# Deriving products, quotients, and chains

***October 15th, 2024***

Here are some key ideas from sections 3.4 and 3.5.

- The derivative of the product of $f(x)$ and $g(x)$ is $\frac{d}{dx} f(x)g(x) =$ _____.
- The derivative of the quotient of $f(x)$ and $g(x)$ is $\frac{d}{dx} \frac{f(x)}{g(x)} =$ _____.
- The derivative of $\tan x$ is $\frac{d}{dx} \tan x =$ _____.
- The chain rule has to do with _____ of functions.
  If $g$ is differentiable at _____ and $f$ is differentiable at _____, then

- The chain rule helps us with implicit differentiation, which is used when we can't isolate the _____ variable. Here's an example:

. . . . . .

**Midterm practice (Paulin MT1 '16):** Use a limit definition of the derivative to find $f'(x)$ for $f(x) = x^{3/2}$. Then find the domain of the derivative.

My Attempt:

Solution:

**Problem 1:** Use the rules we talked about today to find the derivatives of

a) $\sec x$; b) $\csc x$; c) $\cot x$.

My Attempt:

Solution:

**Problem 2:** (Stewart 3.5) Find the derivative of $F(x) = (x^4 + 3x^2 - 2)^5$ (no need to expand).

My Attempt:

Solution:

**Problem 3:** (Stewart 3.5) Find the derivative of $e^{x \cos x}$.

My Attempt:

Solution:

**Problem 4:** (Stewart 3.5)

a) Find the 50th derivative of $y = \cos 2x$.

b) Find the 1000th derivative of $f(x) = xe^{-x}$.

My Attempt:

Solution:

**Problem 5:** (Stewart 3.5) Find an equation of the tangent line to $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$ at the point $\left(0, \frac{1}{2}\right)$. *Fun fact: this curve is called cardioid because it's shaped like a heart!*

My Attempt:

Solution:

**Problem 6:** (Stewart 3.5) Find the derivatives of the following functions using the triple chain rule.

a) $y = e^{e^x}$;

b) $y = \sin(\cos(\tan(x)))$;

My Attempt:

Solution:

**Problem 7:** (Stewart 3.5) Find $\frac{dy}{dx}$.

a) $e^y \cos x = 1 + \sin(xy)$;

b) $4 \cos x \sin y = 1$;

c) $e^{x/y} = x - y$.

My Attempt:

Solution:

**Challenge problem:** Find a parabola that passes through $(1, 4)$ and whose tangent lines at $x = -1$ and $x = 5$ have slopes 6 and $-2$, respectively.