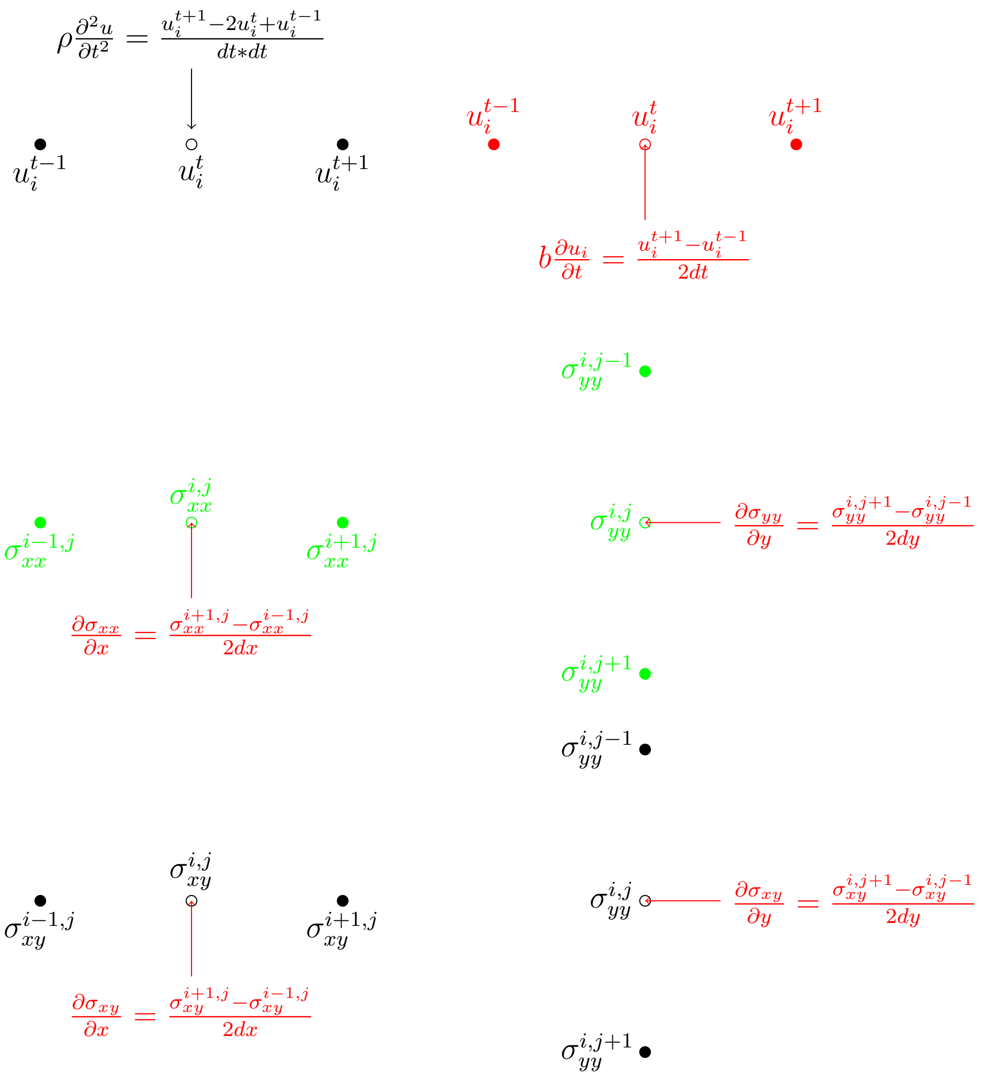
Objective: To transform the frame of reference to normal and tangential coordinates in the diffuse interface region of a 2D circular precipitate.
Why we need to transform?

$$\frac{\partial\sigma}{\partial r} = 0$$
$$\rho\frac{\partial^2 u_i}{\partial t^2} + b\frac{\partial u_i}{\partial t} = \frac{\partial \sigma_{ij}}{\partial x_j}$$

The second equation is solved by:



solved using In the previous report of plane interface inverse plottation modelwe have observed that in order to have a continous stress and strain profile in the system we require the coordinates be in normal and tangential axes,calculation of C_{ijkl} in the diffuse interface by inverse interplotation.

Formulation:

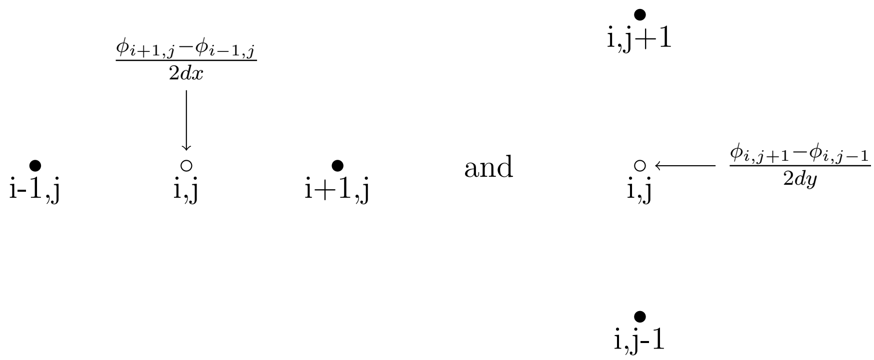
finding normal vector:
normal vector

$$n = -\frac{\nabla\phi}{|\nabla\phi|} \tag{1}$$

$$\nabla\phi_x = -\frac{\phi(x+1) - \phi(x-1)}{2dx} \tag{2}$$

$$\nabla\phi_y = -\frac{\phi(y+1) - \phi(y-1)}{2dy} \tag{3}$$

$$n_{x,y} = \frac{\nabla\phi_{x,y}}{\sqrt{\nabla\phi_x^2 + \nabla\phi_y^2}} \tag{4}$$



Discretization:Central difference is used.
I have stored the index of the points where atleast one of n_x, n_y vector is non-zero in a file named *domain-transform.txt*.
Rotation matrix of transformation:
A 2D array a[2][2] is used to store rotation transformation matrix and it is written at each index of domain in a file called rotation-matrix.txt
Notation used:
$a[0][0] = a_{nx} = n_x$
$a[0][1] = a_{ny} = n_y$
$a[1][0] = a_{tx} = -n_y$
$a[1][1] = a_{ty} = n_x$
C_{ijkl} transformation to C_{ntrs}:
1.Notation: In C_{ijkl} i,j,k,l takes x,y.The reduced form of C_{ijkl} due to symmetry operation is

$$\begin{bmatrix} \$C_{11} & C_{12} & C_{16} \\ C_{21} & C_{22} & C_{26} \\ C_{61} & C_{62} & C_{66}\$ \end{bmatrix} = \begin{bmatrix} C_{xxxx} & C_{xxyy} & C_{xxxy} \\ C_{yyxx} & C_{yyyy} & C_{yyxy} \\ C_{xyxx} & C_{xyyy} & C_{xyxy} \end{bmatrix}$$

only six independent components of C_{ntrs} at each index of domain is stored in a file.

2.$C^{\alpha,\beta}_{ntrs}$ at each index of the domain is obtained from $C^{\alpha,\beta}_{ijkl}$ and stored in files respectively. $C^{\alpha,\beta}_{ntrs} = a_{ni}a_{tj}a_{rk}a_{sl}C^{\alpha,\beta}_{ijkl}$

Stress and strains in the domain calculations: Let $\sigma_{nn}, \sigma_{nt}, \sigma_{tt}, \epsilon_{nn}, \epsilon_{nt}, \epsilon_{tt}$ be the total stress and strain components in the domain region.Let $\epsilon^{\alpha,\beta}_{nn}, \epsilon^{\alpha,\beta}_{nt}$ be the strain of the corresponding components α, β in the diffuse interface region.

First strains in the difffuse interface regio has been transformed into normal coordinates as follows.
$\epsilon_{nt} = a_{ni}a_{tj}\epsilon_{ij}$

$$S^{\alpha,\beta}_{nt} = \begin{bmatrix} C^{\alpha,\beta}_{nnnn} & 2C^{\alpha,\beta}_{nnnt} \\ C^{\alpha,\beta}_{ntnn} & 2C^{\alpha,\beta}_{ntnt} \end{bmatrix}^{-1} \tag{5}$$

Let

$$\begin{bmatrix} \sigma_{nn} \\ \sigma_{nt} \end{bmatrix} = \left(S^{\alpha}_{nt}\phi + S^{\beta}_{nt}\left(1-\phi\right)\right)^{-1} \left(\begin{bmatrix} \epsilon_{nn} \\ \epsilon_{nt} \end{bmatrix} + S^{\alpha}_{nt} \begin{bmatrix} C^{\alpha}_{nntt}\epsilon_{tt} \\ C^{\alpha}_{nttt}\epsilon_{tt} \end{bmatrix} \phi + S^{\beta}_{nt} \begin{bmatrix} C^{\beta}_{nntt}\epsilon_{tt} \\ C^{\beta}_{nnnt}\epsilon_{tt} \end{bmatrix} (1-\phi)\right) \tag{6}$$

$$\begin{bmatrix} \epsilon^{\alpha,\beta}_{nn} \\ \epsilon^{\alpha,\beta}_{nt} \end{bmatrix} = S^{\alpha,\beta}_{nt} \begin{bmatrix} \sigma_{nn} - C^{\alpha,\beta}_{nntt}\epsilon_{tt} \\ \sigma_{nt} - C^{\alpha,\beta}_{nttt}\epsilon_{tt} \end{bmatrix} \tag{7}$$

This is used to calculate σ_{tt} as follows:

$$\sigma_{tt} = \sigma^{\alpha}_{tt}\phi + \sigma^{\beta}_{tt}(1-\phi) \tag{8}$$

$\sigma^{\alpha,\beta}_{tt} = C^{\alpha,\beta}_{ttnn}\epsilon^{\alpha,\beta}_{nn} + 2C^{\alpha,\beta}_{ttnt}\epsilon^{\alpha,\beta}_{nt} + C^{\alpha,\beta}_{tttt}\epsilon_{tt}$