Math 10A Fall 2024 Worksheet 2

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1 Vectors, visually

1. Plot the following vectors in the Cartesian plane.
   (a) $[-1, 2]$ (b) $[3, 0]$ (c) $[5, 5]$
2. Use the head-to-tail method to add the following vectors graphically. Then check your answer by computing the sum directly (by adding coordinates).
   (a) $[1, 0] + [0, 2]$ (b) $[-3, 3] + [1, -2]$, (c) $[5, -2] + [-5, 2]$ (d) $[-1, 4] + [0, 0]$
3. Plot the following vectors in 3D space. Practice drawing 3D coordinate axes and getting comfortable working in 3D!
   (a) $[-1, 2, 0]$ (b) $[3, 1, 3]$ (c) $[1, -4, -2]$ (d) $[-5, 0, 2]$
4. Compute $[-1, 2, 0] + [3, 1, 3]$ using the head-to-tail method to add these vectors graphically. Then check your answer by computing the sum directly (by adding coordinates).

2 Length and unit vectors

1. Rescale the following vectors into unit vectors.
   (a) $[3, 4]$ (b) $[-1, 1]$ (c) $[1, -2, -2]$ (d) $[3, -2, 1]$ (e) $[1, 1, 1, 1, 1, 1, 1]$ (f) $[0, 0, 0]$
2. Rescale each of the vectors in the previous question to a vector of length 3.

3 Angles between vectors

Recall that the angle θ between two nonzero vectors $\vec{v}, \vec{w}$ is given by the formula $\|\vec{v}\| \, \|\vec{w}\| \cos\theta = \vec{v} \cdot \vec{w}$.

1. Compute the angle between the following pairs of vectors. Express your answer in radians and simplify where possible.
   (a) $[0, 1]$ and $[1, 0]$ (b) $[2, -2]$ and $[-4, 4]$ (c) $[1, \sqrt{3}]$ and $[1, 0]$ (d) $[-3, 2]$ and $[1, 2]$
   (e) $[-4, 6]$ and $[-8, 9]$
2. Plot the pairs of vectors from the previous question in the Cartesian plane. Check to see whether the angle you found seems reasonable. (If necessary, use a calculator[1] to evaluate the angle numerically. Convert to degrees if you find degrees more intuitive than radians.)
3. Let $\vec{v}$ and $\vec{w}$ be unit vectors. What is $\vec{v} \cdot \vec{w}$ when these vectors are
   (a) Pointing in the same direction?
   (b) Pointing in opposite directions?
   (c) Pointing orthogonally to each other?

   Check your answer by figuring out what the angle formula says in each of these cases.

[1] Google will compute inverse trig functions for you!

Solutions

1 Vectors, visually

2. (a) $[1,\ 2]$ (b) $[-2, 1]$ (c) $[0, 0]$ (d) $[-1, 4]$

4. $[2, 3, 3]$

2 Length and unit vectors

1. (a) $[3/5, 4/5]$ (b) $[-1/\sqrt{2}, 1/\sqrt{2}]$ (c) $[1/3, -2/3, -2/3]$ (d) $[3/\sqrt{14}, -2/\sqrt{14}, 1/\sqrt{14}]$
   (e) $[1/\sqrt{7}, 1/\sqrt{7}, 1/\sqrt{7}, 1/\sqrt{7}, 1/\sqrt{7}, 1/\sqrt{7}, 1/\sqrt{7}]$ (f) Impossible to make a unit vector

2. (a) $[9/5, 12/5]$ (b) $[-3/\sqrt{2}, 3/\sqrt{2}]$ (c) $[1, -2, -2]$ (d) $[9/\sqrt{14}, -6/\sqrt{14}, 3/\sqrt{14}]$
   (e) $[3/\sqrt{7}, 3/\sqrt{7}, 3/\sqrt{7}, 3/\sqrt{7}, 3/\sqrt{7}, 3/\sqrt{7}, 3/\sqrt{7}]$ (f) Impossible

3 Angles between vectors

1. (a) $\pi/2$ (b) π (c) $\pi/3$ (d) $\arccos(1/\sqrt{65}) \approx 1.45$ rad $\approx 83°$
   (e) $\arccos\left(\frac{86}{2\sqrt{1885}}\right) \approx 0.139$ rad $\approx 7.943°$

3. (a) 1 (b) -1 (c) 0