

Copula Based Model for Representation of Hybrid Power Plants in Non-Sequential Monte Carlo Reliability Evaluation

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Abstract

This article aims to propose models that allow the representation of the correlation between intermittent renewable generation in reliability evaluation based on non-sequential Monte Carlo Simulation (MCS). The models make use of the theory of Copulas, specifically the theory of Archimedean Copulas. To represent hybrid power plants with different types of energy sources (wind and solar, for example), it is used the theory of Hierarchical Archimedean Copulas. The models are validated by calculating generation reliability indices by three different approaches that are then compared: sequential MCS (that uses the chronological simulation), classical non-sequential MCS (that assumes independency between random variables) and the non-sequential MCS incorporating the Copula based models. The results show that the use of Copulas is able to represent accurately the correlation and complementarity between the sources, without the computational burden of the chronological simulation.

Keywords: Reliability Evaluation, Hybrid Power Plant, Correlation, Copulas, Hierarchical Archimedean Copulas, Monte Carlo Simulation

1. INTRODUCTION

The increasing use of renewable energy sources in the electrical system introduces a series of challenges from the point of view of evaluating the

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reliability of power systems. In systems with predominantly thermoelectric or hydroelectric generation with large reservoirs, one can often abstract the underlying energy source from the reliability analysis, since in robust systems and with adequate operational decisions, there is sufficient stock to guarantee the energy supply. Thus, the assessment of generation reliability (HL1) is only concerned with the failure and repair processes of the generating units themselves.

However, certain types of renewable generation, such as wind, photovoltaic and run-of-river hydroelectric plants may not be dispatched by the operator's decision, as they depend on intermittent and non-stockable energy sources. Therefore, in these cases it is necessary to take into account the nature of the energy source and its characteristics regarding availability or unavailability.

Another question that arises concerns the statistical correlation between energy sources. For example, since the value of the power generated in wind or photovoltaic plants depends on the wind speed and irradiance, respectively, it is expected that nearby parks of the same source present a positive correlation regarding the generated value of power, since geographically close regions tend to have similar climatic characteristics. This effect is illustrated in Figure 1, in which it can be seen the power data generated in two wind farms and two solar parks of the same hybrid power plant in a period of 5 days.

It is worth mentioning another type of correlation to be explored in this work, although less obvious than the one verified between values of power generated by two parks that depend on the same energy source, but relevant in the context of hybrid plants. These plants often contain a renewable energy component that is balanced by a second form of generation. In Figure 1, for example, it is possible to observe a certain degree of complementarity (negative correlation) between wind and solar generations, in which solar sources generate more in the periods of the day when wind energy presents a reduction in generation and vice versa.

Therefore, this paper explore the use of Copulas theory and, in particular, the Hierarchical Archimedean Copulas (HAC) model, to guarantee the maintenance of the correlation and complementarity between the energy sources during the sampling stage of the Non-Sequential Monte Carlo simulation (MCS) in reliability evaluation studies.

Copula models are used to generate typical scenarios of joint power output of wind-photovoltaic plants. In [1], different types of copula are explored

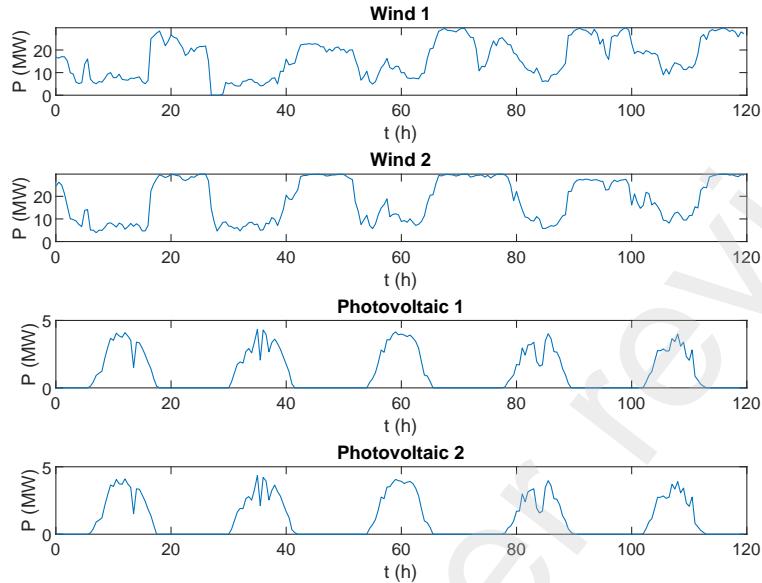


Figure 1: Time Series of Wind and Photovoltaic Generation of the Tacaratu Hybrid Park, Pernambuco - Brazil (5 days).

to establish the most suitable model of joint wind-solar output, a large number of samples is generated using the improved Latin Hypercube and then the K-means clustering algorithm is applied to generate typical scenarios. The joint wind-solar distribution function is established considering the Copula Frank in [2] and the correlations between renewable energy sources are used to generate energy output scenarios and then cubic spline interpolation method is used to fit the cumulative output curve and the backward scenario reduction technique is applied to reduce the sample number. The Frank Copula is also used in [3] for a probabilistic assessment of outgoing transformer operation risk considering solar-wind correlation. Reference [4] proposed a two-stage scenario generation method for wind-solar joint power output considering spatial and temporal correlations through the Markov chain based copula model. In [5] copula theory is used to define the dependence and probability distribution of the wind and the load when using Monte Carlo to generate several possible scenarios. Copula theory is also used in [6] to correlate instantaneous irradiance between different locations, based on the cross-correlation of irradiance indices for the locations.

All articles cited above focus on using copula models for the generation of typical scenarios, while in this paper copula models are used to represent

the correlation as well as complementarity between variable energy sources in reliability studies using Non-Sequential MCS. Therefore, the purpose of using copulas in this paper is to represent the correlation between time series in an state sampling approach that is originally based on the hypothesis of independence between events. The adequate performance of the method is proven by comparing its results with the ones obtained by the Sequential MCS, which is based on the chronological simulation.

The paper is organized as follows: in Section 2, Copulas concepts and main definitions are described, and in Section 3 the HAC is presented; Section 4 describes how the HAC is incorporated into MCS reliability evaluation, Section 5 analyses the time series used in this paper for the Copulas selection, while Section 6 compares the reliability indices calculated using Copulas with the independent and sequential MCS; finally, Section 7 draws the main conclusions of the paper.

2. COPULAS

Copulas are functions whose main application involves using them to model the joint distribution of random variables from the marginal distributions and the structure of dependency between variables. More formally, as per Nelsen [7]:

Definition 1. *A function C , with domain at $[0, 1]^2$ and range at $[0, 1]$, is a two-dimensional copula if it has the following properties:*

1. $C(u, v)$ is increasing on u and v ;
2. $C(0, v) = C(u, 0) = 0$, $C(1, v) = v$ and $C(u, 1) = u$;
3. For $\forall u_1, u_2, v_1, v_2$ in $[0, 1]$ such that $u_1 \leq u_2$ and $v_1 \leq v_2$ we have $C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0$.

From the above definition, if $u = F(x)$ and $v = G(y)$, where F and G are *marginal distribution functions* of the random variables X and Y , respectively, then $C(F(x), G(y))$ is the joint distribution function of the random variables X and Y .

2.1. Archimedean Copulas

Among the several families of existing copula, the only ones to be investigated in the present work are the Archimedean copulas, following the methodology proposed in Yang [4]. First, the following auxiliary definition is necessary, according to Nelsen [7]:

Definition 2 (Copula Generating Function). Let $\varphi(\cdot)$ be a function $[0, 1] \rightarrow [0, \infty)$ twice differentiable in the interval $(0, 1)$ with the following properties:

- $\varphi(1) = 0$, $\varphi'(t) < 0$ (decreasing first derivative) and $\varphi''(t) > 0$ (convex function)
- $\varphi(0) = \infty$

From the function $\varphi(\cdot)$ satisfying these properties, the Archimedean copula is defined:

$$C(u, v) = \varphi^{-1}(\varphi(u) + \varphi(v)) \quad (1)$$

Table 1 shows the three subfamilies of Archimedean copulas used in the present work with their respective generating functions.

Table 1: Archimedean Copulas: Clayton, Gumbel and Frank.

Copula	$C_\theta(u, v)$	$\varphi_\theta(t)$
Gumbel	$\exp(-[(-\ln u)^\theta + (-\ln v)^\theta]^{1/\theta})$	$(-\ln(t))^{-\theta}$
Clayton	$[\max\{u^{-\theta} + v^{-\theta} - 1, 0\}]^{-1/\theta}$	$\theta^{-1}(t^{-\theta} - 1)$
Frank	$-\frac{1}{\theta} \ln \left(1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1} \right)$	$-\ln \left(\frac{e^{-\theta t} - 1}{e^{-\theta} - 1} \right)$

It is worth mentioning that all three subfamilies presented in Table 1 are uniparametric, that is, the dependency structure is controlled by only one parameter, θ , which will be calculated or estimated from Kendall's Tau after choosing an appropriate copula model.

2.2. Correlation Measures

There are several measures of dependence between variables that have been proposed over the years. Not all, however, are suitable for all types of application. The most popular measure in the bivariate case is the Pearson correlation, ρ , which is defined as:

$$\rho = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}} \quad (2)$$

There are some limitations of the Pearson's correlation:

- It is not a sufficiently informative measure in the presence of asymmetric dependence;

- It is not invariant by nonlinear transformations;
- The independence of two random variables implies $\rho = 0$, but on the other hand, if $\rho = 0$ it does not imply independence.

Because of these limitations, some alternative dependency measures have been proposed, such as rank correlation, whose most notorious representatives are Kendall's Tau (τ) and Spearman's Rho (ρ_S). This work makes use of Kendall's Tau, which has the following two definitions, according to Nelsen [7]:

Definition 3 (Concordance and Discordance). Suppose (x_i, y_i) and (x_j, y_j) are two observations of the vector (X, Y) of continuous random variables. We say that (x_i, y_i) and (x_j, y_j) are concordant if $x_i < x_j$ and $y_i < y_j$ or if $x_i > x_j$ and $y_i > y_j$. Similarly, we say that the pairs are discordant if $x_i < x_j$ and $y_i > y_j$, or if $x_i > x_j$ and $y_i < y_j$.

Definition 4 (Kendall's Tau). Let $(x_1, y_1), \dots, (x_n, y_n)$ a sample of random variables with n observations from a vector (X, Y) of continuous random variables. There are $\binom{n}{2}$ distinct pairs (x_i, y_i) and (x_j, y_j) of the observed sample, where each pair can be concordant or discordant, denote c as the number of pairs concordant and d the number of discordant pairs. Then Kendall's tau for a sample is defined as:

$$\tau = \frac{c - d}{c + d} = \frac{c - d}{\binom{n}{2}} \quad (3)$$

Table 2 shows the calculated values of Kendall's Tau for the generation data used in the present work. The values presented serve to provide a quantitative basis for the qualitative analyzes that will be shown latter. For example, it is noted that wind generation data are less strongly correlated than solar generation data. Furthermore, the wind and solar generation data showed a weak correlation, but a negative one.

Table 2: Kendall's τ Correlation Coefficient of the joint distribution.

	Wind 1	Wind 2	Solar 1	Solar 2
Wind 1	1.0000	0.7877	-0.2297	-0.2257
Wind 2	0.7877	1.0000	-0.2918	-0.2866
Solar 1	-0.2297	-0.2918	1.0000	0.9568
Solar2	-0.2257	-0.2866	0.9568	1.0000

The main reason that motivated the calculation of Kendall's Tau for the data is that there are closed formulas that relate Kendall's Tau to the parameter of some Archimedean copula subfamilies.

3. ARCHIMEDIAN HIERARCHICAL COPULA

The definition of copula given in previous section has been made for the bivariate case. However, in general, the definition can be easily expanded to the multivariate case, this definition being particularly simple in the case of Archimedean copulas: $C(u_1, \dots, u_n) = \varphi^{-1}(\varphi(u_1) + \dots + \varphi(u_n))$. Thus one can, for example, build a copula with four variables in order to obtain a single model of correlation between all the generations under consideration in this paper: wind 1, wind 2, photovoltaic 1, photovoltaic 2.

The approach of direct construction of a single multivariate Archimedean copula (tetravariate in this case), however, has certain disadvantages. First, Archimedean copulas are uniparametric, that is, a single parameter would have to be adjusted for an aggregate model of four variables that store different degrees of dependence, which would represent a simplifying hypothesis. Second, if a single tetravariate model were proposed, a single copula class would have to be selected. However, different copula models may be more appropriate to represent each specific type of correlation between two variables, might they be, for example, wind-wind, photovoltaic-photovoltaic or wind-photovoltaic. Thus, the ideal would be to be able to separately model the correlations that maintain specific characteristics among themselves and then have a way to aggregate them all into a final model.

Some limitations should be highlighted in the HAC approach. First, each copula chosen must necessarily belong to the Archimedean copula family. Although this is the approach adopted in the present paper, it may be of interest to investigate the suitability of other copula families, such as Gaussian or t-copula. Second, there are theoretical criteria that establish sufficient nesting conditions for the process to be feasible. Finally, it is essential that the variables are aggregated by levels, from the most correlated to the least correlated, as will be later illustrated.

4. INCORPORATION OF HAC INTO RELIABILITY EVALUATION

The HAC is incorporated into the reliability evaluation in order to represent the correlation between energy sources. The procedure through which

the Copulas models are used in the non-sequential MCS will be explained for the bivariate case, for simplicity.

In the classical non-sequential MCS (without considering the correlation), two random numbers between 0 and 1 are independently generated, which are then used in the CDFs of the random variables to select the state (level) of each generation (MW). Similarly, from the copula model, a pair of random numbers between 0 and 1 is generated, which will be used in the CDFs of the different energy sources to determine the generation state. The crucial difference lies in the fact that the copula model does not give rise to two numbers independently, but in a correlated way. There are several algorithms for sampling copulas, and reference [8] shows some of those for different applications (mainly in finance). The sampling process will be further illustrated in the reliability evaluation results section.

In general terms, the conceptual algorithm for reliability evaluating based on non-sequential MCS adopting the Copula models to represent correlation between time series is detailed below.

- *Time Series Analysis*

1. Analyze each pair of time series to identify their correlations and select the best Copula to represent them;
2. Create the HAC model of the multivariate problem in hierarchical levels from the most correlated to the least ones;
3. Construct the CDF associated with each time series.

- *Reliability Evaluation*

1. Sample a pair of pseudo-random numbers [0,1] from the appropriate Copula model;
2. Use the sampled numbers in the CDFs of each energy source to determine the generation state (Inverse Transformation);
3. Repeat this process in accordance with the HAC model for all sources;
4. Calculate the reliability indices (there is load shed if the sum of available generation is lower than the load value);
5. If the coefficient of variation (β) of the reliability indices are higher than the required value, return to step 1.

5. TIME SERIES ANALYSIS

The time series data used in this paper (shown in Figure 1) are from the Tacaratu Hybrid Power Plant, located in Pernambuco, Brazil, and are publicly available at the National System Operator website [9]. The terminologies wind 1, wind 2, photovoltaic 1 and photovoltaic 2 will be used throughout this work for simplicity, but refer to the following complexes within the Tacaratu Hybrid Plant: Pau Ferro (wind 1), Pedra do Gerônimo (wind 2), Fontes Solar I (photovoltaic 1) and Fontes Solar II (photovoltaic 2). These data are arranged in four time series with 39 months of duration in an hourly time basis. The tool used for implementation of the models was MATLAB.

5.1. Correlation Between Time Series

This section presents a qualitative and quantitative analysis of the correlation between the various combinations of pairs of variables involved, that is, correlations between the same energy source (wind-wind and photovoltaic-photovoltaic) and between different energy sources (wind-photovoltaic).

5.1.1. Wind-Wind Correlation

In Figure 2, a scatter plot of the wind generation series data and a frequency histogram of these same data are presented. From the scatter plot, the existing positive correlation can be seen, since pairs are not observed in the upper left or lower right corners, which would correspond to scenarios in which one of the wind farms was producing high output power, while the other would produce little or almost nothing.

Furthermore, from the side histograms and the joint histogram of Figure 2, there is a dependence on both tails. According to Joe [10], this concept of dependence on the tails is related to the amount of dependence on the tail of the upper right quadrant or the tail of the lower left quadrant of a bivariate distribution. This observation will be relevant later when choosing the appropriate copula model to represent the specific type of wind-wind correlation.

5.1.2. Photovoltaic-Photovoltaic Correlation

Similarly to the previous case, in Figure 3, a scatter plot of the data from the solar generation series and a frequency histogram of these same data are presented. A clear positive correlation can be observed from the scatter plot

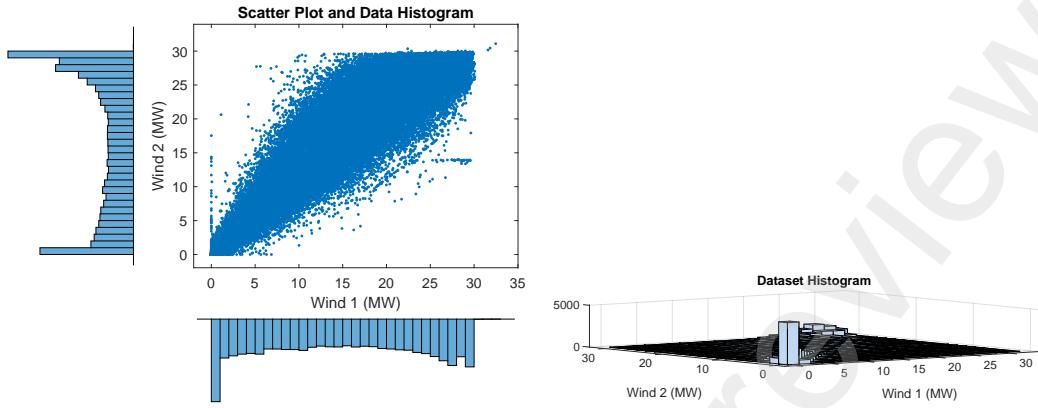


Figure 2: Scatter Plot and Histogram of Wind-Wind Correlation.

and, in addition, solar generation data seem to present a smaller dispersion in relation to wind generation, which indicates that they are more strongly correlated.

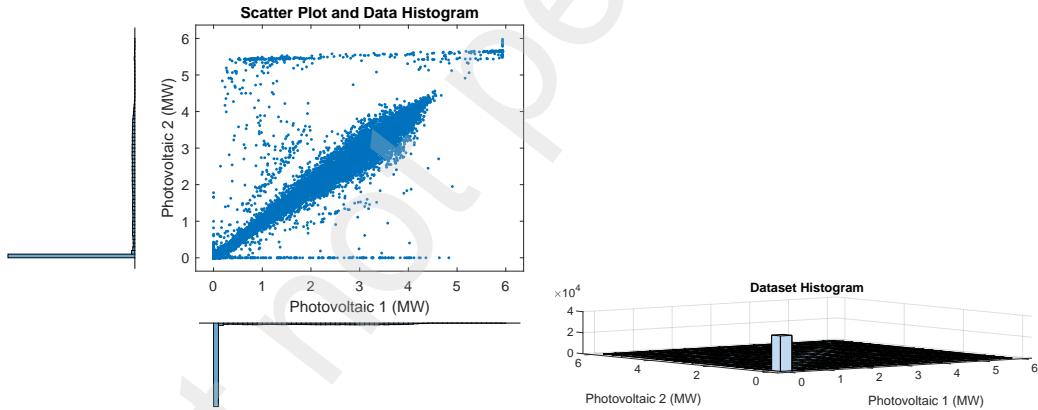


Figure 3: Scatter Plot and Histogram of Photovoltaic-Photovoltaic Correlation.

Unlike the wind-wind case, the lateral histograms and the joint histogram of Figure 3 indicate only a strong dependence on the lower tail, which reflects the fact that during all nights there is no solar generation, causing the most frequent value to appear in both series of solar generation to be null. Again, this observation will be relevant later when choosing the appropriate copula model to represent the specific type of photovoltaic-photovoltaic correlation.

5.1.3. Wind-Photovoltaic Correlation

Although there is a low and negative correlation between wind and solar generations, as already pointed out and shown in Table 2, Figure 4 do not seem to indicate any kind of clear correlation. Thus, the scatter plot seems to visually indicate that the wind and solar generations are not correlated, although a negative correlation exists.

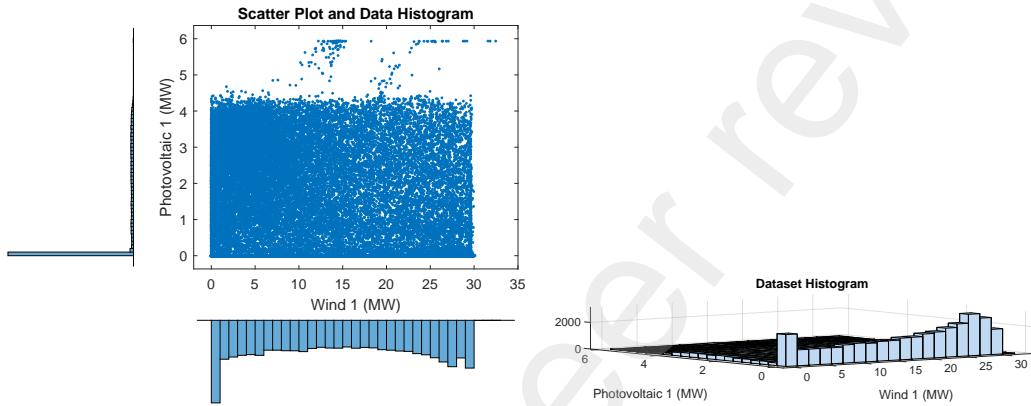


Figure 4: Scatter Plot and Histogram of Wind-Photovoltaic Correlation.

5.2. Copulas Selection

This section presents the selection of the copula models used to represent the different types of correlation between the time series. The methodology and justifications closely follow those presented in Yang [4] and Pan [11].

5.2.1. Wind 1 – Wind 2 Copula

As presented in Yang [4], the Gumbel copula is selected to represent the correlation model between wind generation sources. Although it does not allow adjustment of the lower tail by parameter, this copula model allows not only to represent the positive correlation, but also to adjust the dependence on the upper tail. The adjusted Gumbel copula is shown in Figure 5, where the mentioned characteristics can be observed. The value of the parameter θ estimated from Kendall's Tau for wind1-wind2 correlation for copula fitting was $\theta_{W-W} = 4.7112$, since C_θ is a member of the Gumbel family and therefore the equality $\tau_\theta = \frac{\theta-1}{\theta}$ is valid [7].

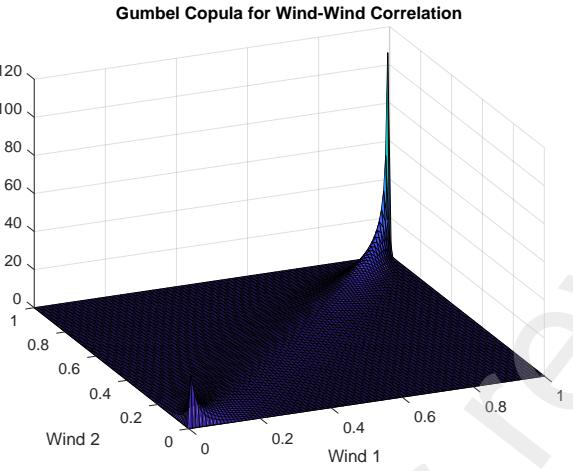


Figure 5: Gumbel Copula.

5.2.2. Photovoltaic 1 – Photovoltaic 2 Copula

As the Clayton copula allows to represent dependence on the lower tail, which is the main characteristic of the frequency histogram of the joint distribution between solar generations, it is selected as the model to represent the photovoltaic1-photovoltaic2 correlation and has its parameter θ adjusted to $\theta_{P-P} = 44.3349$, the equality being $\tau_\theta = \frac{\theta}{\theta+2}$, since C_θ is a member of the Clayton family [7].

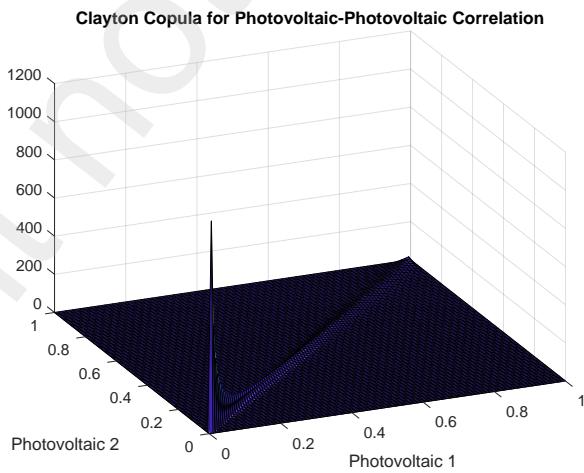


Figure 6: Clayton Copula.

5.2.3. Wind 1 – Photovoltaic 1 Copula

According to Yang [4], the Frank copula is appropriate to represent the wind-photovoltaic correlation due to the fact that it represents well the negative correlation between variables. In this way, it efficiently models the complementary behavior existing between these two energy sources. For this model, the adjusted θ value is $\theta_{W-P} = -2.1607$, since C_θ is a member of the Frank family and therefore the equality $\tau_\theta = 1 - \frac{4}{\theta}[1 - D_1(\theta)]$ is valid, where D_1 is the Debye function[7]. The Frank Copula can be seen in Figure 7.

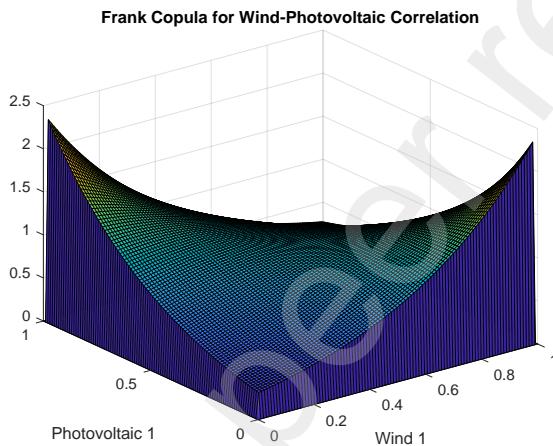


Figure 7: Frank Copula.

5.3. HAC model

The Archimedean hierarchical model has the approach of aggregating different copulas by levels as illustrated in Figure 8. Considering w_1, w_2 as wind 1, wind 2 and p_1, p_2 as photovoltaic 1, photovoltaic 2, in the first hierarchical level the Gumbel $C_G(w_1, w_2)$ and Clayton $C_C(p_1, p_2)$ copulas would be constructed to represent the respective correlations. Then, at the second hierarchical level, these would be aggregated in the construction of a Frank copula that is a function of the others $C(C_G(w_1, w_2), C_C(p_1, p_2))$. The methodology outlined here is fully exposed in Yang [4].

This structure is flexible enough to allow the use of different subfamilies of copulas, in addition to the fact that the parameter adjustment is done individually. Thus, the structure of Figure 8, in which the wind-wind and photovoltaic-photovoltaic copulas are first constructed and then aggregated later, is not arbitrary and the order of realization cannot be changed.

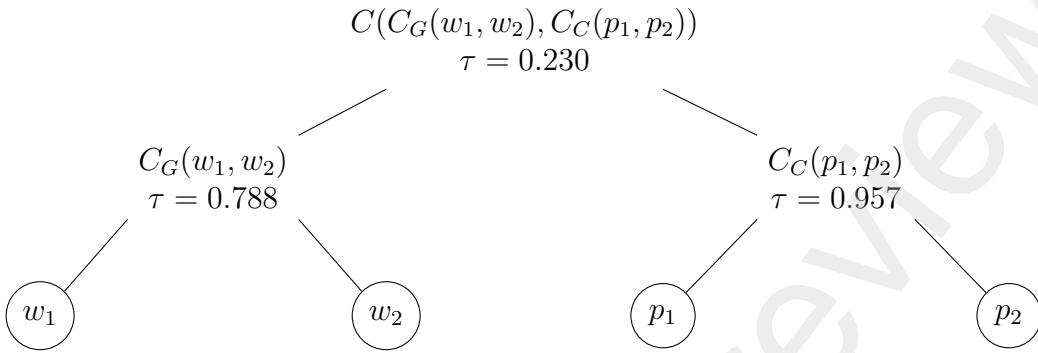


Figure 8: Proposed Structure - HAC Model.

Figure 9 shows all the bimarginal distributions (for each pair of variables) obtained by the HAC model, where U1, U2, U3, U4 refers to w1, w2, p1, p2, respectively.

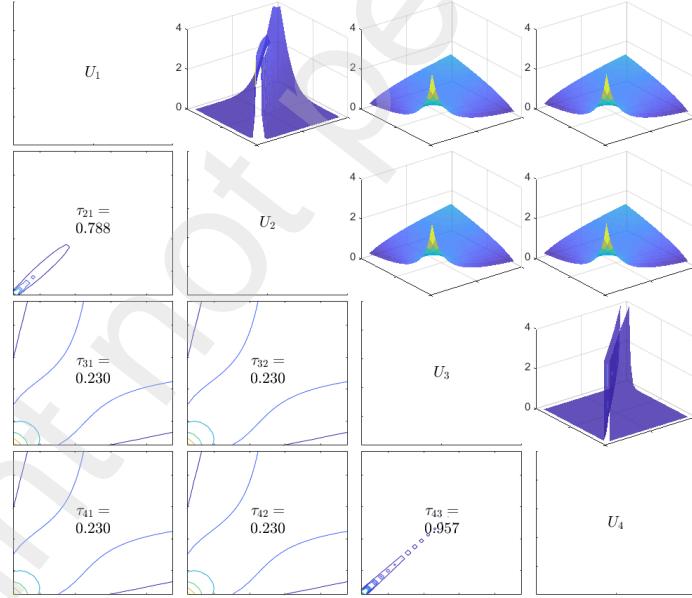


Figure 9: Bim marginal Distribution of HAC Model.

5.4. Cumulative Distribution Functions

It is necessary to construct the cumulative probability distributions (CDF) of the generation data, as shown in Figure 10. These will be used later in

the non-sequential MCS in the sampling process for reliability evaluation.

In Figure 10, two curves are present in each graph. The curves in blue, called empirical, are the cumulative probability distributions calculated from the frequency of occurrence of the different generation values in the respective historical series; the curves in red are cumulative probability distributions estimated using the Kernel Density Estimation technique. The purpose of showing both curves in these graphics is to indicate the flexibility of using the proposed approach even under conditions of scarce data, as done in [12].

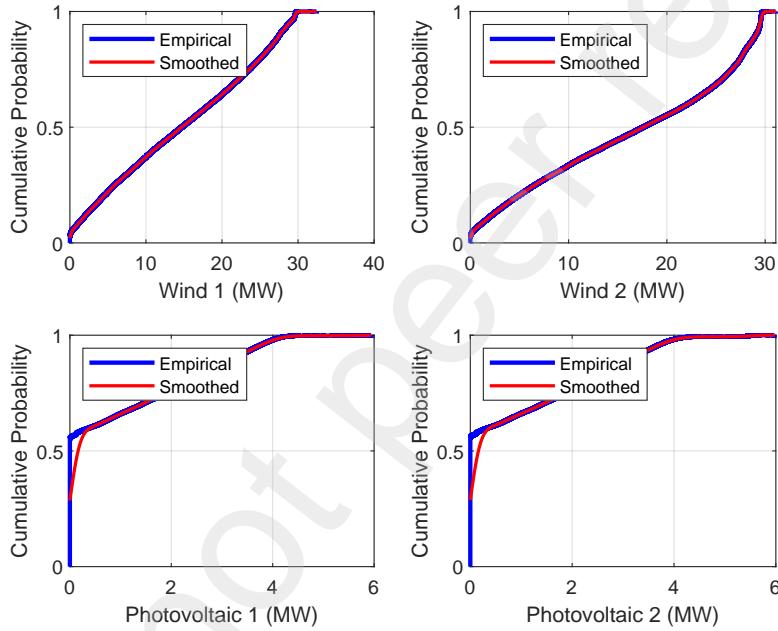


Figure 10: Cumulative Probability Distributions.

6. RELIABILITY EVALUATION

To validate the models presented above within the scope of their respective uses in power system reliability evaluation, four case studies were conducted. Considering the system generally used for generation reliability evaluation shown in Figure 11, fixed load levels are considered (but different between cases) and the reliability indices (LOLP - Loss of Load Probability, LOLE - Loss of Load Expectation [h], EPNS - Expected Power Not Supplied [MW], EENS - Expected Energy Not Supplied [MWh]) are calculated for the following four cases:

- Case A - Complete generation of the hybrid power plant (Wind 1 + Wind 2 + Photovoltaic 1 + Photovoltaic 2), with correlation represented by HAC model and load of 24MW;
- Case B - Partial generation of the hybrid plant (Wind 1 + Photovoltaic 1), with correlation represented by Frank copula and load of 12MW;
- Case C - Exclusively wind generation (Wind 1 + Wind 2), with correlation represented by Gumbel copula and load of 20MW;
- Case D - Exclusively solar generation (Photovoltaic 1 + Photovoltaic 2), with correlation represented by Clayton copula and load of 2MW.

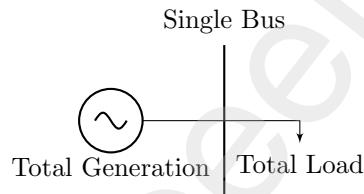


Figure 11: Test System for Generation Reliability Evaluation.

For the four cases analyzed, three MCS were performed: sequential MCS, which by its very nature preserves the correlation between the data; independent non-sequential MCS (without taking into account the correlation between the data) and non-sequential MCS with copula model to represent correlation.

6.1. Sampling Processes

The outcomes of the sampling process is illustrated in Figure 12 for the independent sampling, and in Figure 13 for the copula approach, both for the wind 1 - wind 2 case. For this joint distribution, 1000 pairs of random numbers between 0 and 1 were generated in both cases. In the case of Figure 12, the pairs of random numbers were generated by a traditional algorithm for generating (pseudo) random numbers uniformly distributed between 0 and 1. In the case of Figure 13, these numbers were sampled from the Gumbel copula with a parameter calculated to represent the specific type of correlation existing between wind generation series.

In Figure 12, it is immediately observed that the data are spread out completely randomly, as if there was no correlation between the wind 1 and wind 2 generation values. In Figure 13, on the other hand, the data are spread out in the scatter plot with the same pattern observed in the scatter plot of the real data of Figure 2. Furthermore, the calculated Kendall's Tau for the sets of samples from these figures are -0.007251 and 0.787591, respectively. It is then confirmed that the data generated without considering the correlation are in fact uncorrelated (or very poorly correlated), with Kendall's Tau close to zero. It is worth mentioning that the value of 0.787591 obtained for Kendall's Tau among the data sampled using Copula is equal, up to the third decimal, to that shown in Table 2 (calculated for the joint empirical distribution of the data).

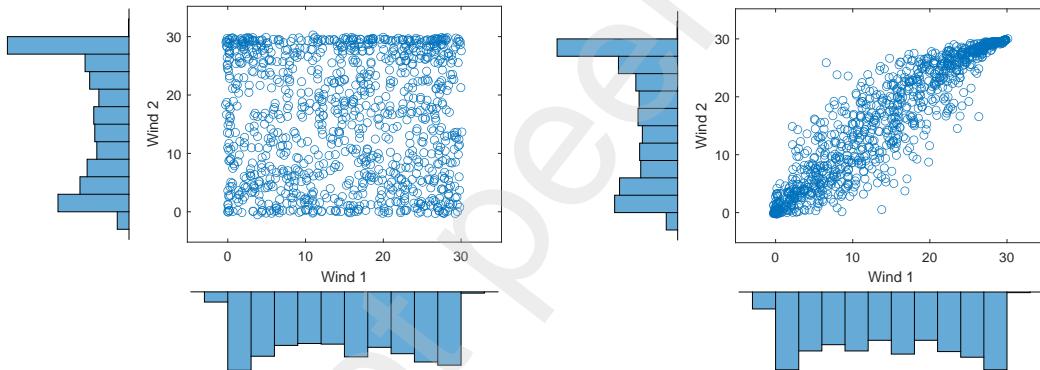


Figure 12: Generation of 1000 random samples from wind-wind without correlation.

Figure 13: Generation of 1000 random samples from wind-wind copula.

6.2. Reliability Indices

The simulation results for Case A, B, C and D are presented, respectively, in Tables 3, 4, 5 and 6. In these tables, N refers to the number of samples required to reach a coefficient of variation $\beta = 5\%$ and each index is shown with its correspondent confidence interval. Comparing the results of the two non-sequential simulations with the result of the sequential one, it is possible to evaluate the quality and adequacy of the models proposed and developed in this paper.

Table 3: Case A Reliability Indices.

Index	Non-Sequential (Independent)	Non-Sequential (with Copulas)	Sequential
	N = 1853	N = 1018	
LOLP (%)	25.63 ± 0.01	36.74 ± 0.02	36.32 ± 1.47
LOLE (h)	2440.37 ± 1.64	3497.52 ± 2.30	3457.67 ± 140.14
EPNS (MW)	2.36 ± 0.003	4.95 ± 0.004	4.51 ± 0.16
EENS (MWh)	22505.78 ± 27.93	47160.71 ± 37.68	42890.09 ± 1512.46

Table 4: Case B Reliability Indices.

Index	Non-Sequential (Independent)	Non-Sequential (with Copulas)	Sequential
	N = 984	N = 999	
LOLP (%)	38.01 ± 0.03	39.04 ± 0.02	39.66 ± 1.38
LOLE (h)	3618.37 ± 2.46	3716.52 ± 2.37	3775.67 ± 115.74
EPNS (MW)	2.23 ± 0.002	2.42 ± 0.002	2.41 ± 0.08
EENS (MWh)	21191.88 ± 17.22	22992.22 ± 17.09	22973.71 ± 716.96

Table 5: Case C Reliability Indices.

Index	Non-Sequential (Independent)	Non-Sequential (with Copulas)	Sequential
	N = 2033	N = 1120	
LOLP (%)	23.36 ± 0.04	33.66 ± 0.08	34.49 ± 1.21
LOLE (h)	2224.30 ± 5.60	3204.50 ± 7.87	3283.17 ± 94.18
EPNS (MW)	1.92 ± 0.01	3.87 ± 0.01	4.07 ± 0.11
EENS (MWh)	18265.11 ± 80.02	36835.50 ± 107.69	38788.65 ± 880.30

Table 6: Case D Reliability Indices.

Index	Non-Sequential (Independent)	Non-Sequential (with Copulas)	Sequential
	N = 472	N = 241	
LOLP (%)	55.08 ± 0.21	65.84 ± 0.38	65.94 ± 1.74
LOLE (h)	5244.07 ± 23.40	6268.31 ± 36.42	6277.67 ± 135.14
EPNS (MW)	0.78 ± 0.01	1.16 ± 0.01	1.22 ± 0.03
EENS (MWh)	7390.26 ± 49.75	11043.2 ± 71.82	11617.84 ± 234.05

Case A is the more complex system, since it has four different energy sources, and the use of the HAC structure proved to capture the dependencies properly. The calculated indices are within the confidence interval of those calculated by the sequential MCS.

The reliability indices calculated for Case B are even closer to the ones calculated by Sequential simulation, and this is due to the fact that it utilizes

the appropriate Frank Copula to model the correlation between wind and solar energy sources.

Both in Case C and Case D, the copula model also represented the correlation well, having performed properly. The indices calculated are within the confidence interval of the sequential MCS.

Concerning the independent non-sequential simulation, the calculated indices in Case B are less discrepant than for all other cases. This is due to the fact that the correlation between wind and photovoltaic generation is already low. Therefore, not considering the correlation reduces the errors in the results. On the other hand, in Case C and Case D there are large discrepancies when comparing the independent non-sequential Monte Carlo indices to the real ones. This is due to the high correlation between the variables and the need to somehow represent them in the simulation.

In general terms, it can be concluded that the non-sequential simulation with copula models produce accurate reliability indices, meaning with a fair representation of the correlation between the energy sources. In most cases there was an intersection of confidence intervals between the non-sequential and sequential simulations, allowing the conclusion that the values obtained are statistically the same.

Regarding the independent non-sequential MCS, all calculated indices are inaccurate and outside the confidence interval of the sequential simulation. It is worth noting that in all cases the calculated reliability indices were more optimistic than the true indices, which leads to erroneous conclusions about the reliability of the system.

7. CONCLUSION

This paper presented a model based on Copulas to incorporate the correlation between the different energy sources that compose an hybrid power plant in reliability evaluation based in state space sampling approach, like the non-sequential MCS. The proposed approach calculates reliability indices with the same accuracy of the sequential MCS, but avoids the computational burden of the chronological simulation. The tests with different case studies showed that ignoring the correlation between the sources may lead to an overestimation of the availability of power from the hybrid plant (or an underestimation in other cases, depending on the characteristics of the power sources). This leads to the conclusion that incorporating the Copula model

into the classical non-sequential MCS produces accurate indices when dealing with renewable energy that are correlated.

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