Planes and projections

September 3rd, 2024

Here are some key ideas from section 8.3.

- Dot products have some cool properties!
  - $\vec{a} \cdot \vec{a} =$ ________.
  - $\vec{0} \cdot \vec{a} =$ ________.
  - $\vec{a} \cdot (\vec{b} + \vec{c}) =$ ______________.
  - $(c\vec{a}) \cdot \vec{b} =$ ______________ $=$ ______________.
- The dot product helps us write equations of **planes**. A plane is determined by a ____________________ and a vector $\vec{n} = [a, b, c]$ that is __________________________ to the plane.
- An equation of the plane passing through the point $P_0(x_0, y_0, z_0)$ and perpendicular to the vector $[a, b, c]$ is
- The **vector projection** of $\vec{b}$ onto $\vec{a}$ is given by ______________________________________. You can think of it as the shadow that $\vec{b}$ casts on $\vec{a}$.
- The **scalar projection** (component) is the __________________________ of the vector projection, and is given by ______________________________________.

Problem 1: (LibreTexts) For the vectors $\vec{u} = \langle 4, 3 \rangle$ onto $\vec{v} = \langle 2, 8 \rangle$, first sketch and then calculate the vector projection of $\vec{u}$ onto $\vec{v}$. Then calculate the scalar projection.

My Attempt:	Solution:

Problem 2: (LibreTexts) Find the scalar projection and vector projection of $\vec{b} = \langle 0, 1, \frac{1}{2} \rangle$ onto $\vec{a} = \langle 2, -1, 4 \rangle$.

My Attempt:	Solution:

Problem 3: (Stewart & Day 8.3) Suppose that $\vec{a}$ and $\vec{b}$ are nonzero vectors. When does $\text{comp}_{\vec{a}}\vec{b} = \text{comp}_{\vec{a}}\vec{b}$? When does $\text{proj}_{\vec{a}}\vec{b} = \text{proj}_{\vec{a}}\vec{b}$?

My Attempt:	Solution:

Problem 4: (Stewart & Day 8.3) Find the equation of the plane that passes through the origin and is perpendicular to the vector $\langle 1, -2, 5 \rangle$.

My Attempt:	Solution:

Problem 5: (Stewart & Day 8.3) The orthogonal projection of $\vec{b}$ onto $\vec{a}$ is $\text{orth}_{\vec{a}}\vec{b} = \vec{b} - \text{proj}_{\vec{a}}\vec{b}$. Show that the orthogonal projection is, in fact, orthogonal to $\vec{a}$.

My Attempt:	Solution:

Problem 6: (Stewart & Day 8.3) For $\vec{a} = \langle 1, 2, 0 \rangle$, find a vector $\vec{b}$ such that $\text{comp}_{\vec{a}}\vec{b} = 2$. Then describe the set of all vectors $\vec{w}$ such that $\text{proj}_{\langle 1,2,0 \rangle}(\vec{w}) = \vec{0}$.

My Attempt:	Solution:

Problem 7: Muscle fibers contract in various directions. Suppose a muscle fiber contracts along a vector $\vec{v}$ with a magnitude of 5 cm. The direction of contraction makes an angle of $\pi/4$ with a bone. Find the scalar projection of the contraction vector $\vec{v}$ onto the bone's direction, which represents the effective shortening of the muscle along the bone's axis.

My Attempt:	Solution:

Problem 8: Find an equation of the plane containing the points $(1, 0, 0)$, $(1, 1, 1)$, and $(1, 1, 0)$.

My Attempt:	Solution:

Challenge Problem: (Stewart & Day Chapter 8) Use a scalar projection to show that the distance from a point $P_1(x_1, y_1)$ to the line $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$