Quiz 10 study guide

November 24th, 2024

General information

Quiz 10 covers sections 5.3, 5.4, and 5.5. Here are some general things of note about the quiz:

- You should review the hardest problems on the **homework** when you study for this quiz.
- Technically, any concept in the book in the sections we covered is fair game. However, you should focus on concepts that were covered in lecture and discussion.

Here are some things you should know for the quiz (feel free to use this as a checklist):

- ☐ The first part of the Fundamental Theorem of Calculus (section 5.3)
- ☐ The second part of the Fundamental Theorem of Calculus, also known as the Evaluation Theorem (section 5.3)
- ☐ Finding definite integrals using the evaluation theorem (section 5.3)
- ☐ The definition of an indefinite integral (section 5.3)
- ☐ Indefinite integrals of common functions (see page 345 of section 5.3)
- ☐ The substitution rule (section 5.4)
- ☐ Solving indefinite and definite integration problems with substitution (section 5.4)
- ☐ Integrals of even and odd functions (section 5.4)
- ☐ Integration by parts (section 5.5)
- ☐ Solving indefinite and definite integration problems with integration by parts (section 5.5)
- ☐ Solving problems with both substitution and integration by parts (section 5.5)

Help! I'm stuck on....

- ...understanding the first part of the Fundamental Theorem: check out this 11 minute video
- ...understanding the Evaluation Theorem: check out this 9 minute video
- ...working through problems about the Fundamental Theorem: check out this 41 minute video with lots and lots of examples
- ...understanding the substitution rule: check out this 10 minute video
- ...examples with the substitution rule: check out this 20 minute video
- ...understanding integration by parts: check out this 13 minute video
- ...solving problems with integration by parts: check out this 32 minute video with lots of examples

Practice problems

1. Evaluate $\int_0^{\frac{\pi}{3}} \sec\theta \tan\theta \, d\theta$.
2. If $f(x) = \int_0^x (1-t^2)e^{t^2} \, dt$, find the intervals on which f is increasing.
3. Evaluate $\int_0^1 (3t-1)^{50} \, dt$.
4. Evaluate $\int \frac{dt}{(1-6t)^4}$.
5. Evaluate $\int \cos^3\theta \sin\theta \, d\theta$.
6. Evaluate $\int \frac{(x+1) \, dx}{(x^2+2x+2)^3}$.
7. Evaluate $\int (2+3x) \sin 5x \; sdx$.
8. Let $\int_0^2 f(t) \, dt = x^2(1+x)$. Assuming f is continuous, compute $f(2)$.

Solutions

1. To evaluate the integral $\int_0^{\frac{\pi}{3}} \sec\theta \tan\theta \, d\theta$, we can use the fact that the derivative of $\sec\theta$ is $\sec\theta\tan\theta$. The integral becomes:

$$\int_0^{\frac{\pi}{3}} \sec\theta \tan\theta \, d\theta = [\sec\theta]_0^{\frac{\pi}{3}} = 2 - 1 = 1$$

   Therefore, the value of the given integral is 1.

2. To find the intervals on which $f(x) = \int_0^x (1-t^2)e^{t^2} \, dt$ is increasing, we analyze the sign of its derivative $f'(x)$. If $f'(x) > 0$, then $f(x)$ is increasing on that interval. The derivative is given by:

$$f'(x) = (1-x^2)e^{x^2}$$

   Now, we need to determine the intervals where $f'(x) > 0$. The sign of $1 - x^2$ depends on the values of x. The critical points occur where $1 - x^2 = 0$, which are $x = -1$ and $x = 1$. We can create a sign chart to find the intervals:

x	$(-\infty, -1)$	$(-1, 1)$	$(1, +\infty)$
$f'(x)$	$-$	$+$	$-$

   Therefore, $f'(x) > 0$ when $x \in (-1, 1)$.

3. To evaluate the integral $\int_0^1 (3t-1)^{50} \, dt$ using u-substitution, let $u = 3t - 1$. Then, $du = 3dt$, and we have:

$$\begin{aligned}
\int_0^1 (3t-1)^{50} \, dt &= \frac{1}{3}\int_{u=-1}^{u=2} u^{50} \, du \\
&= \frac{1}{3}\left[\frac{1}{51}u^{51}\right]_{-1}^{2} \\
&= \frac{1}{3}\left(\frac{2^{51}}{51} - \frac{(-1)^{51}}{51}\right) \\
&= \frac{1}{3}\left(\frac{2^{51}+1}{51}\right).
\end{aligned}$$

   Therefore, $\int_0^1 (3t-1)^{50} \, dt = \frac{1}{3}\left(\frac{2^{51}+1}{51}\right)$.

4. Using substitution, let $u = 1 - 6t$, then $du = -6\,dt$. The integral becomes:

$$\int \frac{dt}{(1-6t)^4} = \int \frac{-1/6\,du}{u^4} = -\frac{1}{6}\int u^{-4}\,du = -\frac{1}{6}\cdot\frac{u^{-3}}{-3} + C = \frac{1}{18(1-6t)^3} + C.$$

5. Using substitution, let $u = \cos\theta$, then $du = -\sin\theta\,d\theta$. The integral becomes:

$$\int \cos^3\theta \sin\theta \, d\theta = -\int u^3\,du = -\frac{u^4}{4} + C = -\frac{\cos^4\theta}{4} + C.$$

6. Let $u = x^2 + 2x + 2$, so $du = (2x+2)\,dx$ and $x + 1 = \frac{1}{2}(2x+2)$. The integral becomes:

$$\int \frac{(x+1)\,dx}{(x^2+2x+2)^3} = \int \frac{\frac{1}{2}(2x+2)\,dx}{u^3} = \frac{1}{2}\int \frac{du}{u^3}.$$

   Simplify and evaluate:

$$\frac{1}{2}\int u^{-3}\,du = \frac{1}{2}\cdot\frac{u^{-2}}{-2} = -\frac{1}{4}u^{-2} + C.$$

Substitute back $u = x^2 + 2x + 2$:

$$\int \frac{(x+1)\,dx}{(x^2+2x+2)^3} = -\frac{1}{4}\frac{1}{(x^2+2x+2)^2} + C.$$

7. Use integration by parts. Let $u = 2 + 3x$ and $dv = \sin 5x\,dx$. Then $du = 3\,dx$ and $v = -\frac{\cos 5x}{5}$. The integral becomes:

$$\int (2+3x)\sin 5x\,dx = -(2+3x)\frac{\cos 5x}{5} + \int \frac{3}{5}\cos 5x\,dx.$$

Simplify:

$$= -\frac{(2+3x)\cos 5x}{5} + \frac{\sin 5x}{25} + C.$$

8. Differentiate both sides with respect to x using the Fundamental Theorem of Calculus:

$$\frac{d}{dx}\int_0^x f(t)\,dt = f(x).$$

Differentiate the right-hand side:

$$f(x) = \frac{d}{dx}\left(x^2(1+x)\right) = \frac{d}{dx}\left(x^3 + x^2\right) = 3x^2 + 2x.$$

At $x = 2$, we find:

$$f(2) = 3(2^2) + 2(2) = 12 + 4 = 16.$$