PY. The effect of the yaw on the standard basis vectors $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ is to rotate them to the new directions $\mathbf{y}_1$, $\mathbf{y}_2$, and $\mathbf{y}_3$. So the vectors $\mathbf{y}_1$, $\mathbf{y}_2$, and $\mathbf{y}_3$ will define the directions of the x, y, and z axes when we do the pitch. The desired pitch transformation is then a rotation about the new y-axis (i.e., the axis in the direction of the vector $\mathbf{y}_2$). The vectors $\mathbf{y}_1$ and $\mathbf{y}_3$ form a plane, and when the pitch is applied, they are both rotated by an angle v in that plane. The vector $\mathbf{y}_2$ will remain unaffected by the pitch, since it lies on the axis of rotation. Thus, the composite transformation L has the following effect on the standard basis vectors.

$$\begin{aligned}
\mathbf{e}_1 &\xrightarrow{\text{yaw}} \mathbf{y}_1 \xrightarrow{\text{pitch}} \cos v\,\mathbf{y}_1 + \sin v\,\mathbf{y}_3 \\
\mathbf{e}_2 &\xrightarrow{\text{yaw}} \mathbf{y}_2 \xrightarrow{\text{pitch}} \mathbf{y}_2 \\
\mathbf{e}_3 &\xrightarrow{\text{yaw}} \mathbf{y}_3 \xrightarrow{\text{pitch}} -\sin v\,\mathbf{y}_1 + \cos v\,\mathbf{y}_3
\end{aligned}$$

The images of the standard basis vectors form the columns of the matrix representing the composite transformation:

$$\begin{aligned}
(\cos v\,\mathbf{y}_1 + \sin v\,\mathbf{y}_3,\ \mathbf{y}_2,\ -\sin v\,\mathbf{y}_1 + \cos v\,\mathbf{y}_3) &= (\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3)\begin{Bmatrix} \cos v & 0 & -\sin v \\ 0 & 1 & 0 \\ \sin v & 0 & \cos v \end{Bmatrix} \\
&= YP
\end{aligned}$$

It follows that matrix representation of the composite is a product of the two individual matrices representing the yaw and the pitch, but the product must be taken in the reverse order, with the yaw matrix Y on the left and the pitch matrix P on the right. Similarly, for a composite transformation of a yaw with angle u, followed by a pitch with angle v, and then a roll with angle w, the matrix representation of the composite transformation would be the product YPR.

SECTION 4.2 EXERCISES

1. Refer to Exercise 1 of Section 4.1. For each linear transformation L, find the standard matrix representation of L.

2. For each of the following linear transformations L mapping $\mathbb{R}^3$ into $\mathbb{R}^2$, find a matrix A such that $L(\mathbf{x}) = A\mathbf{x}$ for every $\mathbf{x}$ in $\mathbb{R}^3$:
   (a) $L((x_1, x_2, x_3)^T) = (x_1 + x_2, 0)^T$
   (b) $L((x_1, x_2, x_3)^T) = (x_1, x_2)^T$
   (c) $L((x_1, x_2, x_3)^T) = (x_2 - x_1, x_3 - x_2)^T$

3. For each of the following linear operators L on $\mathbb{R}^3$, find a matrix A such that $L(\mathbf{x}) = A\mathbf{x}$ for every $\mathbf{x}$ in $\mathbb{R}^3$:
   (a) $L((x_1, x_2, x_3)^T) = (x_3, x_2, x_1)^T$
   (b) $L((x_1, x_2, x_3)^T) = (x_1, x_1 + x_2, x_1 + x_2 + x_3)^T$
   (c) $L((x_1, x_2, x_3)^T) = (2x_3, x_2 + 3x_1, 2x_1 - x_3)^T$

4. Let L be the linear operator on $\mathbb{R}^3$ defined by
$$L(\mathbf{x}) = \begin{Bmatrix} 2x_1 - x_2 - x_3 \\ 2x_2 - x_1 - x_3 \\ 2x_3 - x_1 - x_2 \end{Bmatrix}$$
   Determine the standard matrix representation A of L, and use A to find $L(\mathbf{x})$ for each of the following vectors $\mathbf{x}$:
   (a) $\mathbf{x} = (1, 1, 1)^T$ **(b)** $\mathbf{x} = (2, 1, 1)^T$
   (c) $\mathbf{x} = (-5, 3, 2)^T$

5. Find the standard matrix representation for each of the following linear operators:
   (a) L is the linear operator that rotates each $\mathbf{x}$ in $\mathbb{R}^2$ by 45° in the clockwise direction.

(b) L is the linear operator that reflects each vector $\mathbf{x}$ in $\mathbb{R}^2$ about the x_1 axis and then rotates it 90° in the counterclockwise direction.

(c) L doubles the length of $\mathbf{x}$ and then rotates it 30° in the counterclockwise direction.

(d) L reflects each vector $\mathbf{x}$ about the line $x_2 = x_1$ and then projects it onto the x_1-axis.

6. Let

$$\mathbf{b}_1 = \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}, \quad \mathbf{b}_2 = \begin{Bmatrix} 1 \\ 0 \\ 1 \end{Bmatrix}, \quad \mathbf{b}_3 = \begin{Bmatrix} 0 \\ 1 \\ 1 \end{Bmatrix}$$

and let L be the linear transformation from $\mathbb{R}^2$ into $\mathbb{R}^3$ defined by

$$L(\mathbf{x}) = x_1\mathbf{b}_1 + x_2\mathbf{b}_2 + (x_1 + x_2)\mathbf{b}_3$$

Find the matrix A representing L with respect to the ordered bases $\{\mathbf{e}_1, \mathbf{e}_2\}$ and $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$.

7. Let

$$\mathbf{y}_1 = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}, \quad \mathbf{y}_2 = \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}, \quad \mathbf{y}_3 = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

and let $\mathcal{I}$ be the identity operator on $\mathbb{R}^3$.

(a) Find the coordinates of $\mathcal{I}(\mathbf{e}_1)$, $\mathcal{I}(\mathbf{e}_2)$, and $\mathcal{I}(\mathbf{e}_3)$ with respect to $\{\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3\}$.

(b) Find a matrix A such that $A\mathbf{x}$ is the coordinate vector of $\mathbf{x}$ with respect to $\{\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3\}$.

8. Let $\mathbf{y}_1, \mathbf{y}_2$, and $\mathbf{y}_3$ be defined as in Exercise 7, and let L be the linear operator on $\mathbb{R}^3$ defined by

$$\begin{aligned} &L(c_1\mathbf{y}_1 + c_2\mathbf{y}_2 + c_3\mathbf{y}_3) \\ &= (c_1 + c_2 + c_3)\mathbf{y}_1 + (2c_1 + c_3)\mathbf{y}_2 - (2c_2 + c_3)\mathbf{y}_3 \end{aligned}$$

(a) Find a matrix representing L with respect to the ordered basis $\{\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3\}$.

(b) For each of the following, write the vector $\mathbf{x}$ as a linear combination of $\mathbf{y}_1, \mathbf{y}_2$, and $\mathbf{y}_3$ and use the matrix from part (a) to determine $L(\mathbf{x})$:

(i) $\mathbf{x} = (7, 5, 2)^T$ **(ii)** $\mathbf{x} = (3, 2, 1)^T$

(iii) $\mathbf{x} = (1, 2, 3)^T$

9. Let

$$R = \begin{Bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{Bmatrix}$$

The column vectors of R represent the homogeneous coordinates of points in the plane.

(a) Draw the figure whose vertices correspond to the column vectors of R. What type of figure is it?

(b) For each of the following choices of A, sketch the graph of the figure represented by AR and describe geometrically the effect of the linear transformation:

(i) $A = \begin{Bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{Bmatrix}$

(ii) $A = \begin{Bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{Bmatrix}$

(iii) $A = \begin{Bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{Bmatrix}$

10. For each of the following linear operators on $\mathbb{R}^2$, find the matrix representation of the transformation with respect to the homogeneous coordinate system:

(a) The transformation L that rotates each vector by 120° in the counterclockwise direction

(b) The transformation L that translates each point 3 units to the left and 5 units up

(c) The transformation L that contracts each vector by a factor of one-third

(d) The transformation that reflects a vector about the y-axis and then translates it up 2 units

11. Determine the matrix representation of each of the following composite transformations.

(a) A yaw of 90°, followed by a pitch of 90°

(b) A pitch of 90°, followed by a yaw of 90°

(c) A pitch of 45°, followed by a roll of −90°

(d) A roll of −90°, followed by a pitch of 45°

(e) A yaw of 45°, followed by a pitch of −90° and then a roll of −45°

(f) A roll of −45°, followed by a pitch of −90° and then a yaw of 45°

12. Let Y, P, and R be the yaw, pitch, and roll matrices given in equations (1), (2), and (3), respectively, and let $Q = YPR$.

(a) Show that Y, P, and R all have determinants equal to 1.

(b) The matrix Y represents a yaw with angle u. The inverse transformation should be a yaw with angle $-u$. Show that the matrix representation of the inverse transformation is Y^T and that $Y^T = Y^{-1}$.

(c) Show that Q is nonsingular and express Q^{-1} in terms of the transposes of Y, P, and R.

13. Let L be the linear transformation mapping P_2 into $\mathbb{R}^2$ defined by

$$L(p(x)) = \begin{bmatrix} \int_0^1 p(x)\,dx \\ p(0) \end{bmatrix}$$

Find a matrix A such that

$$L(\alpha + \beta x) = A \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

14. The linear transformation L defined by

$$L(p(x)) = p'(x) + p(0)$$

maps P_3 into P_2. Find the matrix representation of L with respect to the ordered bases $[x^2, x, 1]$ and $[2, 1 - x]$. For each of the following vectors $p(x)$ in P_3, find the coordinates of $L(p(x))$ with respect to the ordered basis $[2, 1 - x]$:

(a) $x^2 + 2x - 3$ **(b)** $x^2 + 1$

(c) $3x$ **(d)** $4x^2 + 2x$

15. Let S be the subspace of $C[a, b]$ spanned by e^x, xe^x, and x^2e^x. Let D be the differentiation operator of S. Find the matrix representing D with respect to $[e^x, xe^x, x^2e^x]$.

16. Let L be a linear operator on $\mathbb{R}^n$. Suppose that $L(\mathbf{x}) = \mathbf{0}$ for some $\mathbf{x} \neq \mathbf{0}$. Let A be the matrix representing L with respect to the standard basis $\{\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n\}$. Show that A is singular.

17. Let L be a linear operator on a vector space V. Let A be the matrix representing L with respect to an ordered basis $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ of V [i.e., $L(\mathbf{v}_j) = \sum_{i=1}^{n} a_{ij}\mathbf{v}_i, j = 1, \ldots, n$]. Show that A^m is the matrix representing L^m with respect to $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$.

18. Let $E = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ and $F = \{\mathbf{b}_1, \mathbf{b}_2\}$, where

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

and

$$\mathbf{b}_1 = (1, -1)^T, \qquad \mathbf{b}_2 = (2, -1)^T$$

For each of the following linear transformations L from $\mathbb{R}^3$ into $\mathbb{R}^2$, find the matrix representing L with respect to the ordered bases E and F:

(a) $L(\mathbf{x}) = (x_3, x_1)^T$

(b) $L(\mathbf{x}) = (x_1 + x_2, x_1 - x_3)^T$

(c) $L(\mathbf{x}) = (2x_2, -x_1)^T$

19. Suppose that $L_1: V \to W$ and $L_2: W \to Z$ are linear transformations and E, F, and G are ordered bases for V, W, and Z, respectively. Show that, if A represents L_1 relative to E and F and B represents L_2 relative to F and G, then the matrix $C = BA$ represents $L_2 \circ L_1$: $V \to Z$ relative to E and G. *Hint*: Show that $BA[\mathbf{v}]_E = [(L_2 \circ L_1)(\mathbf{v})]_G$ for all $\mathbf{v} \in V$.

20. Let V and W be vector spaces with ordered bases E and F, respectively. If $L: V \to W$ is a linear transformation and A is the matrix representing L relative to E and F, show that

(a) $\mathbf{v} \in \ker(L)$ if and only if $[\mathbf{v}]_E \in N(A)$.

(b) $\mathbf{w} \in L(V)$ if and only if $[\mathbf{w}]_F$ is in the column space of A.

4.3 Similarity

If L is a linear operator on an n-dimensional vector space V, the matrix representation of L will depend on the ordered basis chosen for V. By using different bases, it is possible to represent L by different $n \times n$ matrices. In this section, we consider different matrix representations of linear operators and characterize the relationship between matrices representing the same linear operator.

Let us begin by considering an example in $\mathbb{R}^2$. Let L be the linear transformation mapping $\mathbb{R}^2$ into itself defined by

$$L(\mathbf{x}) = (2x_1, x_1 + x_2)^T$$

Since

$$L(\mathbf{e}_1) = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \text{and} \quad L(\mathbf{e}_2) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Thus, the matrix representing L with respect to $\{\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3\}$ is

$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

We could have found D by using the transition matrix $Y = (\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3)$ and computing

$$D = Y^{-1}AY$$

This was unnecessary due to the simplicity of the action of L on the basis $\{\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3\}$. ■

In Example 2, the linear operator L is represented by a diagonal matrix D with respect to the basis $\{\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3\}$. It is much simpler to work with D than with A. For example, it is easier to compute $D\mathbf{x}$ and $D^n\mathbf{x}$ than $A\mathbf{x}$ and $A^n\mathbf{x}$. Generally, it is desirable to find as simple a representation as possible for a linear operator. In particular, if the operator can be represented by a diagonal matrix, this is usually the preferred representation. The problem of finding a diagonal representation for a linear operator will be studied in Chapter 6.

SECTION 4.3 EXERCISES

1. For each of the following linear operators L on $\mathbb{R}^2$, determine the matrix A representing L with respect to $\{\mathbf{e}_1, \mathbf{e}_2\}$ (see Exercise 1 of Section 1.2) and the matrix B representing L with respect to $\{\mathbf{u}_1 = (1, 1)^T, \mathbf{u}_2 = (-1, 1)^T\}$:

(a) $L(\mathbf{x}) = (-x_1, x_2)^T$ **(b)** $L(\mathbf{x}) = -\mathbf{x}$

(c) $L(\mathbf{x}) = (x_2, x_1)^T$ **(d)** $L(\mathbf{x}) = \frac{1}{2}\mathbf{x}$

(e) $L(\mathbf{x}) = x_2\mathbf{e}_2$

2. Let $\{\mathbf{u}_1, \mathbf{u}_2\}$ and $\{\mathbf{v}_1, \mathbf{v}_2\}$ be ordered bases for $\mathbb{R}^2$, where

$$\mathbf{u}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \mathbf{u}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

and

$$\mathbf{v}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Let L be the linear transformation defined by

$$L(\mathbf{x}) = (-x_1, x_2)^T$$

and let B be the matrix representing L with respect to $\{\mathbf{u}_1, \mathbf{u}_2\}$ [from Exercise 1(a)].

(a) Find the transition matrix S corresponding to the change of basis from $\{\mathbf{u}_1, \mathbf{u}_2\}$ to $\{\mathbf{v}_1, \mathbf{v}_2\}$.

(b) Find the matrix A representing L with respect to $\{\mathbf{v}_1, \mathbf{v}_2\}$ by computing SBS^{-1}.

(c) Verify that

$$L(\mathbf{v}_1) = a_{11}\mathbf{v}_1 + a_{21}\mathbf{v}_2$$
$$L(\mathbf{v}_2) = a_{12}\mathbf{v}_1 + a_{22}\mathbf{v}_2$$

3. Let L be the linear transformation on $\mathbb{R}^3$ defined by

$$L(\mathbf{x}) = \begin{pmatrix} 2x_1 - x_2 - x_3 \\ 2x_2 - x_1 - x_3 \\ 2x_3 - x_1 - x_2 \end{pmatrix}$$

and let A be the standard matrix representation of L (see Exercise 4 of Section 4.2). If $\mathbf{u}_1 = (1, 1, 0)^T$, $\mathbf{u}_2 = (1, 0, 1)^T$, and $\mathbf{u}_3 = (0, 1, 1)^T$, then $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is an ordered basis for $\mathbb{R}^3$ and $U = (\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)$ is the transition matrix corresponding to a change of basis from $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ to the standard basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$. Determine the matrix B representing L with respect to the basis $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ by calculating $U^{-1}AU$.

4. Let L be the linear operator mapping $\mathbb{R}^3$ into $\mathbb{R}^3$ defined by $L(\mathbf{x}) = A\mathbf{x}$, where

$$A = \begin{pmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{pmatrix}$$

and let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

Find the transition matrix V corresponding to a change of basis from $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ to $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$, and use it to determine the matrix B representing L with respect to $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

5. Let L be the operator on P_3 defined by

$$L(p(x)) = xp'(x) + p''(x)$$

(a) Find the matrix A representing L with respect to $[1, x, x^2]$.

(b) Find the matrix B representing L with respect to $[1, x, 1 + x^2]$.

(c) Find the matrix S such that $B = S^{-1}AS$.

(d) If $p(x) = a_0 + a_1 x + a_2(1 + x^2)$, calculate $L^n(p(x))$.

6. Let V be the subspace of $C[a, b]$ spanned by $1, e^x, e^{-x}$, and let D be the differentiation operator on V.

(a) Find the transition matrix S representing the change of coordinates from the ordered basis $[1, e^x, e^{-x}]$ to the ordered basis $[1, \cosh x, \sinh x]$. [$\cosh x = \frac{1}{2}(e^x + e^{-x})$, $\sinh x = \frac{1}{2}(e^x - e^{-x})$.]

(b) Find the matrix A representing D with respect to the ordered basis $[1, \cosh x, \sinh x]$.

(c) Find the matrix B representing D with respect to $[1, e^x, e^{-x}]$.

(d) Verify that $B = S^{-1}AS$.

7. Prove that if A is similar to B and B is similar to C, then A is similar to C.

8. Suppose that $A = S\Lambda S^{-1}$, where Λ is a diagonal matrix with diagonal elements $\lambda_1, \lambda_2, \ldots, \lambda_n$.

(a) Show that $A\mathbf{s}_i = \lambda_i \mathbf{s}_i$, $i = 1, \ldots, n$.

(b) Show that if $\mathbf{x} = \alpha_1\mathbf{s}_1 + \alpha_2\mathbf{s}_2 + \cdots + \alpha_n\mathbf{s}_n$, then

$$A^k\mathbf{x} = \alpha_1\lambda_1^k\mathbf{s}_1 + \alpha_2\lambda_2^k\mathbf{s}_2 + \cdots + \alpha_n\lambda_n^k\mathbf{s}_n$$

(c) Suppose that $|\lambda_i| < 1$ for $i = 1, \ldots, n$. What happens to $A^k\mathbf{x}$ as $k \to \infty$? Explain.

9. Suppose that $A = ST$, where S is nonsingular. Let $B = TS$. Show that B is similar to A.

10. Let A and B be $n \times n$ matrices. Show that if A is similar to B then there exist $n \times n$ matrices S and T, with S nonsingular, such that

$$A = ST \quad \text{and} \quad B = TS$$

11. Show that if A and B are similar matrices, then $\det(A) = \det(B)$.

12. Let A and B be similar matrices. Show that

(a) A^T and B^T are similar.

(b) A^k and B^k are similar for each positive integer k.

13. Show that if A is similar to B and A is nonsingular, then B must also be nonsingular and A^{-1} and B^{-1} are similar.

14. Let A and B be similar matrices and let λ be any scalar. Show that

(a) $A - \lambda I$ and $B - \lambda I$ are similar.

(b) $\det(A - \lambda I) = \det(B - \lambda I)$.

15. The *trace* of an $n \times n$ matrix A, denoted $\text{tr}(A)$, is the sum of its diagonal entries; that is,

$$\text{tr}(A) = a_{11} + a_{22} + \cdots + a_{nn}$$

Show that

(a) $\text{tr}(AB) = \text{tr}(BA)$

(b) if A is similar to B, then $\text{tr}(A) = \text{tr}(B)$.

Chapter Four Exercises

MATLAB EXERCISES

1. Use MATLAB to generate a matrix W and a vector $\mathbf{x}$ by setting

$$W = \texttt{triu}(\texttt{ones}(5)) \quad \text{and} \quad \mathbf{x} = [1 : 5]'$$

The columns of W can be used to form an ordered basis

$$F = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, \mathbf{w}_4, \mathbf{w}_5\}$$

Let $L\colon \mathbb{R}^5 \to \mathbb{R}^5$ be a linear operator such that

$$L(\mathbf{w}_1) = \mathbf{w}_2, \quad L(\mathbf{w}_2) = \mathbf{w}_3, \quad L(\mathbf{w}_3) = \mathbf{w}_4$$

and

$$L(\mathbf{w}_4) = 4\mathbf{w}_1 + 3\mathbf{w}_2 + 2\mathbf{w}_3 + \mathbf{w}_4$$
$$L(\mathbf{w}_5) = \mathbf{w}_1 + \mathbf{w}_2 + \mathbf{w}_3 + 3\mathbf{w}_4 + \mathbf{w}_5$$

(a) Determine the matrix A representing L with respect to F, and enter it in MATLAB.

(b) Use MATLAB to compute the coordinate vector $\mathbf{y} = W^{-1}\mathbf{x}$ of $\mathbf{x}$ with respect to F.