Entry, Exit and the Shape of Aggregate Fluctuations in a General Equilibrium Model with Capital Heterogeneity

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ABSTRACT

We study the cyclical implications of endogenous firm-level entry and exit decisions in a dynamic, stochastic general equilibrium model wherein firms face persistent shocks to both aggregate and individual productivity. The model we explore is in the spirit of Hopenhayn (1992). Firms' decisions regarding entry into production and their subsequent continuation are affected not only by their expected productivities, but also by the presence of convex and nonconvex capital adjustment costs, and hence their existing stocks. Our model is unique relative to other DSGE settings in that our incumbent firms face two discrete choices, one regarding continuation and one regarding capital adjustment. As such, we can explore how age, size and selection reshape macroeconomic fluctuations in an equilibrium environment with realistic firm life-cycle dynamics and investment patterns.

Examining standard business cycle moments and impulse responses, we find that changes in entry and exit rates and the age-size composition of firms amplify responses over a typical business cycle driven by a disturbance to aggregate productivity and, to a lesser extent, protract them. Both results stem from an endogenous drag on TFP induced by a *missing generation effect*, whereby an usually small number of entrants fails to replace an increased number of exitors. This effect is most injurious several years out as the reduced cohorts of young firms approach maturity. Declines in the number of firms, and most notably in the numbers of young firms, were dramatic over the U.S. 2007 recession. In an exercise designed to emulate that unusual episode, we consider a second shock that more directly affects entry and the exit decisions of younger firms. We find that it sharpens the missing generation effect, delivering far more anemic recovery, and thus closer resemblance to the U.S. post-2009Q2 experience.

Keywords: entry & exit, selection, (S,s) policies, capital reallocation, propagation, business cycles

1 Introduction

It is well understood that the dynamics of capital investment have enormous implications for an economy's business cycle fluctuations. When endogenous capital accumulation is introduced into a typical equilibrium business cycle model, the consequences of temporary disturbances are amplified and propagated in quantitatively important ways. Given this observation, one might expect that the dynamics of other forms of investment would also be important in shaping the size and persistence of aggregate fluctuations. When viewed from an aggregate perspective, microeconomic decisions that influence the number and characteristics of an economy's firms have the capacity to generate such alternative investment dynamics.

How do endogenous movements in the number of firms and their age, size and productivity composition affect macroeconomic fluctuations? To explore this question, we design a dynamic stochastic general equilibrium model with endogenous entry and exit and firm-level capital accumulation. Our firms have persistent differences in idiosyncratic productivity, they face fixed costs to enter production and fixed operating costs to continue, and capital reallocation across them is hindered by microeconomic adjustment frictions. Thus, we can consider how age, size and selection reshape macroeconomic fluctuations in a general equilibrium environment disciplined by realistic firm life-cycle dynamics and investment patterns.

Examining standard business cycle moments and impulse responses, we find that changes in firms' entry and exit decisions amplify ordinary business cycles driven by shocks to aggregate productivity and, to a lesser extent, protract them. Both results stem from an endogenous downward pull on TFP induced by a *missing generation* effect, whereby an usually small number of entrants fails to replace an increased number of exitors. In anticipation of this TFP drag, employment and investment fall more than otherwise, amplifying the fall in total production. The missing generation effect is most prominent several years out as the reduced cohorts of young firms approach maturity and would ordinarily account for a large share of aggregate production. That episode persists over several years, gradualizing the recovery in GDP.

The effects of an aggregate productivity shock are inherently uniform, in that they directly scale all firms' productivities. We also consider the macroeconomic response to a shock that has an asymmetric impact on the distribution of firms and emulates some aspects of the Great Recession. Declines in the number of firms, the number of young firms, and the overall employment share of small firms were dramatic over the U.S. 2007-9 recession. Our second shock induces such unusual

changes through a rise in firms' operating costs. Because the payment of such costs is a discrete decision determined by firm value, this shock most directly affects entry and the exit decisions of younger firms. As such, it sharpens the missing generation effect described above, delivering a far more anemic recovery relative to that following a typical recession.

To be informative about the ways in which firms' entry and exit decisions shape aggregate fluctuations in actual economies, it is essential that our theoretical environment generate firm life-cycle dynamics resembling those in the data. Our model reproduces a key set of stylized facts about the characteristics of new firms, incumbent firms in production, and those exiting the economy. At the core of our setting, we have in essence Hopenhayn's (1992) model of industry dynamics. Potential firms receive informative signals about their future productivities and determine whether to pay fixed costs to become startups. Startups and incumbent firms have productivities affected by a persistent common component and a persistent idiosyncratic component, and they decide whether to pay fixed costs to operate or leave the economy. This set of assumptions immediately implies a selection effect whereby the average productivity, size and value of surviving members within a cohort rise as that cohort ages. Firms that have recently entered production are, on average, smaller, less productive and more likely to exit than are older firms, as consistent with the observations of Dunne, Roberts and Samuelson (1989) and other studies. Moreover, all else equal, large firms are those that have relatively high productivities, so mean-reversion in productivity delivers the unconditional negative relationships between size and growth and between age and growth.

One limitation of the original Hopenhayn framework is its perfect mapping between productivity, size and growth. After controlling for size, this leaves no independent negative relationship between age and growth, in contrast to evidence presented by Evans (1987) and Hall (1987). As in Clementi and Palazzo (2010), we overcome this problem by including capital in the production function and imposing frictions on capital reallocation, so that idiosyncratic productivity and capital become separately evolving state variables for a firm. Because firms cannot immediately adjust their capital stocks following changes in their productivities, those observed to be large in the usual employment-based sense need not be firms with high productivity; some may be large by virtue of their accumulated capital stocks..

Consider a group of firms of common size. Given one-period time-to-build in capital, those among them with the smallest stocks and highest idiosyncratic productivities will exhibit the

fastest growth between this period and the next, as they raise their capital toward a level consistent with their high relative productivity. By contrast, those with large stocks and low productivity will shrink as they shed excess capital. To be in the latter position, a firm must have experienced a sufficiently long episode of high productivity to have accumulated a large stock. Such firms are more likely to be old than young, particularly given micro-level investment frictions that gradualize firms' capital adjustments.

Given its success in reproducing the essential aspects of firm life-cycle dynamics, the model of Clementi and Palazzo (2010) serves as our starting point.[1] There, changes in entry and exit over the cycle are seen to not only amplify the unconditional variation of aggregate series such as GDP and employment, but also generate greater persistence in the economy's responses to shocks. We revisit the findings there, extending the environment to general equilibrium by explicit introduction of a representative household supplying labor and savings to firms. One problem we confront in doing so is the fact that aggregate excess demand moves discontinuously in a search for an equilibrium interest rate path if small changes in prices induce sharp changes in the number of operating firms. We overcome this obstacle by introducing randomness in the fixed costs of both entry and operation.

We calibrate the parameters of our model using long-run observations on aggregate and firm-level variables, including a series of moments on age, size and survival rates drawn from the BDS and a separate set of observations from Cooper and Haltiwanger (2006) regarding the average distribution of firm-level investment rates. Next, we verify that our model is a useful laboratory in which to explore that aggregate implications of selection and reallocation by confirming that its microeconomic predictions are consistent with the above-mentioned regularities. Next, we solve the model using a nonlinear method similar to that in Khan and Thomas (2008).

Nonlinearities are absent in representative agent models, which necessarily abstract from binary decisions. By contrast, our setting has three sets of such decisions characterized by (S,s) thresholds. When the common exogenous component of TFP is unusually low, a potential firm that might otherwise pay its fixed entry cost sees its expected value reduced. At any given idiosyncratic productivity signal, the set of entry costs a potential firm is willing to accept shrinks. Thus, at the onset of a recession, the number of new startups falls, while their mean expected

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[1]Lee and Mukoyama (2009) also consider the implications of entry and exit in a model based on the Hopenhayn framework. Aside from the fact that ours is a general equilibrium study, a primary distinction between our work and theirs is our inclusion of capital.

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productivity rises. Next, there are the operating decisions determining which new firms actually enter into production and which incumbent firms remain. Given the drop in all firms' values at the onset of a TFP-led recession, the willingness to pay operating costs to produce and continue in the economy falls rises at each capital and idiosyncratic productivity pair, implying reduced entry and raised exit. Fewer incumbents remain in production, and they are more selective than usual about continuing from relatively low individual productivity levels. Because similar mechanics deter entry, our model delivers both countercyclical exit and procyclical entry. As noted above, these forces amplify the responses in aggregate production, employment and investment following an aggregate productivity shock. Third, given micro-level capital adjustment frictions, we also have extensive margins decisions involving investment. However, in keeping with results in Khan and Thomas (2003, 2008), we find these have negligible impact for macroeconomic fluctuations in our model.

As noted above, changes in firm startup, entry and exit decisions imply greater persistence in aggregate fluctuations, due to a missing generation effect. Following a negative TFP shock, an unusually small number of young firms are in production. Over subsequent periods, as aggregate productivity begins to revert toward its mean, the typical surviving member of this smaller-than-average group of young firms grows in productivity and size, so the cohort's reduced membership hinders aggregate productivity and production.[2]

There is, by now, a mounting body of firm-level evidence that the most recent U.S. recession had disproportionate negative effects on young firms (Sedlacek (2013), Sedlacek and Sterk (2014)) and on small firms (Khan and Thomas (2013), Siemer (2013)). Indirect evidence suggests that this recession originated with a shock in the financial sector (Almeida et al. (2009), Duchin et al. (2010)). Khan and Thomas (2013) examines a shock to the availability of credit in an equilibrium model where a fixed measure of heterogenous firms face real and financial frictions. Predictions there match the 2007 recession well, but the model fails to deliver the subsequent anemic recovery. Several recent equilibrium studies have considered whether changes in the number and composition of firms may have contributed to this. Sedlacek (2013) examines a search and matching model with multi-worker firms and endogenous entry and exit following a TFP shock, while Siemer (2013) considers a credit crunch in a setting where new firms must finance a fraction of their startup costs with debt. Both models predict a missing (or lost) generation

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[2]As such, the findings of Clementi and Palazzo (2010) are supported by our general equilibrium results.

effect that propagates the effects of an aggregate shock; however, both abstract from capital and thus its reallocation. Khan, Senga and Thomas (2014) considers a shock to default recovery rates in a model with endogenous default, entry and exit and finds endogenous destruction to the stock of firms slows the recovery; however, because that model abstracts from micro-level investment frictions, it is not so tightly calibrated to firm life-cycle data as the setting we consider here.

Drawing on evidence from the BDS, three striking observations distinguish the Great Recession relative to a typical recession. First, the total number of firms fell by 5 percent (Siemer (2013)). Second, the number of young (age 5 and below) firms fell by 15 percent (Sedlacek (2013)). Third, total employment among small (fewer than 100 employees) firms fell more than twice as much as it did among large (more than 1000 employees) firms (Khan and Thomas (2013)). When our model is confronted with a shock raising firms' operating costs, we find that its asymmetric effect generates these sorts of effects. As noted above, the disparate impact of this shock on young firms sharpens the missing generation effect in our model, and delivers an anemic recovery in GDP.

The remainder of the paper is organized as follows. Section 2 describes our model. Next, section 3 analyzes the three sets of threshold policy rules arising therein and derives a series of implications useful in developing a numerical algorithm to solve for competitive equilibrium. Section 4 discusses our model's calibration to moments drawn from postwar U.S. aggregate and firm-level data and thereafter describes the solution method we adopt. Section 5 presents results, first exploring aspects of our model's steady state, then considering aggregate fluctuations. Section 6 concludes.

2   Model

Our model economy builds on Clementi and Palazzo (2010), extending their setting to general equilibrium.[3] We have three groups of decision makers: households, firms and potential firms. Households are identical and own all firms. Potential firms face fixed entry costs to access the opportunity to produce in the next period. Firms face fixed operating costs as well as both convex and nonconvex costs of capital adjustment. These costs compound the effects of persistent

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[3]Beyond our explicit treatment of households, the main departure in extending that environment to general equilibrium is the introduction of idiosyncratic randomness to fixed costs associated with firm entry and continuation. Given discrete firm-specific productivity shocks, this modification serves to smooth the responses in aggregate excess demand to changes in prices, faciliating the search for equilibrium.

differences in total factor productivities, yielding substantial heterogeneity in production. We begin this section with a summary of the problems facing firms and potential firms, then follow with a brief discussion of households and a description of equilibrium.

2.1 Firms

Our economy houses a large, time-varying number of firms. Conditional on survival, each firm produces a homogenous output using predetermined capital stock k and labor n, via an increasing and concave production function F. Each such firm's output is $y = z\varepsilon F(k, n)$, where z is exogenous stochastic total factor productivity common across firms, and ε is a persistent firm-specific counterpart. For convenience, we assume that ε is a Markov chain; $\varepsilon \in \{\varepsilon_1, \ldots, \varepsilon_{N_\varepsilon}\}$, where $\Pr(\varepsilon' = \varepsilon_m \mid \varepsilon = \varepsilon_l) \equiv \pi_{lm}^\varepsilon \geq 0$, and $\sum_{m=1}^{N_\varepsilon} \pi_{lm}^\varepsilon = 1$ for each $l = 1, \ldots, N_\varepsilon$. Similarly, $z \in \{z_1, \ldots, z_{N_z}\}$ with $\Pr(z' = z_j \mid z = z_i) \equiv \pi_{ij} \geq 0$, and $\sum_{j=1}^{N_z} \pi_{ij} = 1$ for each $i = 1, \ldots, N_z$.

At the beginning of any period, each firm is defined by its predetermined stock of capital, $k \in \mathcal{K} \subset \mathbf{R}_+$, and by its current idiosyncratic productivity level, $\varepsilon \in \{\varepsilon_1, \ldots, \varepsilon_{N_\varepsilon}\}$. We summarize the start-of-period distribution of firms over (k, ε) using the probability measure μ defined on the Borel algebra for the product space $\mathcal{K} \times \mathcal{E}$; $\mu : \mathcal{B}(\mathcal{K} \times \mathcal{E}) \rightarrow [0, 1]$. The aggregate state of the economy will be fully described by (z, μ), with the distribution of firms evolving over time according to an equilibrium mapping, Γ, from the current state; $\mu' = \Gamma(z, \mu)$. The evolution of the firm distribution is determined in part by the actions of continuing firms and in part by the startups of potential firms to be described below.[4]

On entering a period, any given firm (k, ε) observes the economy's aggregate state (hence equilibrium prices) and also observes an output-denominated fixed cost it must pay to remain in operation, φ. This operating cost is individually drawn each period from a time-invariant distribution $H(\varphi)$ with bounded support $[\varphi_L, \varphi_U]$. The firm can either pay its φ to enter current production, or it can immediately and permanently exit the economy. If it chooses to exit, it sells its capital to recover a scrap value $(1 - \lambda)k$, where $\lambda \in [0, 1]$.

If a firm pays its operating cost, it then chooses its current level of employment, n, undertakes production, and pays its wage bill. Next, it observes its realization of a fixed cost associated with capital adjustment, $\xi \in [\xi_L, \xi_U]$, which is denominated in units of labor and individually

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[4]Our distribution μ includes new business startups (described in the section below). When comparing to data, we define entrants in our model as those startups that choose to produce; we exclude those that never produce from all measures of exit.

drawn each period from the time-invariant distribution $G(\xi)$. At that point, the firm chooses its investment in capital for the next period, given the standard accumulation equation,

$$k' = (1 - \delta) k + i, \tag{1}$$

where $\delta \in (0,1)$ is the rate of capital depreciation, and primes indicate one-period-ahead values.

The firm can avoid capital adjustment costs by undertaking zero investment. However, if it chooses to set $i \neq 0$, then it must hire ξ units of labor at equilibrium wage rate $\omega(z, \mu)$ to manage the activity, and it must also suffer a convex output-disruption cost $c_q(\frac{i}{k})^2 k$, where $c_q > 0$. This binary choice is summarized below. We will return to consider the resulting two-sided (S,s) investment rules below in section 3.

investment	adjustment costs	future capital
$i \neq 0$	$\omega(z, \mu)\xi + c_q \frac{i^2}{k}$	any $k' \in \mathcal{K}$
$i = 0$	0	$k' = (1 - \delta)k$

The optimization problem facing each of the economy's firms may be described as follows. Given the current aggregate state, (z_i, μ), let $v^1(k, \varepsilon_l, \varphi; z_i, \mu)$ denote the expected discounted value of a firm that enters the period with capital k and idiosyncratic productivity ε_l just after it observes its current operating cost φ. Let $v^0(k, \varepsilon_l; z_i, \mu)$ be its expected value just beforehand;

$$v^0(k, \varepsilon_l; z_i, \mu) \equiv \int_{\varphi_L}^{\varphi_U} v^1(k, \varepsilon_l, \varphi; z_i, \mu) H(d\varphi). \tag{2}$$

The first decision the firm faces is whether to operate or exit. Defining the flow profit function,

$$\pi(k, \varepsilon; z, \mu) \equiv \max_n \left[z\varepsilon F(k, n) - \omega(z, \mu)n \right], \tag{3}$$

the firm solves the following binary maximization problem at the start of the period.

$$v^1(k, \varepsilon_l, \varphi; z_i, \mu) = \max\left\{ (1 - \lambda)k, \ \pi(k, \varepsilon_l; z_i, \mu) - \varphi + \int_{\xi_L}^{\xi_U} v^2(k, \varepsilon_l, \xi; z_i, \mu) G(d\xi) \right\} \tag{4}$$

Since the firm cannot observe its fixed capital adjustment cost until it produces, the ex-production continuation value in (4) computed at the start of the period involves an expectation over the possible realizations of ξ. In some places below, we find it convenient to represent the continuation decision of an incumbent firm using an indicator function χ.

$$\chi(k, \varepsilon, \varphi; z, \mu) = \begin{cases} 1 \text{ if } \pi(k, \varepsilon; z, \mu) - \varphi + \int_{\xi_L}^{\xi_U} v^2(k, \varepsilon, \xi; z, \mu) G(d\xi) \geq (1 - \lambda)k \\ 0 \text{ otherwise} \end{cases}$$

The value function v^2 represents an operating firm's discounted continuation value net of investment and capital adjustment costs. The firm faces a second binary decision at the end of the current period as it chooses its investment. Let $d_j(z_i, \mu)$ represent the discount factor each firm applies to its next-period value conditional on $z' = z_j$ and the current aggregate state (z_i, μ). Taking as given the evolution of ε and z according to the transition probabilities defined above, and taking as given the evolution of the firm distribution, $\mu' = \Gamma(z, \mu)$, the firm solves the optimization problem in (5) - (6) to determine its future capital.

$$v^2(k, \varepsilon_l, \xi; z_i, \mu) = \max\Big\{\sum_{j=1}^{N_z}\sum_{m=1}^{N_\varepsilon} \pi_{ij}\pi_{lm}^\varepsilon d_j(z_i, \mu) v^0((1-\delta)k, \varepsilon_m; z_j, \mu'), \tag{5}$$

$$-\omega(z_l, \mu)\xi + e(k, \varepsilon_l; z_i, \mu)\Big\}, \text{ where}$$

$$e(k, \varepsilon_l; z_i, \mu) = \max_{k' \in \mathcal{K}}\Big[-[k' - (1-\delta)k] - \frac{c_q}{k}[k' - (1-\delta)k]^2 \tag{6}$$

$$+ \sum_{j=1}^{N_z}\sum_{m=1}^{N_\varepsilon} \pi_{ij}\pi_{lm}^\varepsilon d_j(z_i, \mu) v^0(k', \varepsilon_m; z_j, \mu')\Big]$$

The firm can select line 1 of (5), avoiding all capital adjustment costs, and continue to the next period with the remains of its current capital after depreciation. Alternatively, by selecting line 2, it can pay its random fixed cost ξ (converted to output units by the wage) and select a k' that maximizes its continuation value net of investment and convex adjustment costs.

In section 3, we will revisit the incumbent firm problem from (2) - (6) and characterize the resulting decision rules. For now, note that there is no friction associated with a firm's employment choice, since the firm pays its current wage bill after production takes place, and its capital choice for next period also has no implications for current production. Thus, conditional on paying the fixed costs to operate, firms sharing in common the same (k, ε) combination select a common employment and output, which we denote by $n(k, \varepsilon; z, \mu)$ and $y(k, \varepsilon; z, \mu)$, respectively. By contrast, they make differing investment decisions, given differences in their fixed capital adjustment costs. We denote their choices of next-period capital by $g(k, \varepsilon, \xi; z, \mu)$.[5]

2.2 Potential firms

There is a fixed stock of blueprints in the economy, Q. Any blueprint not in use by operating firms (one blueprint per firm) may be used to create a potential firm. Thus, in any date t, there

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[5]Absent the convex cost of capital adjustment, the same k' would solve (6) for all firms sharing the same current productivity, ε. In that case, an operating firm of type (k, ε) would adopt either $k'(\varepsilon; z, \mu)$ or $(1-\delta)k$.

are M_t potential firms, where:

$$M_t \equiv M(z, \mu) = Q - \int\limits_{\mathcal{K} \times \mathcal{E}} \int_{\varphi_L}^{\varphi_U} \chi(k, \varepsilon, \varphi; z, \mu) H(d\varphi) \mu(d\,[k \times \varepsilon]). \tag{7}$$

Each potential firm draws a productivity signal and chooses whether to pay a fixed entry cost to become a startup firm. Any such startup chooses a capital stock with which it will appear in the firm distribution at the start of next period.

A potential firm observes the current aggregate state, its output-denominated fixed entry cost, γ, and its productivity signal, s_l. Entry costs are individually drawn from the time-invariant distribution $H_e(\gamma)$ with bounded support $[\gamma_L, \gamma_U]$. Signals are individually drawn from a distribution with the same support as incumbent firm productivities, $\{s_1, \ldots, s_{N_\varepsilon}\} = \{\varepsilon_1, \ldots, \varepsilon_{N_\varepsilon}\}$, and with probability weights $\pi^e(s_l) \equiv \Pr(s = s_l)$. The transition probabilities from signals to future productivities match those for incumbent firms: $\Pr\left(\varepsilon' = \varepsilon_m \mid s = s_l\right) = \pi_{lm}^\varepsilon$, and startups choose their capital stocks accordingly.

Equations 8 - 9 describe the optimization problem for a potential firm identified by $(s_l, \gamma; z_i, \mu)$. The first line reflects a binary choice of whether to become a startup. In the second line, a startup firm selects capital for the next period, when it will have its first opportunity to produce.

$$v^p(s_l, \gamma; z_i, \mu) = \max\left\{0, -\gamma + v^e(s_l; z_i, \mu)\right\} \tag{8}$$

$$v^e(s_l; z_i, \mu) = \max_{k' \in \mathcal{K}}\left[-k' + \sum_{j=1}^{N_z} \sum_{m=1}^{N_\varepsilon} \pi_{ij} \pi_{lm}^\varepsilon d_j\,(z_i, \mu)\, v^0(k', \varepsilon_m; z_j, \mu')\right] \tag{9}$$

We let $g^e(s_l; z_i, \mu)$ denote the capital solving (9).[6] At points below, we reflect the entry decision of a potential firm using the indicator function χ^e.

$$\chi^e(s_l, \gamma; z_i, \mu) = \begin{cases} 1 \text{ if } -\gamma + v^e(s_l; z_i, \mu) \geq 0 \\ 0 \text{ otherwise} \end{cases}$$

2.3 Households

The economy is populated by a unit measure of infinitely-lived, identical households. Household wealth is held as one-period shares in firms, which we denote using the measure λ.[7] Given

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[6] If incumbent firms faced no convex costs of capital adjustment ($c_q = 0$), any entrant with signal s_l would select the same k' as every incumbent firm with productivity ε_l currently undertaking nonzero investment. That convenient result does not hold for the current model, since $c_q > 0$ implies incumbents' intensive margin investment decisions are affected by their current capital levels.

[7] Households also have access to a complete set of state-contingent claims. As there is no heterogeneity across households, these assets are in zero net supply in equilibrium, so we do not explicitly model them here.

the prices they receive for their current shares, $\rho_0\left(k,\varepsilon;z_i,\mu\right)$, and the real wage they receive for their labor, $\omega\left(z_i,\mu\right)$, households determine their current consumption, c, hours worked, n^h, and the numbers of new shares, $\lambda'\left(k',\varepsilon'\right)$, to purchase at prices $\rho_1\left(k',\varepsilon';z_i,\mu\right)$. The lifetime expected utility maximization problem of the representative household is listed below.

$$W\left(\lambda;z,\mu\right)=\max_{c,n^h,\lambda'}\left[U\left(c,1-n^h\right)+\beta\sum_{j=1}^{N_z}\pi_{ij}W\left(\lambda';z_j,\mu'\right)\right] \qquad (10)$$

subject to

$$c+\int_{\mathcal{K}\times\mathcal{E}}\rho_1\left(k',\varepsilon';z,\mu\right)\lambda'\left(d\left[k'\times\varepsilon'\right]\right)\leq\omega\left(z,\mu\right)n^h+\int_{\mathcal{K}\times\mathcal{E}}\rho_0\left(k,\varepsilon;z,\mu\right)\lambda\left(d\left[k\times\varepsilon\right]\right).$$

Let $C\left(\lambda;z,\mu\right)$ describe the household consumption choice, and let $N\left(\lambda;z,\mu\right)$ be its choice of hours worked. Finally, let $\Lambda\left(k',\varepsilon',\lambda;z,\mu\right)$ be the quantity of shares purchased in firms that will begin the next period with k' units of capital and idiosyncratic productivity ε'.

2.4   Recursive equilibrium

A *recursive competitive equilibrium* is a set of functions,

$$\left(\omega,\left(d_j\right)_{j=1}^{N_z},\rho_0,\rho_1,v^1,n,g,\chi,v^p,g^e,\chi^e,W,C,N,\Lambda\right),$$

that solve firm and household problems and clear the markets for assets, labor and output, as described by the following conditions.

(i) v^1 solves (4) - (6), given the definitions in (2) and (3), and (χ,n,g) are the associated policy functions for firms

(ii) v^p solves (8) - (9), and χ^e and g^e are the resulting policy functions for potential firms

(iii) W solves (10), and (C,N,Λ) are the associated policy functions for households

(iv) $\Lambda\left(k',\varepsilon',\mu;z,\mu\right)=\mu'\left(k',\varepsilon';z,\mu\right)$, for each $(k',\varepsilon')\in\mathcal{K}\times\mathcal{E}$

(v) $N\left(\mu;z,\mu\right)=\displaystyle\int_{\mathcal{K}\times\mathcal{E}}\int_{\varphi_L}^{\varphi_U}\chi(k,\varepsilon,\varphi;z,\mu)\left[n\left(k,\varepsilon;z,\mu\right)+\int_{\xi_L}^{\xi_U}\xi\mathcal{J}\Big(g\left(k,\varepsilon,\xi;z,\mu\right)\right.$

$$\left.-\left(1-\delta\right)k\Big)G\left(d\xi\right)\right]H(d\varphi)\mu(d\left[k\times\varepsilon\right]),$$

where $\mathcal{J}(x)=0$ if $x=0$; $\mathcal{J}(x)=1$ otherwise.

(vi) $C(\mu; z, \mu) = \int_{\mathcal{K} \times \mathcal{E}} \int_{\varphi_L}^{\varphi_U} \chi(k, \varepsilon, \varphi; z, \mu) \Big[z \varepsilon F(k, n(k, \varepsilon; z, \mu)) - \varphi - \int_{\xi_L}^{\xi_U} \Big[g(k, \varepsilon, \xi; z, \mu) - (1-\delta) k$

$$+ \frac{c_q}{k} \Big(g(k, \varepsilon, \xi; z, \mu) - (1-\delta) k \Big)^2 \Big] \mathcal{J} \Big(g(k, \varepsilon, \xi; z, \mu) - (1-\delta) k \Big) G(d\xi) \Big] H(d\varphi) \mu(d[k \times \varepsilon])$$

$$+ \int_{\mathcal{K} \times \mathcal{E}} \int_{\varphi_L}^{\varphi_U} \Big[1 - \chi(k, \varepsilon, \varphi; z, \mu) \Big] \Big[(1-\lambda) k \Big] H(d\varphi) \mu(d[k \times \varepsilon])$$

$$- M(z, \mu) \sum_{l=1}^{N_\varepsilon} \pi^e(s_l) \int_{\gamma_L}^{\gamma_U} \chi^e(s_l, \gamma; z, \mu) \Big[\gamma + g^e(s_l; z, \mu) \Big] H_e(d\gamma),$$

where $\mathcal{J}(x) = 0$ if $x = 0$; $\mathcal{J}(x) = 1$, and $M(z, \mu)$ is given by (7).

(vii) $\mu'(D, \varepsilon_m) = \int_{\{(k, \varepsilon_l, \xi) \,|\, g(k, \varepsilon_l, \xi; z, \mu) \in D\}} \chi(k, \varepsilon_l, \varphi; z, \mu) \pi_{lm}^\varepsilon G(d\xi) H(d\varphi) \mu(d[\varepsilon_l \times k])$

$$+ M(z, \mu) \sum_{\{s_l \,|\, g^e(s_l; z, \mu) \in D\}} \pi^e(s_l) \pi_{lm}^\varepsilon \int_{\gamma_L}^{\gamma_U} \chi^e(s_l, \gamma; z, \mu) H_e(d\gamma),$$

for all $(D, \varepsilon_m) \in \mathcal{K} \times \mathcal{E}$, defines Γ

Let C and N represent the market-clearing values of household consumption and hours worked satisfying conditions (v) and (vi) above. It is straightforward to show that market-clearing requires that (a) the real wage equal the household marginal rate of substitution between leisure and consumption, $\omega(z, \mu) = D_2 U(C, 1-N) / D_1 U(C, 1-N)$, and that (b) firms' (and potential firms') state-contingent discount factors agree with the household marginal rate of substitution between consumption across states. Letting C'_{ij} denote household consumption next period given current state (z_i, μ) and future state $(z_j, \mu'(z_i, \mu))$ and with N'_{ij} as the corresponding labor input, the resulting discount factors are: $d_j(z_i, \mu) = \beta D_1 U\Big(C'_{ij}, 1 - N'_{ij}\Big) / D_1 U(C, 1-N)$.

3   Analysis

We may compute equilibrium by solving a single Bellman equation that combines the firm profit maximization problem with the equilibrium implications of household utility maximization from above. Here, we effectively subsume households' decisions into the problems faced by firms.

Without loss of generality, we assign $p(z, \mu)$ as an output price at which firms and potential firms value current profits and payments, and we correspondingly impose that their future values are discounted by the household subjective discount factor. Given this alternative means of expressing equilibrium discount factors, the following two conditions ensure all markets clear in our economy.

$$p(z, \mu) = D_1 U(C, 1 - N) \tag{11}$$

$$\omega(z, \mu) = D_2 U(C, 1 - N) / p(z, \mu) \tag{12}$$

To develop a tractable numerical algorithm with which to solve our economy, it is useful to characterize the optimizing decisions of incumbent and potential firms in ways convenient for aggregation. As we consider firms' and potential firms' binary choice problems, we find it convenient to start with the continuous decision problems contingent on each action, then work backward to the binary choice. Throughout this section, we suppress aggregate state arguments in the p and ω functions to shorten the equations, and we continue abbreviating $\mu'(z, \mu)$ by μ'.

We begin by reformulating (2) - (6) to describe each firm's value in units of marginal utility, with no change in the resulting decision rules. Exploiting the fact that the choice of n is independent of the k' choice, suppressing the indices for current aggregate and idiosyncratic productivity, and defining $V^0(k, \varepsilon; z, \mu) \equiv \int_{\varphi_L}^{\varphi_U} V^1(k, \varepsilon, \varphi; z, \mu) H(d\varphi)$, we have the following recursive representation for the start-of-period value of a type (k, ε) firm drawing operating cost φ.

$$V^1(k, \varepsilon, \varphi; z, \mu) = \max\left\{ p(1 - \lambda)k, \ p[\pi(k, \varepsilon; z, \mu) - \varphi] + \int_{\xi_L}^{\xi_U} V^2(k, \varepsilon, \xi; z, \mu) G(d\xi) \right\} \tag{13}$$

$$V^2(k, \varepsilon, \xi; z, \mu) = \max\left\{ \beta \sum \sum \pi_{ij} \pi_{lm}^{\varepsilon} V^0((1 - \delta)k, \varepsilon_m; z_j, \mu'), \ -p\omega\xi + E(k, \varepsilon; z, \mu) \right\} \tag{14}$$

$$E(k, \varepsilon; z, \mu) = \max_{k' \in \mathcal{K}} \left[-p[k' - (1 - \delta)k] - \frac{pc_q}{k}[k' - (1 - \delta)k]^2 \right. \tag{15}$$
$$\left. + \beta \sum \sum \pi_{ij} \pi_{lm}^{\varepsilon} V^0(k', \varepsilon_m; z_j, \mu') \right]$$

The problem of a potential firm from (8) - (9) is analogously reformulated.

$$V^p(s_l, \gamma; z_i, \mu) = \max\left\{ 0, \ -p\gamma + V^e(s_l; z_i, \mu) \right\} \tag{16}$$

$$V^e(s_l; z_i, \mu) = \max_{k' \in \mathcal{K}} \left[-pk' + \beta \sum_{j=1}^{N_z} \sum_{m=1}^{N_\varepsilon} \pi_{ij} \pi_{lm}^{\varepsilon} V^0(k', \varepsilon_m; z_j, \mu') \right] \tag{17}$$

3.1 Continuing firms' investment decisions

Consider first the end-of-period decision made by a continuing firm that has chosen to pay its adjustment cost and undertake a nonzero investment. Any such firm will adopt a *target capital* consistent with its current productivity and the aggregate state, which we denote by $k^*(k, \varepsilon; z, \mu)$.

$$k^*(k, \varepsilon; z, \mu) \equiv \arg\max_{k' \in \mathcal{K}} \left[-pk' - \frac{pc_q}{k}[k' - (1-\delta)k]^2 + \beta \sum_{j=1}^{N_z} \sum_{m=1}^{N_\varepsilon} \pi_{ij} \pi_{lm}^\varepsilon V^0(k', \varepsilon_m; z_j, \mu') \right] \quad (18)$$

The gross adjustment value associated with this action is $E(k, \varepsilon; z, \mu)$ from equation 15.

If there were no convex adjustment costs, notice that the target capital choice would be independent of a firm's current capital, since the price of investment goods (p) is unaffected by its level of investment and the current capital adjustment cost draw ξ carries no information about future draws (and thus does not enter V^0). In that case, all firms with the same current productivity level undertaking nonzero investment would move to the next period with a common capital stock, and their gross adjustment values would be linear in k; both observations could be used to expedite model solution. However, given $c_q > 0$, the scale of adjustment affects the level of adjustment costs; hence, target capitals depend on not only ε but also k.

Next, we turn to the binary capital adjustment decision. For a continuing firm of type (k, ε), the ex-production value of undertaking no adjustment is $\beta \sum \sum \pi_{ij} \pi_{lm}^\varepsilon V^0((1-\delta)k, \varepsilon_m; z_j, \mu')$, while the value of adjustment is $-p\omega\xi + E(k, \varepsilon; z, \mu)$. The firm pays its capital adjustment cost only if the net benefit of doing so is non-negative, i.e., if:

$$[-p\omega\xi + E(k, \varepsilon; z, \mu)] - \beta \sum \sum \pi_{ij} \pi_{lm}^\varepsilon V^0((1-\delta)k, \varepsilon_m; z_j, \mu') \geq 0.$$

The firm's capital decision rule can be described as a threshold policy. Define $\widetilde{\xi}(k, \varepsilon; z, \mu)$ as the fixed cost that leaves the firm indifferent to adjustment, and define $\xi^T(k, \varepsilon; z, \mu)$ as the resulting threshold cost confined to the support of the cost distribution.

$$\begin{aligned} \widetilde{\xi}(k, \varepsilon; z, \mu) &= \frac{E(k, \varepsilon; z, \mu) - \beta \sum \sum \pi_{ij} \pi_{lm}^\varepsilon V^0((1-\delta)k, \varepsilon_m; z_j, \mu')}{p\omega} \\ \xi^T(k, \varepsilon; z, \mu) &= \max\{\xi_L, \min\{\xi_U, \widetilde{\xi}(k, \varepsilon; z, \mu)\}\} \end{aligned} \quad (19)$$

If the firm draws a fixed cost at or below its threshold, ξ^T, it pays that cost and adopts the target $k^*(k, \varepsilon; z, \mu)$. Otherwise, it undertakes zero investment. The resulting capital decision rule is listed below.

$$g(k, \varepsilon, \xi; z, \mu) = \begin{cases} k^*(k, \varepsilon, z, \mu) & \text{if} \quad \xi \leq \xi^T(k, \varepsilon; z, \mu) \\ (1-\delta)k & \text{otherwise} \end{cases}$$

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All else equal, a firm tends to be more willing to pay adjustment costs when its existing stock is farther away from its target. When this is so, the threshold cost is higher, which in turn implies a greater likelihood that the firm will adopt its k^*. Thus, our model implies (S,s) capital decisions and rising adjustment hazards as in Caballero and Engel (1999), Khan and Thomas (2003, 2008) and other studies involving nonconvex microeconomic investment decisions.

Observe from (19) that all firms of type (k, ε) share in common the same threshold cost ξ^T. Thus, each of them has the same probability of capital adjustment and hence the same expected ex-production continuation value before the individual ξ draws have been realized. Let $\alpha^k(k, \varepsilon; z, \mu)$ denote any such firm's probability of capital adjustment, which is simply the probability of drawing $\xi \leq \xi^T$, and let $\Phi^k(k, \varepsilon; z, \mu)$ denote the conditional expectation of the fixed cost to be paid.

$$\alpha^k(k, \varepsilon; z, \mu) \equiv G\Big(\xi^T(k, \varepsilon; z, \mu)\Big) \tag{20}$$

$$\Phi^k(k, \varepsilon; z, \mu) \equiv \int_{\xi_L}^{\xi^T(k,\varepsilon;z,\mu)} \xi G(d\xi) \tag{21}$$

3.2   Operating decisions

As firms make their operating decisions at the start of a period, recall that they do not yet know their current fixed capital adjustment costs. As such, they use (18) - (21) from above to compute their expected ex-production continuation values.

$$\int_{\xi_L}^{\xi_U} V^2(k, \varepsilon, \xi; z, \mu) G(d\xi) = [1 - \alpha^k(k, \varepsilon; z, \mu)]\beta \sum_{j=1}^{N_z} \sum_{m=1}^{N_\varepsilon} \pi_{ij} \pi_{lm}^\varepsilon V^0((1-\delta)k, \varepsilon_m; z_j, \mu') \tag{22}$$
$$+ \alpha^k(k, \varepsilon; z, \mu) E(k, \varepsilon; z, \mu) - p\omega \Phi^k(k, \varepsilon; z, \mu)$$

Given the expected continuation value from equation 22, we can solve any firm's start-of-period operating decision. If the firm exits the economy, it achieves a scrap value $p(1 - \lambda)k$. If it operates, it achieves the flow profits $\pi(k, \varepsilon; z, \mu)$ from (3) and the expected continuation value from (22). The firm continues into production only if the value of its current operating cost does not exceed the net benefit of doing so;

$$\left[p\pi(k, \varepsilon; z, \mu) + \int_{\xi_L}^{\xi_U} V^2(k, \varepsilon, \xi; z, \mu) G(d\xi) \right] - p(1 - \lambda)k \geq p\varphi.$$

The firm's binary operating decision can be described as a threshold policy. Define $\widetilde{\varphi}(k, \varepsilon; z, \mu)$ as the cost that leaves the firm indifferent to continuing, and define $\varphi^T(k, \varepsilon; z, \mu)$ as the resulting

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threshold cost confined to the support of H.

$$\widetilde{\varphi}(k,\varepsilon;z,\mu) = \pi(k,\varepsilon;z,\mu) - (1-\lambda)k + \frac{1}{p}\int_{\xi_L}^{\xi_U} V^2(k,\varepsilon,\xi;z,\mu)G(d\xi)$$

$$\varphi^T(k,\varepsilon;z,\mu) = \max\{\varphi_L, \min\{\widetilde{\varphi}(k,\varepsilon;z,\mu),\varphi_U\}\} \tag{23}$$

If the firm realizes a φ above the threshold, φ^T, it exits the economy. Otherwise, it hires and produces according to the decision rules $n(k,\varepsilon;z,\mu)$ and $y(k,\varepsilon;z,\mu)$ that maximize its current flow profits (see equation 3).

Before leaving this subsection, note that (23) implies that all firms entering the period with the same (k,ε) pair have the same threshold operating cost. This means that, as they are entering the period, each of them has equal probability of survival, α^c, and equal conditional expectation of the operating costs they will pay, Φ^c;

$$\alpha^c(k,\varepsilon;z,\mu) = H\Big(\varphi^T(k,\varepsilon;z,\mu)\Big)$$

$$\Phi^c(k,\varepsilon;z,\mu) = \int_{\varphi_L}^{\varphi^T(k,\varepsilon;z,\mu)} \varphi H(d\varphi).$$

Combining the results above (and recalling that $V^0(k,\varepsilon;z,\mu) \equiv \int_{\varphi_L}^{\varphi_U} V^1(k,\varepsilon,\varphi;z,\mu) H(d\varphi)$), we can compute the start-of-period expected value of any firm as it enters a period:

$$
\begin{aligned}
V^0(k,\varepsilon;z,\mu) &= [1 - \alpha^c(k,\varepsilon;z,\mu)]p(1-\lambda)k - p\Phi^c(k,\varepsilon;z,\mu) \\
&\quad + \alpha^c(k,\varepsilon;z,\mu)[p(z,\mu)\pi(k,\varepsilon;z,\mu) - p\omega\Phi^k(k,\varepsilon;z,\mu)] \\
&\quad + \alpha^c(k,\varepsilon;z,\mu)\alpha^k(k,\varepsilon;z,\mu)E(k,\varepsilon;z,\mu) \\
&\quad + \alpha^c(k,\varepsilon;z,\mu)[1 - \alpha^k(k,\varepsilon;z,\mu)]\beta\sum_{j,m}\pi_{ij}\pi_{lm}^\varepsilon V^0((1-\delta)k,\varepsilon_m;z_j,\mu'),
\end{aligned}
$$

where $E(k,\varepsilon;z,\mu)$ is defined in (15).

3.3 Entry decisions

Conditional on paying its entry cost to become a startup, a potential firm with productivity signal s_l adopts the capital stock solving (17) above. We denote that choice by $k_e^*(s_l;z,\mu)$ here forward. The potential firm pays its entry cost, γ, if:

$$\beta\sum_{j=1}^{N_z}\sum_{m=1}^{N_\varepsilon}\pi_{ij}\pi_{lm}^\varepsilon V^0(k_e^*(s_l;z,\mu),\varepsilon_m;z_j,\mu') - pk_e^*(s_l;z,\mu) \geq p\gamma.$$

Define $\widetilde{\gamma}(\varepsilon_l;z,\mu)$ as the entry cost implying indifference, and define $\gamma^T(\varepsilon_l;z,\mu)$ as the associated threshold entry cost confined to the support of H_e.

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$$\widetilde{\gamma}(\varepsilon_l; z, \mu) = \frac{\beta}{p} \sum \sum \pi_{ij} \pi_{lm}^{\varepsilon} V^0(k_e^*(s_l; z, \mu), \varepsilon_m; z_j, \mu') - k_e^*(s_l; z, \mu)$$

$$\gamma^T(\varepsilon_l; z, \mu) = \max\{\gamma_L, \min\{\widetilde{\gamma}(\varepsilon_l; z, \mu), \gamma_U\}\}$$

Only if the potential firm draws an entry cost at or below γ^T will it become a startup. Thus, we have the fraction of potential firms with signal s_l that will choose to invest toward next period entry, as well as the expected cost paid by each.

$$\alpha^e(s_l; z, \mu) = H\Big(\gamma^T(\varepsilon_l; z, \mu)\Big)$$

$$\Phi^e(s_l; z, \mu) = \int_{\gamma_L}^{\gamma^T(\varepsilon_l; z, \mu)} \gamma H_e(d\gamma)$$

3.4 Aggregation

Given the probabilities of entry, continuation, and capital adjustment from above, alongside the conditional fixed cost expectations, and the accompanying labor, output and capital decision rules, aggregation is straightforward. Aggregate production and employment are

$$Y(z, \mu) = \int_{\mathcal{K} \times \mathcal{E}} \Big[\alpha^c(k, \varepsilon; z, \mu) y(k, \varepsilon; z, \mu)\Big] \mu(d[k \times \varepsilon])$$

$$N(z, \mu) = \int_{\mathcal{K} \times \mathcal{E}} \Big[\alpha^c(k, \varepsilon; z, \mu) n(k, \varepsilon; z, \mu)\Big] \mu(d[k \times \varepsilon]) + \Psi_n^k(z, \mu),$$

where Ψ_n^k is total labor-denominated fixed costs associated with capital adjustment;

$$\Psi_n^k(z, \mu) = \int_{\mathcal{K} \times \mathcal{E}} \Big[\alpha^c(k, \varepsilon; z, \mu) \Phi^k(k, \varepsilon; z, \mu)\Big] \mu(d[k \times \varepsilon]).$$

Aggregate investments across incumbent firms (I^c) and startup firms (I^e) are

$$I^c(z, \mu) = \int_{\mathcal{K} \times \mathcal{E}} \alpha^c(k, \varepsilon; z, \mu) \alpha^k(k, \varepsilon; z, \mu) \Big[k^*(\varepsilon; z, \mu) - (1 - \delta)k\Big] \mu(d[k \times \varepsilon])$$

$$- \int_{\mathcal{K} \times \mathcal{E}} \Big[\Big(1 - \alpha^c(k, \varepsilon; z, \mu)\Big)(1 - \lambda)k\Big] \mu(d[k \times \varepsilon])$$

$$I^e(z, \mu) = M(z, \mu) \sum_{l=1}^{N_\varepsilon} \pi^e(s_l) \alpha^e(s_l; z, \mu) k^*(s_l; z, \mu),$$

with the measure of potential firms given by $M(z, \mu) = Q - \int_{\mathcal{K} \times \mathcal{E}} \alpha^c(k, \varepsilon; z, \mu) \mu(d[k \times \varepsilon])$. House-hold consumption is

$$C(z, \mu) = Y(z, \mu) - [I^c(z, \mu) + I^e(z, \mu)] - [\Psi^e(z, \mu) + \Psi^c(z, \mu) + \Psi_y^k(z, \mu)],$$

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where Ψ^e, Ψ^c, and Ψ_y^k are the total output-denominated costs associated with startup entry, firm operations, and capital adjustment (Ψ_y^k), respectively.

$$\Psi^e(z, \mu) = M(z, \mu) \sum_{l=1}^{N_\varepsilon} \pi^e(\varepsilon_l) \Phi^e(s_l; z, \mu)$$

$$\Psi^c(z, \mu) = \int_{\mathcal{K} \times \mathcal{E}} \Phi^c(k, \varepsilon; z, \mu) \mu(d[k \times \varepsilon])$$

$$\Psi_y^k(z, \mu) = \int_{\mathcal{K} \times \mathcal{E}} \left[\alpha^c(k, \varepsilon; z, \mu) \alpha^k(k, \varepsilon; z, \mu) \right] \cdot$$
$$\left[\frac{c_q}{k} \left(k^*(k, \varepsilon; z, \mu) - (1 - \delta)k \right)^2 \right] \mu(d[k \times \varepsilon]).$$

Finally, before turning to the calibration, we identify *incumbents*, *entrants*, and *exitors* in our model for comparison with firm-level data. Here forward, an incumbent is a firm that produced in the previous period, an entrant is a firm that has not produced before and does so in the current period, and an exitor is an incumbent that does produce in the current period. Given current aggregate state (z, μ) and next period state (z', μ'), the number of producers next period will be $\int_{\mathcal{K} \times \mathcal{E}} \alpha^c(k, \varepsilon; z, \mu) \mu(d[k \times \varepsilon])$, the number of entrants will be:

$$M(z, \mu) \sum_{l=1}^{N_\varepsilon} \pi^e(s_l) \alpha^e(s_l; z, \mu) \sum_{m=1}^{N_\varepsilon} \pi_{lm}^\varepsilon \alpha^c \left(k^*(s_l; z, \mu), \varepsilon_m; z', \mu' \right),$$

and incumbent producers will be the difference between these two. To compute total exit next period, we count all potential producers leaving the economy, then discard those that have never produced (startups that do not enter):

$$\int_{\mathcal{K} \times \mathcal{E}} [1 - \alpha^c(k, \varepsilon; z, \mu)] \mu(d[k \times \varepsilon]) - M(z, \mu) \sum_{l=1}^{N_\varepsilon} \pi^e(s_l) \alpha^e(s_l; z, \mu) \sum_{m=1}^{N_\varepsilon} \pi_{lm}^\varepsilon \left[1 - \alpha^c \left(k^*(s_l; z, \mu), \varepsilon_m; z', \mu' \right) \right].$$

We define the entry rate in our model as the ratio of entrants to incumbents, and the exit rate as the ratio of exitors to incumbents.

4   Calibration and solution

In the sections to follow, we will at points consider how the mechanics of our model compare to those in a reference model with an exogenously fixed measure of firms. Aside from the changes noted here for that reference, we will select a common parameter set by targeting our full model economy at a series of moments drawn from postwar U.S. aggregate and firm-level data discussed

below. To construct our no-entry/exit reference, we then reset the upper bounds on entry and continuation costs to 0, and reduce the fixed stock of blueprints Q to imply a number of firms matching that in the start-of-period distribution of our full model.

4.1 Functional forms and aggregate targets

We assume that the representative household's period utility is the result of indivisible labor (Rogerson (1988)): $u(c, L) = \log c + \theta L$. Firm-level production is Cobb-Douglas: $z \varepsilon F(k, n) = z \varepsilon k^\alpha n^\nu$. In specifying our exogenous stochastic process for aggregate productivity, we begin by assuming a continuous shock following a mean zero AR(1) process in logs: $\log z' = \rho_z \log z + \eta'_z$ with $\eta'_z \sim N\left(0, \sigma^2_{\eta_z}\right)$. Next, we estimate the values of ρ_z and σ_{η_z} from Solow residuals measured using NIPA data on US real GDP and private capital, together with the total employment hours series constructed by Cociuba, Prescott and Ueberfeldt (2012) from CPS household survey data over 1959-2002. Next, we discretize the productivity process using a grid with 5 shock realizations to obtain (z_i) and (π_{ij}). We determine the firm-specific productivity shocks (ε_l) and the Markov Chain governing their evolution (π^ε_{lm}) similarly by discretizing a log-normal process, $\log \varepsilon' = \rho_\varepsilon \log \varepsilon + \eta'_\varepsilon$ using 15 values, and we assign the initial distribution of productivity signals, $T(s)$, as a discretized Pareto distribution with curvature parameter $\mathfrak{p}$.

We set the length of a period to correspond to one year, and we determine the values of β, ν, δ, α, and θ using moments from the aggregate data as follows. First, we set the household discount factor, β, to imply an average real interest rate of 4 percent, consistent with recent findings by Gomme, Ravikumar and Rupert (2008). Next, we set the production parameter ν to imply an average labor share of income at 0.60 (Cooley and Prescott (1995)). The depreciation rate, δ, is taken to imply an average investment-to-capital ratio at 0.069, corresponding to the average value for the private capital stock between 1954 and 2002 in the U.S. Fixed Asset Tables, controlling for growth. Given that value, we determine capital's share, α, so that our model matches the average private capital-to-output ratio over the same period, at 2.3, and we set the parameter governing the preference for leisure, θ, to imply an average of one-third of available time is spent in market work. The parameter set obtained from this part of our calibration exercise is summarized below.

β	ν	δ	α	θ	ρ_z	σ_{η_z}
0.962	0.60	0.069	0.26	2.58	0.852	0.014

4.2 Firm-level targets

The remaining parameters are jointly determined using moments from U.S. firm- and establishment-level data. Most of our target moments are drawn from the Business Employment Dynamics (BDS) database constructed from the Quarterly Census of Employment and Wages and maintained by the Bureau of Labor Statistics for the period 1977-2011. Beyond its public availability, an advantage of this annual data set relative to the establishment data in the Longitudinal Research Database (LRD) is that it includes all firms covered by state unemployment insurance programs, which accounts for roughly 98 percent of all nonfarm payrolls. We target the average exit rate among firms in the BDS over 1979-2007, which is 8.7 percent. We also target the BDS employment share of firms aged at and below 5 years over the same period (17.1 percent), and the cumulative survival rate for young firms over their first 5 years of production (45 percent). Finally, we target the exit rates among firms aged 1 and 2 years (roughly 21 and 16 percent, respectively), and the population shares of each of those groups (roughly 8.7 and 7.4 percent, respectively).

For further discipline on the extent of idiosyncratic volatility in our model, and to select the capital adjustment parameters, we also target some establishment-level investment moments reported by Cooper and Haltiwanger (2006) from the LRD.[8] These include the average mean investment rate (0.122), standard deviation of investment rates (0.337), serial correlation of investment rates (0.058) and fraction of establishments with investment rates exceeding 20 percent (0.186). While our model has life-cycle aspects affecting firms' investments, the Cooper and Haltiwanger (2006) dataset includes only large manufacturing establishments that remain in operation throughout their sample period. Thus, in undertaking this part of our calibration, we must select an appropriate model sample for comparability. This we do by simulating a large number of firms for 30 years, retaining only those firms that survive throughout, and then restricting the dates over which investment rates are measured to eliminate life-cycle effects.

Our firm-level calibration exercise is still in progress. Here, we explore an economy that roughly matches the BDS targets in most respects: overall exit rate (8.5 percent), cumulative survival rate of young firms (40 percent), age 1 and 2 exit rates (17 and 11 percent), age 1 and 2 population shares (7 and 6.2), share of employment in young firms (26 percent). The

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[8]The distinction between firm and establishment may be relatively unimportant here; over 95 percent of firms in the BDS have fewer than 50 employees. A more problematic distinction is the fact that the LRD includes only manufacturing establishments.

parameterization is listed below. In this example, we assume that the random costs of operation, entry and capital adjustment (φ, γ and ξ) are each drawn from uniform distributions. The upper support on the adjustment cost distribution and the persistence and volatility of idiosyncratic productivities are taken from Khan and Thomas (2008).

Q	$\mathfrak{p}$	$[\gamma_L, \gamma_U]$	$[\varphi_L, \varphi_U]$	$[\xi_L, \xi_U]$	c_q	λ	ρ_ε	$\sigma_{\eta_\varepsilon}$
30	20	$[.01, .06]$	$[0, .26]$	$[0, 0.008]$	.08	.05	.653	.138

4.3   Numerical method

The distribution μ in the aggregate state vector of our model economy is a large object. In general, discrete choices imply that this distribution is highly non-parametric. For each level of productivity, we store the conditional distribution using a fine grid defined over capital. However, firms' choices of investment are not restricted to conform to this grid. To allow the possibility that nonconvex capital adjustment may interact with endogenous entry and exit over the business cycle in a way that delivers aggregate nonlinearities, we adopt a nonlinear solution method. Given μ, an exact solution is obviously numerically intractable; thus, we use selected moments of μ as a proxy for the distribution in the aggregate state vector when computing expectations.

Our solution method is adaptation of that in Khan and Thomas (2008). Following the approach developed by Krusell and Smith (1997, 1998), we assume that firms approximate the distribution in the aggregate state vector with a vector of moments, $m = (m_1, ..., m_I)$, drawn from the true distribution. Because our model implies a discrete distribution over k and over ε, conditional means from I equal-sized partitions of the capital distribution work well, implying small forecasting errors.

As in Krusell and Smith (1997), we solve our model by iterating between an inner loop step and an outer loop step until we isolate forecasting rules satisfyingly consistent with equilibrium outcomes. In the inner loop, we take as given a current set of forecasting rules for p and m' and use them to solve incumbent firms' expected value functions V^0 (from section 3). This we do by combining value function iteration with multivariate piecewise polynomial cubic spline interpolation allowing firms to evaluate and select off-grid options. We next move to the outer loop to simulate the economy for 1000 periods. The current set of m' forecast rules are used in the outer loop, while p is endogenously determined in each date. Each period in the simulation begins with the actual distribution of firms over capital and productivity implied by the decisions

of the previous date. Given incumbent firms' value functions from the most recent inner loop, and the aggregation of section 3.4, we determine equilibrium prices and quantities, and thus the subsequent period's distribution. Once the simulation has finished, we use the resulting data to update the forecasting rules, with which we return to the inner loop.

5 Results

5.1 Steady state

In this section, we explore aspects of our model in its steady state and briefly consider how our setting compares to an otherwise identical reference model without entry and exit. In the reference model, an exogenous stock of firms produces each period exempt from fixed costs of operation. As noted above, the stock of firms there is fixed at the steady state number of firms at the start of each period in our full model.

On average, our model economy forfeits roughly 12 percent of its GDP to operating costs. However, the average level of consumption is 96 percent that in the reference model with no such costs. This is achieved in part by the fact that households work roughly 17.5 percent more in our economy. However, the more direct explanation lies in the distribution of firms over productivity levels, which encourages this higher work effort and supports 12.3 percent more investment.

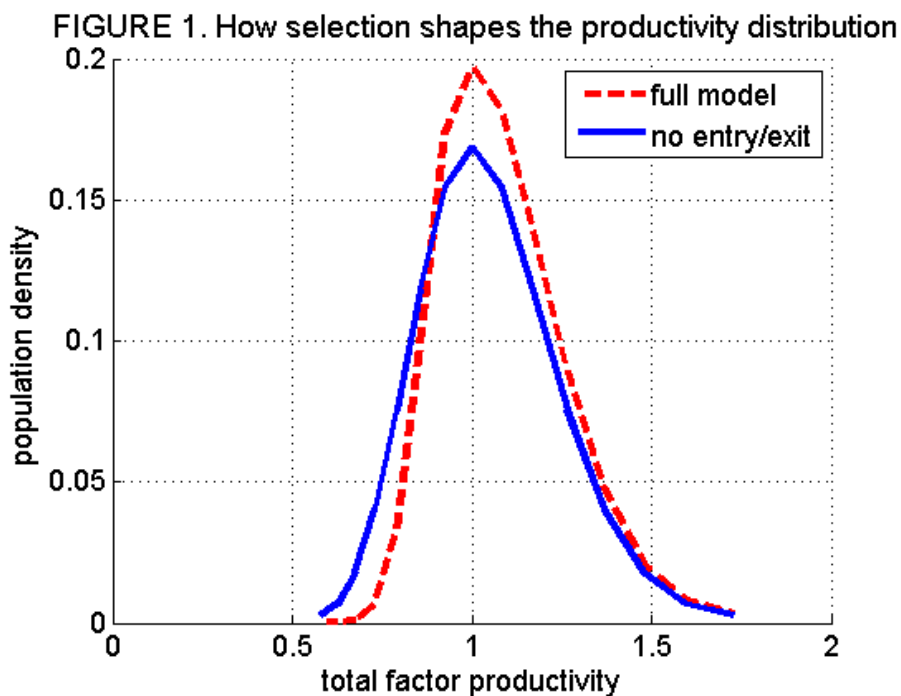FIGURE 1. How selection shapes the productivity distribution

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Figure 1 compares the stationary distribution of firms over total factor productivity in our model economy to that in the model without entry and exit. All else equal, firms with relatively low productivities are induced to exit our model economy by the costs they must pay to remain. Furthermore, fixed entry costs induce those potential firms with relatively low productivity signals to stay out. As such, the typical exiting firm is replaced by an entrant with higher productivity. Given these aspects of selection, the stationary distribution of firms in our model economy has less mass over lower productivity levels and more mass in higher regions of productivity than does the reference model. This raises average productivity by 4.2 percent, and thus encourages households to work and save more.

FIGURE 2. Start of period potential producer distribution
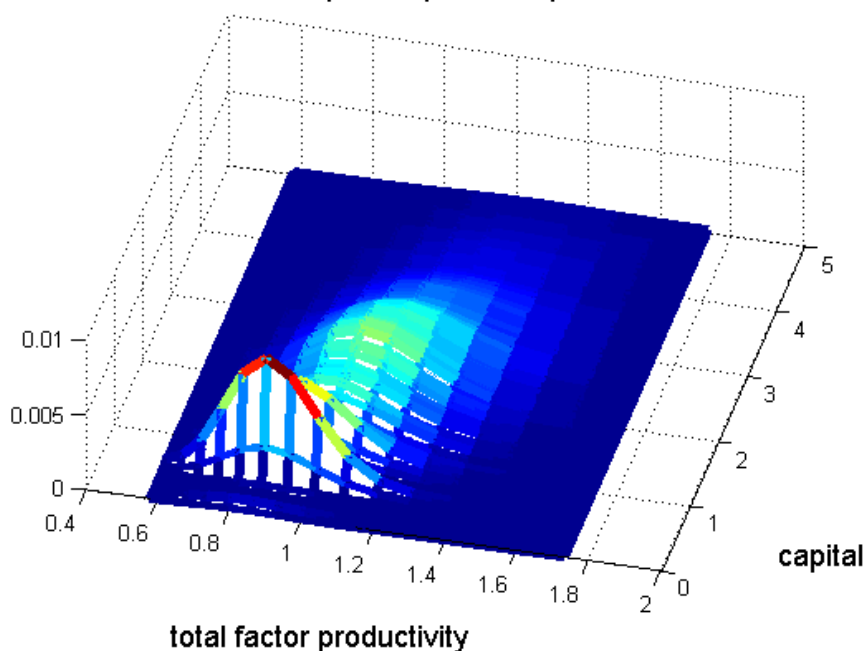
Figures 2 and 4 (below) display the stationary distributions of firms in our economy at the start of a period and at the time of production, respectively. In each of these figures, population density increases as one looks toward the back right corner representing the highest levels of capital and productivity. Comparing the start-of-period distribution to that remaining at production time, we see how selection generates these shapes.

Next, comparing Figures 3 and 4 gives a glimpse into our model's firm life-cycle dynamics. In Figure 3, we have the steady state distribution of entrants in their first year of production. These are the startups from the foreground of Figure 2 that selected to enter production; they are mostly concentrated in the lower ranges of productivity and capital. As we look to the distribution of

all operating firms in Figure 4, we see the mass of firms expanding into higher productivities and capital levels. This indicates that our model is consistent with the empirical evidence that young firms are smaller and less productive than the typical firm. Conditional on survival, young firms become more productive and larger over time as they gradually move toward maturity.

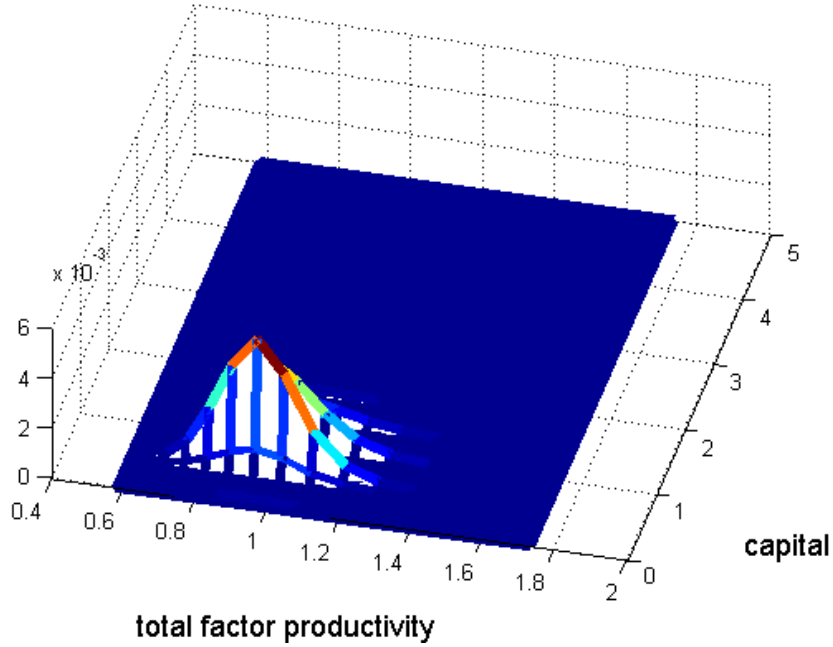FIGURE 3. Production-time distribution: New firms
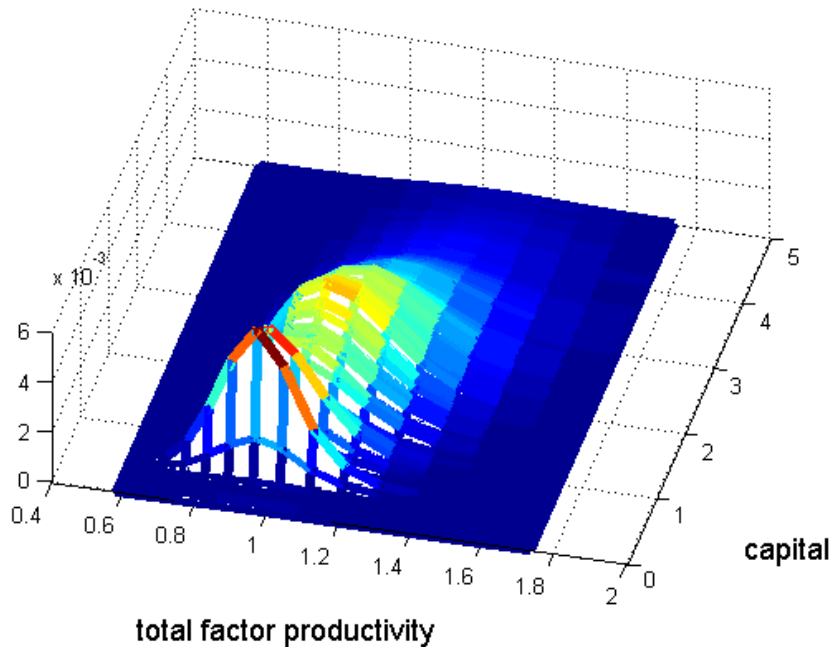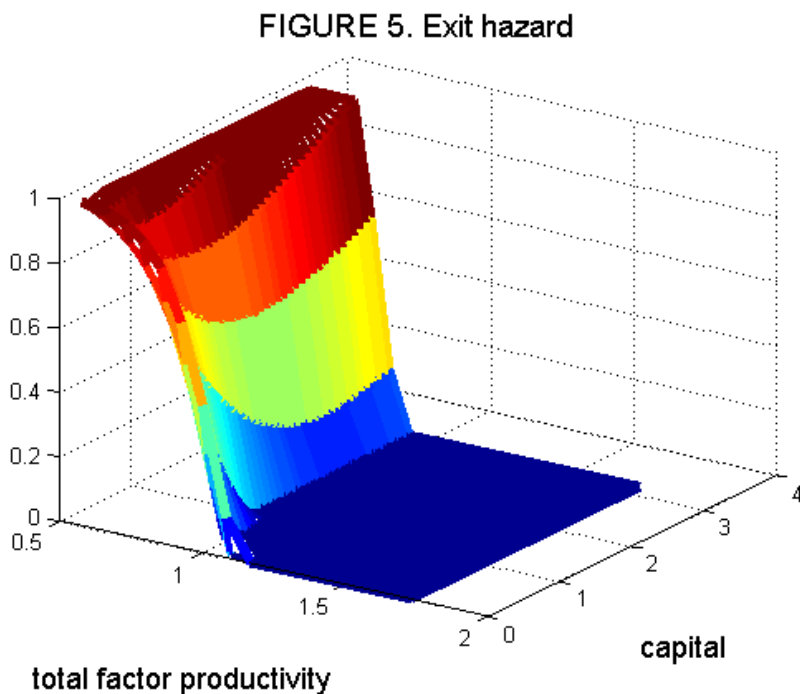

FIGURE 4. Production-time distribution: All Firms

Figure 5 displays our steady state exit hazard for startups and incumbent firms. The patterns here arise naturally from two facts: (i) firm values are increasing in both capital and productivity, while (ii) convex and nonconvex adjustment costs distort optimal capital reallocation.

FIGURE 5. Exit hazard

At productivity ranges above 1.2, irrespective of capital, all firms are willing to pay the highest operating costs; so no firm exits. Elsewhere, for any given capital stock, selection implies that exit probabilities rise as TFP falls. On the other hand, if we condition on productivity, the probability of exit is non-monotone in firm size. Absent costs of capital adjustment, the hazard would always fall in capital (given higher flow profits and the fact that a fraction of the firm's capital is lost when it exits). Here, however, some firms with large capital stocks and low productivity prefer to exit rather than pay operating costs and also suffer adjustment costs to shed their excess capital. As a result, at firm-TFP levels below around 0.8, the exit hazard grows increasingly u-shaped.

We conclude this section with a more direct look at firm life-cycle dynamics. Figure 6 tracks an initially large cohort of startup firms as it ages across 20 years in our model's steady state. By allowing mean-reverting idiosyncratic productivities alongside fixed entry and operating costs, our model obtains the selection-based successes of Hopenhayn's (1992) original model of industry dynamics. Here, as there, the average productivity and value of surviving members within a cohort rise as the cohort ages, so exit rates fall with age.

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FIGURE 6. Productivity, Exit, Population and Output By Age
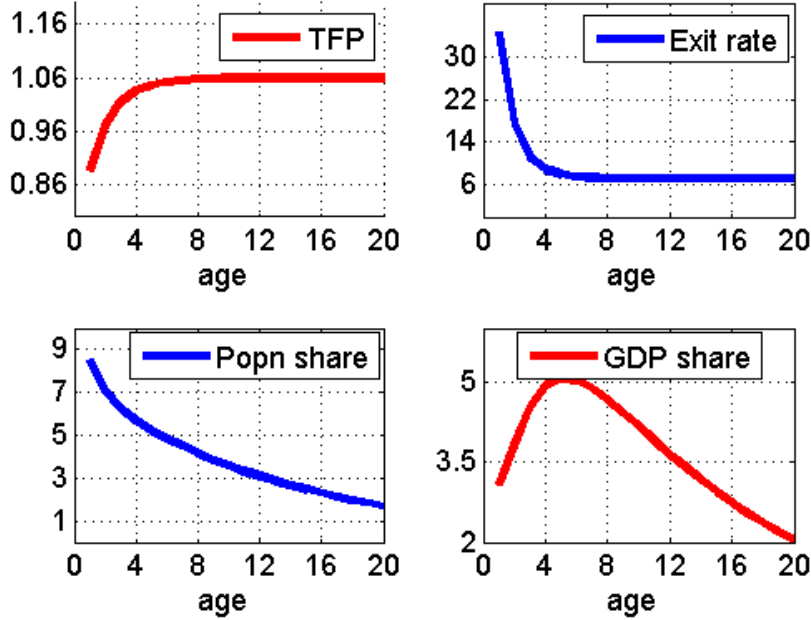
From the top right panel of Figure 6, notice that exit rates fall off sharply from the 34 percent failure rate of startups, to roughly 17 percent failure among one-year old firms, reaching about 10 percent in age 3. At the heart of these mechanics, it takes the typical firm roughly 7 years to reach its ultimate productivity in the top left panel of the figure, although the half-life from its first date of production is only about 3 years. Our older firms tend to have higher productivity than young firms, and they experience mean-reversion in their productivities. Thus, we easily obtain an unconditional negative relationship between firm size and growth, and between firm age and growth, as found in the data by Dunne, Roberts and Samuelson (1989), Haltiwanger, Jarmin and Miranda (2013) and other studies. However, our inclusion of one-period time-to-build capital stocks breaks the perfect mapping between firm productivity and size inherent in the Hopenhayn model. Thus, our model is capable of a negative correlation between age and growth conditional on size consistent with the empirical findings of Evans (1987), Hall (1987) and Haltiwanger, Jarmin and Miranda (2013). Because firms cannot immediately adjust their capital inputs in response to changes in their productivities, firms with large employment levels need not necessarily have high productivity. This muddying effect is compounded by our inclusion of capital adjustment frictions.

Finally, consider the implications of Figure 6 for when the greatest losses from a 'lost generation' of entrants might be felt in our economy. In ordinary times, a young cohort closes roughly

two-thirds of the gap to its ultimate productivity by age 4, when its population share is still relatively high. Taking into account the gradualism implied by firm-level capital accumulation, the cohort has its greatest contribution to aggregate production (cohort output/GDP) at ages 5 and 6 (lower right panel). Thus, to the extent that an aggregate shock to our economy causes a large reduction in firm entry, we may expect to see the largest effects of those losses roughly five years on.

5.2 Aggregate fluctuations

We begin this section by considering how endogenous firm life-cycle dynamics alter the cyclical movements in GDP, employment and other series when fluctuations are driven solely by aggregate productivity shocks. In response to a fall in the exogenous component of aggregate TFP, potential firms realizing any given firm-level productivity signal anticipate lower value relative to an ordinary date. Thus, fewer among them choose to invest toward becoming startups in the next period. For the same reasons, the numbers of startups choosing to pay their operating costs to enter production in the current period also fall, while the numbers of incumbent firms exiting rise. We will see below that these choices drive procyclical entry and countercyclical exit in our model, as in the data. Such changes have the potential to exacerbate the movements in employment and GDP; however, note that the most affected firms are those with low relative productivities, so selection should have some stabilizing influence. Among firms with the same productivity, one might expect that larger firms would be more likely to survive. Recall from Figure 5 that this need not be the case, however, given the implications of micro-level capital adjustment frictions.

To consider how entry and exit decisions reshape the typical business cycle, we first compare HP-filtered moments from our model to those from the reference model described above wherein a fixed set of firms lives forever. Table 1 examines volatility and contemporaneous comovement in the two settings. Despite hindrances to capital reallocation, both economies have the usual traits of an equilibrium business cycle model in terms of their relative volatilities and contemporaneous correlations with GDP. Relative to the reference model, our economy has a bit more volatility in overall GDP and consumption. The differences in employment and capital investment are more pronounced. Changes in the number and composition of firms drive higher volatility in both series and weaken the correlation between investment and GDP.

TABLE 1. Volatilities and contemporaneous correlations with GDP

	GDP	Consumption	Investment	Employment
A: RELATIVE STD. DEV.				
No Entry/Exit	(2.085)	0.457	4.000	0.605
Full Model	(2.159)	0.449	4.135	0.610
B: CORRELATION				
No Entry/Exit	1.000	0.924	0.972	0.957
Full Model	1.000	0.924	0.954	0.960

Table 2 presents the cross-date correlations of entry, exit and the measure of operating firms with contemporaneous GDP. There, our model delivers the promised signs with respect to the contemporaneous entry and exit correlations. Entry is procyclical, while exit is strongly counter-cyclical. Because there is a one-period time-to-build nature in the creation of entering firms, the strongest relationship between entry and GDP is with a one-year lag. Thus, movements in the number of producers are protracted; the contemporaneous correlation with GDP is lower than the correlations at both date t+1 and date t+2.

TABLE 2. Cross-date correlations with current GDP

	$t+0$	$t+1$	$t+2$	$t+3$	$t+4$
Entry	0.579	0.769	−0.031	−0.377	−0.458
Exit	−0.832	−0.038	0.426	0.387	0.301
Firms	0.580	0.822	0.655	0.388	0.126

Across Tables 1 and 2, we have seen that cyclical changes in firms' entry and exit decisions amplify business cycles. Given the usual firm life-cycle patterns presented in Figure 6, such changes also have the potential to add persistence to the movements in GDP. Consider the fact that an unusually small number of new firms enters into production following a negative TFP shock. Over subsequent periods, the typical surviving member of this smaller-than-average cohort of young firms grows in productivity and size. We noted in closing section 5.1 that, in ordinary times, a young cohort contributes increasingly to GDP as it nears age 5. Thus, early reductions in the numbers of entering firms in response to a negative aggregate TFP shock can hold aggregate production down at later dates, even as the exogenous component of aggregate productivity reverts toward its mean, thereby protracting a TFP-driven recession. We will see some evidence

of this phenomenon in Figure 8 below. However, Table 3 reveals that it is largely absent in the HP-filtered GDP series at short lag lengths. The first- and second- order autocorrelations of GDP are marginally weaker in our model than in the reference model, which recall has no life-cycle dynamics. Consistent with our reasoning to this point, the negative correlations of GDP with itself and exogenous TFP at lags 3 and 4 are reduced.

TABLE 3. Persistence and the propagation of shocks

	GDP at t+1	GDP at t+2	GDP at t+3	GDP at t+4
A: CORRNS WITH GDP(t)				
No Entry/Exit	0.444	0.077	−0.117	−0.232
Full Model	0.392	0.061	−0.093	−0.190
B: CORRNS WITH Z(t)				
No Entry/Exit	0.451	0.087	−0.106	−0.222
Full Model	0.431	0.093	−0.061	−0.163

For some further insight into the moments reported in Tables 1 - 3, we examine impulse responses following a one standard deviation negative productivity shock. We begin with the reference model in Figure 7. There, GDP falls roughly 4 percent at the shock's impact, while employment and investment fall roughly 2.4 percent and 16 percent, respectively. Thereafter, these series display the usual mean-reversion seen in business cycle models driven by AR(1) shocks. Consumption and the real wage also exhibit the usual u-shape of a business cycle model with indivisible labor preferences.

Examining the lower right panel of Figure 7, notice that there is virtually no difference in the response of measured versus exogenous productivity in the reference model despite the presence of capital reallocation frictions. This is consistent with results in Khan and Thomas (2003, 2008), although those studies did not include convex adjustment costs. The payment of such costs is included in broad investment (as will be entry and operation costs in our full model below); however these are sufficiently minor and unaffected by the shock as to imply no distinction between the narrow and broad investment responses.[9]

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[9]The same is true of the labor-denominated nonconvex adjustment costs in both models. Thus, we do not report responses for the narrow employment series.
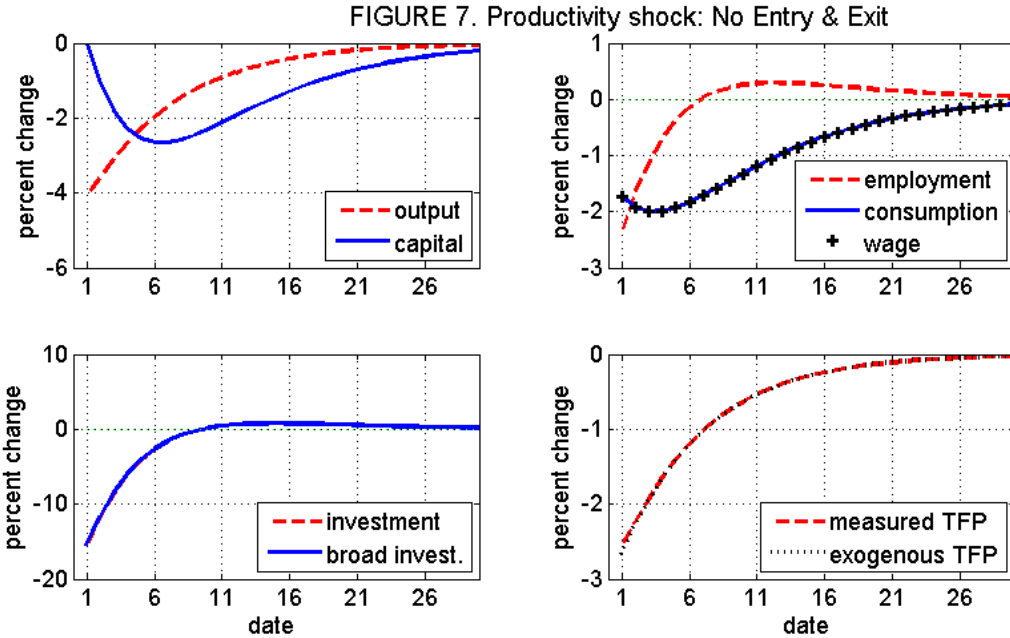
FIGURE 7. Productivity shock: No Entry & Exit

Figure 8 shows responses to the same productivity shock in our full model economy, holding the axes fixed from Figure 7 for easy comparability. GDP falls about 0.4 percent more in Figure 8, and there is a similar difference in the employment response. Capital investment falls about 2 percent further relative to the reference model, whereas broad investment falls 5 percent less. The latter simply reflects the fact that firms' investment in the form of fixed entry and operating costs is far less responsive to the shock than is their investment in capital. The former arises from an anticipated endogenous drag on aggregate productivity evident in the lower right panel. Its absence from Figure 7 implies that it is driven entirely by the disruption in firm-level entry and exit decisions.

Beyond amplification, our model also implies some increased persistence. Our GDP half-life is a little less than one-year greater than in the reference model. The remainder of the recovery is more appreciably gradualized; for example, GDP reaches 1 percent below normal about 3 years later in our model. This is a direct result of the TFP wedge in our model, and the fact that it gains prominence over the first 5 dates of the response.

To understand why the overall propagation of a TFP shock is altered in our model, we turn to the responses in market participation that distinguish it in Figure 9. Recall again from Figure 6 that the essential mechanism we anticipated would offset mean-reversion in exogenous TFP to hold aggregate production down longer was a missing generation effect, the growth phase of a smaller-than-usual cohort of young firms following the shock. Figure 9 shows that entry falls

roughly 2.5 percent below normal at the date of the shock and recovers about half way in the following year. Thus, two cohorts of young firms are appreciably reduced. These new cohorts fail to replace an initially large number of exiting firms. Exit rises about 8 percent at date 1, and many young firms fail; hence, we see a u-shaped response in the measure of operating firms.

FIGURE 8. Productivity shock: Full Model

FIGURE 9. TFP shock: Entry, Exit and Firms
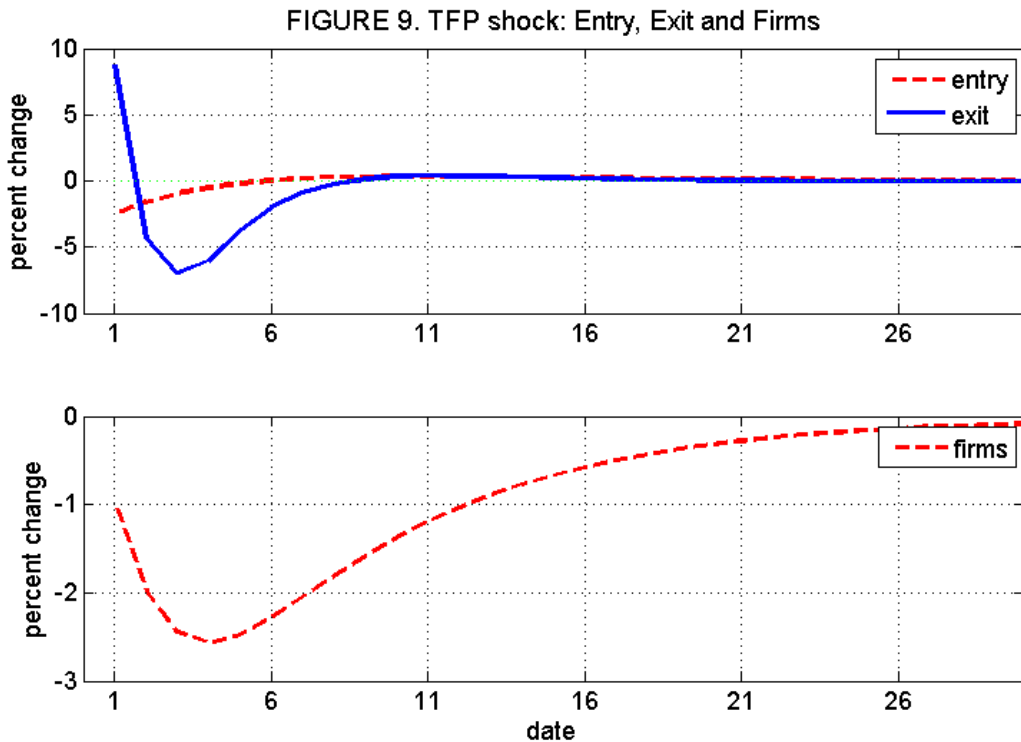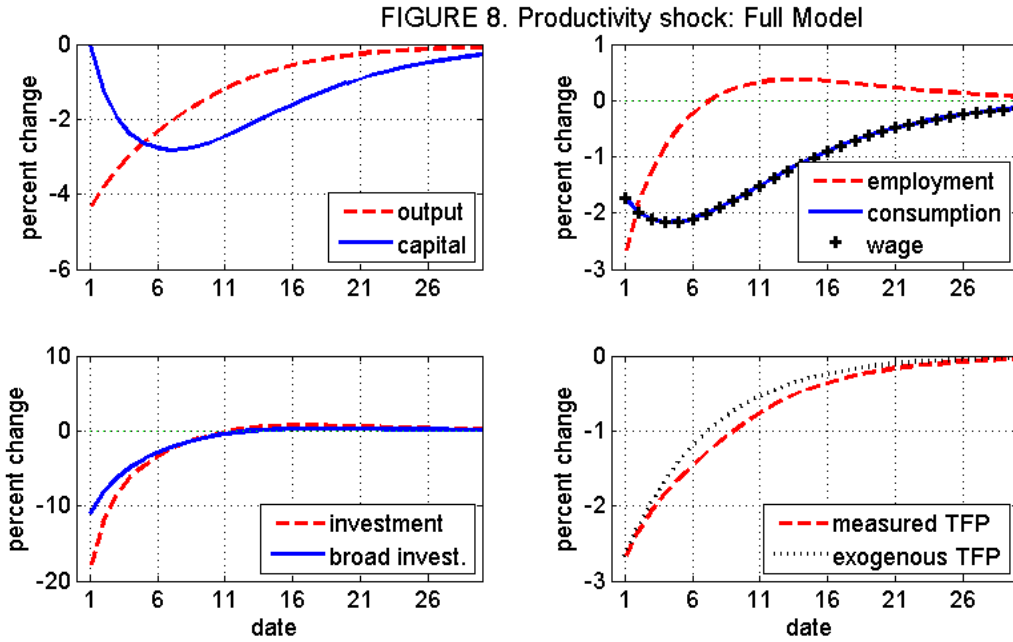
So far, we have studied how entry, exit and selection contribute to the mechanics of a typical recession. We next consider their role in a Great Recession such as the U.S. 2007-9 experience. There is, by now, a mounting body of evidence from the BDS data that this particular downturn had disproportionate negative effects on young firms (Sedlacek (2013), Sedlacek and Sterk (2014)) and on small firms (Khan and Thomas (2013), Siemer (2013)). Indirect evidence suggests that the recession originated in a shock in the financial sector (Almeida et al. (2009), Duchin et al. (2010)). Khan and Thomas (2013) examines a shock to the availability of credit in an equilibrium model where a fixed measure of heterogenous firms face real and financial frictions. Predictions there match the 2007-9 recession well, but the model fails to deliver the subsequent anemic recovery. Several recent equilibrium studies have considered whether changes in the number and composition of firms may have contributed to this. Sedlacek (2013) examines a search and matching model with multi-worker firms and endogenous entry and exit following a TFP shock, while Siemer (2013) considers a credit crunch in a setting where new firms must finance a fraction of their startup costs with debt. Both models predict a missing (or lost) generation effect that propagates the effects of an aggregate shock; however, both abstract from capital and thus its reallocation. Khan, Senga and Thomas (2014) considers a financial shock affecting balance sheets and default recovery rates in a model with endogenous default, entry and exit and finds that endogenous destruction to the stock of firms slows the recovery. However, given the challenge of solving for individual equilibrium loan rate schedules, that study omits capital adjustment frictions we find useful for a tight calibration to firm life-cycle data.

Drawing on evidence from the BDS, three striking observations distinguish the Great Recession relative to a typical recession. First, the total number of firms fell 5 percent (Siemer (2013)). Second, the number of young (age 5 and below) firms fell 15 percent (Sedlacek (2013)). Third, the relative employment decline among small firms (those with less than 100 employees) was disproportionate in comparison to such firms' usual employment share (Khan and Thomas (2013)). To the extent that this was led by a financial shock, it distorted firm life-cycle dynamics substantially. Here, we proxy for the implications of financial disruption by adding a shock that, by virtue of the ordinary life-cycle exit and productivity patterns seen above in Figure 6, should disproportionately affect small and young firms. Specifically, alongside the TFP shock displayed above, we consider a 5 percent rise in the upper support of the operating cost distribution; φ_U rises to .273 and thereafter exhibits an AR(1) return with persistence $\rho = \rho_z$.

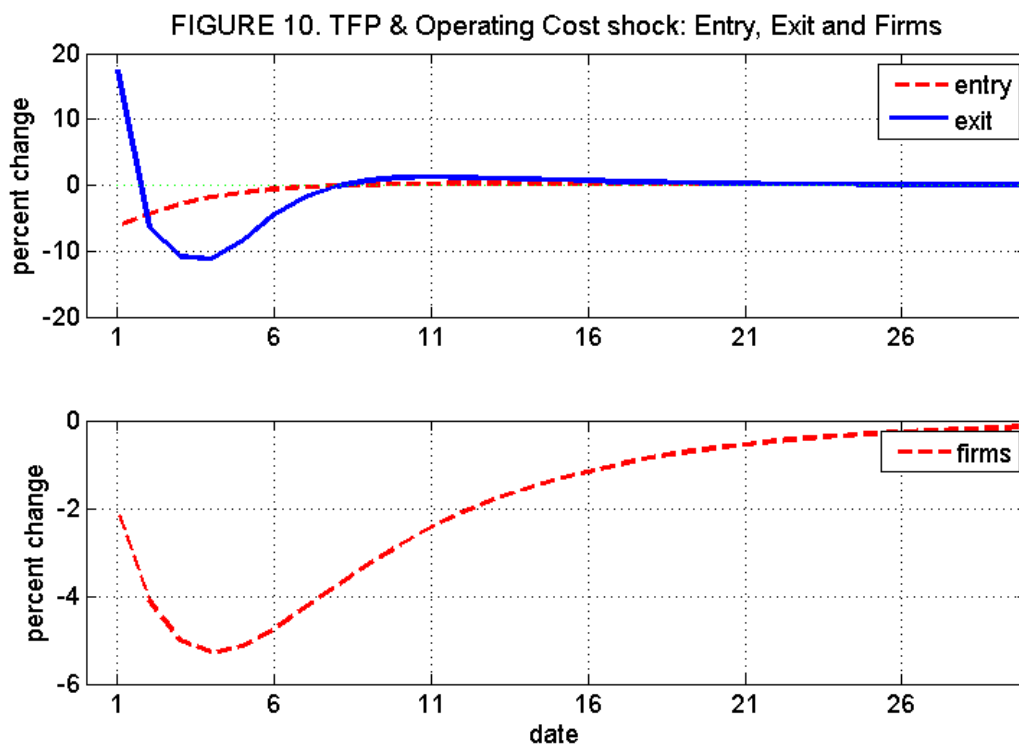FIGURE 10. TFP & Operating Cost shock: Entry, Exit and Firms

Figure 10 shows the overall effect our second shock has on entry, exit and the number of firms. With an increase to the costs of operating, the rise in exit and the fall in entry roughly double in comparison to Figure 9. This carries over into the number of firms in the bottom panel, and generates an ultimate drop roughly matching the 5 percent fall over the Great Recession.

The overall number of operating firms is an important input into aggregate production, of course, given decreasing returns at the firm. However, recall from Figure 6 that firms of different ages are not equally valuable. The greatest contributions to GDP come from firms aged 4-6 as they move toward maturity with growing productivity and size, while still relatively large in their numbers. (NOTE: Add composition figure following the cohort of startups from date 1 for comparison to Figure 6 above.) Thus, an important aspect of our proxy financial shock is the fact that it disproportionately eliminates young firms, strengthening the missing generation effect we saw following the TFP shock alone.

Figure 11 is our model counterpart to the Great Recession. Notice that, in comparison to Figure 9, the more direct destruction of smaller, younger firms here amplifies the downturn, because it yields an endogenous drop in aggregate TFP at its impact. Larger differences are seen in later dates as the aggregate shocks mean revert. The consequences of the missing generation begin to cumulate in measured TFP after only one or two periods. By year 6, we see a roughly 1

percent endogenous drag on aggregate productivity, which fails to diminish over many subsequent dates. This slows GDP's return markedly, lengthening its half-life by more than a year.

FIGURE 11. TFP & Operating Cost shock: Full Model
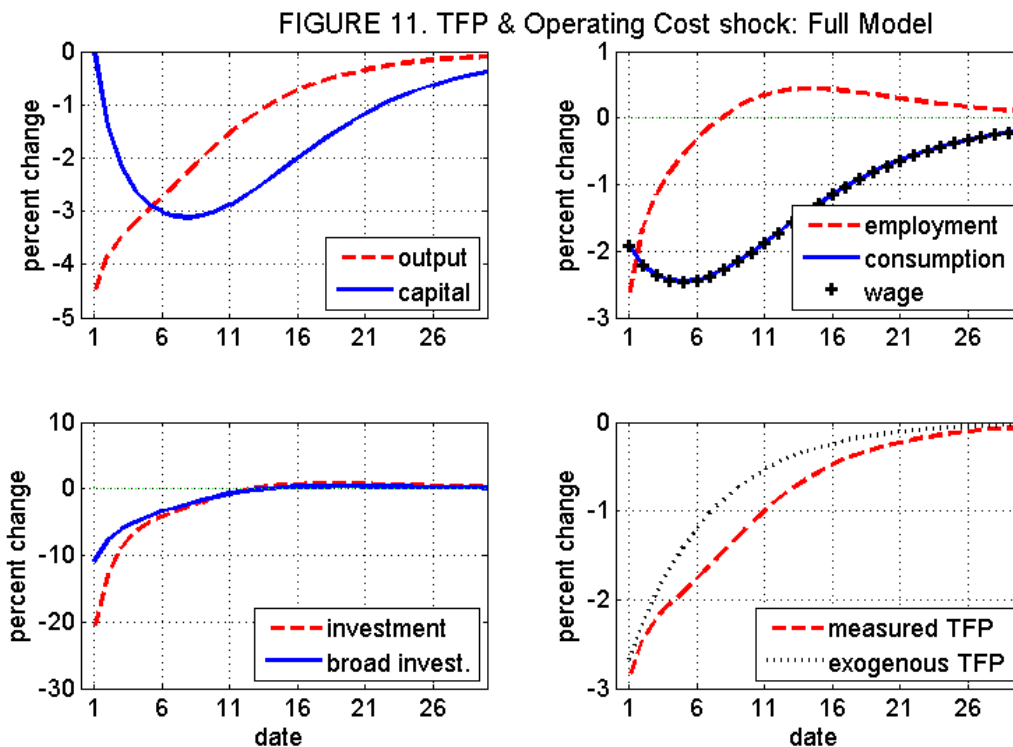
6   Concluding remarks

In the sections above, we have developed a dynamic, stochastic general equilibrium model allowing for time-varying entry and exit in a setting where firms face persistent shocks to aggregate and individual productivity, and they must pay fixed costs to enter and to continue in production. Our firms' decisions regarding entry and their subsequent continuation are affected not only by their expected productivities, but also by capital reallocation frictions, and thus by their existing stocks. We have explored this model toward arriving at a better understanding of whether and how changes in firm entry and exit rates and the composition of firms affects aggregate fluctuations in an environment with realistic firm-level investment patterns and life-cycle dynamics. Based on an examination of standard second moments and impulse responses, we have seen that such changes amplify responses over a typical business cycle driven by a disturbance to aggregate productivity and, to a lesser extent, protract them.

Our model amplifies a standard recession because it delivers an endogenous TFP wedge

through procyclical movements in entry and countercyclical exit. Recovery is gradualized because the endogenous productivity effects grow over time. That, in turn, happens because young firms fail to replace a raised number of exiting firms in early dates following a shock, and the overall measure of producers falls over time. This missing generation effect is most prominent in GDP at the time when the reduced young cohorts are nearing maturity, and lingers many years thereafter.

Changes in the number of firms, and more particularly in the numbers of young firms, were dramatic over the U.S. 2007 recession. In an exercise designed to emulate this unusual episode, we have also considered a shock to firms' fixed operating costs. This might be interpreted as a loose proxy for a disruption to external finance in that it most directly affects entry decisions and the exit decisions of younger firms, given selection and their relatively low average productivity levels. We have seen that such a shock sharpens the missing generation effect, delivering a more pronounced cumulating drag on aggregate productivity and thus a more anemic recovery.

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