Characteristic polynomials, iteration

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Here are some key ideas from sections 8.7 and 8.8.

- One way we can solve for eigenvalues is by using **characteristic polynomials**. Recall that to have an eigenvector (which is nonzero), we need that

  where λ is an ______________________________. Now consider the matrix $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$. Solving, we get:

- Matrices express transformations. If we start with a vector $\vec{n}_0$, then we can keep applying the transformation:

- The **recursion** for this formula is __________________________. Now let $\vec{v}_1$ be an eigenvalue with eigenvector λ_1. Likewise, let $\vec{v}_2$ be an eigenvalue with eigenvector λ_2. For

$$P = \begin{bmatrix} \quad\quad\quad\quad \end{bmatrix}; \quad \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \quad\quad\quad\quad\quad ;$$

  the solution to the recursion is:

. . . .

Problem 1: (Stewart & Day 8.7) For the example matrix $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$, solve for the eigenvalues and the corresponding eigenvectors.

My Attempt:	Solution:

Problem 2: (Stewart & Day 8.7) Suppose that $A^2 = 0$ for some matrix A. Show that the only possible eigenvalue of A is 0.

My Attempt: | Solution:

Problem 3: (Stewart & Day 8.8) Show that $A = PDP^{-1}$, where P is a diagonal matrix whose columns are the eigenvectors of A and D is a diagonal matrix with the corresponding eigenvalues.

a) $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$.

My Attempt: | Solution:

Problem 4: (Stewart & Day 8.8) Suppose $\vec{n}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Express the solution to the recursion $\vec{n}_{t+1} = A\vec{n}_t$ in terms of the eigenvectors and eigenvalues of A.

My Attempt: | Solution:

Problem 5: (Stewart & Day 8.7) Suppose that $\vec{v}$ is an eigenvector of matrix A with eigenvalue λ_A, and it is also an eigenvector of matrix B with eigenvalue λ_B. Show that $\vec{v}$ is an eigenvector of $A+B$ and find its associated eigenvalue. Then show that $\vec{v}$ is an eigenvector of AB and find its associated eigenvalue.

My Attempt:	Solution:

Problem 6: (Stewart & Day 8.7) (Stewart & Day 8.8) Suppose $\vec{n}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & a \\ 0 & b \end{bmatrix}$, for $a \neq 0$ and $b \neq 1$. Express the solution to the recursion $\vec{n}_{t+1} = A\vec{n}_t$ in terms of the eigenvectors and eigenvalues of A.

My Attempt:	Solution:

Challenge Problem: (Stewart & Day 8.8) Suppose that $T = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, with $a \neq 0$ and $b \neq 0$. Show that the eigenvalues of T are $\lambda = a \pm bi$. Then show that T can be written as $T = r \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.