Linear approximations, Taylor polynomials

October 22nd, 2024

Here are some key ideas from section 3.8.

- The linearization (also known as the tangent line approximation) of $f(x)$ at $x = a$ is $L(x) = f(a) + f'(a)(x - a)$.
- Newton's method can be used to find a root, or zero, of a function $f(x)$:
  1. Make a guess for the root, and call it x_1.
  2. Successively calculate $x_{n+1} = x_n - \frac{x_n}{f'(x_n)}$.
  3. After enough iterations, and with an appropriate initial guess, x_n gets closer to a zero.
- The nth degree Taylor polynomial is $T_n(x) = c_0 + c_1(x - a) + c_2(x - a)^2 + \cdots + c_n(x - a)^n$, where $c_0 = f(a)$, $c_1 = f'(a)$, $c_2 = \frac{1}{2}f''(a)$, and so on.

. . . .

Problem 1: Use a linear approximation to estimate $(2.001)^5$.

My Attempt:	Solution:

Problem 2: (Stewart Chapter 3) At what point on the curve $y = [(\ln(x + 4)]^2$ is the tangent line horizontal?

My Attempt:	Solution:

Problem 3: (Stewart Chapter 3) Find the derivative of $\sin^2\left(\cos\sqrt{\sin \pi x}\right)$.

My Attempt:

Solution:

Problem 4: (Stewart 3.8) Find the first three Taylor polynomial of degree n for $f(x) = e^x$, centered at a.

My Attempt:

Solution:

Problem 5: (Stewart 3.8) Find an initial value of x_1 such that Newton's method fails on the function $x^3 - 3x + 6$.

My Attempt:

Solution:

Problem 6: ★ (Stewart Chapter 3) Find h' in terms of f' and g'.

$$h(x) = \frac{f(x)g(x)}{f(x) + g(x)}$$

My Attempt:

Solution:

Problem 7: ★ (Stewart Chapter 3) Evaluate

$$\lim_{x \to 0} \frac{\sqrt{1 + \tan x} - \sqrt{1 + \sin x}}{x^3}.$$

My Attempt:

Solution:

Problem 8: ★ (Stewart Chapter 3) Show that

$$\frac{d}{dx}\left(\frac{\sin^2 x}{1+\cot x}+\frac{\cos^2 x}{1+\tan x}\right)=-\cos 2x.$$

My Attempt:

Solution:

Problem 9: ★ (Stewart Chapter 3) For what values of c does the equation $\ln x = cx^2$ have exactly one solution?

My Attempt:

Solution: