Finite limits, properties of limits

October 3rd, 2024

Here are some key ideas from sections 2.3 and 2.4.

- We write $\lim_{x\to a} f(x) = L$ to mean the function $f(x)$ approaches _____ as x approaches _____.
- If $\lim_{x\to a} f(x) = L$, then $f(x)$ [has to/might not] equal L.
- We write $\lim_{x\to a^+} f(x) = L$ to mean $f(x)$ approaches L from _____.
  Likewise, $\lim_{x\to a^-} f(x) = L$ means $f(x)$ approaches L from _____.
- The line $x = a$ is a vertical asymptote if

- If $f(x) \le g(x)$ when x is near a, and if the limits both exist as x approaches a, then $\lim_{x\to a} f(x)$ _____ $\lim_{x\to a} g(x)$.
- **Squeeze theorem**: If $f(x) \le g(x) \le h(x)$ when x is near a, and $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} g(x) =$ _____.
- Important limit that you should know: $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} =$ _____.

. . . . . .

Midterm practice:

(a) For what values of m are the vectors $[-6, m, 2]$ and $[m, m^2, m]$ orthogonal?

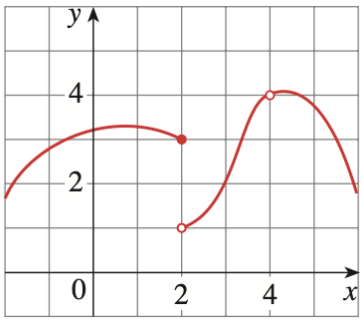
(b) Find two unit vectors that make an angle of $60°$ with $\mathbf{v} = [3, 4]$.

My Attempt:

Solution:

Problem 1: (Stewart 2.3) For the given graph of the function f, state the value of each quantity if it exists. If it does not, explain why.

a) $\lim_{x\to 2^-} f(x)$ b) $\lim_{x\to 2^+} f(x)$ c) $\lim_{x\to 2} f(x)$ d) $f(2)$ e) $\lim_{x\to 4} f(x)$ f) $f(4)$



My Attempt:

Solution:

Problem 2: (Stewart 2.3) Determine

$$\lim_{x\to -3^+} \frac{x+2}{x+3}.$$

Hint: plug in values to see which infinity it approaches.

My Attempt:

Solution:

Problem 3: (Stewart 2.3) Find $\lim_{x\to 3^+} \ln(x^2-9)$.

My Attempt:

Solution:

Problem 4: (Stewart 2.3) Find $\lim_{x\to 2\pi^-} x\csc x$. *Hint:* $\csc(x)$ *is the cosecant function, which is* $1/\sin(x)$.

My Attempt:

Solution:

Problem 5: (Stewart 2.4) Find

$$\lim_{x \to 2^-} \frac{x^2 - 2x}{x^2 - 4x + 4}.$$

My Attempt:

Solution:

Problem 6: (Stewart 2.4) Evaluate

$$\lim_{t \to 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right).$$

My Attempt:

Solution:

Problem 7: (Stewart 2.4) Find

$$\lim_{x \to 0} \frac{\sin 4x}{\sin 6x}.$$

My Attempt:

Solution:

Challenge problem: (Stewart 2.4) Show that $\lim_{x \to 0} x^4 \cos \frac{2}{x} = 0$.