MA 304 EXAM 2 ——-spring 2019

REMARKS There are 8 problems. Problems 1-4 are each worth 12 points while problems 5-8 are each worth 13 points. Show all *relevant* work. **NO CALCULATORS, NO CELL PHONES.**

1.

a. Let S be the subspace of P_3 (quadratics) consisting of all polynomials p(x) such that p(0) = 0, and let T be the subspace of all polynomials q(x) such that q(1) = 0. Find bases for T and $S \cap T$.

b. Given

$$\mathbf{v_1} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}, \quad \mathbf{v_2} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} 4 & 1 \\ 2 & 1 \end{pmatrix}$$

find vectors $\mathbf{u_1}$ and $\mathbf{u_2}$ so that **S** will be the transition matrix from $[\mathbf{v_1}, \mathbf{v_2}]$ to $[\mathbf{u_1}, \mathbf{u_2}]$.

2.

a. How many solutions will the linear system $A\mathbf{x} = \mathbf{b}$ have if **b** is in the column space of A and the column vectors of A are linearly dependent? Explain!

b. Let **A** be an $m \times n$ matrix with rank equal to n. Show that if $\mathbf{x} \neq 0$ and $\mathbf{Ax} = \mathbf{y}$, then $\mathbf{y} \neq 0$.

3. Find the kernel and range of each operator on P_3.

a. $\mathrm{Lp(x)} = xp'(x)$

b. $\mathrm{Lp(x)} = \mathrm{p(0)x} + \mathrm{p(1)}$

4.

a. If $L(x_1, x_2, x_3) = (2x_3, x_2+3x_1, 2x_1-x_3)$ is a linear operator on R^3 find a matrix A such that $\mathrm{L}(\mathbf{x}) = \mathrm{A}\mathbf{x}$ for every $\mathbf{x}$ in R^3.

b. Find the standard matrix representation for the linear operator L that reflects each vector $\mathbf{x}$ in R^2 about the x-axis and then rotates it 90° in the counterclockwise direction.

5.

a. Let L be the operator on P_3 defined by $L(p(x)) = xp'(x) + p''(x)$.

Find the matrix A representing L with respect to $[1, x, 1 + x^2]$

b. Show that if A and B are similar matrices, then det(A) = det(B).

6.

a. Find the point on the line y = 2x + 1 that is closest to the point (5, 2).

b. Find the distance from $(2, 1, -2)$ to the plane $6(x - 1) + 2(y - 3) + 3(z + 4) = 0$.

7.

a. Find the orthogonal complement of the subspace of R^3 spanned by $(1, 2, 1)$ and $(1, -1, 2)$.

b. Is it possible for a matrix to have the vector (3, 1, 2) in its row space and $(2, 1, 1)^T$ in its null space? Explain.

8.

a. Find the best least squares fit to the data $(-1, 0), (0, 1), (1, 3), (2, 9)$ by a quadratic po;ynomial.

b. Let A be an 8×5 matrix of rank 3, and let $\mathbf{b}$ be a nonzero vector in $N(A^T)$. How many least squares solutions will the system $A\mathbf{x} = \mathbf{b}$ have? Explain!