Ore's Theorem

CS 111 Presentation

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Questions

Introduction: Background Knowledge

To understand **Ore's theorem**, we must briefly recapitulate what it means for a graph to be **Hamiltonian**.

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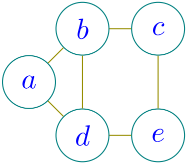

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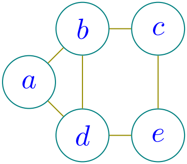
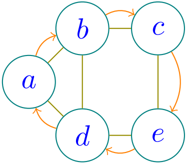




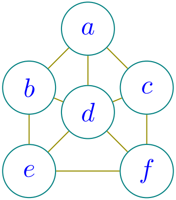
$T = \{a, b, c, e, d, a\}$

Introduction: Ore's Theorem Statement

Example I

First Case

$G_1(V, E)$ **satisfies** Ore's theorem Ω and **has** a Hamiltonian cycle T.



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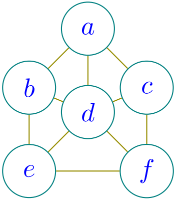

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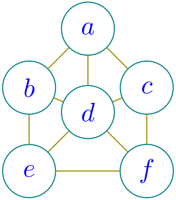

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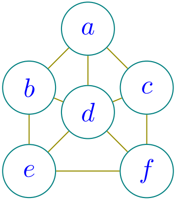

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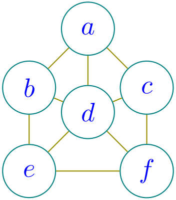

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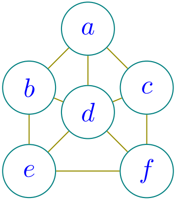

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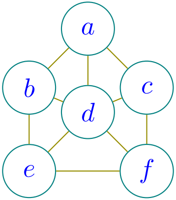

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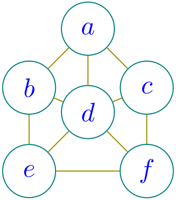

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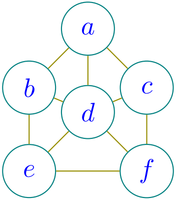


$$|V| = 6 \geq 3 \quad \vdash \quad \Omega_1$$

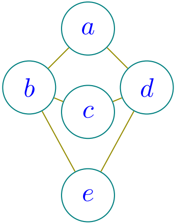
$$\begin{gathered} \deg(a) + \deg(e) = 6 \\ \deg(a) + \deg(f) = 6 \\ \deg(b) + \deg(c) = 6 \\ \deg(b) + \deg(f) = 6 \\ \deg(c) + \deg(e) = 6 \\ \vdash \Omega_2 \end{gathered}$$

This graph **satisfies** Ω and **has**
a Hamiltonian cycle $T = \{d, a, b, e, f, c, d\}$.

Example II

Second Case

$G_2(V, E)$ **does not satisfy** Ω and **does not have** a Hamiltonian cycle T.



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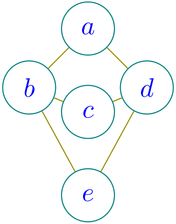

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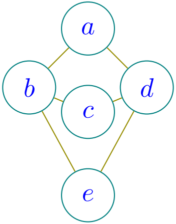

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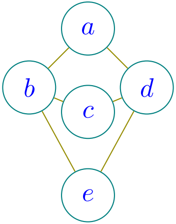

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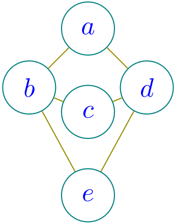

$$|V| = 5 \geq 3 \quad \vdash \quad \Omega_1$$

$$\deg(a) + \deg(c) = 4$$
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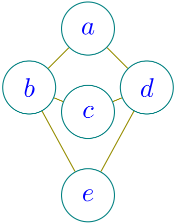

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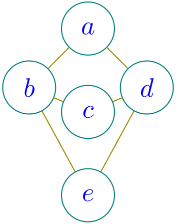

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$$\not\vdash \Omega_2$$

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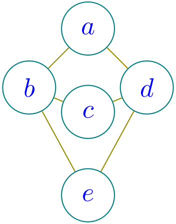

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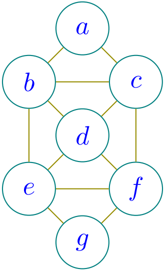
$$\nvdash \Omega_2$$

This graph **does not satisfy** Ω and **does not have** a Hamiltonian cycle.

Example III

Third Case

$G_3(V, E)$ **does not satisfy** Ω and **has** a Hamiltonian cycle T.



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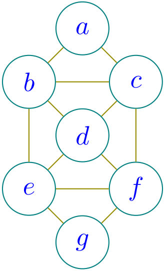

$$|V| = 7 \geq 3 \quad \vdash \quad \Omega_1$$

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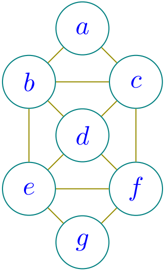

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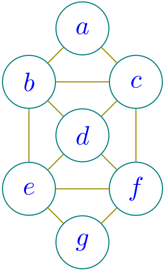

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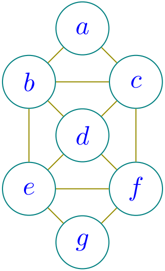

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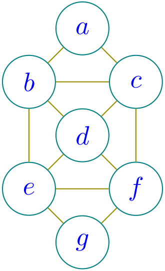

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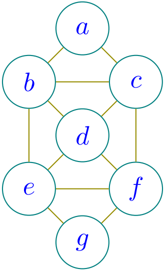

$$|V| = 7 \geq 3 \quad \vdash \quad \Omega_1$$

$$\begin{array}{ccc}
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\deg(c) + \deg(e) = 8 & \deg(c) + \deg(g) = 8 & \deg(d) + \deg(g) = 6
\end{array}$$

$$\nvdash \Omega_2$$

This graph **does not satisfy** Ω but still **has** a Hamiltonian cycle $T = \{a, b, d, e, g, f, c, a\}$.

Applications

Hamiltonian Paths is crucial in the real world

Network Routing :

- Finding the fastest path from A to B

- Improve network traffic and delays from client to server.

- improvement of network runtime.

Ex : when typing elearnhome.ucr.edu(canvas site), your computer internet immediately determines the fastest path to reach the cite

Applications

Navigation Systems :

Navigation is another example of Hamiltonian paths

-Finding a path that travelers can take to reduce traveling time

-Navigation technology uses algorithms that prevents travelers from revisiting destinations and redundant paths

For example : Traffic in the US 60W freeway to Los Angeles is congested

Conclusion: Summary

Conclusion: Questions