Continuity, the definition of the derivative

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Here are some key ideas from sections 2.5, 3.1, and 3.2.

- A function is **continuous** at a if
$$\lim_{x \to a} f(x) =$$
- If you zoom in on the graph of a **tangent line** to the curve at a, you will see that it touches $f(a)$ at exactly ________ point.
- The slope of the tangent line to the curve f at a is the ______________________________ at a, written as $f'(a)$. This can be thought of as the ____________________ rate of change.
- We can use ____________ lines (intersect at ________ points) to approximate tangent lines.
- Mathematically, this idea is called the **derivative**, and can be written as
$$f'(a) = \lim_{h \to 0}$$
- Another valid expression is
$$f'(a) = \lim_{x \to a}$$
- We can replace x with a in the two equations above to express the derivative as a function.
- A function is differentiable if the derivative ____________________

. . . .

Midterm practice (Persson '14 MT1): Find each of the following limits.

(a) $\lim_{x \to -\infty} \frac{x(3x-4)+2}{5x^2-10}$

(b) $\lim_{x \to -3} \frac{x^2-9}{x^2+2x-3}$

(c) $\lim_{x \to 1} \frac{\frac{1}{1+x^4} - \frac{1}{2}}{x-1}$

My Attempt:	Solution:

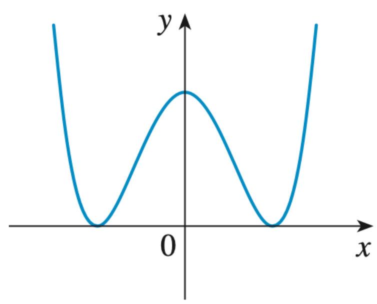
Problem 1: (Stewart 3.1) A curve has equation $y = f(x)$.

(a) Write an expression for the slope of the secant line through the points $P(3, f(3))$ and $Q(x, f(x))$.

(b) Write an expression for the slope of the tangent line at P.

My Attempt:	Solution:

Problem 2: (Stewart 3.1) For the given graph of $f(x)$ below, sketch a graph of $f'(x)$.



My Attempt:	Solution:

Problem 3: Use a limit definition of the derivative to find the equation of the tangent line to $y = \frac{2x+1}{x+2}$ at $(1, 1)$.

My Attempt:	Solution:

Problem 4: (Stewart 3.1) Use a limit definition of the derivative to find the equation of the tangent line to $y = 4x - 3x^2$ at the point $(2, -4)$.

My Attempt:

Solution:

Problem 5: (Stewart 3.1) For the function $f(x) = x^{-2}$, find $f'(a)$ using a limit definition of the derivative.

My Attempt:

Solution:

Problem 6: (Stewart 3.2) State the domain of $f(x) = x + \sqrt{x}$ and the domain of its derivative.

My Attempt:

Solution:

Challenge problem: Assume that

$$f(x) = \begin{cases} \frac{1}{4}x^3 - \frac{1}{2}x^2, & \text{if } x \geq 2 \\ \frac{6x-6}{x^2+2}, & \text{if } x < 2 \end{cases}$$

Determine if f is differentiable at $x = 2$, i.e., determine if $f'(2)$ exists.