Claim. $\left.\begin{array}{l} A=\{3n : n\in\mathbb{Z}\} \\ B=\{6n : n\in\mathbb{Z}\}\end{array}\right] \Rightarrow B \subsetneq A.$

Proof. Given $A=\{3n : n\in\mathbb{Z}\}$
$B=\{6n : n\in\mathbb{Z}\}$.

① $\boxed{B\subseteq A}$ We show: $x\in B \Rightarrow x\in A$.

Given $x\in B$.

$x = 6k,\ k\in\mathbb{Z}$ — by defn of membership in B

(*) $= 3\cdot 2k$

$= 3n,\ n\in\mathbb{Z}$. — by closure of mult in $\mathbb{Z}$ $(2,k\in\mathbb{Z} \Rightarrow 2k\in\mathbb{Z})$

Hence x satisfies membership rules for A
$\Rightarrow x\in A$.

By defn of subset, $B\subseteq A$. $\boxed{1}$

② $\boxed{\exists\, x\in A \text{ s.t. } x\notin B}$ We find an elt of A not in B. Observe that all $x\in B$ are even (by defn of even):

$x\in B \Rightarrow x = 3\cdot 2k = 2\cdot 3k$ — by ⊛ and comm of mult in $\mathbb{Z}$

$= 2m,\ m\in\mathbb{Z}$ — by closure of mult in $\mathbb{Z}$ $(3,k\in\mathbb{Z} \Rightarrow 3k\in\mathbb{Z})$

But there are members of A that are odd, e.g., $3, 6, 15, \ldots$. These members of A are not in B. $\boxed{2}$

B and A satisfy the defn of strict subset $\Rightarrow B\subsetneq A$. $\square$