Average values, volumes

December 5th, 2024

Here are some key ideas from sections 6.2 and 6.4.

- We define the average value of a function on the interval $[a, b]$ to be $f_{\text{ave}} =$ _____.
- The Mean Value Theorem for integrals says _____.
- Suppose we have a solid whose cross-sectional area at x is given by $A(x)$. Then the volume of the solid between $x = a$ and $x = b$ is

$$V = \lim_{n\to\infty} \qquad\qquad = \int$$

.....

Trig practice: (Stewart) If $\sin x = \dfrac{1}{3}$ and $\sec y = \dfrac{5}{4}$, where x and y lie between 0 and $\pi/2$, evaluate $\sin(x + y)$.

Problem 1: (Stewart 6.2) Find the average value of $f(x) = 4x - x^2$ on the interval $[0, 4]$.

My Attempt:

Solution:

Problem 2: (Stewart 6.2) Find the average value of $\cos^4 x \sin x$ on $[0, \pi]$.

My Attempt:

Solution:

Problem 3: (Stewart 6.2) Find the average value of $(3 - 2u)^{-1}$ on the interval $[-1, 1]$.

My Attempt:

Solution:

Problem 4: Consider the region under the curve $y = \sqrt{x}$ from 0 to 1. This problem will walk you through the process of finding the volume of the solid generating by rotating this region about the x axis.

1. Sketch the solid of revolution. What shape is the cross-section?
2. Find the area of a cross-section for a fixed x. This will be your function $A(x)$.
3. Integrate $A(x)$ between the given bounds to find the volume.
4. You did it! Celebrate!

My Attempt:

Solution:

Problem 5: (Stewart 6.4) Find the volume of the solid obtained by rotating the region $y = 1 - x^2$ and $y = 0$ about the x-axis.

My Attempt:

Solution:

Problem 6: (Stewart 6.4) Find the volume of the solid obtained by rotating the region bound by $y = \sqrt{25 - x^2}$, $y = 0, x = 2$, and $x = 4$ about the x-axis.

My Attempt:

Solution:

Problem 7: (Stewart 6.4) Consider the solid obtained by rotating the region bounded by $x = 2\sqrt{y}$, $x = 0$, and $y = 9$ about the y-axis. Sketch the solid, and then find its volume. *Hint: try integrating with respect to y instead of x.*

My Attempt:

Solution:

Problem 8: (Stewart 6.4) Use a definite integral to find the volume of a right circular cone with height h and base radius r.

My Attempt:

Solution:

Challenge problem: (Stewart 6.4) Find the volume common to two spheres, each with radius r, if the center of each sphere lies on the surface of the other sphere.

You made it to the very end! Thanks for coming to discussion this semester :)