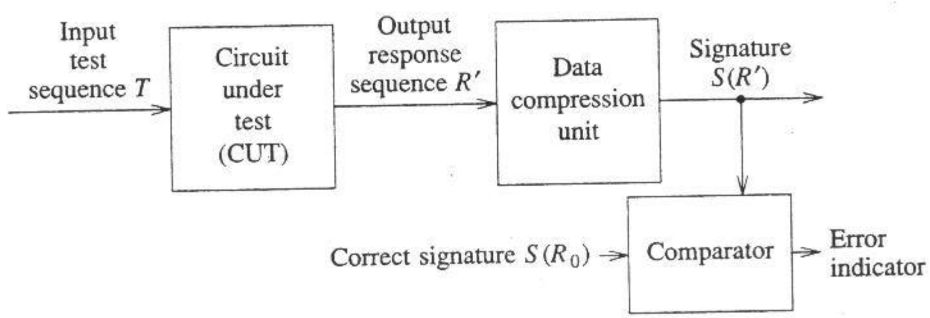
Compression Techniques & BIST (CH.10)

- Overview:
  - Signature / Response Compacting
  - Ones-Count Compression (10.2)
  - Transition-Count Compression (10.3)
  - Parity-Check Compression (10.4)
  - Syndrome Testing (10.5)
    - Special case of Ones-counting (Normalized)
  - Signature Analysis (LFSRs) (10.6)

Compression Techniques & BIST (CH.10)

- **Objective:** Reduce the memory storage requirements for the CUT output response
- ***Response Compression*** ==> ***signature***

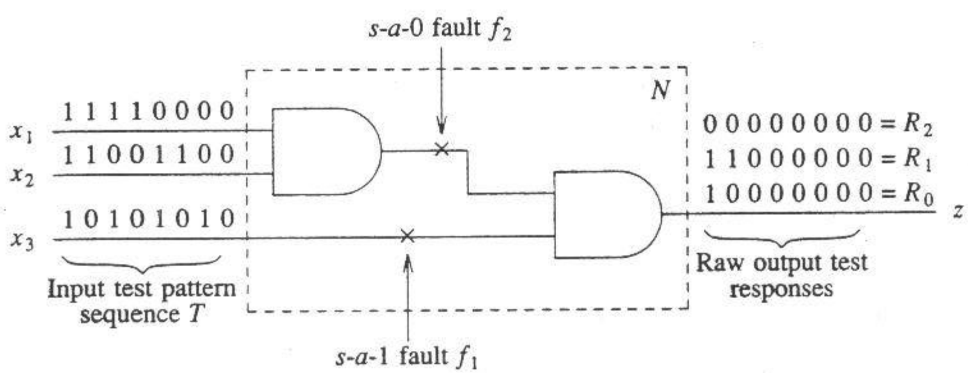


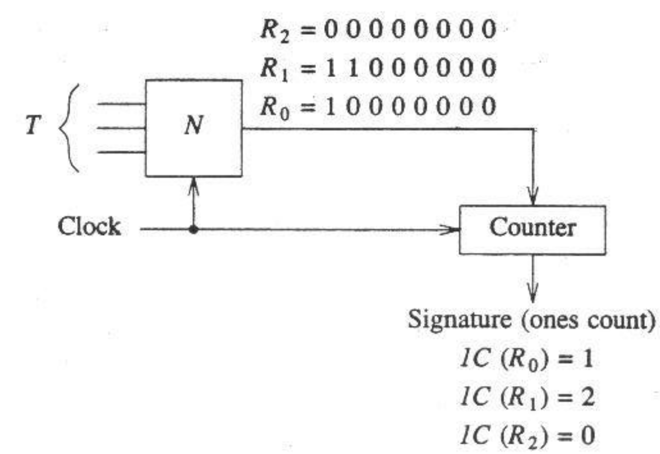
General Aspects of Compression Techniques (CH.10.1)

- Compression circuitry must be:
  - reliable
  - simple (low cost, small size)
  - must <u>not</u> introduce significant delays
  - capable of *significantly* reducing the output response
- Difficulties:
  - *Error Masking* => *alias*
  - How to determine the correct signature
    - simulation can be too expensive for long patterns
    - possibly use "Golden Device"
    - or find circuits which produce common signatures

Ones-Count Compression (CH.10.2)

- In this method, the number of 1's in the output signature is counted.
- The signature is denoted: $1C(R) = \sum_i r_i$
  - and the summation is from 1 to m
  - where m is the length of the output pattern





Ones-Count Compression (CH.10.2)

- The probability of masking depends on the number of 1's in a given response
  - If the number of 1's is close to 0 or to m, then the chances of masking are reduced
  - (assuming that all output patterns are equally likely)
- A fault that causes an odd number of errors will be detected (an even number might not)
- Inverting the output will not effect the test
- For combinational circuits, the *order* of the tests is not important.

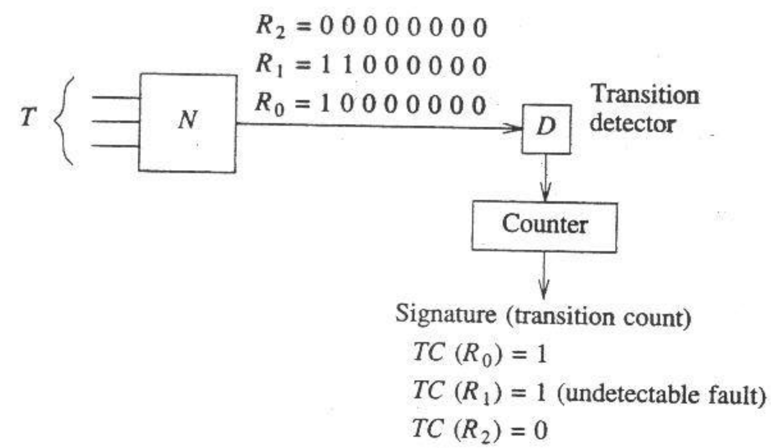
Ones-Count Compression (CH.10.2)

- Theorem 10.1 - For lengthy patterns, the masking probability approaches $(\pi m)^{-1/2}$
- Theorem 10.2 - We can devise a set of tests which will not exhibit error masking:
  - let $T = \{T^0, T^1\}$ be m test vectors that detect faults F.
  - T^0 are those tests with output 0
  - T^1 are those tests with output 1
  - Then let $T'(1C)$ = one copy of T^0 and $|T^0|+1$ copies of T^1
  - then, $T'(1C)$ detects all faults F with no masking.
  - the worst-case length of $T'(1C)$ is m^2

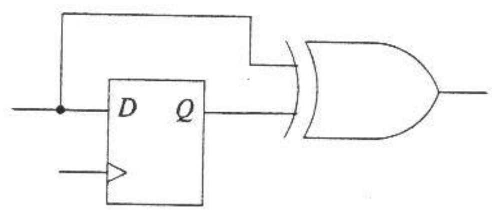
Transition-Count Compression (CH.10.3)

- In this method, we count the number of 0-1 and 1-0 *transitions* in the output response.
- Clearly the order of the tests is important.
- The compression factor is greater for patterns that have long sequences of repeated 1's and/or 0's.
- The T-C response signature is: $TC(R) = \sum_{i=1}^{m-1} (r_i \oplus r_{i+1})$

Transition-Count Compression (CH.10.3)



(a)



(b)

(a) Transition-count testing (b) A transition detector

Transition-Count Compression (CH.10.3)

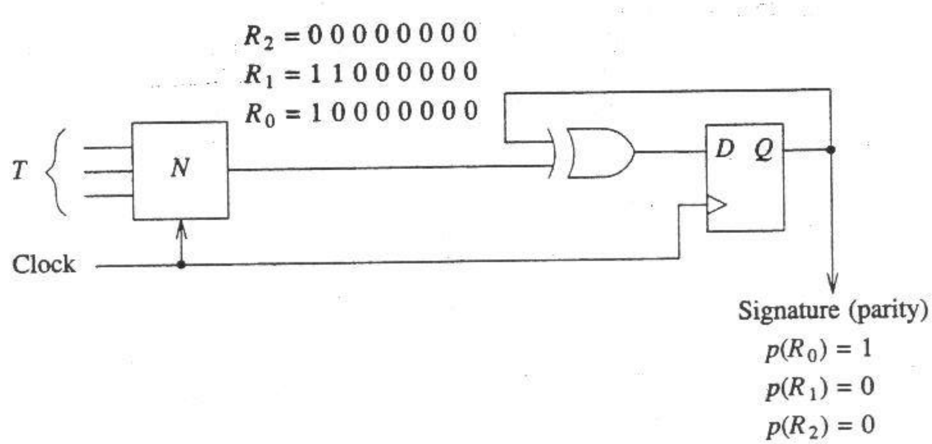
- **Theorem 10.3** - With an arbitrary m-bit sequence, the probability of a single-bit error being masked is (m-2)/2m
  - Note that for large m, this approaches 1/2
- **Theorem 10.4** - For combinational circuits, the masking probability appraches $(\pi m)^{-1/2}$

Transition-Count Compression (CH.10.3)

- **Theorem 10.5** - Let T be a SSF test set for an irredundant single-output combinational circuit. Then we construct a TC test, $X^* = t(1)t(2)t(3)\ldots t(p)$ as follows:
  - X^* contains every test in T
  - X^* alternates between tests from T^0 and T^1

Parity-Check Compression (CH.10.4)

- Actually this is a special case of LFSR (see section 10.6) where the primitive polynomial is just $G(x) = x+1$
- The signature is the parity of the response
  - $S = 0$ for even parity
  - $S=1$ for odd parity (assuming initial state is 0)

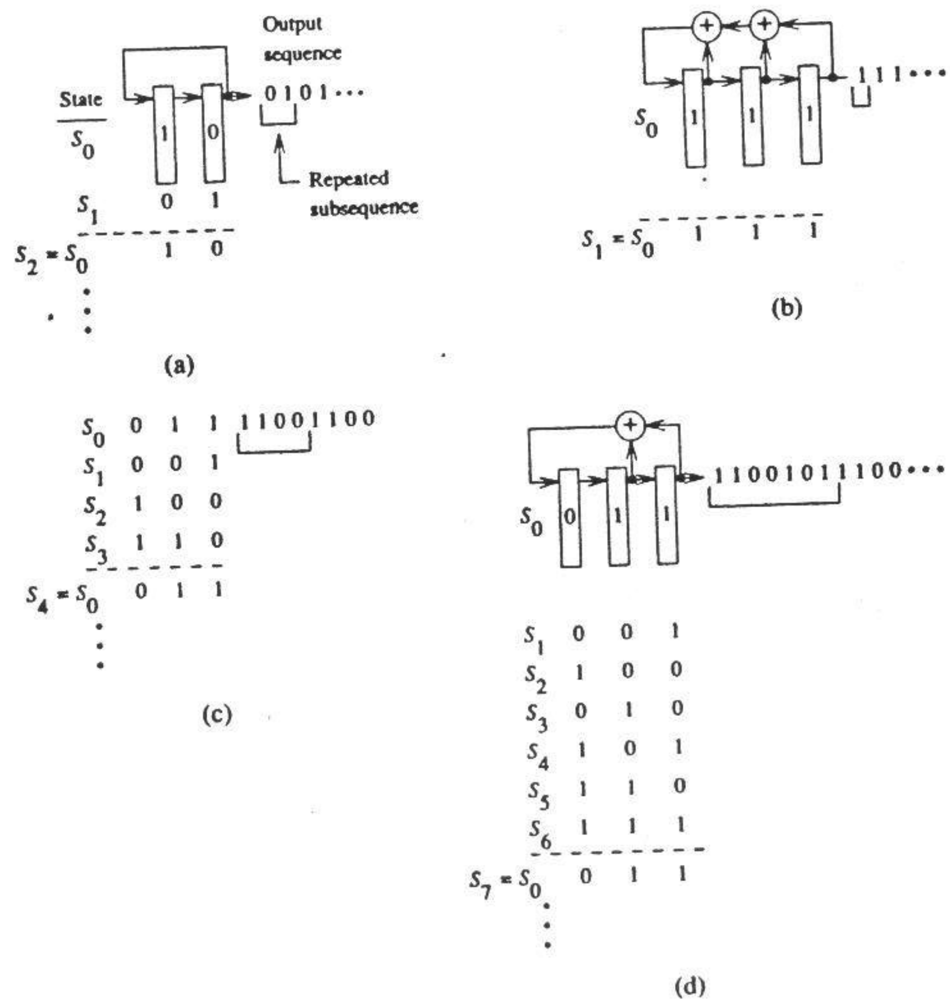


Signature Analysis (CH.10.6)

- Linear Feedback Shift Registers (10.6.1)
  - LFSRs can be used as pseudo-random pattern generators for input stimuli
  - and/or for output compression
  - Note that modulo-2 addition/subtraction can be implemented using XOR gates
  - Note: $x+x = -x-x = x-x = 0$ for Mod-2 addition

Signature Analysis (CH.10.6)

- Examples of Feedback Shift Registers



Signature Analysis (CH.10.6)

- Two types of LFSRs:

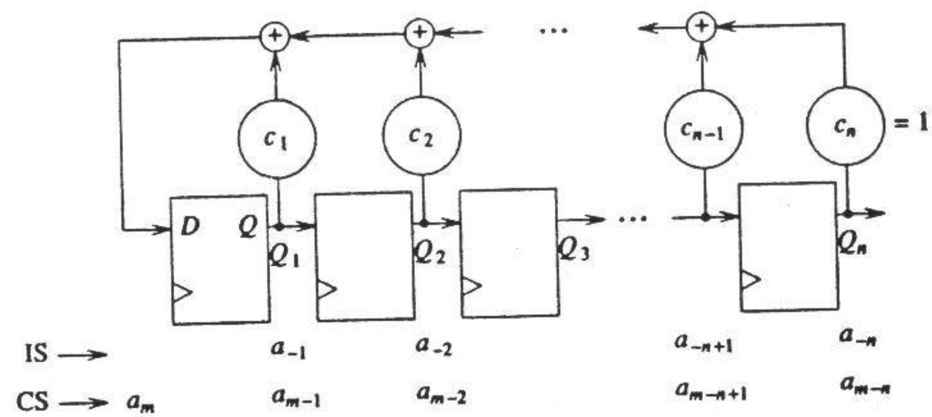


Figure 10.10 Type 1 LFSR

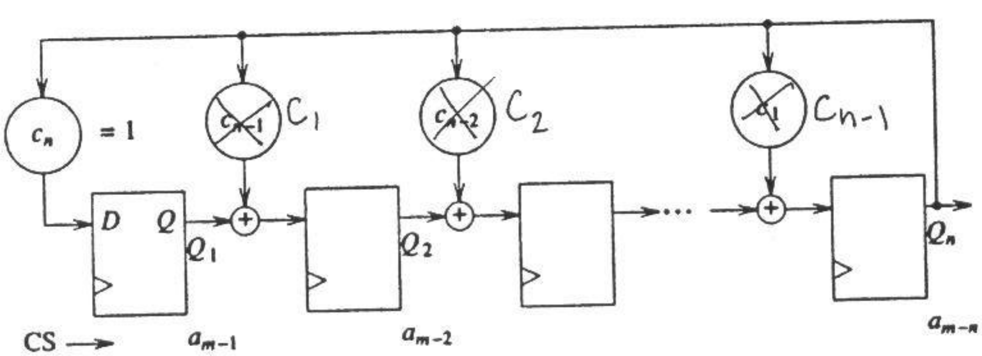


Figure 10.11 Type 2 LFSR

Signature Analysis (CH.10.6)

- Polynomial Division - Examples

Example 10.1:

(a)

$$
\begin{array}{rrrrrr}
 & & & x^2 & +\,x & +\,1 \\
\times & & & & x^2 & +\,1 \\
\hline
 & & & x^2 & +\,x & +\,1 \\
(+) & x^4 & +\,x^3 & +\,x^2 & & \\
\hline
 & x^4 & +\,x^3 & & +\,x & +\,1
\end{array}
$$

since $x^2 + x^2 = 0$.

(b)

$$
\begin{array}{r|l}
 & x^2 + x + 1 \\
\cline{2-2}
x^2 + 1 & x^4 + x^3 + x + 1 \\
 & (-)\; x^4 + x^2 \\
\cline{2-2}
 & \quad x^3 + x^2 + x + 1 \quad (\text{note } x^2 = -x^2) \\
 & \quad (-)\; x^3 + x \\
\cline{2-2}
 & \qquad x^2 + 1 \\
 & \qquad (-)\; x^2 + 1 \\
\cline{2-2}
 & \qquad\qquad 0
\end{array}
$$

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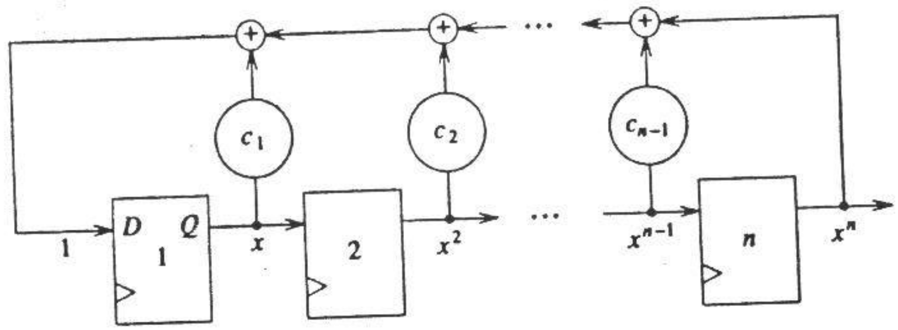
Signature Analysis (CH.10.6)

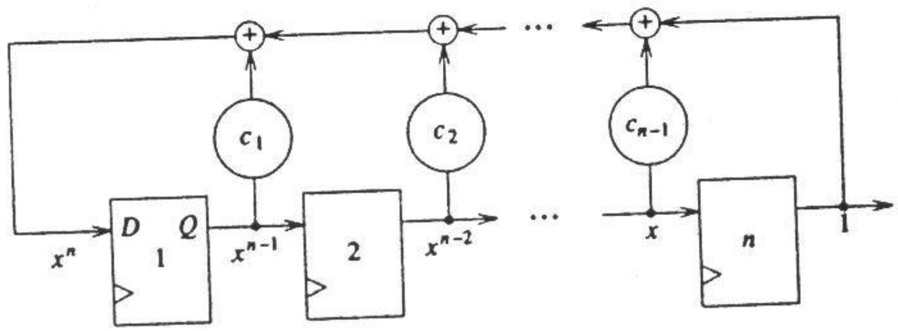
- LFSR Polynomials - Definitions / relations
  - Characteristic Polynomial:
    - $P(x) = 1 + c_1x + c_2x^2 + c_2x^3 + \ldots + c_nx^n$
  - Reciprocal Polynomial:
    - $P^*(x) = c_n + c_{n-1}x^1 + c_{n-2}x^2 + \ldots + c_1x^{n-1} + x^n$
  - Note: $P^*(x) = x^n P(1/x)$

Signature Analysis (CH.10.6)

- LFSR implementations:



(a)



(b)

Figure 10.12 Reciprocal characteristic polynomials
(a) $P(x) = 1 + c_1 x + c_2 x_2 + \ldots + c_{n-1}x^{n-1} + x^n$
(b) $P^*(x) = 1 + c_{n-1} x + c_{n-2} x^2 + \ldots + c_1 x^{n-1} + x^n$

Signature Analysis (CH.10.6)

- Periodicity of LFSRs
  - **Theorem 10.6** - If the initial state (IS) of the LFSR is all zeros except $a_{-n}=1$, then the LFSR sequence is periodic with a period that is the smallest integer k for which p(x) divides $(1-x^k)$.
  - An n-stage LFSR with period 2^n-1 produces a *maximum-length sequence.*
  - **Theorem 10.8** - A primitive polynomial is irreducible if the smallest positive integer k that allows the polynomial to divide evenly into $1+x^k$ occurs for $k=2^n-1$, where n is the degree of the polynomial.

Signature Analysis (CH.10.6)

- Number of primitive polynomials:

n	$\lambda_2(n)$
1	1
2	1
4	2
8	16
16	2048
32	67108864

Signature Analysis (CH.10.6)

- Examples of exponents for primitive polynomials:

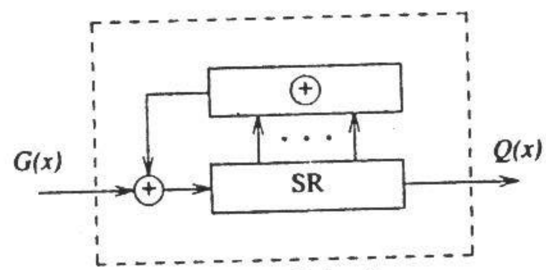
1:	0				13:	4	3	1	0	25:	3	0		
2:	1	0			14:	12	11	1	0	26:	8	7	1	0
3:	1	0			15:	1	0			27:	8	7	1	0
4:	1	0			16:	5	3	2	0	28:	3	0		
5:	2	0			17:	3	0			29:	2	0		
6:	1	0			18:	7	0			30:	16	15	1	0
7:	1	0			19:	6	5	1	0	31:	3	0		
8:	6	5	1	0	20:	3	0			32:	28	27	1	0
9:	4	0			21:	2	0			33:	13	0		
10:	3	0			22:	1	0			34:	15	14	1	0
11:	2	0			23:	5	0			35:	2	0		
12:	7	4	3	0	24:	4	3	1	0	36:	11	0		

Signature Analysis (CH.10.6)

- Characteristics of maximum-length sequences
  - Patterns are pseudorandom
  - The number of 1s differs from the number of 0s by 1
  - The number runs of 1s and 0s is equal
  - Half the runs have length 1, 1/4 have length 2, 1/8 have length 4, etc.

Signature Analysis (CH.10.6)

- LFSRs as Signature Analyzers (10.6.2)
  - Based on Cyclic Reduncancy Checking (CRC)



Initial state: $I(x) = 0$

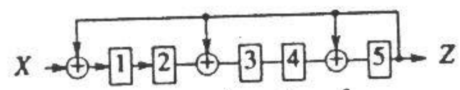
Final state: $R(x)$

$$\frac{G(x)}{P(x)} = Q(x) + \frac{R(x)}{P(x)}$$

or $G(x) = Q(x)\,P(x) + R(x)$

Signature Analysis (CH.10.6)

- Example 10.2



$P(x) = 1 + x^2 + x^4 + x^5$

(a)

Input sequence: 1 1 1 1 0 1 0 1 (8 bits)

$G(x) = x^7 + x^6 + x^5 + x^4 + x^2 + 1$

(b)

Time	Input stream	Register contents	Output stream
		1 2 3 4 5	
0	1 0 1 0 1 1 1 1	0 0 0 0 0 ←	Initial state
1	1 0 1 0 1 1 1	1 0 0 0 0	
⋮	⋮	⋮	
5	1 0 1	0 1 1 1 1	
6	1 0	0 0 0 1 0	1
7	1	0 0 0 0 1	0 1
8	Remainder →	0 0 1 0 1	1 0 1
		Remainder $R(x) = x^2 + x^4$	Quotient $1 + x^2$

(c)

Signature Analysis (CH.10.6)

- Error Masking
  - The chance of NOT detecting an error approaches 2^{-n}
  - see Theorem 10.10.

Signature Analysis (CH.10.6)

- Multiple-input signature register (MISR) 10.6.3