# Antiderivatives

***November 7th, 2024***

Here are some key ideas from sections 4.6.

- We say that $F(x)$ is an antiderivative of $f(x)$ if _____.
- If $F(x)$ is one of the antiderivatives of $f(x)$, then all of the antiderivatives of $f(x)$ have the form _____.
- Let's do an example. One of the antiderivatives of $f(x) = \sin x$ is:

  The most general antiderivative of $\sin x$ is:

.....

**Trig practice:** Find $\sec(225°)$ and $\tan(225°)$.

**Problem 1:** (Stewart 4.6) Find the most general antiderivatives of the following functions. Remember that the derivative of the antiderivative is the original function.

a) $cf(x)$; b) $f(x) + g(x)$; c) $x^n$, for $n \neq -1$; d) $1/x$;
e) $e^x$; f) $e^{cx}$; g) $\cos x$; h) $\sin x$;
i) $\sec^2 x$; j) $\sec x \tan x$ k) $\frac{1}{1+x^2}$.

My Attempt:

Solution:

**Problem 2:** (Stewart 4.6) Find the most general antiderivative of $\frac{3}{t^2}$, assuming $t > 0$.

My Attempt:

Solution:

**Problem 3:** (Stewart 4.6) Find the most general antiderivative of $f(x) = x(2-x)^2$.

My Attempt:

Solution:

**Problem 4:** (Stewart 4.6) Find the most general antiderivative of $f(x) = 2\sqrt{x} + 6\cos x$.

My Attempt:

Solution:

**Problem 5:** (Stewart 4.6) Suppose $\frac{dr}{d\theta} = \cos\theta + \sec\theta\tan\theta$, where $0 < \theta < \pi/2$ and $r(\pi/3) = 4$. Find $r(\theta)$. *Hint: this is an inital value problem—find the general antiderivative first and then use the provided "initial value."*

My Attempt:

Solution:

**Problem 6:** (Stewart 4.6) For each of the functions $f''(x)$ below, find the most general expression for $f(x)$. *Hint: you will need to "undo" the derivative twice.*

a) $f''(x) = 6x + 12x^2$

My Attempt:

Solution:

b) $f''(x) = 6x + \sin x$.

**Problem 7:** (Stewart 4.6) Suppose a sample of a radiocative substance with initial mass of 75 mg decays $t$ years later at a rate of $1.7325e^{-0.0231t}$ mg/year. Find the mass of the sample after 20 years.

My Attempt:

Solution:

**Challenge problem:** (Stewart 4.6) Assume that a snowball melts so that its volume decreases at a rate proportional to its surface area. If it takes three hours for the snowball to decrease to half its original volume, how much longer will it take for the snowball to melt completely?