$$\text{Forward Difference of } f'(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\text{Backward Difference of } f'(x) = \frac{f(x) - f(x-\Delta x)}{\Delta x}$$

$$\text{Central Difference of } f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$

$$\text{Taylor Series Expansion of } f(x+\Delta x)$$

$$f(x+\Delta x) = f(x) + \Delta x \frac{df}{dx} + \frac{\Delta x^2}{2!}\frac{d^2 f}{dx^2} + \ldots + \frac{\Delta x^n}{n!}\frac{d^n f}{dx^n}$$

$$\text{Rearranging the terms}$$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{df}{dx} + \frac{\Delta x}{2!}\frac{d^2 f}{dx^2} + \ldots + \frac{\Delta x^{n-1}}{n!}\frac{d^n f}{dx^n}$$

$$\mathit{Since}\ \Delta x\ \mathit{is\ small,\ the\ right\ hand\ side\ is\ dominated\ by}\ \frac{\Delta x}{2!}\frac{d^2 f}{dx^2}$$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{df}{dx} + \frac{\Delta x}{2!}\frac{d^2 f}{dx^2}$$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} = f'(x) + \frac{\Delta x}{2!} f''(x)$$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} - f'(x) = \frac{\Delta x}{2!} f''(x)$$

First term of left hand side is forward difference approximation of $f'(x)$

Right hand side is the approximation error which is proportional to Δx

$$\text{Forward difference of } f''(x) = \frac{2f(x+\Delta x) - 2f(x)}{\Delta x^2} - \frac{2f'(x)}{\Delta x}$$

Likewise by doing Taylor Series Expansion of $f(x-\Delta x)$, we obtain

$$\text{Backward difference of } f''(x) = \frac{2f(x-\Delta x) - 2f(x)}{\Delta x^2} + \frac{2f'(x)}{\Delta x}$$

$$\text{Central difference of } f''(x) = \frac{\mathit{Forward\ Difference}\ + \mathit{Backward\ Difference}}{2}$$

$$\frac{\frac{2f(x+\Delta x) - 2f(x)}{\Delta x^2} - \frac{2f'(x)}{\Delta x} + \frac{2f(x-\Delta x) - 2f(x)}{\Delta x^2} + \frac{2f'(x)}{\Delta x}}{2}$$

$$\text{Central difference of } f''(x) = \frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{\Delta x^2}$$

In terms of the given variables, our final approximation of x''(t)

$$\frac{x(t+\Delta t) - 2x(t) + x(t-\Delta t)}{\Delta t^2}$$