Implicit differentiation

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Here are some key ideas from section 3.5.

- The chain rule says

- The chain rule helps us with implicit differentiation, which is used when we can't isolate the ________________________________ variable.

- Here's an example:

Midterm practice (Vojta MT2 '22): Find the following derivatives.

a) $\frac{d}{dx} e^x \cos x$

b) $\frac{d}{dx} \frac{\sin x}{x^2+1}$.

My Attempt:	Solution:

Problem 1: Suppose $x^3 + y^3 = 1$. In this problem, we will find $\frac{dy}{dx}$ by implicit differentiation.

a) Write $\frac{d}{dx}$ on both sides.
Simplify the right hand side, and expand the left hand side with the sum rule.

b) Use a technique from the example to simplify $\frac{d}{dx}y^3$.

c) Solve for $\frac{dy}{dx}$.

My Attempt:	Solution:

Problem 2: (Stewart 3.5) Suppose $2\sqrt{x} + \sqrt{y} = 3$. Use the same steps as in the previous problem to find $\frac{dy}{dx}$ by implicit differentiation.

My Attempt:	Solution:

Problem 3: (Stewart 3.5) Suppose $e^{x/y} = x - y$. Find $\frac{dy}{dx}$ by implicit differentiation.

My Attempt:	Solution:

Problem 4: (Stewart 3.7) Two important properties are

$$\frac{d}{dx}\log_b x = \frac{1}{x\ln b} \qquad \frac{d}{dx}\ln x = \frac{1}{x}.$$

Find

$$\frac{d}{dz}\ln\sqrt{\frac{a^2 - z^2}{a^2 + z^2}}.$$

My Attempt:

Solution:

Problem 5: (Stewart 3.7) Logarithmic differentiation is helpful when want to take the derivative of a complicated function (i.e. lots of products, fractions, exponents, etc.). Here are the steps we can use:

1. Take natural logarithms of both sides and use logarithm laws to simplify.
2. Implicitly differentiate with respect to x.
3. Solve for y'.

Use these steps to differentiate

$$y = \frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^2}.$$

My Attempt:

Solution:

Problem 6: If $y = \tan^{-1} x$, then $\tan y = x$. Use implicit differentiation to find the derivative of the inverse tangent function.

My Attempt: | Solution:

Problem 7: (Stewart 3.7) Find $\frac{d}{dx} x \arctan \sqrt{x}$.

My Attempt: | Solution:

Challenge problem: Show that

$$\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x$$

for any $x > 0$.