Types of functions, inverses

September 26th, 2024

Here are some key ideas from sections 1.2, 1.3, 1.4, and 1.5.

Type of function	**Function definition**
Linear	Expression:
Polynomial	General expression:
Power	General expression:
Rational	Expression:
Trigonometric	Three main examples:
Exponential	Expression:
Logarithmic	Expression:

- Exponential functions take the form ________________. These are the important laws of exponents: $b^{x+y} =$ ____________, $b^{x-y} =$ ____________, $(b^x)^y =$ ____________, $(ab)^x =$ ____________.
- A function f is called *one-to-one* if it never takes the same value twice: ____________________________.
- The following are the steps to find the inverse of a one-to-one function f.
  1.
  2.
  3.

Problem 0: Draw a unit circle!

My Attempt: | Solution:

Problem 1: (Stewart 1.5) Find the inverse function f^{-1} of $f(x) = \frac{1}{3}\sqrt{7 + e^{5x}}$. *Hint: the inverse of* e^x *is* $\ln x$.

My Attempt: | Solution:

Problem 2: (Stewart Section 1.4) Simplify $27^{2/3}$.

My Attempt: | Solution:

Problem 3: (Stewart Section 1.3) Describe the symmetry of $f(x) = \frac{1-e^{1/x}}{1+e^{1/x}}$. Is it even, odd, or neither?

My Attempt: | Solution:

Problem 4: (Stewart Section 1.4) If $f(x) = 5^x$, show that

$$\frac{f(x+h) - f(x)}{h} = 5^x \left(\frac{5^h - 1}{h} \right).$$

My Attempt: | Solution:

Problem 5: (Stewart Section 1.4) Find the domain of the function below.

$$f(x) = \frac{1 - e^{x^2}}{1 - e^{1-x^2}}$$

My Attempt: | Solution:

Problem 6: (Stewart Section 1.3) Express $R(x) = \sqrt{\sqrt{x} - 1}$ in the form $f \circ g \circ h$ (this can also be written as $f(g(h(x)))$.

My Attempt: | Solution:

Problem 7: (Stewart Section 1.3) Under ideal conditions, a certain bacteria population is known to double every 2 hours. Suppose there are initially 700 bacteria. What is the size of the population after t hours?

My Attempt: | Solution:

Problem 8: (Bamler Fall '18 Final Exam) Simplify $\sin(\tan^{-1}(x))$ by drawing a triangle.

My Attempt: | Solution:

Problem 9: (Borcherds '05 Midterm 1) Sketch the graph of $y = |x^2 - 2x|$.

My Attempt: | Solution:

Challenge Problem: Solve the inequality $\ln(x^2 - 2x - 2) \leq 0$.