Improper integrals, areas between curves

December 3rd, 2024

Here are some key ideas from sections 5.8 and 6.1.

- An improper integral is when we have _____ as one (or both!) of the bounds of integration. We write

$$\int_a^{\infty} f(x)\,dx =$$
$$\int_{-\infty}^{b} f(x)\,dx =$$
$$\int_{-\infty}^{\infty} f(x)\,dx =$$

- An improper integral is called _____ if the limit exists and _____ otherwise.
- If the curve of $f(x)$ is _____ the curve of $g(x)$ on the interval $[a, b]$, then the area between f and g on the interval is

$$A =$$

.....

Trig practice: Find all real x such that

a) $\sin x = 1$; b) $\cos x = 1$; c) $\sin x = -1$; d) $\cos x = -1$.

Final exam practice: (Apostol 1.26) Find a quadratic function f such that $f(0) = f(1) = 0$ and $\int_0^1 f(x)\,dx = 1$.

My Attempt:

Solution:

Problem 1: (Stewart Chapter 5) Write the following improper integrals as limits.

a) $\int_a^{\infty} f(x)\,dx$; b) $\int_{-\infty}^{b} f(x)\,dx$; c) $\int_{-\infty}^{\infty} f(x)\,dx$.

My Attempt:

Solution:

Problem 2: (Stewart 5.8) For each integral below, either evaluate it or show that it is divergent.

a) $\int_1^\infty \frac{1}{(2x+1)^3}\,dx$ b) $\int_0^\infty \frac{\ln x}{x^4}\,dx$ c) $\int_{-\infty}^0 e^{-2x}\,dx.$

My Attempt:

Solution:

Problem 3: (Stewart 5.8) For each integral below, either evaluate it or show that it is divergent.

a) $\int_3^\infty \frac{1}{(x-2)^{3/2}}\,dx$ b) $\int_0^\infty \frac{e^x}{e^{2x}+3}\,dx$ c) $\int_{-\infty}^\infty x^3 e^{-x^4}\,dx.$

My Attempt:

Solution:

Problem 4: (Stewart 6.1) Find the area of the regions bounded by the given curves.

a) $y = x^2, \quad y = 2x - x^2$

b) $y = x^2, \quad y = 4x - x^2$

My Attempt:

Solution:

.

Problem 5: (Stewart 6.1) Find the area of the regions bounded by the given curves.

a) $y = 1/x, y = x^2, y = 0, x = e$

b) $x + y = 0, x = y^2 + 3y$

c) $y = \cos x, y = 2 - \cos x, [0, 2\pi]$

My Attempt:

Solution:

Challenge problem: Find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $(1, 1)$, and the x-axis.