L'Hôpital's rule, optimization

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Here are some key ideas from sections 4.3 and 4.4.

- Suppose f and g are differentiable and $g'(x) \neq 0$ near a. L'Hôpital's rule says

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a}$$

only when $\lim_{x \to a} \frac{f(x)}{g(x)}$ is ______________________________ in the form ______________ or ______________.

- Optimization is about finding absolute extrema. Here are some pointers for optimization problems.
  1. If possible, draw a diagram and identify given quantities on the diagram.
  2. Assign a variable to the value that is to be maximized or minimized, and express it in terms of

     ____
  3. Find the domain of the independent variable and proceed to find the ______________________________ extrema.

  ⋆ If the domain is an **open** interval or **half-open** interval, then you must compute the value of f at open endpoints as well. If the absolute extremum appears at the value of an open endpoint, then ____.

Trig practice:

Problem 1: (Stewart 4.4) Find two numbers whose difference is 100 and whose product is a minimum.

My Attempt:	Solution:

Problem 2: (Stewart 4.4) Find two positive numbers whose product is 100 and whose sum is a minimum.

My Attempt:	Solution:

Problem 3: (Stewart 4.3) Find $\lim_{x \to 1} \frac{\ln x}{\sin \pi x}$.

My Attempt: | Solution:

Problem 4: (Stewart 4.3) Rank the following functions in order of how quickly they grow as $x \to \infty$:

$$y = 2^x, \quad y = 3^x, \quad y = e^{x/2}, \quad y = e^{x/3}.$$

My Attempt: | Solution:

Problem 5: (Stewart 4.4) A box with a square base and open top must have a volume of 32,000 cm^3. Find the dimensions of the box that minimizes the amount of material used.

My Attempt: | Solution:

Problem 6: (Stewart 4.3) Find $\lim_{x \to \infty} (xe^{1/x} - x)$.

My Attempt:

Solution:

Problem 7: (Stewart 4.4) Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$.

My Attempt:

Solution:

Challenge problem: Show that of all the isosceles triangles with a given perimeter, the one with the greatest area is equilateral.