Substitution, integration by parts

November 21st, 2024

Here are some key ideas from sections 5.3 and 5.4

- The substitution method is a way to undo the ___________________ rule. First, some motivation:

  $$\frac{d}{dx} f(g(x)) =$$

  Now for the rule: suppose $u = g(x)$ is the "inside function." If u is differentiable, then:

⋆ We can use the substitution method for definite integrals, but we would need to change the bounds of integration!

- Integration by parts is a way to undo the ___________________ rule. First, some motivation:

  $$\frac{d}{dx} f(x)g(x) =$$

  Now for the rule: if $u = f(x)$ and $v = g(x)$, then $du = f'(x)\,dx$ and $dv = g'(x)\,dx$, so

. . . .

Trig practice: Solve $2\sin^2 x - \sin x - 1 = 0$ for $x \in [0, 2\pi]$.

Problem 1: (Stewart 5.4) This problem will walk you through evaluating $\int 2x\sqrt{1+x^2}\,dx$ using **substitution**.

1. For the substitution method to work, we need the integrand (stuff inside the integral) to have a composition of functions and the derivative of the inside function. What is the composition $f(g(x))$? What is $g'(x)$?
2. Let u be the inside function $g(x)$. Then $du = g'(x)\,dx$. Find du.
3. Rewrite the integral in terms of u, and evaluate the antiderivative (still in terms of u).
4. Substitute $g(x)$ for u. Celebrate, you did it!

My Attempt:	Solution:

Problem 2: (Stewart 5.5) This problem will walk you through evaluating $\int x \sin x \, dx$ using **integration by parts**.

1. For integration by parts to work, we need a product of functions in the integrand. We will let u correspond to the function that is "easier" to differentiate, and dv will be the other one. Find u and dv.
2. Using your choice of dv from the previous part, find v and du.
3. Use the formula $\int u \, dv = uv - \int v \, du$ to find the antiderivative. Yay, you did it again!

My Attempt: | Solution:

Problem 3: (Stewart 5.4) Evaluate the following integrals. *Hint: remember to change the bounds of integration accordingly!*

a) $\int_0^1 \cos(\pi t/2) \, dt$; b) $\int_0^1 (3t+1)^5 \, dt$; c) $\int_0^1 \frac{e^z + 1}{e^z + z} \, dz$.

My Attempt: | Solution:

Problem 4: (Stewart 5.4)

a) If f is continuous and $\int_0^9 f(x) \, dx = 4$, find $\int_0^3 x f(x^2) \, dx$.

b) If f is continuous and $\int_0^4 f(x) \, dx = 10$, find $\int_0^2 f(2x) \, dx$.

My Attempt: | Solution:

Problem 5: (Stewart 5.5) Evaluate the following integrals.

a) $\displaystyle\int_1^2 \frac{\ln x}{x^2}\,dx$; b) $\displaystyle\int_0^\pi e^{\cos t}\sin 2t\,dt$; c) $\displaystyle\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3\cos(\theta^2)\,d\theta$; d) $\displaystyle\int \sin(\ln x)\,dx$.

My Attempt: | Solution:

Problem 6: (Stewart 5.4) Evaluate $\displaystyle\int_{-2}^2 (x+3)\sqrt{4-x^2}\,dx$.

My Attempt: | Solution:

Problem 7: (Stewart 5.5) Use integration by parts to prove the reduction formula $\displaystyle\int x^n e^x\,dx = x^n e^x - n\int x^{n-1}e^x\,dx$.

My Attempt: | Solution:

Challenge problem: If a and b are positive numbers, show that $\displaystyle\int_0^1 x^a(1-x)^b\,dx = \int_0^1 x^b(1-x)^a\,dx$.