Shapes of graphs

October 31st, 2024

Here are some spooky key ideas from section 4.2.

The Mean Value Theorem says that if f is ______________________ on the interval $[a, b]$, then there exists some c between a and b such that

$$f'(c) =$$

On a given interval, we say f is $\begin{cases} \rule{6cm}{0.4pt} & \text{if } f'(x) > 0 \\ \rule{6cm}{0.4pt} & \text{if } f'(x) < 0 \end{cases}$ on that interval.

The **first derivative test** is used to find extrema. Recall that c is a critical number if $f'(c)$ is 0 or undefined.

We say f has a $\begin{cases} \rule{6cm}{0.4pt} & \text{at } c \text{ if } f' \text{ changes from positive to negative} \\ \rule{6cm}{0.4pt} & \text{at } c \text{ if } f' \text{ changes from negative to positive} \end{cases}$ at c.

But if f' does not change sign at c, then ____.

We can also define concavity in terms of second derivatives.

On a given interval, we say f is $\begin{cases} \rule{6cm}{0.4pt} & \text{if } f''(x) > 0 \\ \rule{6cm}{0.4pt} & \text{if } f''(x) < 0 \end{cases}$ on that interval.

The second-derivative analogue for critical points is *inflection points*. A point P on a curve $f(x)$ is an inflection point if f is continuous at P and ____________________________.

Here is the **second derivative test** for a function f continuous near c:

We say f has a $\begin{cases} \rule{6cm}{0.4pt} & \text{at } c \text{ if } f'(c) = 0 \text{ and } f''(c) > 0 \\ \rule{6cm}{0.4pt} & \text{at } c \text{ if } f'(c) = 0 \text{ and } f''(c) < 0 \end{cases}$ at c.

....

Trig practice: Show that $(\sin^4\theta - \cos^4\theta + 1)\csc^2\theta = 2$.

Problem 1: (Stewart 4.2) Suppose that the derivative of a function f is $f'(x) = (x+1)^2(x-3)^5(x-6)^4$. On what interval is f increasing?

My Attempt:	Solution:

Problem 2: (Stewart 4.2) Find the intervals of increase and the intervals of decrease of $f(x) = \sin x + \cos x$, for $0 \leq x \leq 2\pi$.

My Attempt: | Solution:

Problem 3: (Stewart 4.2) Find the intervals of concavity and the inflection points of $f(x) = 4x^3 + 3x^2 - 6x + 1$.

My Attempt: | Solution:

Problem 4: (Stewart 4.2) Suppose that $3 \leq f'(x) \leq 5$ for all x. Show that $18 \leq f(8) - f(2) \leq 30$. *Hint: use the Mean Value Theorem!*

My Attempt: | Solution:

Problem 5: (Stewart 4.2) For what values of a and b does the function

$$f(x) = axe^{bx^2}$$

have the maximum value $f(2) = 1$?

My Attempt:

Solution:

Problem 6: (Stewart 4.2) Find the intervals of concavity and the inflection points of $f(x) = \frac{x^2}{x^2+3}$.

My Attempt:

Solution:

Challenge problem: (Stewart 4.2) Find the x-coordinate of the inflection point of a cubic function with real roots x_1, x_2, and x_3.