

MA 304 EXAM 1 ——-spring 2019

REMARKSThere are 8 problems. Problems 1-4 are each worth 12 points while problems 5-8 are each worth 13 points. Show all relevant work. NO CALCULATORS.

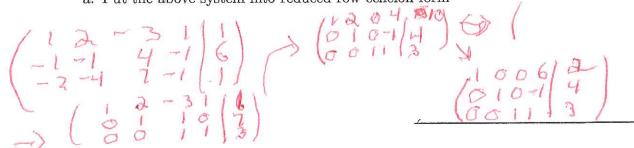
1. Given the following matrix equation

$$w + 2x - 3y + z = 1$$

$$-w - x + 4y - z = 6$$

$$-2w - 4x + 7y - z = 1$$

a. Put the above system into reduced row echelon form



b. Write an expression for all solutions to the matrix equation and write it as an equation of some plane, i.e., $(w, x, y, z)^T =$

2. Let

$$A = \begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$$

a. Write b as a linear combination of the columns of A.

$$\begin{pmatrix} 4 \\ 0 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 1 \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

b. Use the results from a. to determine a solution of the linear system Ax = b.

c. Let A be a 5×3 matrix. If a_1, a_2, a_3 represent the columns of A and

$$b = a_1 + a_2 = a_2 + a_3$$

then what can you conclude about the number of solutions of the linear system Ax = b? Explain!

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$$\binom{1}{i}$$
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3. Consider the 3×3 Vandermonde matrix:

$$V = \begin{pmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^3 \end{pmatrix}$$

b. What conditions must the scalars x_1, x_2, x_3 satisfy for V to be nonsingular?

4. Let A be the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & 3 \\ 1 & 2 & 2 \end{pmatrix}$$

Find A^{-1} by computing the reduced row echelon form of A.

- 5.
- a. Compute the LU factorization of

$$A = \begin{pmatrix} 2 & 4 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -21 \end{pmatrix}$$

$$(-1) \begin{pmatrix} 2 & 4 \\ 0 & 5 \end{pmatrix}$$

b. Let

$$A = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

Find a 2×2 matrix X which solves AX + B = X.

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6.

a. Is the following set a spanning set for \mathbb{R}^3 . Justify your answer.

$$A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & -2 \end{pmatrix}$$
 {(2,1,-2),(3,2,-2),(2,2,0)} $\begin{pmatrix} 2 & 2 & 0 \\ 3 & 2 & -2 \end{pmatrix}$ $\begin{pmatrix} 2 & 2 & 0 \\ 0 & 1 & -2 \end{pmatrix}$ rows not L. I.

b. Is the following a spanning set for P_3 (quadratic polynomials).

$$B = \begin{cases} 0 & 1 & 2 \\ 1 & 3 & -1 \end{cases}, \quad |B| = |1| = -1 = -1 = 0$$

$$B = \begin{cases} 0 & 1 & 2 \\ 1 & 3 & -1 \end{cases}, \quad |B| = |1| = -1 = -1 = 0$$

$$B = \begin{cases} 0 & 1 & 2 \\ 1 & 3 & -1 \end{cases}, \quad |B| = |1| = -1 = -1 = 0$$

$$E = \begin{cases} 0 & 1 & 2 \\ 1 & 3 & -1 \end{cases}$$

7.

a. Use Cramer's rule to solve the following system

b. Evaluate the determinant of the following matrix. Write you answer as a polynomial in x.

$$A = \begin{pmatrix} a + x & b & c \\ 1 & -x & 0 \\ 0 & 1 & -x \end{pmatrix}$$

$$(a - x)[x^{2}] - (-bx^{2})$$

$$-x^{2} + ax^{2} + bx + c$$

8.

a. Let x_1, x_2 and x_3 be linearly independent vectors in \mathbb{R}^n and let

$$y_1 = x_2 + x_1, \ y_2 = x_3 + x_2, \ y_3 = x_3 + x_1$$

Are y_1, y_2 and y_3 linearly independent? Prove your answer.

a (x2+x1)+b(x3+x2)+c(x3+x1) 50
a (x2+x1)+b(x3+x2)+c(x3+x1) 50
(b+c)x3+(a+b)x2+(a+c)x1 50
(b+c)x3+(a+b)x2+(a+c)x1 50

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b. Let $\mathbf{x_1}, \mathbf{x_2}, \cdots, \mathbf{x_k}$ be linearly independent vector in R^n , and let A be a nonsingular $n \times n$ matrix. Define $\mathbf{y_j} = \mathbf{Ax_j}$ for $j = 1, \dots, k$. Show that $\mathbf{y_1}, \mathbf{y_2}, \dots, \mathbf{y_k}$ are linearly independent.

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