# Ore's Theorem

## CS 111 Presentation

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# Questions

# Introduction: Background Knowledge

To understand **Ore's theorem**, we must briefly recapitulate what it means for a graph to be **Hamiltonian**.

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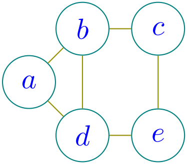
Example:



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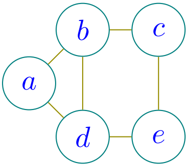
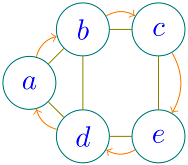


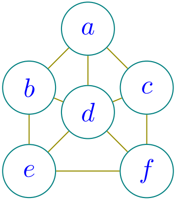

$T = \{a, b, c, e, d, a\}$

# Introduction: Ore's Theorem Statement

# Example I

*First Case*

$G_1(V, E)$ **satisfies** Ore's theorem $\Omega$ and **has** a Hamiltonian cycle $T$.



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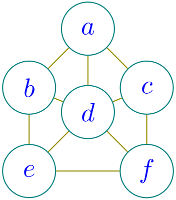


$$|V| = 6 \geq 3 \quad \vdash \quad \Omega_1$$

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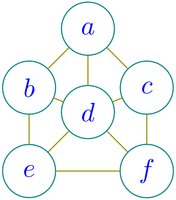


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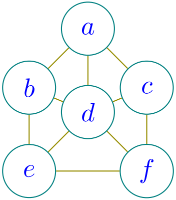


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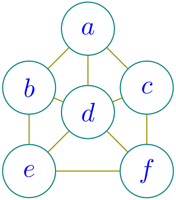


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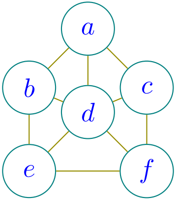


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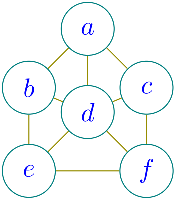


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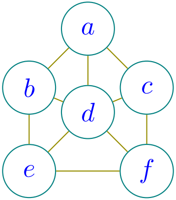


$$|V| = 6 \geq 3 \quad \vdash \quad \Omega_1$$

$$\begin{gathered}
\deg(a) + \deg(e) = 6 \\
\deg(a) + \deg(f) = 6 \\
\deg(b) + \deg(c) = 6 \\
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\deg(c) + \deg(e) = 6 \\
\vdash \Omega_2
\end{gathered}$$

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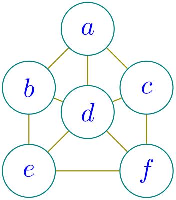


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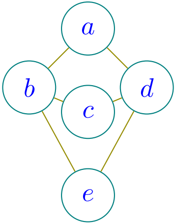
$$\begin{gathered} \deg(a) + \deg(e) = 6 \\ \deg(a) + \deg(f) = 6 \\ \deg(b) + \deg(c) = 6 \\ \deg(b) + \deg(f) = 6 \\ \deg(c) + \deg(e) = 6 \\ \vdash \Omega_2 \end{gathered}$$

This graph **satisfies** $\Omega$ and **has**
a Hamiltonian cycle $T = \{d, a, b, e, f, c, d\}$.

# Example II

*Second Case*

$G_2(V, E)$ **does not satisfy** $\Omega$ and **does not have** a Hamiltonian cycle $T$.



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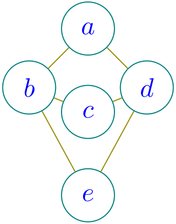


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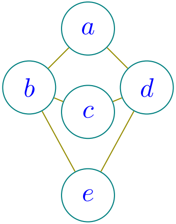


$$|V| = 5 \geq 3 \quad \vdash \quad \Omega_1$$

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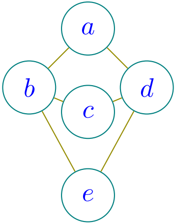


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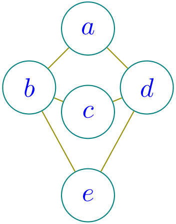


$$|V| = 5 \geq 3 \quad \vdash \quad \Omega_1$$

$$\deg(a) + \deg(c) = 4$$
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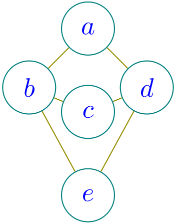

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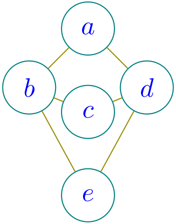


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$$\begin{aligned} \deg(a) + \deg(c) &= 4 \\ \deg(a) + \deg(e) &= 4 \\ \deg(b) + \deg(d) &= 6 \\ \deg(c) + \deg(e) &= 4 \end{aligned}$$

$$\nvdash \Omega_2$$

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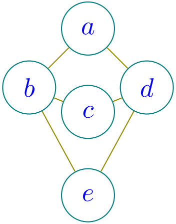


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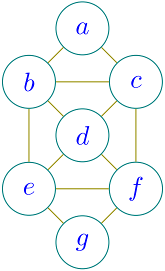
This graph **does not satisfy** $\Omega$ and **does not have** a Hamiltonian cycle.

# Example III

*Third Case*

$G_3(V, E)$ **does not satisfy** $\Omega$ and **has** a Hamiltonian cycle $T$.



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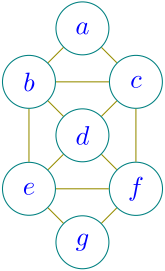


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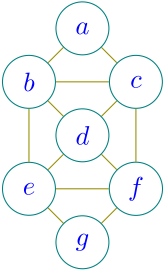


$$|V| = 7 \geq 3 \quad \vdash \quad \Omega_1$$

$$\deg(a) + \deg(d) = 6 \qquad \deg(a) + \deg(e) = 8 \qquad \deg(a) + \deg(f) = 8$$

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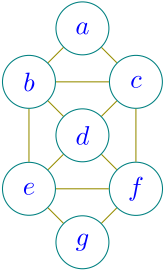


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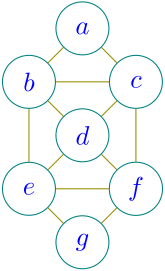


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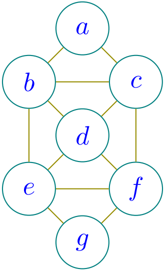


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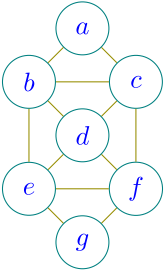


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This graph **does not satisfy** $\Omega$ but still **has** a Hamiltonian cycle $T = \{a, b, d, e, g, f, c, a\}$.

# Applications

# Conclusion: Summary

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