Midterm study guide: calculus

October 22nd, 2024

The calculus content so far is from Chapters 1, 2, and 3 of the textbook. Here are some things you should know for the exam (feel free to use this as a checklist):

- ☐ The unit circle
- ☐ The definitions of functions, domains, ranges, increasing, decreasing, even/odd functions (1.1)
- ☐ The general form of linear functions, polynomials, power functions, rational functions, algebraic functions, trigonometric functions, exponential functions, and logarithmic functions (1.2)
- ☐ How to find the domains and ranges of the above functions (1.2)
- ☐ The exponent rules (e.g. $x^a x^b = x^{a+b}$), logarithm rules (e.g. $\log(a^b) = b \log a$), and trigonometry rules (1.4, 1.5)
- ☐ The general form of horizontal shifts to the left and to the right, vertical shifts up and down, horizontal stretches/squeezes, vertical stretches/squeezes (1.3)
- ☐ Obtaining the equation of the transformation of a function from the graph of its transformation (1.3)
- ☐ How to compose several functions (1.3)
- ☐ Once again, the unit circle
- ☐ The definition of a one-to-one function, and that one-to-one functions have inverses (1.3)
- ☐ How to find the inverse of a function (1.5)
- ☐ Limit laws for adding, subtracting, multiplying, dividing, and exponentiating sequences (2.1)
- ☐ Limit laws for r^n (2.1)
- ☐ The definitions of convergent and divergent sequences (2.1)
- ☐ Finding $\lim_{n\to\infty} r^n$ for various values of n (2.1)
- ☐ The definition of a limit at infinity, and how limits at infinity relate to horizontal asymptotes (2.2)
- ☐ Using limit laws with limits at infinity (2.2)
- ☐ Strategies for evaluating limits (combining fractions, dividing the numerator/denominator by the highest power of x, multiplying/dividing by the conjugate) (2.2, 2.4)
- ☐ The unit circle!!!
- ☐ The limit laws for sums, differences, products, quotients, and exponentiation (2.3)
- ☐ The direct substitution property for polynomials: If f is a polynomial, then $\lim_{x\to a} f(x) = f(a)$ (2.4)
- ☐ If direct substitution gives us an indeterminate form (i.e. $\frac{0}{0}, \frac{\infty}{\infty}, \infty \cdot 0$, etc), we must modify the function somehow using the limit laws or other properties (2.4)
- ☐ The definition of a finite limit, and how finite limits connect to vertical asymptotes (2.3)
- ☐ Solving limits using the squeeze theorem (2.4)
- ☐ The limit definition of continuity, and how to find where functions are continuous (by first finding all of their discontinuities) (2.5)
- ☐ The limit definition of continuity from the left, right, and on an interval (2.5)
- ☐ If f and g are continuous at a, then so are $f + g$, $f - g$, fg, and f/g (assuming $g(a) \neq 0$). (2.5)

- ☐ Where polynomial, rational, root, exponential, power, logarithmic, trigonometric, and inverse trigonometric functions are continuous (2.5)
- ☐ Continuity of compositions of functions (2.5)
- ☐ The statement of the Intermediate Value Theorem (IVT) (2.5)
- ☐ Finding the instantaneous rate of change (e.g. velocity given the change in position function) (3.1)
- ☐ The **two different** limit definitions of a derivative: $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ and $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$
- ☐ The definition of differentiability (the limit must exist and cannot be $\pm\infty$) (3.2)
- ☐ If something is differentiable at a, it is continuous at a, and the reverse **does not** necessarily hold! (3.2)
- ☐ The power rule (derivative of x^n) (3.2)
- ☐ The derivative of $c \cdot f(x)$, $f(x)+g(x)$, and $f(x)-g(x)$ (3.3)
- ☐ The derivative of $\sin x$, $\cos, x$, $\tan x$, and e^x (3.3)
- ☐ The product rule (derivative of $f(x)\cdot g(x)$) (3.4)
- ☐ Applying the product rule to a product of three functions $f(x)g(x)h(x)$ (3.4)
- ☐ The quotient rule (derivative of $f(x)/g(x)$) (3.4)
- ☐ The derivatives of $\arctan, \tan, \sec, \csc, \cot$ (3.4)
- ☐ The chain rule and the conditions for it to be used (3.5)
- ☐ Applying the chain rule to a composition of three functions $f(g(h(x))$ (3.5)
- ☐ The derivative of e^x (3.5)
- ☐ The dervative of b^x for arbitrary b (3.5)
- ☐ How to use implicit differentiation (3.5)
- ☐ How and when to use logarithmic differentiation (3.5)
- ☐ The derivative of $\log_b x$ (3.7)
- ☐ The derivatives of $\ln x$ and $\ln|x|$ (3.7)
- ☐ For the last time, the unit circle
- ☐ ★ The two ways to write e as a limit: $\lim_{x\to 0}(1+x)^{1/x}$ and $e=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$
- ☐ ★ The linear approximation (tangent line approximation) for $f(x)$ at $x=a$ (3.8)
- ☐ ★ Newton's method for approximating roots (3.8)
- ☐ ★ How to find the nth degree Taylor polynomial centered at a (3.8)

Help! I'm stuck on....

- ...finding limits of sequences: check out this 30 minute video (lots of examples!)
- ...the definition of a **function**: check out this 14 minute video
- ...the **domains and ranges** of different types of functions: check out this 18 minute video, or if you'd like a ton of problems you can skip around this very long video
- ...solving problems with **increasing and decreasing functions**: check out this 11 minute video
- ...determining if a function is **even or odd**: check out this 12 minute video

Visit tinyurl.com/sections10a for my discussion resources.

- ...using **exponent laws**: check out this 13 minute video
- ...using **logarithm laws**: check out this 5 minute video
- ...**squeeze/stretch** transformations: check out this 8 minute video
- ...**compositions** of functions: check out this 5 minute video
- ...solving problems about **limits at infinity**: check out this 13 minute video (tons of examples)
- ...solving problems with the **limit laws**: check out this 12 minute video
- ...intution for the **Squeeze Theorem**: check out this 11 minute video
- ...intuition for **continuity**: check out this 13 minute video
- ...how to tell **where a function is continuous**: check out this 10 minute video
- ...what continuity from the **left/right** means: check out this 3 minute video
- ...what the limits of **compositions** of functions are: check out this 5 minute video
- ...what the **Intermediate Value Theorem** (IVT) says: check out this 8 minute video
- ...how to use IVT to find **roots**: check out this 13 minute video
- ...how to evaluate **limits at infinity**/horizontal asymptotes: check out this 19 minute video
- ...how to find the **instantaneous rate of change**: this 4 minute video
- ...the **limit definition** of a derivative: check out this 23 minute video (watch at 2x)
- ...the **other limit definition** of a derivative: check out this 8 minute video
- ...what **differentiability** means and how it relates to continuity: check out this 9 minute video
- ...the **product rule** and how to use it: check out this 11 minute video
- ...the **quotient rule** and how to use it: check out this 11 minute video
- ...motivation for the **chain rule**: check out this 15 minute video
- ...**examples** with the chain rule: check out this 18 minute video
- ...understanding **implicit differentiation**: check out this 11 minute video
- ...how and when to use **logarithmic differentiation**: check out this 13 minute video
- ...how and when to use **linear approximation**: check out parts of this 54 minute video with tons of examples
- ...motivation for **Taylor series**: check out this 22 minute video