Global Topology Of Networks -6.1

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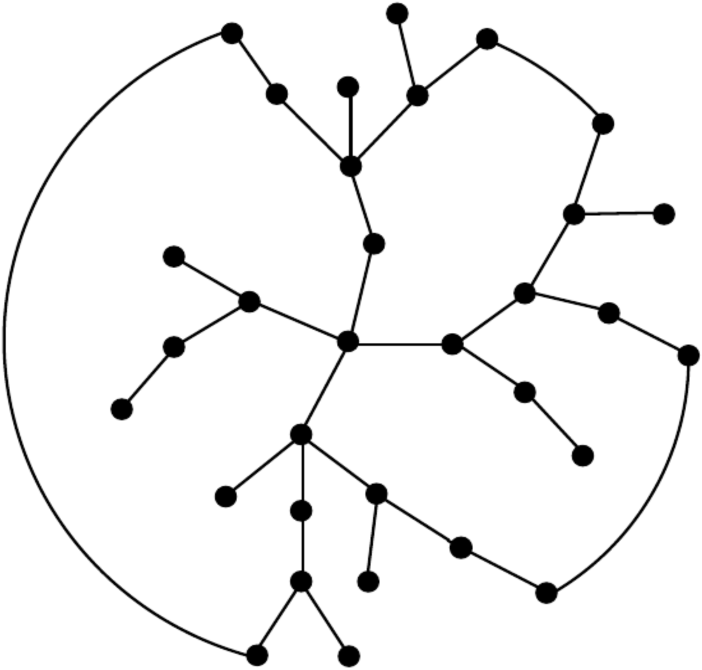
1. What the network looks like
2. Search in networks
3. Average shortest-path length
4. Birth of the giant component
5. Core of a net
6. Distribution of finite connected components



the distribution of the number of connections of a randowmly chosen end vertex of a randomly chosen edge:

$$\frac{kP(k)}{\bar{k}}$$

$$
\begin{aligned}
C &= \sum_{k_1,k_2} \frac{k_1 P(k_1)}{\bar{k}} \frac{k_2 P(k_2)}{\bar{k}} \frac{(k_1-1)(k_2-1)}{N\bar{k}} \\
&= \frac{\bar{k}}{N} \frac{(< k^2 > - \bar{k})^2}{\bar{k}^2}
\end{aligned}
\tag{6.1}
$$

the average degree of a randowmly chosen end vertex of a randomly chosen edge:

$$\sum_k k\frac{kP(k)}{\bar{k}} = \frac{< k^2 >}{\bar{k}} \tag{6.2}$$

the average number of the second-nearest neighbours of a vertex:

$$z_2 = < k^2 > - \bar{k} \tag{6.3}$$

In a finite net,the degree distribution has a cut-off.The special case of the power-law degree distribution.(m of the moment):

$$k_{cut} \sim k_0^m N^{(m-1)/(\gamma-1)-(\gamma-2)/(\gamma-1)} \tag{6.4}$$

γ < 3:

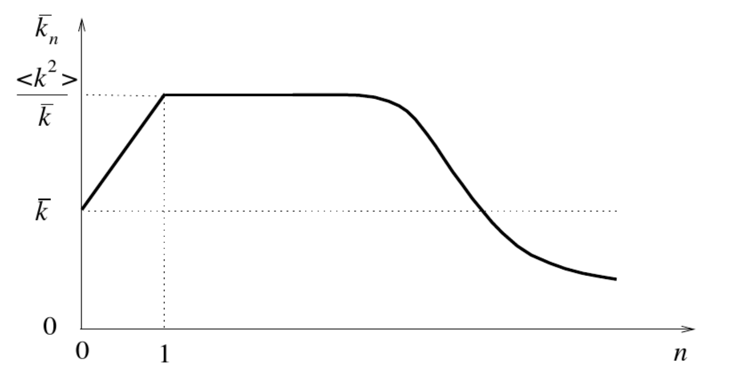
$$z_2 \cong < k^2 > \sim k_0^2 N^{(3-\gamma)/(\gamma-1)}$$

γ = 3:

$$z_2 \cong < k^2 > \approx k_0^2 \ln N$$

$$C = \frac{z_2^2}{N z_1^3} \tag{6.6}$$

$$C \sim k_0 N^{(7-3\gamma)/(\gamma-1)} \tag{6.7}$$



$\bar{k}_n$
$\frac{<k^2>}{\bar{k}}$
$\bar{k}$
0
0
1
n

$$t_s(N) = \sim \frac{1}{\bar{k}} N^2 (\gamma - 2)(\gamma - 1)$$

$$z_n = (\frac{z_2}{z_1})^{n-1} z_1$$

$$z_1 + z_1(z_2/z_1)^2 + ... + z_1(z_2/z_1)^{\bar{l}-1} \sim N_G \sim N$$

$$\bar{l} \approx \frac{ln(N/z_1) + ln[(z_2 - z_1)/z_1]}{ln(z_2/z_1)}$$

Reference:1+$\sum_{n=1}^{-l} z_m$ = N(Newman,Storgatz,and Watts 2001)

$$z_2 > z_1$$

$$W = 1 - \sum_{k=0}^{\infty} P(k) x^k$$

$$x = \sum_{k=0}^{\infty} \frac{kP(k)}{\bar{k}} x^{k-1}$$