

Math 10A Fall 2024 Worksheet 8

September 24 2024

1. Determine if the following functions are even, odd, or neither.

- (a) $f(x) = x^2 - 1$
- (b) $f(x) = \sin(x)$
- (c) $f(x) = 42$
- (d) $f(x) = x + \sin(x)$
- (e) $f(x) = x^2 - 3\cos(x)$
- (f) $f(x) = x^2 + x + 1$

2. Find the domain of the following functions.

- (a) $f(x) = x$
- (b) $f(x) = \sqrt{x+3}$
- (c) $f(x) = \sqrt{4-\sqrt{x}}$
- (d) $f(x) = \frac{1}{x} + \frac{1}{x-1} + \frac{1}{x-2} + \cdots + \frac{1}{x-10}$

3. If

$$f(x) = \frac{x^3 - 1}{x - 1}, \quad g(x) = x^2 + x + 1$$

is it true $f = g$?

4. Let $f(x) = \sqrt{x}$. Plot the graphs of the following functions. Also describe the domains.

- (a) $y = f(x)$
- (b) $y = f(x+1)$
- (c) $y = 3f(x)$
- (d) $y = f(x) - 3$
- (e) $y = 2f(x-1) + 2$

5. Let

$$f(x) = x + \frac{1}{x}, \quad g(x) = \frac{x+1}{x+2}.$$

Find $f \circ g$ and $g \circ f$. What are the domains of these functions?

1 Solutions

1. (a) even
(b) odd
(c) even
(d) odd
(e) even
(f) neither

Note that (even) + (even) = (even) and (odd) + (odd) = (odd).

2. **Key principle:** The two main conditions to a number being in the domain are:

- i) Avoid dividing by zero.
 - ii) Avoid taking the square root, or any even power root, of a negative number. Odd power roots of negatives are okay, for example $\sqrt[3]{-1} = -1$.
- (a) All $x \in \mathbb{R}$.
 - (b) $x \geq -3$
 - (c) $0 \leq x \leq 16$. All the values in square roots should be nonnegative, which gives $x \geq 0$ and $4 - \sqrt{x} \geq 0 \Leftrightarrow x \leq 16$.
 - (d) All the values in the denominator should be nonzero. Hence $x \in \mathbb{R} \setminus \{0, 1, 2, \dots, 10\}$. This is a notation that means “the set \mathbb{R} (all real numbers), except for the set $\{0, 1, 2, \dots, 10\}$ ”. It is also correct to write the domain as a bunch of intervals, but it’s shorter to write it like this.

3. No, because $f(x)$ is not defined at $x = 1$ but $g(x)$ is, even they agree on any other $x \neq 1$.

4. For the graphs, visit the following link: <https://www.desmos.com/geometry/lshqfmuch>

- (a) $x \geq 0$
- (b) $x \geq -1$
- (c) $x \geq 0$
- (d) $x \geq 0$
- (e) $x \geq 1$

5.

$$(f \circ g)(x) = f(g(x)) = \frac{x+1}{x+2} + \frac{x+2}{x+1} = \frac{2x^2 + 6x + 5}{(x+2)(x+1)}$$
$$(g \circ f)(x) = g(f(x)) = \frac{x + \frac{1}{x} + 1}{x + \frac{1}{x} + 2} = \frac{x^2 + x + 1}{x^2 + 2x + 1} = \frac{x^2 + x + 1}{(x+1)^2}$$

The domains are $x \neq -1, -2$ and $x \neq -1$, respectively.