Partial fractions

November 26th, 2024

Here are some key ideas from sections 5.6.

a) Here's a video on partial fraction decomposition.

b) Here's a video on integrating partial fractions.

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Trig practice: Find all values of $\sin, \cos$, and $\tan$ for for the following values of θ: $0, \pi/6, \pi/4, \pi/3, \pi/2$.

Problem 1: (Stewart 5.6) This problem will walk you through evaluating $\displaystyle\int \frac{x+5}{x^2+x-2}\,dx$ using **partial fractions**.

1. Factor the denominator! You should get $(x-a)(x-b)$, so find a and b.
2. Each linear term will be the denominator of a fraction in your partial fraction decomposition. You should get something that looks like
$$\frac{x+5}{x^2+x-2} = \frac{A}{x-a} + \frac{B}{x-b}.$$
Find A and B.
3. Integrate your partial fraction decomposition term by term! Don't forget to add a constant C.
4. Give yourself a well-earned pat on the back.

My Attempt:

Solution:

Problem 2: (Stewart 5.6) Write each fraction as a sum of partial fractions.

a) $\frac{1}{x^2-1}$; b) $\frac{2}{x^2+x}$ c) $\frac{2-x}{x^2-2x+8}$ d) $\frac{x}{x^2+x-2}$.

My Attempt:

Solution:

Problem 3: Find the antiderivative of each fraction in the previous problem using the decompositions you found.

My Attempt:

Solution:

Problem 4: (Stewart 5.6) Evaluate the following integrals.

a) $\int \frac{ax}{x^2 - bx}\, dx$

b) $\int \frac{1}{(x+a)(x+b)}\, dx.$

c) $\int_0^1 \frac{2}{2x^2 + 3x + 1}\, dx.$

My Attempt:

Solution:

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Problem 5: (Stewart 5.6) Use both the substitution rule and partial fractions to evaluate the following integrals.

a) $\int_9^1 6\frac{\sqrt{x}}{x-4}\, dx;$

b) $\int \frac{\cos x}{\sin^2 x + \sin x}\, dx.$

My Attempt:

Solution:

Challenge problem: Evaluate $\int \frac{x^2 - 5x + 16}{(2x+1)(x-2)^2}\, dx.$