MA 304 EXAM 1 ——-spring 2019

REMARKS There are 8 problems. Problems 1-4 are each worth 12 points while problems 5-8 are each worth 13 points. Show all *relevant* work. **NO CALCULATORS.**

1. Given the following matrix equation

$$\begin{aligned} w + 2x - 3y + z &= 1 \\ -w - x + 4y - z &= 6 \\ -2w - 4x + 7y - z &= 1 \end{aligned}$$

a. Put the above system into reduced row echelon form

b. Write an expression for **all** solutions to the matrix equation and write it as an equation of some plane, i.e., $(w, x, y, z)^T =$

2. Let

$$A = \begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$$

a. Write **b** as a linear combination of the columns of **A**.

b. Use the results from a. to determine a solution of the linear system $\mathbf{Ax} = \mathbf{b}$.

c. Let A be a 5×3 matrix. If a_1, a_2, a_3 represent the columns of A and

$$b = a_1 + a_2 = a_2 + a_3$$

then what can you conclude about the number of solutions of the linear system $Ax = b$? Explain!

3. Consider the 3×3 Vandermonde matrix:

$$V = \begin{pmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^{\cancel{3}\,2} \end{pmatrix}$$

a. Show that $det(V) = (x_2 - x_1)(x_3 - x_1)(x_3 - x_2)$.

4 b. What conditions must the scalars x_1, x_2, x_3 satisfy for V to be nonsingular?

4. Let A be the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 4 & 3 \\ 1 & 2 & 2 \end{pmatrix}$$

Find A^{-1} by computing the reduced row echelon form of A.

5.

a. Compute the LU factorization of

$$A = \begin{pmatrix} 2 & 4 \\ -2 & 1 \end{pmatrix}$$

b. Let

$$A = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

Find a 2×2 matrix X which solves AX + B = X.

6.

a. Is the following set a spanning set for R^3. Justify your answer.

$$\{(2,1,-2),(3,2,-2),(2,2,0)\}$$

b. Is the following a spanning set for P_3 (quadratic polynomials).

$$\{x+2, x+1, x^2-1\}$$

7.

a. Use Cramer's rule to solve the following system

$$\begin{aligned} 2x+3y&=2 \\ 3x+2y&=5 \end{aligned}$$

b. Evaluate the determinant of the following matrix. Write you answer as a polynomial in x.

$$A = \begin{pmatrix} a+x & b & c \\ 1 & -x & 0 \\ 0 & 1 & -x \end{pmatrix}$$

8.

a. Let $\mathbf{x_1}, \mathbf{x_2}$ and $\mathbf{x_3}$ be linearly independent vectors in R^n and let

$$\mathbf{y_1} = \mathbf{x_2} + \mathbf{x_1}, \quad \mathbf{y_2} = \mathbf{x_3} + \mathbf{x_2}, \quad \mathbf{y_3} = \mathbf{x_3} + \mathbf{x_1}$$

Are $\mathbf{y_1}, \mathbf{y_2}$ and $\mathbf{y_3}$ linearly independent? Prove your answer.

b. Let $\mathbf{x_1}, \mathbf{x_2}, \cdots, \mathbf{x_k}$ be linearly independent vector in R^n, and let A be a nonsingular $n \times n$ matrix. Define $\mathbf{y_j} = A\mathbf{x_j}$ for $j = 1, \cdots, k$. Show that $\mathbf{y_1}, \mathbf{y_2}, \cdots, \mathbf{y_k}$ are linearly independent.