Claim. $\left.\begin{array}{l} A = \{3n : n \in \mathbb{Z}\} \\ B = \{6n : n \in \mathbb{Z}\} \end{array}\right] \Rightarrow B \subsetneq A.$

Proof. Given $A = \{3n : n \in \mathbb{Z}\}$
$B = \{6n : n \in \mathbb{Z}\}$.

① $\boxed{B \subseteq A}$ We show: $x \in B \Rightarrow x \in A$.

Given $x \in B$.

$x = 6k,\ k \in \mathbb{Z}$ by defn of membership in B

(*) $= 3 \cdot 2k$

$= 3n,\ n \in \mathbb{Z}$. by closure of mult in $\mathbb{Z}$ $(2, k \in \mathbb{Z} \Rightarrow 2k \in \mathbb{Z})$

Hence x satisfies membership rules for A $\Rightarrow x \in A$.

By defn of subset, $B \subseteq A$. $\boxed{1}$

② $\boxed{\exists x \in A \text{ s.t. } x \notin B}$ We find an elt of A not in B. Observe that all $x \in B$ are even (by defn of even):

$x \in B \Rightarrow x = 3 \cdot 2k = 2 \cdot 3k$ by (*) and comm of mult in $\mathbb{Z}$

$= 2m,\ m \in \mathbb{Z}$ by closure of mult in $\mathbb{Z}$ $(3, k \in \mathbb{Z} \Rightarrow 3k \in \mathbb{Z})$

But there are members of A that are odd, e.g., 3, 6, 15, ... These members of A are not in B. $\boxed{2}$

B and A satisfy the defn of strict subset $\Rightarrow B \subsetneq A$. $\square$