Matrix algebra

September 10th, 2024

Here are some key ideas from sections 8.4 and 8.5.

- A **matrix** is an array of vectors. For a given matrix A having m rows and n columns, the entry a_{ij} is in the _____th row and _____th column.

$$A = \begin{bmatrix} 0 & 7 & 1 \\ 2 & 9 & 2 \end{bmatrix} = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \end{bmatrix}.$$

- If the number of rows is the same as the number of columns, we say the matrix is _____.
- The **transpose** of A is written as _____ and is obtained by interchanging the rows and columns. The transpose of the above matrix is

$$A^T =$$

- When we **multiply** two matrices A and B, we need that the number of _____ in A is equal to the number of _____ in B.

  The entry in the ith row and jth column of AB is the dot product of the _____ of A and the _____ of B. For example,

$$A = \begin{bmatrix} 2 & 7 \\ 9 & -3 \end{bmatrix}, \; B = \begin{bmatrix} 3 & -7 & 2 \\ 1 & 5 & 9 \end{bmatrix}, \; AB = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \end{bmatrix}.$$

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Problem 1: (Stewart & Day 8.4) Just as with vectors, adding matrices and multiplying by scalar are done coordinate-wise. Suppose we have

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 7 \end{bmatrix}, \; B = \begin{bmatrix} 7 & x \\ a & 5 \end{bmatrix}, \; C = \begin{bmatrix} 9 & 2 \\ 7 & 10 \end{bmatrix}.$$

Find $A - 3C$ and $5B - A$.

My Attempt:

Solution:

Problem 2: (Stewart & Day 8.4) For a matrix C, the quantity C^n represents multiplying C by itself n times.

$$C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

find C^2.

My Attempt:

Solution:

Problem 3: (Stewart & Day 8.4) For the same matrix C as in Problem 2, find C^3, C^4, and C^5.

My Attempt:

Solution:

Problem 4: (Stewart & Day 8.4) For arbitrary 2×2 matrices A, B, and C, verify the following identity:

$$A(B + C) = AB + AC.$$

Hint: set the entries equal to variables.

My Attempt:

Solution:

Problem 5: (Stewart & Day 8.4) For

$$F = \begin{bmatrix} y & 0 & y \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 4 & 6 \\ 9 & 7 & 8, \end{bmatrix}$$

find FG.

My Attempt:

Solution:

Problem 6: (Stewart & Day 8.4) Find the transposes of the following matrices.

a) $\begin{bmatrix} 3X \\ 1 \\ 2 \end{bmatrix}$ b) $\begin{bmatrix} 3 & 3 & 9 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 1 & 7 \\ 8 & 3 & 6 \end{bmatrix}$ d) $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$.

My Attempt:

Solution:

Challenge Problem: (Stewart & Day 8.4) Show that for any $n \times n$ matrices A and B,

$$(AB)^T = B^T A^T.$$