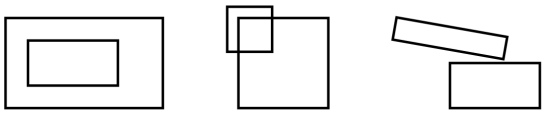
Mathematics Of Doing, Understand, Learning, and Educating Secondary Schools

MODULE(S^2): Algebra for Secondary Mathematics Teaching

Adapted for MODULE(S^2)

Version Spring 2018



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Part I

Introduction to Fields

1 Fields and Other Algebraic Structures

In this section we will begin our study of **fields**. You've already encountered fields in your mathematical studies: the set of rational numbers $\mathbb{Q}$ and the set of real numbers $\mathbb{R}$ are fields, as is the set of complex numbers $\mathbb{C}$. The sets $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$ are different in many ways, but here we will focus on the ways in which they are similar. We will also see that there are fields that are different from these three in some very important ways.

Consider the equation

$$3x + 8 = 14.$$

It's not hard to see that the solution to this equation is $x = 2$: $3(2) + 8 = 14$. Let's us solve this equation step-by-step, justifying each step along the way. First we will subtract 8 from both sides:

$$(3x + 8) - 8 = 14 - 8.$$

(Note that we could also view this as adding -8 to both sides. The number -8 is known as the **additive inverse** of 8.) Applying the associative law on the left-hand side gives

$$3x + (8 - 8) = 6.$$

We know that $8 - 8 = 0$ so we have

$$3x + 0 = 6.$$

The number 0 is an **additive identity**. That means adding 0 returns the value we added it to. So we have

$$3x = 6.$$

We now multiply each side by $1/3$ to obtain

$$\frac{1}{3}(3x) = \frac{1}{3} \cdot 6.$$

Multiplication is associative, so we can write this as

$$\left(\frac{1}{3} \cdot 3\right) x = 2.$$

The number $1/3$ is the **multiplicative inverse** of 3, meaning that $\frac{1}{3} \cdot 3$ is equal to the **multiplicative identity**; that is, $\frac{1}{3} \cdot 3 = 1$. Thus we have

$$1x = 2.$$

The number 1 is the **multiplicative identity** meaning that $1x = x$. So we conclude that

$$x = 2.$$

Part II

Constructible Numbers

Part III

Three Famous Problems