$$Given:$$

$$(1)\, x''(t) + \frac{kx(t)}{m} = 0$$

$$(2)\, x(t) = x_0 cos(\sqrt{\frac{k}{m}}t)$$

$$Starting\, from\, (2)$$

$$x(t) = x_0 cos(\sqrt{\frac{k}{m}}t)$$

$$x'(t) = -x_0\sqrt{\frac{k}{m}} sin(\sqrt{\frac{k}{m}}t)$$

$$x''(t) = -\frac{kx_0}{m} cos(\sqrt{\frac{k}{m}}t)$$

$$Verifying\ that\, (2)\, satisfies\, (1)\, by\, substituting\, x''(t)\, and\, x(t)$$

$$(1)\, x''(t) + \frac{kx(t)}{m} = 0$$

$$-\frac{kx_0}{m} cos(\sqrt{\frac{k}{m}}t) + \frac{kx_0}{m} cos(\sqrt{\frac{k}{m}}t) = 0$$

$$Verifying\ initial\ conditions\ x(0) = x_0\ and\ x'(0) = 0$$

$$x(t) = x_0 cos(\sqrt{\frac{k}{m}}t)$$

$$x(0) = x_0 cos(0) = x_0$$

$$x'(t) = -x_0\sqrt{\frac{k}{m}} sin(\sqrt{\frac{k}{m}}t)$$

$$x'(0) = -x_0\sqrt{\frac{k}{m}} sin(0) = 0$$