

The Banker's Sequence

Corin Lawson

December 19, 2011

1 Recursive function

Initial value of the Banker's sequence is a bit string of zeros with length n .

Let n , y and z be such that

$$b = \underbrace{b_0 b_1 b_2 \dots 1}_{n} \overbrace{00 \dots 0}^z \overbrace{11 \dots 1}^y.$$
$$next(b) = b_0 b_1 \dots b_{n-y-z-2} 0 \overbrace{11 \dots 1}^{y+1} \overbrace{00 \dots 0}^{z-1} \quad (1)$$

2 Inverse function

Let c_i be the cardinality (number of ones) of the first $i + 1$ bits in the binary representation of b (i.e. bit string $b_0 b_1 b_2 \dots b_i$), where $0 \leq i < n$ and (as a convenience) let the cardinality of b be $c = c_{n-1}$.

$$a = \sum_{i=0}^{n-1} \begin{cases} \binom{n}{c_i-1}, & b_i = 1 \\ \binom{n-i-1}{c-c_i-1}, & b_i = 0 \text{ and } c_i < c \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

3 Non-recursive function

Let c_i be the cardinality of bit string $b_0b_1b_2 \dots b_i$, where $0 \leq i < n$ and let the cardinality of b be $c = c_{n-1}$.

Let

$$e_0 = a - \sum_{i=0}^c \binom{n}{i} \quad (3)$$

and

$$b_0 = \begin{cases} 1, & \binom{n-1}{c-1} > e_0 \\ 0, & \text{otherwise} \end{cases}. \quad (4)$$

1. Choose c for max c such that

$$\sum_{i=0}^c \binom{n}{i} \leq a. \quad (5)$$

2. For $0 < i < n$,

$$e_{i+1} = \begin{cases} e_i - \binom{n-i-1}{c-c_i-1}, & b_i = 0 \\ e_i, & b_i = 1 \end{cases} \quad (6)$$

$$b_{i+1} = \begin{cases} 1, & \binom{n-i-2}{c-c_i-1} > e_{i+1} \text{ and } c \geq c_i \\ 1, & e_{i+1} = 0 \text{ and } c \geq c_i \\ 0, & \text{otherwise} \end{cases} \quad (7)$$