Notes for Modellering og løsning af optimeringsproblemer

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1 Week 1

1.1 Key features of a Linear Programming model

There are are three main feature of importance in t linear programming (LP) model. These are:

- A linear objective function that needs to be maximized or minimized.
- A set of linear constraints that restrict the values of the decision variables. These maybe equalities or inequalities.
- Determination of the coefficients in the objective function.

Models has to reflect the system that they are trying to represent. It is important to consider things like non-negativity constraints or if the variables can take on fractional values, as these often reflect real-world limitations.

1.2 Motivations for using a model

- **Costs:** Experimenting with reality can be very expensive, e.g., regarding the location of a factory.
- **Time:** Conducting experiments in practice is very time-consuming.
- **Repetition:** It may be necessary to conduct many repetitions to reduce statistical uncertainty. This is both expensive and time-consuming.
- **Danger:** There may be significant dangers in the modeled reality, e.g., bridge collapses or aircraft collisions.
- **Legality:** A model can, for example, be used to predict the effects of changing legislation.

1.3 Formation of a slack variable

A slack variable is introduces to simplify inequalities in constraints. For example, suppose we have the following constraint:

$$\sum_{i=1}^{n} a_i x_i \leq b_1 \tag{1}$$

and

$$\sum_{i=1}^{n} a_i x_i \geq b_2. \tag{2}$$

This yields the combined constraint:

$$b_2 \leq \sum_{i=1}^{n} a_i x_i \leq b_1. \tag{3}$$

To convert this into an equation, we can introduce a slack variable $s \geq 0$. We first look at the upper bound, which we can transform into and equality by defining the slack variable as:

$$s = b_1 - \sum_{i=1}^{n} a_i x_i. \tag{4}$$

Then, we take the lower bound inequality and use that $\sum_{i=1}^{n} a_i x_i = b_1 - s$, by definition, to get:

$$b_2 \leq b_1 - s \implies 0 \leq s \leq b_1 - b_2. \tag{5}$$

This forms the equivalent constraints:

$$\sum_{i=1}^{n} a_i x_i + s = b_1, \tag{6}$$

with

$$0 \leq s \leq b_1 - b_2. \tag{7}$$

Then the slack variable s can be treated as an additional decision variable in the linear programming model. And the inequality can be incorporated into the objective function as an equality equation.

2 Week 2

2.1 Importance of Linearity

Linearity of the objective function is important since it restricts the solutions to the vertices of the feasible region. Even when alternative solutions exist along and edge, at least one vertex will always be an optimal solution. This property simplifies the search for optimal solutions, as algorithms can focus on evaluating vertices rather than the entire feasible region.

2.2 Defining Objectives

Sometimes the solutions are constrained only by the constraints and not by the objective function. Since solutions will always lie in the vertices of the feasible region, we can end up in situations where the objective function is irrelevant, since there is perhaps only one existing vertex. In these cases, the objective function can be defined arbitrarily, as it will not affect the solution.

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