The definite ~~integral~~ squiggle!

November 14th, 2024

Here are some key ideas from section 5.2.

- Last time, we learned that a Riemann sum can be used to approximate the area bounded between a curve and the x-axis on the interval $a \leq x \leq b$
  - So suppose we use n rectangles, each with width ________.
  - Moreover, suppose we use ______________________________ to represent the *sample points* (for example, left endpoints, right endpoints, or midpoints).
  - Then the exact area under the curve is
- If the limit exists, we say the function is ______________________________, and the definite integral is
- When can we integrate a function (when can we find the area under the curve)?
  - A ___________________________________ function is always integrable.
  - A function with ____ jump discontinuities is always integrable.

. . . .

$\star$ In this worksheet, you will need these summation laws to solve problems.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Trig practice: Sketch the graphs of $\tan x$ and $\arctan x$. On what interval is $\arctan x$ a function?

Problem 1: (Stewart 5.2) Evaluate the left Riemann sum for $f(x) = 3 - \frac{1}{2}x$ on the interval $2 \leq x \leq 14$, with six subintervals. Use left endpoints as sample points.

My Attempt:	Solution:

Problem 2: (Stewart 5.2) Express the following limits as definite integrals.

a) $\lim_{n\to\infty} \sum_{i=1}^{n} x_i \ln(1 + x_i^2)\, \Delta x$, on the interval $[2, 6]$;

b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\cos x_i}{x_i}\, \Delta x$, on the interval $[\pi, 2\pi]$;

c) $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{2x_i^*(x_i^*)^2}\, \Delta x$, on the interval $[\pi, 2\pi]$.

My Attempt:

Solution:

Problem 3: (Stewart 5.2) Geometrically find $\int_{-1}^{5} (1 + 3x)\, dx$ by sketching a graph of the function and dividing the area into known and friendly shapes.

My Attempt:

Solution:

Problem 4: (Stewart 5.2) Evaluate $\int_{-3}^{0} \left(1 + \sqrt{9 - x^2}\right)\, dx$ by interpreting in terms of areas.

My Attempt:

Solution:

Problem 5: (Stewart 5.2) Use the limit definition to evaluate $\int_0^5 (1 + 2x^3)\, dx$.

My Attempt:

Solution:

Problem 6: (Stewart 5.2) Using lower and upper bounds for $\int_0^2 \frac{1}{1+x^2}\, dx$, estimate the value of the integral.

My Attempt:

Solution:

Problem 7: (Stewart 5.2) Evaluate $\int_{-2}^2 \sin(x)x^3\, dx$.

My Attempt:

Solution:

Challenge problem: (Stewart 5.2) If

$$\int_0^4 e^{(x-2)^4}\, dx = k,$$

find the value of

$$\int_0^4 xe^{(x-2)^4}\, dx.$$