Vectors and dot products

September 3rd, 2024

Here are some key ideas from sections 8.2 and 8.3.

- When we **scale** a vector by a constant c, we multiply its coordinates by a factor of _____ and multiply its magnitude by a factor of _____.
- For a vector $\vec{a} = [a_1, a_2, \cdots a_n]$, its **magnitude** is given by

- For two vectors $\vec{a} = [a_1, a_2, a_3]$ and $\vec{b} = [b_1, b_2, b_3]$, their **dot product** is _____.
- Let θ be the angle between the two vectors lying in the interval $0 \leq \theta \leq \pi$ (in other words, the smaller of the two angles we could draw). Then we can calculate θ using the formula

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Problem 1: (Stewart & Day 8.2) Find a unit vector in the same direction as $[8, -1, 4]$.

My Attempt:

Solution:

Problem 2: (Stewart & Day 8.2) Find a vector that has the same direction as $[-2, 4, 2]$ but has length 6.

My Attempt:

Solution:

Problem 3: (LibreTexts) A methane molecule has a carbon atom situated at the origin and four hydrogen atoms located at points $P(1,1,-1), Q(1,-1,1), R(-1,1,1)$, and $S(-1,-1,-1)$. Let O be the origin. Find the angle between vectors OS and OR (both beginning at O) that connect the carbon atom with the hydrogen atoms located at S and R. This is also called the bond angle.

My Attempt:

Solution:

Problem 4: (Stewart & Day 8.3) For what values of b are $[-6, b, 2]$ and $[b, b^2, b]$ perpendicular?

My Attempt:

Solution:

Problem 5: (Stewart & Day 8.3) Find the three angles of the triangle with the vertices $(1, 0)$, $(3, 6)$, and $(-1, 4)$.

My Attempt:

Solution:

Problem 5: Use a formula we discussed today to derive new ones.

1. Write a formula for the dot product of two parallel vectors.
2. Write a formula for the dot product of two vectors pointing in opposite directions.
3. Write a formula for the dot product of two perpendicular vectors.

My Attempt:

Solution:

Problem 6: (Stewart & Day 8.3) Let $\vec{u}$ be a diagonal of some cube (going through the cube), and let $\vec{v}$ be a diagonal of one of its faces. Find the angle between $\vec{u}$ and $\vec{v}$. *Hint: what are the vectors?*

My Attempt:

Solution:

Problem 7: (Apostol 12.8) Prove that for two vectors $\vec{a}$ and $\vec{b}$ in $\mathbb{R}^n$, we have

$$|\vec{a}+\vec{b}|^2 + |\vec{a}-\vec{b}|^2 = 2|\vec{a}|^2 + 2|\vec{b}|^2.$$

My Attempt:

Solution:

Challenge Problem: (Stewart & Day 8.3) Show that if $\vec{u}+\vec{v}$ and $\vec{u}-\vec{v}$ are orthogonal, then the vectors $\vec{u}$ and $\vec{v}$ must have the same length.