Solving systems, eigenstuff

September 17th, 2024

Here are some key ideas from sections 8.6 and 8.7.

- We can write systems of equations in matrix notation as $A\vec{x} = \vec{b}$. A is a ________________, while $\vec{x}$ and $\vec{b}$ are both ________________. If $\vec{b}$ is not the 0 vector, we say the system is ________________.
  For the system of equations given by
  $$3x_1 - 2x_2 = -4$$
  $$7x_1 + x_2 = 19,$$
  the corresponding matrix equation is

- Fill out the following table with the number of solutions to the matrix equation.

	A is invertible	A is not invertible
Homogeneous ($\vec{b} = \vec{0}$)		
Inhomogeneous ($\vec{b} \neq \vec{0}$)		

- An ________________ is a nonzero vector $\vec{v}$ satisfying the equation below.
  $$A\vec{v} = \lambda\vec{v}.$$
  The corresponding ________________ is λ.

- We can rewrite to get

  which tells us that there are nonzero eigenvectors if and only if ________________.

. . . .

Problem 1: (Stewart & Day 8.6) Solve the discussed system of equations using matrices. *Hint: multiply by* A^{-1}.

$$3x_1 - 2x_2 = -4$$
$$7x_1 + x_2 = 19,$$

My Attempt:

Solution:

Problem 2: (Stewart & Day 8.7) Which of the following scalars k are eigenvalues of their corresponding matrices?

a) $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $k = 3$; b) $A = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$, $k = 0$; c) $A = \begin{bmatrix} 5 & 2 \\ 0 & 1 \end{bmatrix}$, $k = 2$.

My Attempt:

Solution:

Problem 3: (Stewart & Day 8.7) Consider the following system of equations:

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2.$$

Suppose that the matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is nonsingular. Derive expressions for x_1 and x_2.

My Attempt:

Solution:

Problem 4: (Stewart & Day 8.7) Find the eigenvalues of each matrix.

a) $\begin{bmatrix} 2 & 0 \\ 3 & 0 \end{bmatrix}$; b) $\begin{bmatrix} 5 & -4 \\ 6 & -5 \end{bmatrix}$; c) $\begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix}$.

My Attempt:

Solution:

Problem 5: (Stewart & Day 8.7) Find an eigenvector associated with the given eigenvalue of A.

a) $A = \begin{bmatrix} 9 & 0 \\ 2 & 3 \end{bmatrix}$, $\lambda = 9$; b) $A = \begin{bmatrix} 1 & 5 \\ 2 & 7 \end{bmatrix}$, $\lambda = 4 + \sqrt{19}$; c) $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $\lambda = \frac{1+\sqrt{5}}{2}$.

My Attempt:

Solution:

Challenge Problem: (Stewart & Day 8.7) Derive a general formula for the eigenvalues of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.