

The Fundamental Theorem of Calculus

November 19th, 2024

Here are some key ideas from section 5.3.

- The indefinite integral is $\int f(x) dx$. If $\int f(x) dx = F(x)$, then _____.
 - A definite integral looks like $\int_a^b f(x) dx$ and is a _____.
 - An indefinite integral looks like $\int f(x) dx$ and is a _____.
- **FTC 1:** Also known as the Evaluation Theorem. If f is continuous on the interval $[a, b]$, then $\int_a^b f(x) dx =$ _____, where F is _____ antiderivative of f . For example:

$$\int_0^1 x^2 dx =$$

- **FTC 2:** First, some motivation:

Now, the theorem. Suppose f is continuous on $[a, b]$. Let $g(x) = \int_a^x f(t) dt$ on $[a, b]$. Then $g'(x) =$ _____ on (a, b) . In other words, g is an _____ of f .

Trig practice: Find all values of $\arcsin \frac{1}{\sqrt{2}}$ and $\arccos \frac{\sqrt{3}}{2}$ on $[0, 2\pi]$.

Problem 1: (Stewart 5.3) Why is this process incorrect?

$$\int_{-1}^3 \frac{1}{x^2} dx = \left. \frac{x^{-1}}{-1} \right|_{-1}^3 = -\frac{4}{3}.$$

My Attempt:

Solution:

Problem 2: (Stewart 5.3) Find $\int (1-t)(2+t^2) dt$.

My Attempt:

Solution:

Problem 3: (Stewart 5.3) Evaluate the following integrals.

a) $\int_{-2}^3 (x^2 - 3) dx;$

b) $\int_{-5}^5 e dx;$

c) $\int_0^1 10^x dx.$

My Attempt:

Solution:

Problem 3: (Stewart 5.3) Find the following indefinite integrals.

a) $\int (1 + \tan^2 \alpha) d\alpha;$

b) $\int \frac{\sin x}{1 - \sin^2 x} dx;$

c) $\int v(v^2 + 2)^2 dv.$

My Attempt:

Solution:

Problem 4: (Stewart 5.3) Find the first derivatives of the following functions.

a) $g(x) = \int_1^x \frac{1}{t^3 + 1} dt;$

b) $g(y) = \int_3^y e^{t^2 - t} dt;$

c) $g(r) = \int_0^4 \sqrt{x^2 + 4} dx.$

My Attempt:

Solution:

Problem 5: (Stewart 5.3) Find the first derivatives of the following functions.

a) $h(x) = \int_2^{1/x} \arctan t \, dt;$

b) $h(x) = \int_0^{x^2} \sqrt{1+r^3} \, dr;$

c) $y = \int_{e^x}^0 \sin^3 t \, dt.$

My Attempt:

Solution:

Problem 6: (Stewart 5.3) Find a function f and a number a such that, for all $x > 0$,

$$6 + \int_a^x \frac{f(t)}{t^2} \, dt = 2\sqrt{x}.$$

My Attempt:

Solution:

Problem 7: (Apostol 5.5) A function f is continuous everywhere and satisfies the equation

$$\int_0^x f(t) \, dt = -\frac{1}{2} + x^2 + x \sin 2x + \frac{1}{2} \cos 2x.$$

for all x . Compute $f(\pi/4)$ and $f'(\pi/4)$.

My Attempt:

Solution:

Challenge problem: (Apostol 5.5) Show that, for all real x ,

$$\int_0^x (t + |t|^2) \, dt = \frac{2x^2}{3}(x + |x|).$$