Initiation to research

Simulation of cells in water with Boltzmann lattice

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Simulation of cells in water with Boltzmann lattice

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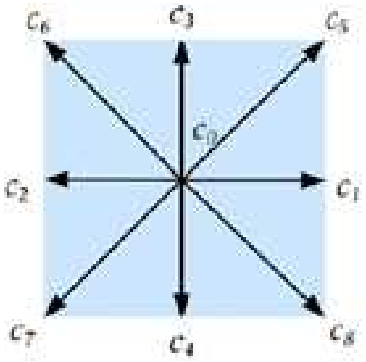
The problem

- Boltzmann lattice simulates fluides thanks to a discretisation of space
- Could we do the same for cells ?
- A lot of possibilities of scales and models

Boltzmann lattice

Collision : $f_i^*(\vec{x}, t+\Delta t) = f_i(\vec{x}, t) + \frac{1}{\tau}(f_i^{eq} - f_i)$

Propagation : $f_i(\vec{x} + \vec{v}_i \Delta t, t + \Delta t) = f_i^t(\vec{x}, t + \Delta t)$



Macroscopic fluid density and fluid speed :

$$\rho(\vec{x}, t) = \sum f_i(\vec{x}, t), \quad \vec{j}(\vec{x}, t) = \rho(\vec{x}, t)\vec{u}(\vec{x}, t) = \sum \vec{v}_i f_i(\vec{x}, t$$

The Physics

Particles are represented in discrete places (buckets) and in discrete directions

$$f_k(x + c_k\Delta t, t + \Delta t) = f_k(x,t) + \frac{\Delta t}{\tau}\left[f_k^{eq}(x,t) - f_k(x,t)\right] + \Delta t F_k$$

$$f_k^{eq}(x,t) = \omega_k \rho \left[1 + \frac{c_k.U}{c_s^2} + 0.5\frac{(c_k.U)^2}{c_s^4} - 0.5\frac{U.U}{c_s^2}\right]$$

$$g_{lk}(x + c_k\Delta t, t + \Delta t) = g_{lk}(x,t) + \frac{\Delta t}{\tau_l}\left[g_{lk}^{eq}(x,t) - g_{lk}(x,t)\right] + \Delta t \omega_k S_l$$

$$g_{lk}^{eq}(x,t) = \omega_k C_l \left[1 + \frac{c_k.U}{c_s^2}\right]$$

Bounce on walls are total
One can define an arbitrary number of species and an arbitrary number of reactions.

The Code

This projects uses python, numpy, matplotlib and numba

```
Initialisation of the lattice
  for each step
        simulate the flow : drift particles
        apply the boundaries
        apply the collisions between particles
        apply the chemical reactions
        plot every ten steps
```

The second method

- **Purpose :** modelize adhesion, modelize cells as individuals
- **What is implemented :**

- Action of the force caused by density of water on cells
- Action of the speed of water on cells (fluid friction)
- Adhesion of cells to obstacles
- Adhesion of cells together
- Resistance of borders and obstacles
- Impossibility for cells to go were there are already too many cells
- Action of cells on water

Formulas for the second method

- **Fluid friction:**

$$\vec{F} = -3\pi\,\mu\,D\,\vec{v}$$

For a sphere of diameter D, fluid of dynamic viscosity μ and low speed :

- **Force density:**

$$P = \rho g h$$

$$\vec{F_d}^{\,i} = \frac{\vec{v_i}}{|\vec{v_i}|} \times \frac{P(x) - P(x+dx)}{dx}$$

$$\vec{F_d} = \sum_i \vec{F_d}^{\,i}$$

Formulas for the second method

- **Feedback on density:**

From n to n' cells: $\rho_{n'} = \rho_n \times \frac{V - nv}{V - n'v}$

where V = volume of a square, v = volume of a cell, m_e = water quantity in the square, ρ = density if there is no cell in the square, ρ_n = density if there are n cells in the square

Justification :

$$\rho = \frac{m_e}{V}$$

$$\rho_n = \frac{m_e}{V - nv} = \frac{V\rho}{V - nV}$$

$$\rho = \rho_n \times \frac{V - nv}{V}$$