3×3 eigenstuff, representing functions

September 24th, 2024

Here are some key ideas from sections 1.1 and 1.3.

- A **function** is a rule that assigns each element x in its domain to [more than one / exactly one / less than one] element in its range.
- The _____ is a way to tell whether or not a graph in the xy-plane is a function. It says that an xy-curve is the graph of a function **if and only if** no vertical line intersects the curve more than once.
- A function f is **even** if _____. A function f is **odd** if _____. These must hold for all x.
- A function f is called *increasing* on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.
- A function f is called *decreasing* on an interval I if _____.

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Problem 1: (Stewart & Day 8.8) But first...two more matrix problems. This time, we'll work with 3×3 ones.

(a) Let A be the matrix $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$. Write an expression for the determinant of A.

(b) Let B be the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$. Find the eigenvalues of B.

(c) Find an eigenvector for each eigenvalue of B.

My Attempt:

Solution:

Problem 2: (Stewart & Day 8.8) Find the eigenvalues of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 7 \\ 0 & 2 & 1 \end{bmatrix}$, and find one eigenvector for each eigenvalue.

My Attempt:

Solution:

Problem 3: (Borcherds '05 Midterm 1) Find the domain of the function $g(u) = \sqrt{u} + \sqrt{2-u}$.

My Attempt:

Solution:

Problem 4: (Stewart 1.1) Recall that a *piecewise function* splits its domain into pieces and is defined by different formulas for each piece. Sketch the graph of the following piecewise function:

$$f(x) = \begin{cases} x+1 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}.$$

My Attempt:

Solution:

Problem 5: (Stewart 1.1) Determine whether $f(x) = x|x|$ is even, odd, or neither.

My Attempt:

Solution:

Problem 6: (Stewart 1.1) Does $x^2 + (y-3)^2 = 5$ define a function? Explain why or why not.

My Attempt:

Solution:

Problem 7: Consider the function $f(x) = 4 + 3x - x^2$. Evaluate the difference quotient given by

$$\frac{f(3+h) - f(3)}{h}.$$

My Attempt:

Solution:

Problem 8: (Stewart 1.1) Solve $|x-3| + |x+2| < 11$ mathematically (don't guess and check values).

My Attempt:

Solution:

Challenge Problem: (Stewart 1.1) Sketch the region in the plane consisting of all points (x, y) such that $|x-y| + |x| - |y| \leq 2$.