Types of functions, inverses

September 26th, 2024

Here are some key ideas from sections 1.2, 1.3, 1.4, and 1.5.

Type of function	Function definition
Linear	Expression: $f(x) = mx + b$
Polynomial	General expression: $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$
Power	General expression: $f(x) = x^n$
Rational	Expression: $f(x) = p(x) / q(x)$ where $p(x)$ and $q(x)$ are polynomials
Trigonometric	Three main examples: $\sin(x)$, $\cos(x)$, $\tan(x)$
Exponential	Expression: $f(x) = b^x$
Logarithmic	Expression: $f(x) = \log_b x$

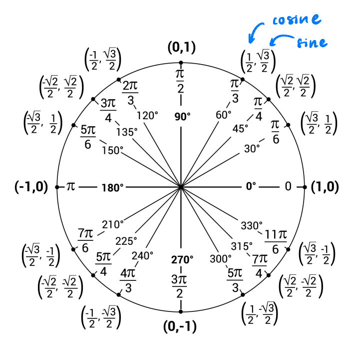
- Exponential functions take the form $f(x) = b^x$. These are the important laws of exponents: $b^{x+y} = b^x b^y$, $b^{x-y} = b^x / b^y$, $(b^x)^y = b^{xy}$ $\neq b^{x^y}$, $(ab)^x = a^x b^x$.
- A function f is called *one-to-one* if it never takes the same value twice: $a \neq b \Rightarrow f(a) \neq f(b)$.
- The following are the steps to find the inverse of a one-to-one function f.
  1. switch x & y
  2. solve for y
  3. set $y = f^{-1}(x)$

. . . .

Problem 0: Draw a unit circle!

My Attempt:

Solution:



Problem 1: (Stewart 1.5) Find the inverse function f^{-1} of $f(x) = \frac{1}{3}\sqrt{7 + e^{5x}}$. *Hint: the inverse of* e^x *is* $\ln x$.

My Attempt:

Solution:

$y = \frac{1}{3}\sqrt{7 + e^{5x}}$

$x = \frac{1}{3}\sqrt{7 + e^{5y}}$ (switch x & y)

$3x = \sqrt{7 + e^{5y}}$ (solve for y)

$9x^2 = 7 + e^{5y}$

$9x^2 - 7 = e^{5y}$

$\frac{1}{5}\ln(9x^2 - 7) = y$

$f^{-1}(x) = \frac{1}{5}\ln(9x^2 - 7)$ $y = f^{-1}(x)$

Problem 2: (Stewart Section 1.4) Simplify $27^{2/3}$.

My Attempt:

Solution:

$(3^3)^{2/3} = 3^2 = \boxed{9}$ both ways are ok

$\sqrt[3]{27^2} = \sqrt[3]{3\cdot3\cdot3\cdot3\cdot3\cdot3} = 3\cdot3 = \boxed{9}$

Problem 3: (Stewart Section 1.3) Describe the symmetry of $f(x) = \frac{1-e^{1/x}}{1+e^{1/x}}$. Is it even, odd, or neither?

My Attempt:

Solution:

$f(-x) = \dfrac{1-e^{1/-x}}{1+e^{1/-x}}$

even? $\dfrac{1-e^{1/-x}}{1+e^{1/-x}} \stackrel{?}{=} \dfrac{1-e^{1/x}}{1+e^{1/x}}$ ✗

odd? $\dfrac{1-e^{1/-x}}{1+e^{1/-x}} = -\dfrac{1-e^{1/x}}{1+e^{1/x}}$ ✗

so neither!

Problem 4: (Stewart Section 1.4) If $f(x) = 5^x$, show that

$$\frac{f(x+h)-f(x)}{h} = 5^x\left(\frac{5^h-1}{h}\right).$$

My Attempt:

Solution:

$f(x+h) = 5^{x+h} = 5^x 5^h$

$\dfrac{f(x+h)-f(x)}{h} = \dfrac{5^x5^h - 5^x}{h} = 5^x\dfrac{(5^h-1)}{h}$ ✓

Problem 5: (Stewart Section 1.4) Find the domain of the function below.

$$f(x) = \frac{1-e^{x^2}}{1-e^{1-x^2}}$$

My Attempt:

Solution:

We need $1-e^{1-x^2} \neq 0$

$e^{1-x^2} \neq 1$

$1-x^2 \neq 0$

$x^2 \neq 1$

so $\boxed{x \neq 1, -1}$

Problem 6: (Stewart Section 1.3) Express $R(x) = \sqrt{\sqrt{x} - 1}$ in the form $f \circ g \circ h$ (this can also be written as $f(g(h(x)))$.

My Attempt:

Solution:

Break it into parts.

$f(x) = \sqrt{x}$
$g(x) = x - 1$
$h(x) = \sqrt{x}$

Problem 7: (Stewart Section 1.3) Under ideal conditions, a certain bacteria population is known to double every 2 hours. Suppose there are initially 700 bacteria. What is the size of the population after t hours?

My Attempt:

Solution:

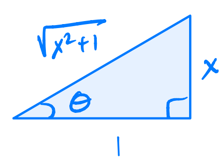
t	Population (P)
0	700
2	$700 \cdot 2$
4	$700 \cdot 2^2$
	$\vdots$

$P = 700(2)^{t/2}$

Problem 8: (Bamler Fall '18 Final Exam) Simplify $\sin(\tan^{-1}(x))$ by drawing a triangle.

My Attempt:

Solution:



Notice $\tan^{-1}(x) = \theta$
means $\tan\theta = x$
so $\frac{opp}{adj} = x = \frac{x}{1}$

Then hyp $= \sqrt{x^2+1^2}$

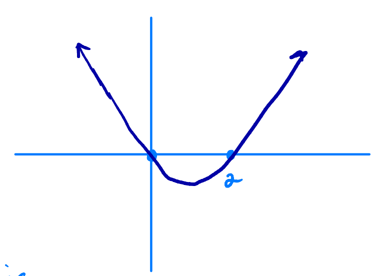
$\sin(\tan^{-1}(x)) = \sin(\theta) = \frac{opp}{hyp} = \frac{x}{\sqrt{x^2+1}}$

Problem 9: (Borcherds '05 Midterm 1) Sketch the graph of $y = |x^2 - 2x|$.

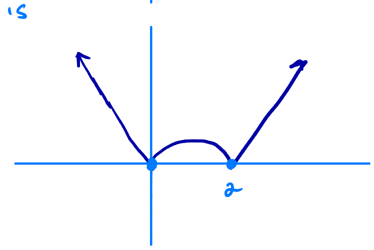
My Attempt:

Solution:

$y = x^2 - 2x$ is



so $y = |x^2 - 2x|$ is



Challenge Problem: Solve the inequality $\ln(x^2 - 2x - 2) \leq 0$.

$x^2 - 2x - 2 \leq e^0 = 1$
$x^2 - 2x - 3 \leq 0$
$(x-3)(x+1) \leq 0$

$x \in (-1, 3)$

Visit tinyurl.com/sections10a for my discussion resources.