The Banker's Sequence

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1 Recursive function

Initial value of the Banker's sequence is a bit string of zeros with length n.

Let n, y and z be such that

$$b = \underbrace{b_0 b_1 b_2 \dots 1 \overbrace{00\dots0}^{z}\overbrace{11\dots1}^{y}}_{n}.$$

$$next(b) = b_0 b_1 \dots b_{n-y-z-2} 0 \overbrace{11\dots1}^{y+1}\overbrace{00\dots0}^{z-1} \tag{1}$$

2 Inverse function

Let c_i be the cardinality (number of ones) of the first $i+1$ bits in the binary representation of b (i.e. bit string $b_0 b_1 b_2 \dots b_i$), where $0 \leq i < n$ and (as a convenience) let the cardinality of b be $c = c_{n-1}$.

$$a = \sum_{i=0}^{n-1} \begin{cases} \binom{n}{c_i - 1}, & b_i = 1 \\ \binom{n-i-1}{c-c_i-1}, & b_i = 0 \text{ and } c_i < c \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

3 Non-recursive function

Let c_i be the cardinality of bit string $b_0b_1b_2\dots b_i$, where $0 \leq i < n$ and let the cardinality of b be $c = c_{n-1}$.

Let

$$e_0 = a - \sum_{i=0}^{c-1} \binom{n}{i} \tag{3}$$

and

$$b_0 = \begin{cases} 1, & \binom{n-1}{c-1} > e_0 \\ 0, & \text{otherwise} \end{cases}. \tag{4}$$

1. Choose c for $\max c$ such that

$$\sum_{i=0}^{c-1} \binom{n}{i} \leq a. \tag{5}$$

2. For $0 \leq i < n-1$,

$$e_{i+1} = \begin{cases} e_i - \binom{n-i-1}{c-c_i-1}, & b_i = 0 \\ e_i, & b_i = 1 \end{cases} \tag{6}$$

$$b_{i+1} = \begin{cases} 1, & \binom{n-i-2}{c-c_i-1} > e_{i+1} \text{ and } c > c_i \\ 1, & e_{i+1} = 0 \text{ and } c > c_i \\ 0, & \text{otherwise} \end{cases} \tag{7}$$