any external voltage. The effect producing this current is termed as the Josephson effect. This current is called the Supercurrent and the device which is flowed across is known as a Josephson Junction.These have important applications in quantum-mechanical circuits, such as superconducting quantum interference device (SQUID) and superconducting qubits.

The following equations are known as the Josephson equations :

$$I(t) = I_c \sin\phi(t)$$

$$\frac{\partial\phi}{\partial t} = \frac{2eV(t)}{\hbar}$$

here V(t) is the voltage across the Josephson Junction and is called the **superconducting phase evolution equation** and I(t) is known as the **weak-link current-phase relation**. I_c is the critical current(it's a parameter of the junction).
Let's rewrite the Josephson relations as -

$$\frac{\partial I}{\partial\phi} = I_c \cos\phi$$

$$\frac{\partial\phi}{\partial t} = \frac{2\pi}{\phi_0}V$$

We'll find the time derivative of current from these two equations using Chain Rule in order to find the Josephson Inductance-

$$\frac{\partial I}{\partial t} = I_c \cos\phi . \frac{2\pi}{\phi_0}V$$

$$V = \frac{\phi_0}{2\pi I_c \cos\phi}\frac{\partial I}{\partial t} = L(\phi)\frac{\partial I}{\partial t}$$

$$L(\phi) = \frac{\phi_0}{2\pi I_c \cos\phi} = \frac{L_J}{\cos\phi}$$

Here, $L_J = L(0) = \frac{\phi_0}{2\pi I_c}$ is the Josephson Inductance.

2.3 Hamiltonian

The self energy from a Cooper pair across a Josephson Junction will be given by

$$E_c = \frac{(2e)^2}{2(C_g + C_J)}$$

where C_j is the junction capacitance and C_g is the gate capacitance.
From here we can get the first unperturbed term of the Hamiltonian. If we consider n as the number of excess Cooper pairs, the unperturbed Hamiltonian will be given by

$$H_0 = \sum_n E_c(n - n_g)^2 \left|n\right\rangle \left\langle n\right|$$

where $n_g = \frac{C_g V_g}{2e}$ is the effective offset charge.
The second interaction term of the Hamiltonian will be obtained using the Jaynes-Cummings model. The term will be :

$$H_i = -\sum_n \frac{1}{2} E_j(\left|n\right\rangle \left\langle n+1\right| + \left|n+1\right\rangle \left\langle n\right|)$$

where E_j is the Josephson energy of the tunneling junction.
The net Hamiltonian will be of the form -

$$H = \sum_n [E_c(n - n_g)^2 \left|n\right\rangle \left\langle n\right| - \frac{1}{2} E_j(\left|n\right\rangle \left\langle n+1\right| + \left|n+1\right\rangle \left\langle n\right|)]$$

3 Engineering Quantum Circuits

3.1 From quantum harmonic oscillator to Transmon qubit

3.1.1 Linear responses

In this section of the paper, we will try to understand the working of a superconducting circuit by drawing parallels with a classical linear LC oscillator circuit. In such a circuit, energy oscillates between the electrical energy that can be stored in the capacitor, with a capacitance C, and the magnetic energy that can be stored in the inductor, with an inductance L. For convenience we can associate the electrical energy with the "kinetic energy" and the magnetic energy with the "potential energy" with the sole purpose of providing an analogy. The general expression for the energy of any element in a circuit can be given by,

$$E(t) = \int_{-\infty}^{t} V(t')I(t')dt'$$

where $V(t')$ and $I(t')$ denote the voltage and current flowing through the element, in our case a capacitor and an inductor, at time t'. When we want to find the *Hamiltonian* of this system in the classical picture, we must find the corresponding *generalized coordinate(s).* For this case, we can choose *magnetic flux* as our generalized coordinate since we arbitrarily chose magnetic energy to be analogous to the kinetic energy (if we had chosen electrical energy, charge would have been the more natural choice for the generalized coordinate). We shall define the flux as the time integral of voltage as given below,

$$\Phi(t) = \int_{-\infty}^{t} V(t')dt'$$

By combining the above two equations and also using them to solve the coupled differential equations, $V(t) = LdI/dT$ and $I = CdV/dt$. We now find the "kinetic" and "potential" energies of the system in terms of the flux,

$$T = \frac{1}{2}LI^2 = \frac{1}{2}C\dot{\Phi}^2$$

$$V = \frac{1}{2L}\Phi^2$$

The *Lagrangian* can then be given by

$$\mathcal{L} = T - V = \frac{1}{2}C\dot{\Phi}^2 - \frac{1}{2L}\Phi^2$$

Using Legendre's transformation, we then write the Hamiltonian, for which we need to the know the corresponding generalized momentum, which turns out to be *electrical charge* in this case.

$$Q = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}} = C\Phi$$

The Hamiltonian is now defined by,

$$\mathcal{H} = Q\dot{\Phi} - \mathcal{L}$$
$$\mathcal{H} = \frac{1}{2}C\dot{\Phi}^2 + \frac{1}{2L}\Phi^2$$
$$\mathcal{H} = \frac{1}{2}CV^2 + \frac{1}{2}LI^2,$$

where we have written it in terms of current and voltage in the last expression. We now see that this Hamiltonian is analogous to that of a classical harmonic oscillator

To obtain the quantum mechanical version of this Hamiltonian, we pay attention to the commutator of Q and Φ. We can find an analogy once again by finding the *Poisson bracket* of these 2 quantities which will be,

$$\{\Phi, Q\} = \frac{\delta\Phi}{\delta\Phi}\frac{\delta Q}{\delta Q} - \frac{\delta\Phi}{\delta Q}\frac{\delta Q}{\delta\Phi} = 1$$

We know that, the Poisson bracket and commutator are linearly proportional to each other with the constant of proportionality $1/i\hbar$. So $[\Phi, Q] = i\hbar$. We can now define the *reduced flux* and *reduced charge* as $\phi = 2\pi\Phi/\Phi_0$ and $n = Q/2e$ respectively, and then write the quantum mechanical Hamiltonian as,

$$H = 4E_C n^2 + \frac{1}{2}E_L\phi^2$$

where $E_C = e^2/2C$ is the energy required to add each electron of the Cooper-pair to the island and $E_L = ((\Phi_0/2\pi)^2/L)$ is the inductive energy and $\Phi_0 = h/2e$ is the superconducting flux quantum. n is a quantum operator which represents the excess number of Cooper-pairs on the island and ϕ can also be interpreted as the gauge invariant phase across the inductor. These two operators do not commute, $[\phi, n] = i$.

Once again, we draw an analogy between the mechanical energies and the circuit energies, we consider ϕ as the generalized coordinate, so E_C will be the kinetic energy and E_L will be the potential energy. We know that the adjacent energy eigenvalues in a quantum harmonic potential are equally spaced apart form each other, mathematically formulating the same, we can write $E_{k+1} - E_k = \hbar\omega_0$ where ω_0 denotes the resonant frequency of the circuit, $\omega_0 = \sqrt{8E_LE_C}/\hbar = 1\sqrt{LC}$ and represent in the familiar quantum mechanical by using the annihilation and creation operators as $H/\hbar = \omega_0(a^\dagger a + 1/2$.

Similarly we define $\phi = \phi_{zpf}(a + a^\dagger)$ and $n = in_{zpf}(a - a^\dagger)$ where $n_{zpf} = [E_L/(32E_C)]^[1/4]$ and $\phi_{zpf} = (2E_C/E_L)^{1/4}$ are the zero-point fluctuations of the charge and phase variables respectively. Wavefunctions are in general functions of n and ϕ, which are the "generalized variables" of this system, which are distributed over a range of values of the parameters. Such ranges have non-zero deviations which are referred to as *quantum fluctuations* and in general they may exist for the ground state as well in which case they'll be *zero-point fluctuations.*

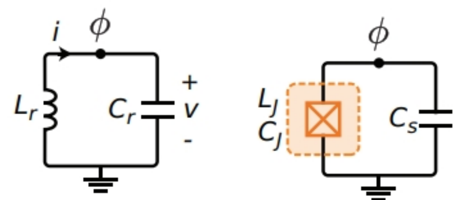


Figure 1: Where the "X" symbol for the second image represents the Josephson junction

3.1.2 Anharmonicities

We noted that the energy levels in a linear response of the harmonic oscillator are equally spaced from each other. So when we want to consider this as a system for a *qubit*, we need to be able to access two energy levels and two only since the dimension of a qubit is 2. So we see that if the energy levels are equally spaced, whatever driving field which would be used to excite the qubit from the ground state to excited state, or in general any state $|1\rangle$ to $|2\rangle$, will also excite other levels of the system. This is a fundamental problem since we want to operate with only two levels.

To counter this problem we see that the most natural way of doing it would be to include *anharmonicities* which would destroy this equal spacing between the energy levels and we must also ensure that the frequencies corresponding to the two pairs of energies are different. For example $\omega^{0\rightarrow 1}$ and $\omega^{1\rightarrow 2}$ must be different enough so that the source which drives $\omega^{0\rightarrow 1}$ should not drive $\omega^{1\rightarrow 2}$. But it turns out that even though strengthening the anharmonicity seems like the better option, it would also mean that there would be a stringer limit on how short the pulses used to drive the qubit can be.

The inclusion of a Josephson junction can account for the non-linearity which is very convenient for multiple reasons because not only does it introduce the non-linearity but is also a dissipationless circuit element. This is what makes the Josephson junction such a key aspect in the superconducting qubits.

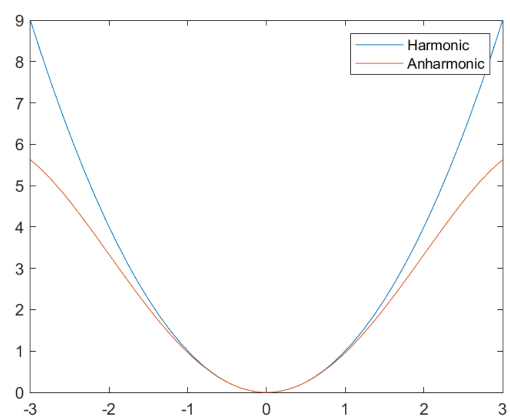
The Josephson relations are given by (as mentioned above),

$$I = I_c sin(\phi), V = \frac{\hbar}{2e}\frac{d\phi}{dt}$$

From these, we get the Hamiltonian as,

$$H = 4E_C n^2 - E_J cos(\phi)$$

where $E_C = e^2/(2C_\Sigma), C_\Sigma = C_s + C_J$ is the total capacitance, including shunt capacitance C_s and the self-capacitance of the junction C_J and $E_J = I_c\Phi_0/2\pi$ is the Josephson energy, I_c is the critical current of the junction. We see that the $cos(\phi)$ factor in E_J introduces the anharmonicity in the potential.



Superconducting qubits have advanced in a way that $E_J >> E_C$, in this case the *flux noise* is more significant compared to the *charge noise.* Recent developments in the field of quantum error correction had made it possible to overcome the flux noise better than the charge noise. To access this regime where $E_J >> E_C$ is to make the qubit less sensitive to charge noise, we make the charging E_C small by shunting junction with a large capacitor, $C_s >> C_J$. Such a circuit is what is called *transmon qubit.* In this limit, the superconducting phase ϕ is a good quantum number i.e. the spread (or quantum fluctuation) of ϕ values represented by the wave-function is small. Thus the lower energy eigenstates can still be approximated as localized states in the potential well. Incorporating these in a mathematical equation we see,

$$E_J cos(\phi) = \frac{1}{2}E_J\phi^2 - \frac{1}{24}E_J\phi^4 + \mathcal{O}(\phi^6)$$

The first term is the harmonic term, the second term is the anharmonicity that we intended to add to destroy the equal spacing of the eigenvalues. The negative coefficient of the second term indicates that the anharmonicity, $\alpha = \omega_q^{1\to 2} - \omega_q^{0\to 1}$ is negative and its limit in magnitude thus this cannot be made arbitrarily large.

In a realistic system, $\alpha = -E_C$ is usually designed to be 100-300 MHz, as required to maintain a desirable a desirable qubit frequency $\omega_q = (\sqrt{8E_JE_C} - E_C)/\hbar = 3 - 6$GHz, with the energy ratio as $E_J/E_C \geq 50$ to suppress charge sensitivity. This works because the charge sensitivity is suppressed with for larger values of this ratio.

Including the first two terms in the expansion as mentioned above, we can effectively modify the Hamiltonian in a way similar to the Duffing Hamiltonian (taking $\hbar = 1$),

$$H = \omega_q a^\dagger a + \frac{\alpha}{2}a^{\dagger 2}a^2$$

With the large difference in α and ω_q the transmon qubit is essentially a weak anharmonic oscillator. The Hamiltonian for the harmonic oscillator is $H = \omega(a^\dagger a + 1/2)$, we can shift the reference energy level and discard the $1/2$ term, and the $a^\dagger 2a^2$ term is the one that corresponds to the non-linear term. Now if we can either have a strong nonlinearity or if we have a good control over the driving field(s), we can effectively treat this as a two-level system although it has to be remembered that other levels physically exist and it turns out that their existence can be exploited in a useful way while creating the circuits and implementing more efficient gates. In this case, we can reformulate the Hamiltonian as,

$$H = \omega_q \frac{\sigma_z}{2}$$

where σ_z is one of the Pauli matrix.

3.2 Interaction Hamiltonian Engineering

The most important and characteristic advantage of quantum algorithms is the property of *entanglement* which is a completely quantum phenomenon. One way to entangle qubits would be to exploit the interactions between the two qubits. In such a case we can write the Hamiltonian in the following form to have a better understanding,

$$H = H_1 + H_2 + H_{int}$$

where $H_i, i = 1, 2$ denotes the Hamiltonians of the individual quantum systems (in this case, the two qubits) and the last term is for the interaction between the two qubits. This interaction Hamiltonian is how the two qubits can be *coupled.* In superconducting qubits, it is the energy of the electric fields or the magnetic fields or even a combination of both of them, which physically represents the coupling energy between the two qubits. These two possibilities correspond to two different broad categories of coupling - *capactitive* coupling and *inductive* coupling. Even this can further be sub-classified as *direct* coupling and *indirect* coupling via a coupler. We will discuss the two of them in the following parts.

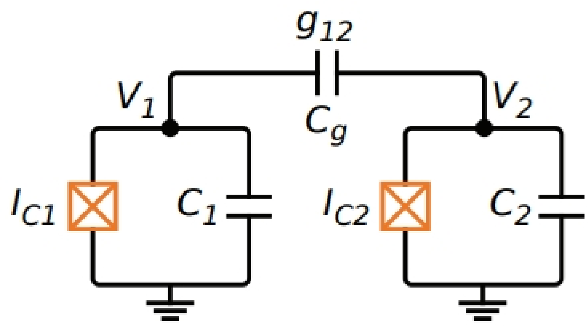


Figure 2: Here the capacitors are directly coupled

To achieve direct capacitive coupling, a capacitor is placed between the two voltage nodes from which we get an energy of the form, $\int_0^{V2} C_g V_1 dV$. The Hamiltonian will then be,

$$H_{int} = C_g V_1 V_2$$

where C_g will be the coupling capacitance and V_1 and V_2 denote the voltages of the two nodes. Further we define $V_i = (2e/C_i)n_i$. Circuit quantization in the limit $C_g << C_1, C_2$ yields

$$H = \sum_{i=1,2} H_i + 4e^2 n_1 n_2 \frac{C_g}{C_1 C_2}$$

$$H_i = 4E_{C,i} n_i^2 - E_{J,i} cos(\phi_i); \quad i = 1, 2$$

Thus the coupling energy depends on both the coupling capacitances and the matrix elements

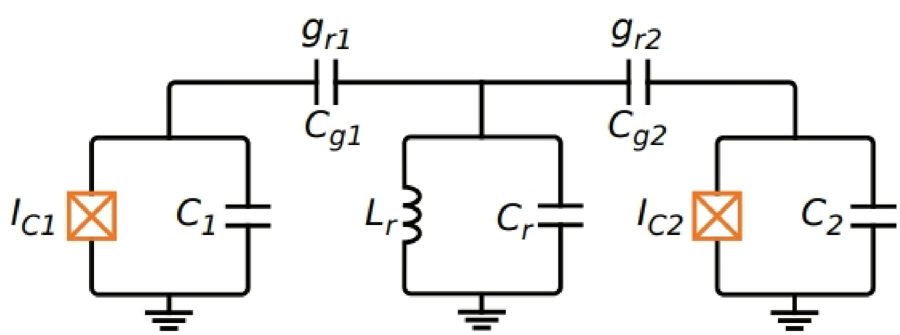


Figure 3: Indirect coupling via a harmonic oscillator circuit

3.3 Coupling axes: transverse and longitudinal

We've talked about the interaction Hamiltonian which was from a classical or a more intuitive perspective, now we consider the quantum picture and attempt to reformulate the Hamiltonian by using operators.

The two individual Hamiltonians belong to the vector spaces V_1 and V_2 respectively, naturally the interaction Hamiltonian has to belong to the tensor product of these two vector spaces $V_1 \otimes V_2$. The total Hamiltonian can now be written as,

$$H = \sum_{i=1,2} \left[\omega_i a_i^\dagger a_i + \frac{\alpha_i}{2} a_i^\dagger a_i^\dagger a_i a_i \right] - g(a_1 - a_1^\dagger)(a_2 - a_2^\dagger)$$

where g represents the coupling energy, the square brackets represent the Duffing oscillator Hamiltonians for the two individual qubits. We've used this particular form of operators for the interaction part deduced from the fact that $V \propto n \propto i(a - a^\dagger)$, this is what leads to the $n1n2$ term, as discussed in the previous section, to take the form as shown above. This form of coupling is what is called *transverse coupling* since the diagonal elements of the interaction Hamiltonian are 0 and only the off-diagonal elements are non-zero.

$$\langle k_i | a_i - a_i^\dagger | k_i \rangle = 0$$

$$\langle k_i | a_i - a_i^\dagger | k_{i\pm 1} \rangle \neq 0$$

Since we earlier considered this as an effective two-level system under certain experimental assumptions, we can write the Hamiltonian in a way similar to the one we write for a system of two spins as shown below,

$$H = \sum_{i=1,2} \frac{1}{2} \omega_i \sigma_{z,i} + g \sigma_{y,1} \sigma_{y,2}$$

This is just a Hamiltonian with 2 spins, coupled by an exchange interaction. This turns out to be a pretty versatile Hamiltonian in the sense that multiple types of two-qubit entangled states can be generated. We get a similar equation in the case of an inductive coupling except that the σ_y is replaced by σ_x. In that case the the current term is the one that contributes to the coupling, which was due to V in this case. $I \propto \phi \propto (a + a^\dagger)$ which gives the σ_x terms after reducing it to a two-level system in the lab frame and both these couplings may be transverse to the qubit.

We had also earlier mentioned about indirect capacitive coupling which can be done by via a *coupler* whose form can be taken as a harmonic oscillator in which case, the Hamiltonian will become,

$$H = \frac{1}{2} \omega_q \sigma_z + \omega_r a^\dagger a + g(\sigma_+ a + \sigma_- a^\dagger)$$

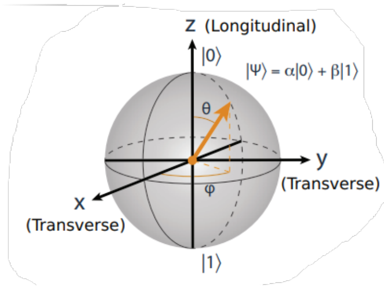
Here ω_q and ω_r are the resonant frequencies of the qubit and the resonator respectively. $\sigma_+ = |1\rangle \langle 0|$ and $\sigma_- = |0\rangle \langle 1|$ are the raising and lowering operators of the qubit. We will actually get a phase factor in all the coupling terms and also two other terms that represent double excitation and de-excitation i.e. $\sigma_+ a^\dagger$ and $\sigma_- a$. For the cases where one mode is excited and the other is de-excited, we can apply the dispersive limit ($g << \omega_q, \omega_r$) which is the familiar *rotating wave approximation* (RWA), and the phase factor can be taken as 1 for those two coupling terms. For the other case where both modes are either excited or de-excited the phase oscillates very rapidly and the effect that we see is really the average of the phase. This average is 0, which is why we consider the other coupling terms to be 0. This Hamiltonian is actually an extension of the one we obtain while dealing with light matter interactions in a cavity. It has many useful applications in superconducting quantum information architectures, such as high-fidelity readout, cavity buses, quantum memory and quantum computation with cat-states to mention a few.

4 Modeling noise and Decoherence

We need the superconducting qubits to be coherent so that there is a greater operational fidelity of these qubits. But there's some random, uncontrollable physical processes leading to decoherence. These processes may be present in qubit control, in the local environment of the quantum processor or in the measurement equipment. First we discuss the basics of noise which lead to decoherence in the superconducting qubits. Finally we study about Bloch-Redfield model of decoherence.

4.1 Bloch sphere representation

A Bloch sphere is a unit sphere which can be used to represent the quantum state of a two level system and a qubit specifically in our case. We first define a Bloch vector which we'll use to visualize the sphere. Its general form is $|\phi\rangle = \alpha |0\rangle + \beta |1\rangle$. The vertical axis is considered as the z-axis and we take the positive side of it as the $|0\rangle$ state and negative as $|1\rangle$. Pure quantum states having unit length i.e. $|\alpha^2| + |\beta^2| = 1$, can be represented as a vector connecting the centre of the sphere to any point on its surface.



For the ease of calculation we can represent all the vectors in polar co-ordinates i.e. $(\sin\theta\cos\phi$, $\sin\theta\sin\phi$, $cos\theta)$. The general state of the form will be -

$$|\phi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = \cos\frac{\theta}{2}\,|0\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|1\rangle$$

We actually choose for the Bloch sphere to be viewed in a reference frame where x and y-axes also rotate around the z-axis at the qubit frequency. In this rotating frame, the vector will appear to be stationary and we can use the equation written above.

The density matrix using the completeness relation for a pure state is written as follows -

$$\rho = |\psi\rangle\langle\psi|$$

$$\rho = \frac{1}{2}(I + \vec{a}.\vec{\sigma}) = \frac{1}{2}\begin{pmatrix} 1+\cos(\theta) & e^{-i\phi}\sin\theta \\ e^{i\phi}sin\theta & 1+\sin(\theta) \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2(\frac{\theta}{2}) & e^{-i\phi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \sin^2(\frac{\theta}{2}) \end{pmatrix}$$

$$= \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \beta\alpha^* & |\beta|^2 \end{pmatrix}$$

here I is the Identity Matrix, and $\vec{\sigma} = [\sigma_x, \sigma_y, \sigma_z]$, which is a vector of Pauli matrices. If $0 \leq \mathrm{Tr}(\rho^2) < 1$ this represents a mixed state and the vector terminates *inside* the sphere.

4.2 Bloch - Redfield Model of decoherence

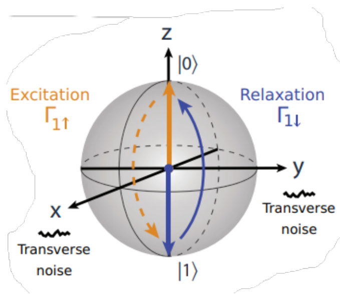
Noise sources weakly coupled to the qubits have short correlation times with respect to the system dynamics, within the standard Bloch-Redfield picture of two-level system dynamics. To the density matrix that we found in the previous section we will make changes to account for the longitudinal and transverse decay function, and explicitly introduce a phase factor.
We will have the following density matrix after we account for all these changes -

$$\rho_{BR} = \begin{pmatrix} 1 + (|\alpha|^2 - 1)e^{-\Gamma_1 t} & \alpha\beta^* e^{i\delta\omega t}e^{-\Gamma_2 t} \\ \beta\alpha^* e^{-i\delta\omega t}e^{-\Gamma_2 t} & |\beta|^2 e^{-\Gamma_1 t} \end{pmatrix}$$

where Γ_1, Γ_1 are the longitudinal and transverse relaxation rates respectively(They are the reciprocals of the respective times T_1 and T_2 respectively). The qubit frequency is ω_q and the rotating-frame frequency is ω_d and $\delta\omega = \omega_q$ - ω_d.
We assume that the environment's temperature is low enough for the qubit to not be excited to higher states through thermal excitations. Hence the first element approaches to 1 while the rest of the elements go to 0 when $t >> (T_1, T_2)$.

Longitudinal Relaxation

The longitudinal relaxation rate Γ_1, describes depolarization along the qubit quantization axis, often referred to as "energy decay" or "energy relaxation." Here we will define the polarization p such that, when its value is 1, the qubit lies entirely in the ground state $|0\rangle$, when -1 it lies in the excited state $|1\rangle$ and p = 0 can be visualized as a depolarized mixed state at the centre of a Bloch sphere.



$\Gamma_{1\uparrow}$ and $\Gamma_{1\downarrow}$ are the *up transition rate* and *down transition rate* respectively i.e. they represent the excitation from $|0\rangle$ to $|1\rangle$ and the relaxation from $|1\rangle$ to $|0\rangle$. The relaxation rate can be written as a sum of these two

$$\Gamma_1 \equiv \frac{1}{T_1} = \Gamma_{1\downarrow} + \Gamma_{1\uparrow}$$

T_1 is the characteristic time scale over which the qubit relaxes to its steady-state value. For superconducting qubits, this steady state value is generally the ground state, due to Boltzmann statistics and typical operating conditions. We get the following relation between the up transition rate and the down transition rate through Boltzmann statistics -

$$\Gamma_{1\uparrow} = \exp(\frac{-\hbar\omega_q}{k_B T})\Gamma_{1\downarrow}$$

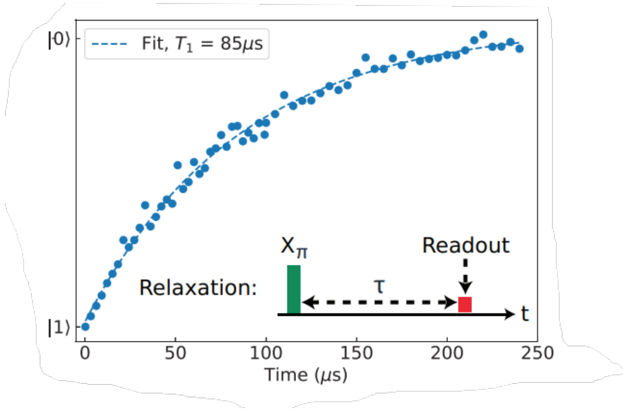
and the equilibrium qubit polarization is -

$$p = \tanh\frac{\hbar\omega_q}{2k_B T}$$

The typical operational frequency of the qubits is nearly 5 GHz and the temperatures of nearly 20 mK. For these values, $\Gamma_{1\uparrow}$ is mostly suppressed by the exponential factor and only the down transition rate $\Gamma_{1\downarrow}$ contributes significantly, which relaxes the population to the ground state. Thus, qubits generally spontaneously lose energy to their cold environment, but the environment rarely introduces a qubit excitation. As a result, the equilibrium polarization approaches 1.

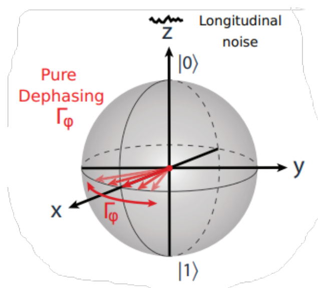
Longitudinal depolarization is generally consistent with the Bloch - Redfield's decoherence model and has similar exponential decay functions. The intuition is that qubit-transition line widths are relatively narrow in frequency, and so the noise generally does not vary much over this narrow frequency range. Hence this is a more *well behaved* noise(short correlation time, many modes weakly coupled to qubit, no divergences).

We'll now look into an example of T_1 measurement. A qubit is first prepared in its excited state $|0\rangle$ by giving an X_π - pulse. This state is left to spontaneously decay to the ground state for a time τ, after which it is measured. Since measuring it will either result in $|0\rangle$ or $|1\rangle$, we repeat the experiment multiple times and obtain corresponding polarization (1 or -1) and plot it. We observe a relaxation time , $T_1 \approx 85ms$.



Pure Dephasing

The pure dephasing rate Γ_ϕ describes depolarization in the x-y plane of the Bloch sphere. It is referred to as *pure dephasing* to distinguish it from other phase breaking processes such as energy excitation or relaxation.



This pure dephasing is caused by the *longitudinal noise* that couples to the qubit via the z - axis. There is a precession induced in the Bloch sphere which arises from the longitudinal noise as the qubit frequency starts fluctuating because of it(due to which $\omega_q \neq \omega_d$). If we consider several instances of the Bloch vector along the x - axis, there will be stochastic fluctuations from each of these which will result in a different precession frequency. This eventually leads to a complete depolarization of the azimuthal angle ϕ. (This stochastic effect is considered in the transverse relaxation time, Γ_2, discussed in the next part)

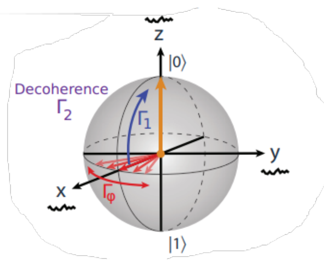
A few important points of differences between pure dephasing and energy relaxation need to be noted here. First, since the pure dephasing is elastic(since there is no energy exchange with the environment), it is in principle *reversible*. The degree to which the quantum information can be retained depends on many factors such as the bandwidth of the noise, the rate of dephasing, the rate at which unitary operations can be performed, etc. The spontaneous energy relaxations on the other hand are very obviously *irreversible*. Quantum information is lost once the qubit emits energy to the environment and can't be recovered. The second difference is that, noise at any frequency can modify the qubit frequency and can cause dephasing. So unlike the energy relaxations, this is *not a resonant phenomenon*.

Transverse Relaxation

We define the transverse relaxation as -

$$\Gamma_2 = \frac{\Gamma_1}{2} + \Gamma_\phi$$

It describes the loss of coherence of a superposition state. See the adjoining figure as an example.



Decoherence in the $(1/\sqrt{2}(|0\rangle + |1\rangle))$ is caused by the longitudinal noise which changes the ω_q and leads to pure dephasing as well(represented in red). It is also caused by transverse noise, which leads to energy relaxation of the excited-state component of the superposition state at a rateΓ_1(in blue). Such a relaxation event is also a phase-breaking process, because once it occurs, the Bloch vector becomes $|0\rangle$, and there is no longer any knowledge of which direction the Bloch vector had been pointing along the equator; the relative phase of the superposition state is lost.

If the decay functions are exponential the dephasing time T_ϕ can be calculated if T_1 and T_2 are known. However, for the usual superconducting qubits, the broadband dephasing noise such as charge noise, flux noise, critical current noise etc. tend to exhibit a 1/f - like power spectrum. These sort of noises does not fall under the Bloch-Redfield model of decoherence and modifications need to be made to understand them better.

5 Summary

In this paper, we have studied how superconducting qubits can be realized in a physical way and how they can be engineered to attain the quantum advantage that is absent in classical computers. Various types of coupling that can be done to realize interactions between the qubits and the corresponding Hamiltonian forms were analyzed. The Bloch sphere turns out to be a convenient form of representing quantum states. We model the noise and decoherence using these and its clear that visualization is quite simplified due to the Bloch spheres and vectors. The Bloch-Redfield model of decoherence was studied to some extent and longitudinal relaxation time was calculated for one such system which we found to be approximately 85 ms.

6 References

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