Limits of sequences, infinite limits

October 1st, 2024

Here are some key ideas from sections 2.1 and 2.2.

- We write $\lim_{n\to\infty} a_n = L$ to mean the sequence a_n approaches ____________. If a_n becomes large as n becomes large, we write _________________.
- If the limit exists, the sequence ______________________. Otherwise, it ______________________.
- A geometric sequence has the form $a, ar, ar^2, \cdots$. If $-1 < r < 1$, the sum of the infinite geometric series is
$$a + ar + ar^2 + \cdots + ar^n + \cdots = \qquad\qquad\qquad .$$
- To evaluate limits of rational sequences, divide by ____.
- The expression $\lim_{x\to\infty} f(x) = L$ can be thought of as the ________________________________ of f. We say $y = L$ is a horizontal asymptote of f if
- To evaluate limits of functions with radical expressions, it may help to multiply by ______________________.

. . . .

Midterm practice:

(a) Diagonalize the following matrix:
$$A = \begin{bmatrix} 5 & -3 \\ 6 & -4 \end{bmatrix}.$$
Explicitly write what P, D, and P^{-1} are.

(b) Consider the vector $\mathbf{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. Compute $A^6\mathbf{v}$.

My Attempt:	Solution:

Problem 1: (Stewart 2.1) Determine if $a_n = \frac{2n^2+n-1}{n^2}$ converges. If it is convergent, find the limit.

My Attempt: | Solution:

Problem 2: (Stewart 2.1) Find the limit of $a_n = \frac{n^2}{\sqrt{n^3+4n}}$.

My Attempt: | Solution:

Problem 3: (Stewart 2.1) Use a series to express $0.\overline{8}$ as a ratio of integers.

My Attempt: | Solution:

Problem 4: (Stewart 2.1) Use a series to express $1.53\overline{42}$ as a ratio of integers.

My Attempt: | Solution:

Problem 5: (Stewart 2.2) Find $\lim_{x\to\infty} \sqrt{x^2+ax} - \sqrt{x^2+bx}$.

My Attempt: | Solution:

Problem 6: (Stewart 2.2) Find $\lim_{x\to\infty} e^{-1/x^2}$.

My Attempt: | Solution:

Problem 7: (Stewart 2.2) Find $\lim_{x\to\infty} \frac{\sqrt{t}+t^2}{2t-t^2}$.

My Attempt: | Solution:

Problem 8: (Stewart 2.2) Find $\lim_{x\to\infty}(e^{-x} + 2\cos 3x)$.

My Attempt: | Solution:

Challenge problem: (Stewart 2.2) Let $f(x) = \left(3^x + 3^{2x}\right)^{\frac{1}{x}}$. Find $\lim_{x\to\infty} f(x)$.