Coordinate systems, vectors

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Here are some key ideas from sections 8.1 and 8.2.

- The **distance formula** in n-dimensions tells us how to get from a point $P_1(a_1, \cdots, a_n)$ to $P_2(b_1, \cdots, b_n)$. It says

$$|P_1P_2| =$$

- The set of points at a constant distance from a given point forms a _____, and its formula is given by

$$r^2 =$$

- **Vectors** have both _____ and _____. When we add vectors, we use the tip-to-tail rule.

- When we **scale** a vector by some number c, we multiply the vector's magnitude by c without changing the _____. A unit vector has magnitude _____.

- If $\vec{a} = [a_1, a_2]$ and $\vec{b} = [b_1, b_2]$, and c is some number, then

$$\vec{a} + \vec{b} =$$

$$c\vec{a} =$$

.....

Problem 1: (Apostol 12.4) Let $\vec{a} = [1, 3, 6]$, $\vec{b} = [4, -3, 3]$, and $\vec{c} = [2, 1, 5]$ be three vectors in $\mathbb{R}^3$. Determine each of the following:

a) $\vec{a} + \vec{b}$; b) $\vec{a} - \vec{b}$; c) $\vec{a} + \vec{b} - \vec{c}$; d) $7\vec{a} - 2\vec{b} - 3\vec{c}$; e) $2\vec{a} + \vec{b} - 3\vec{c}$.

My Attempt:

Solution:

Problem 2: (Stewart & Day 8.1) Sketch the points $(0, 5, 2)$, $(4, 0, -1)$, $(2, 4, 6)$ and $(1, -1, 2)$ on a single set of coordinate axes.

My Attempt:

Solution:

Problem 3: (Stewart & Day 8.1) Find an equation of the sphere with center $(2, -6, 4)$ and radius 5.

My Attempt:

Solution:

Problem 4: (Stewart & Day 8.2) Find an equation of the sphere that passes through the origin and whose center is $(1, 2, 3)$.

My Attempt:

Solution:

Problem 5: (Stewart & Day 8.2) Find a unit vector that has the same direction as $[-3, 7]$. *Hint: what should we scale the vector by so the magnitude is 1?*

My Attempt:

Solution:

Problem 6: (Stewart & Day 8.1) Vaccines tend to protect only against certain pathogens in a defined antigenic space. Suppose the vaccine protects against any strain contained within a sphere of radius 2 centered at $(2, 1, 0)$. For which of the following strains will the vaccine be effective?

a) Strain A at $(0, 0, 0)$; b) Strain B at $(1, 0, 3)$; c) Strain C at $(1, 0, 1)$; d) Strain D at $(1/4, 2, 1)$.

My Attempt:

Solution:

Problem 7: (Stewart & Day 8.1) Describe and sketch the surface in $\mathbb{R}^3$ (three-dimensional space) represented by $x + y = 2$. *Hint: the z-axis is missing from this equation!*

My Attempt:

Solution:

Challenge Problem: (Stewart & Day 8.2) Suppose that some vector in $\mathbb{R}^3$ makes angles θ_1, θ_2, and θ_3 with the x, y, and z-axes respectively. Show that $\cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3 = 1$.