Midterm study guide: linear algebra

October 22nd, 2024

The linear algebra content is from Chapter 8 of the textbook. Here are some things you should know for the exam (feel free to use this as a checklist):

- ☐ Labeling three-dimensional coordinate axes using the right hand rule (8.1)
- ☐ The distance formula in three dimensions (8.1)
- ☐ The equation of a sphere (8.1)
- ☐ The distance formula in n dimensions
- ☐ Vector addition/subtraction algebraically and geometrically (8.2)
- ☐ Scalar multiplication algebraically and geometrically (8.2)
- ☐ How to find the vector between two points (8.2)
- ☐ How to find the length of a two-dimensional vector (8.2)
- ☐ How to find the length of a three dimensional vector (8.2)
- ☐ Algebraic properties of vectors (8.2, last page)
- ☐ The definition of the dot product (8.3)
- ☐ Algebraic properties of the dot product (8.3)
- ☐ The $\cos$ formula for the dot product (8.3)
- ☐ How to find the equation of the plane passing through a given point and perpendicular to a given vector (8.3)
- ☐ Scalar and vector projections (8.3)
- ☐ The definitions of square matrices, column vectors, row vectors (8.4)
- ☐ Forming the transpose of a matrix (8.4)
- ☐ Matrix addition and scalar multiplication (8.4)
- ☐ Associative, commutative, distributive properties of matrix addition (8.4)
- ☐ How to multiply two matrices and when it is possible to do so (8.4)
- ☐ The form of an identity matrix (8.4)
- ☐ How to draw a Leslie diagram (8.5)
- ☐ The definition of the inverse of a matrix and when matrices can be inverted (8.6)
- ☐ The definition of singular and nonsingular matrices (8.6)
- ☐ How to find the inverse of a 2×2 matrix
- ☐ Properties of matrix inverses (8.4, under the definition of the inverse)
- ☐ The determinant of a 1×1 matrix, a 2×2 matrix, and a 3×3 matrix (8.6)
- ☐ Using matrices to represent and solve systems of equations, including determining the possibility of infinite, zero, or no solutions (8.6)
- ☐ If the determinant of A is not zero, then there is a unique solution to $A\vec{x} = \vec{b}$ (8.6)
- ☐ If the determinant of A is zero, then there are either infinitely many solutions or zero solutions to $A\vec{x} = \vec{b}$ (8.6)

- ☐ The definition of an eigenvalue and its corresponding eigenvector (8.7)
- ☐ The definition of a characteristic polynomial (8.7)
- ☐ How to find a characteristic polynomial (8.7)
- ☐ How to find eigenvalues using the characteristic polynomial (8.7)
- ☐ How to find a corresponding eigenvalue given an eigenvector (8.7)
- ☐ How to compute $A\vec{v}$ as a transformation on vectors, especially when the effect of matrix multiplication can be described qualitatively (shears, dilations, rotations, etc.) (8.7)
- ☐ How to write a recursion formula using a matrix (8.8)
- ☐ Diagonalizing matrices, writing $A = PDP^{-1}$ and using this formula to find powers of A(8.8)
- ☐ Using the diagonalization to solve recutsions using $\vec{n}_t = PD^tP^{-1}\vec{n}_0$ (8.8)
- ☐ The general solution for $\vec{n_t}$ in terms of eigenvalues and eigenvectors (8.8)
- ☐ Solving for complex eigenvalues (8.8)

Help! I'm stuck on....

- ...the equation of a **sphere**: check out this 11 minute video
- ...**magnitudes** of vectors: check out this 3 minute video
- ...vector **dot products**: check out this 7 minute video
- ...understanding **matrix definitions** (size, rows, columns, etc.): check out this 11 minute video (watch at 2x)
- ...finding the **transpose** of a matrix: check out this 2 minute video and its chill music
- ...matrix **operations**—adding, subtracting, and scalar multiplying: check out this 9 minute video (watch at 2x)
- ...**Leslie matrices**: check out this 10 minute video
- ...finding the **inverse of a** 2×2 **matrix**: check out this 3 minute video
- ...practicing 2×2 and 3×3 **determinants**: check out this 10 minute video
- ...representing **systems of equations** with matrices: check out this 7 minute video
- ...finding **eigenvalues from characteristic polynomials**: check out this 4 minute video
- ...**solving** for eigenvectors and eigenvalues: check out parts of this 17 minute video
- ...finding **general forms of eigenvectors**: check out this 6 minute video
- ...solving for **corresponding eigenvectors**: check out this 8 minute video