The Fundamental Theorem of Calculus

November 19th, 2024

Here are some key ideas from section 5.3.

- The indefinite integral is $\int f(x)\,dx$. If $\int f(x)\,dx = F(x)$, then ________________________.
  - A definite integral looks like $\int_a^b f(x)\,dx$ and is a ___________________.
  - An indefinite integral looks like $\int f(x)\,dx$ and is a ____.
- **FTC 1**: Also known as the Evaluation Theorem. If f is continuous on the interval $[a,b]$, then $\int_a^b f(x)\,dx =$ ______________________________, where F is ______________ antiderivative of f. For example:

$$\int_0^1 x^2\,dx =$$

- **FTC 2**: First, some motivation:

  Now, the theorem. Suppose f is continuous on $[a,b]$. Let $g(x) = \int_a^x f(t)\,dt$ on $[a,b]$. Then $g'(x) =$ ________ on (a,b). In other words, g is an ________________________ of f.

. . . .

Trig practice: Find all values of $\sin^{-1}\frac{1}{\sqrt{2}}$ and $\cos^{-1}\frac{\sqrt{3}}{2}$ on $[0,2\pi]$.

Problem 1: (Stewart 5.3) Why is this process incorrect?

$$\int_{-1}^{3} \frac{1}{x^2}\,dx = \frac{x^{-1}}{-1}\bigg]_{-1}^{3} = -\frac{4}{3}.$$

My Attempt:	Solution:

Problem 2: (Stewart 5.3) Find $\int (1-t)(2+t^2)\,dt$.

My Attempt:	Solution:

Problem 3: (Stewart 5.3) Evaluate the following integrals.

a) $\int_{-2}^{3} (x^2 - 3)\, dx$; b) $\int_{-5}^{5} e\, dx$; c) $\int_{0}^{1} 10^x\, dx$.

My Attempt: | Solution:

Problem 3: (Stewart 5.3) Find the following indefinite integrals.

a) $\int (1 + \tan^2 \alpha)\, d\alpha$; b) $\int \frac{\sin x}{1 - \sin^2 x}\, dx$; c) $\int v(v^2 + 2)^2\, dv$.

My Attempt: | Solution:

Problem 4: (Stewart 5.3) Find the first derivatives of the following functions.

a) $g(x) = \int_{1}^{x} \frac{1}{t^3 + 1}\, dt$; b) $g(y) = \int_{3}^{y} e^{t^2 - t}\, dt$; c) $g(r) = \int_{0}^{4} \sqrt{x^2 + 4}\, dx$.

My Attempt: | Solution:

Problem 5: (Stewart 5.3) Find the first derivatives of the following functions.

a) $h(x) = \int_2^{1/x} \arctan t \, dt$; b) $h(x) = \int_0^{x^2} \sqrt{1+r^3} \, dr$; c) $y = \int_{e^x}^0 \sin^3 t \, dt$.

My Attempt: | Solution:

Problem 6: (Stewart 5.3) Find a function f and a number a such that, for all $x > 0$,

$$6 + \int_a^x \frac{f(t)}{t^2} \, dt = 2\sqrt{x}.$$

My Attempt: | Solution:

Problem 7: (Apostol 5.5) A function f is continuous everywehre and satisfies the equation

$$\int_0^x f(t) \, dt = -\frac{1}{2} + x^2 + x \sin 2x + \frac{1}{2} \cos 2x.$$

for all x. Compute $f(\pi/4)$ and $f'(\pi/4)$.

My Attempt: | Solution:

Challenge problem: (Apostol 5.5) Show that, for all areal x,

$$\int_0^x (t + |t|)^2 \, dt = \frac{2x^2}{3}(x + |x|).$$