Notes for Modellering og løsning af optimeringsproblemer

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January 23, 2026

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1 Week 1

1.1 Key features of a Linear Programming model

There are are three main feature of importance in t linear programming (LP) model. These are:

- A linear objective function that needs to be maximized or minimized.
- A set of linear constraints that restrict the values of the decision variables. These maybe equalities or inequalities.
- Detemination of the coefficients in the objective function.

Models has to reflect the system that they are trying to represent. It is important to consider things like non-negativity constraints or if the variables can take on fractional values, as these often reflect real-world limitations.

1.2 Motivations for using a model

- **Costs:** Experimenting with reality can be very expensive, e.g., regarding the location of a factory.
- **Time:** Conducting experiments in practice is very time-consuming.
- **Repetition:** It may be necessary to conduct many repetitions to reduce statistical uncertainty. This is both expensive and time-consuming.
- **Danger:** There may be significant dangers in the modeled reality, e.g., bridge collapses or aircraft collisions.
- **Legality:** A model can, for example, be used to predict the effects of changing legislation.

1.3 Formation of a slack variable

A slack variable is introduces to simplify inequalities in constraints. For example, suppose we have the following constraint:

$$\sum_{i=1}^{n} a_i x_i \leq b_1 \tag{1}$$

and

$$\sum_{i=1}^{n} a_i x_i \geq b_2. \tag{2}$$

This yeilds the combined constraint:

$$b_2 \leq \sum_{i=1}^{n} a_i x_i \leq b_1. \tag{3}$$

To convert this into an equation, we can introduce a slack variable $s \geq 0$. We first look at the upper bound, which we can transform into and equality by defining the slack variable as:

$$s = b_1 - \sum_{i=1}^{n} a_i x_i. \tag{4}$$

Then, we take the lower bound inequality and use that $\sum_{i=1}^{n} a_i x_i = b_1 - s$, by definition, to get:

$$b_2 \leq b_1 - s \implies 0 \leq s \leq b_1 - b_2. \tag{5}$$

This forms the equivalent constraints:

$$\sum_{i=1}^{n} a_i x_i + s = b_1, \tag{6}$$

with

$$0 \leq s \leq b_1 - b_2. \tag{7}$$

Then the slack variable s can be treated as an additional decision variable in the linear programming model. And the inequality can be incorporated into the objective function as an equality equation.