Reveiw on Superconducting Qubits

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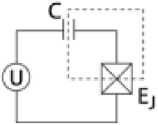
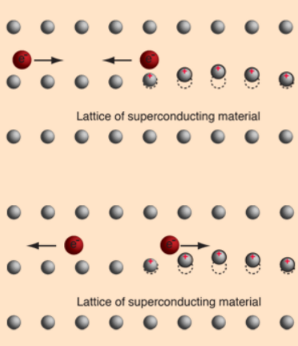


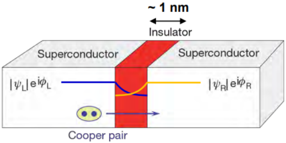
- Qubit having basis states as charge states are said to be Charge qubits.
- Where from Superconductivity? - Cooper pairs and Josephson Junction

Cooper pairs

- Pair of electrons bounded at a low temperature in a certain manner
- How are the electrons bound together?
- Visual model of Cooper pair attraction



Josephson Junction



- Josephson Equations -

$$I(t) = I_c \sin\phi(t)$$

$$\frac{\partial\phi}{\partial t} = \frac{2eV(t)}{\hbar}$$

$$L(\phi) = \frac{\phi_0}{2\pi I_c \cos\phi} = \frac{L_J}{\cos\phi}$$

Hamiltonian of a Charge Qubit

- The Hamiltonian for a charge qubit is of the form -

$$H = \sum_n [E_c(n - n_g)^2 |n\rangle \langle n| - \frac{1}{2}E_j(|n\rangle \langle n+1| + |n+1\rangle \langle n|)]$$

Classical Harmonic Oscillator

- Only considering the linear response in a classical system, we get the *Lagrangian* as

$$\mathcal{L} = T - V = \frac{1}{2}C\dot{\Phi}^2 - \frac{1}{2L}\Phi^2$$

- We then need to use the Legendre transform to find the *Hamiltonian* which will be

$$Q = \frac{\partial \mathcal{L}}{\partial \dot{\Phi}} = C\Phi$$

$$\mathcal{H} = Q\dot{\Phi} - \mathcal{L}$$

$$\mathcal{H} = \frac{1}{2}C\dot{\Phi}^2 + \frac{1}{2L}\Phi^2$$

How to build a superconducting qubit

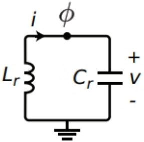


Figure: LC resonant circuit

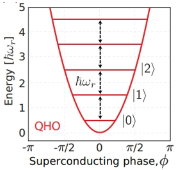


Figure: Linear QHO when $k_B T << \hbar\omega$

Nonlinear circuit

- Energy levels are equally spaced, all transitions have the same frequency.
- Anharmonicity is required. We thus use Josephson junction - a nonlinear inductor. The inductance depends on the current that flows through it.

Transmon Qubit

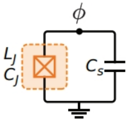


Figure: Josephson junction

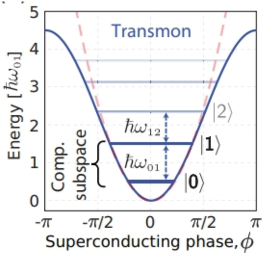


Figure: Anharmonicity included (potential becomes a cosine)

Interaction Hamiltonian

- For weak nonlinearities, we can approximate the cosine potential and the individual Hamiltonian will be

$$H = \omega_q a^\dagger a + \frac{\alpha}{2} a^{\dagger 2} a^2$$

- Total Hamiltonian will be,

$$H = H_1 + H_2 + H_{int}$$

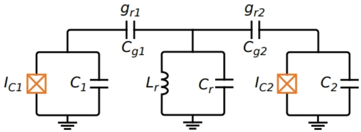


Figure: Capacitive coupling

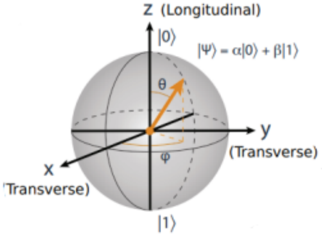
- The Hamiltonian including the coupling energies will be

$$H = \sum_{i=1,2} \left[\omega_i a_i^\dagger a_i + \frac{\alpha_i}{2} a_i^\dagger a_i^\dagger a_i a_i \right] - g(a_1 - a_1^\dagger)(a_2 - a_2^\dagger)$$

- This form of coupling is called transverse coupling because the interaction Hamiltonian has the diagonal elements as 0 and the off-diagonal elements as the only non-zero values.

Noise and Decoherence

- Why coherence?
- Sources of noise -
  - qubit control
  - measurement equipment
  - local environment of the quantum processor
- How to represent and visualize states?



- Density matrix -

$$\rho = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \beta\alpha^* & |\beta|^2 \end{pmatrix}$$

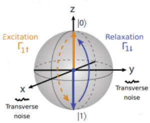
- Pure states and mixed states

Bloch - Redfield Model of Decoherence

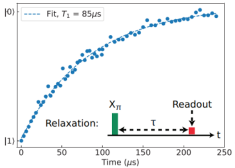
- The modified density matrix according to the Bloch-Redfield model of decoherence is given by -

$$\rho_{BR} = \begin{pmatrix} 1 + (|\alpha|^2 - 1)e^{-\Gamma_1 t} & \alpha\beta^* e^{i\delta\omega t} e^{-\Gamma_2 t} \\ \beta\alpha^* e^{-i\delta\omega t} e^{-\Gamma_2 t} & |\beta|^2 e^{-\Gamma_1 t} \end{pmatrix}$$

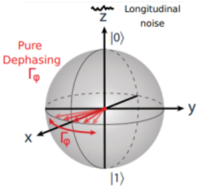
Longitudinal relaxtion



- Results from energy exchange between the qubit and its environment, due to transverse noise that couples to the qubit in the x y plane and drives the transitions.
- Transitions follow Boltzmann statistics
- Example of T_1 measurement.



Pure dephasing

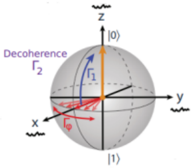


- Precession induced in Bloch sphere due to the longitudinal noise because of which qubit and sphere frequencies will no longer remain equal.
- Points of differnce between dephasing and energy relaxation

Transverse Relaxation

- It is defined as -

$$\Gamma_2 = \frac{\Gamma_1}{2} + \Gamma_\phi$$



- Loss of decoherence using both longitudinal and transverse noises.