Extrema

October 29th, 2024

Here are some key ideas from section 4.1.

- If $f(c)$ is greater than all the other values of $f(x)$, then it is an ____________________ maximum. If it is greater than all the other values in its neighborhood, then it is a ________________ maximum.
- If $f(c)$ is less than all the other values of $f(x)$, then it is an ____________________ minimum. If it is less than all the other values in its neighborhood, then it is a ________________ minimum.
- The Extreme Value Theorem says that if f is ____________________ on a closed interval $[a, b]$, then f attains an absolute maximum value and an absolute minimum on that interval.
- We say that c is a critical value if ____________________ or ____________________.
- Local extrema can only exist at ____________________________.
- Absolute extrema can exist at ____________________ or ____________________.

. . . .

Problem 1: Here's a general algorithm for finding absolute extrema on a closed interval $[a, b]$.

1. Find the critical values of $f(x)$.
2. For each critical value c, find the corresponding y-value given by $f(c)$.
3. Find the values of f at the endpoints of the interval.
4. The largest value in the Steps 2 and 3 is the absolute maximum. The smallest value is the absolute minimum.

(Stewart 4.1) Find the absolute maximum and absolute minimum values of $f(x) = 12 + 4x - x^2$ on $[0, 5]$.

My Attempt:	Solution:

Problem 2:

a) How does the algorithm in Problem 1 change if we are finding absolute extrema on an open interval (a, b)?

b) Draw the graph of a function defined on the open interval $(0, 5)$ that does not have any absolute or local extrema.

My Attempt: | Solution:

Problem 3: (Stewart 4.1) Find the critical values of the following functions.

a) $g(t) = |3t - 4|$;

b) $g(\theta) = 4\theta - \tan\theta$;

c) $f(x) = x^2 e^{-3x}$.

My Attempt: | Solution:

Problem 4: (Stewart 4.1) Find the absolute extrema of $f(t) = t + \cot(t/2)$ on the closed interval $[\pi/4, 7\pi/4]$.

My Attempt: | Solution:

Problem 5: (Stewart 4.1) Find the maximum value of $f(x) = x^a(1-x)^b$ on the interval $[0, 1]$, assuming a and b are both positive numbers.

My Attempt:

Solution:

Problem 6: Find the absolute extrema of $f(x) = \frac{x^2-4}{x^2+4}$ on the interval $[-4, 4]$.

My Attempt:

Solution:

Challenge problem: (Stewart Chapter 4) If x, y, and z are positive numbers, then prove that

$$\frac{(x^2+1)(y^2+1)(z^2+1)}{xyz} \geq 8.$$