L2

Problem 0.1 *Consider the vector field in* $\mathbb{R}^3$

$$F = (x^2 + y^2 + z^2)^2 (X\hat{x} + y\hat{y} + z\hat{z})$$

Is it conserved? What can you say about conserved vector fields? Calculate the curve integral

$$\int_C F \cdot d\vec{r}$$

where C *is a curve starting from* $(1, 0, 0)$ *and ending at* $(2, 2, 1)$.

L3

Problem 0.2 *We have a heavy string with fixed ends which performs transversal oscillations around the x-axis. At t = 0, the string is at rest on the x-axis. We assume that the gravitational force is directed in the negative y direction. Write down, and motivate, the partial differential equation, boundary conditions and initial conditions that allow you to determine the motion of the string for t ¿ 0.*

Hint Your derivation should be 1/2-1 page, including figures, and your motivations are what we are really interested in here. Cf. F2 and Haberman ch. 4.

L4

Problem 0.3 *A rectangular, homogeneous and flexible membrane occupies the square* $x \in [0, 2]$, $y \in [0, 3]$ *in the* x, y *plane. Assume that all boundaries are held fixed, and determine the profile* $u(t, x, y)$ *of the membrane for* $t > 0$ *given the initial conditions*

$$u(0, x, y) = 0, \qquad u_t(0, x, y) = x(2 - x).$$

L6

Problem 0.4 *Solve Laplace's (Poisson) equation inside a cylinder of radius R, and height H, with the following boundary conditions:*

$$u|_{r=R} = 0, \quad u|_{z=0} = 0, \quad u|_{z=H} = f(r)\cos(3\phi).$$

L7

Problem 0.5 *Solve the heat equation*

$$u_t = k\nabla^2 u$$

inside a disk of radius a, subject to the initial condition

$$u|_{t=0} = f(r, \theta),$$

and assuming that the boundary of the disk is kept at constant temperature 0. Give a physical interpretation of the problem and discuss symmetries.

L8

Problem 0.6 *Solve the heat equation*

$$u_t = k\nabla^2 u$$

inside a sphere of radius a, subject to the initial condition

$$u|_{t=0} = r(R - r)\sin\theta\cos\phi,$$

and assuming that the boundary of the sphere is kept at constant temperature 0.

Hint: The angular part of the IC can be recognised to be $Y_{1,\pm 1}$.

L9

Problem 0.7 *Solve the heat equation*

$$u_t = k\nabla^2 u$$

inside a sphere of radius a, but with an inhomogenous BC

$$u|_{r=R} = \sin\theta\cos\phi,$$

and assuming that the boundary of the sphere is kept at constant temperature 0. Write down the solution, you may ignore the IC.

Hint: The angular part of the BC can be recognised to be $Y_{1,\pm 1}$. *You may homogenise the BC by finding a time-independent solution.*

L14

Problem 0.8 *Solve exercise 61 in FMM-2020!*