Dynamics, inverses, determinants

September 12th, 2024

Here are some key ideas from sections 8.5 and 8.6.

- For the given system

$$\begin{aligned} c_{t+1} &= 0c_t + 0a_t + 0.3r_t \\ a_{t+1} &= 0.072c_t + 0.8a_t + 0.88r_t \\ r_{t+1} &= 0c_t + 0.19a_t + 0r_t \end{aligned}$$

  The corresponding **Leslie diagram** is

- The __________________ matrix has 1s along the diagonal and 0s everywhere else.
  Any matrix multiplied by the identity matrix is itself! Mathematically,

- If A is a square matrix and $AB = BA = I$, then B is the __________________ of A. If A has an inverse, we say it is **nonsingular**. Otherwise, it is singular.
- The **determinant** of a 2×2 matrix is

- If A is an $n \times n$ matrix, then A is invertible if and only if __________________.
- The inverse of $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is $A^{-1} =$ __________________________________.

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Problem 1: (Stewart & Day 8.6) Are $A = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$ inverses of each other?

My Attempt:	Solution:

Problem 2: (Stewart & Day 8.6) For the following matrices, calculate the inverse or explain why it's not possible to do so.

a) $\begin{bmatrix} 7 & 9 \\ 5 & 6 \end{bmatrix}$; b) $\begin{bmatrix} 14 & 6 \\ 7 & 3 \end{bmatrix}$; c) $\begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$.

My Attempt:

Solution:

Problem 3: (Stewart & Day 8.6) Find the inverse of $\begin{bmatrix} x^2 & 2x \\ x^3 & x \end{bmatrix}$.

My Attempt:

Solution:

Problem 4: (Stewart & Day 8.6) Use the determinant to decide when $\begin{bmatrix} x & x + x^2 \\ 3x & 0 \end{bmatrix}$ is invertible.

My Attempt:

Solution:

Problem 5: (Stewart & Day 8.6) Find all 2×2 matrices A such that $\det A = 1$ and $A = A^{-1}$.

My Attempt: | Solution:

Problem 6: (Stewart & Day 8.6) Verify that $\det(AB) = \det A \det B$ for arbitrary 2×2 matrices.

My Attempt: | Solution:

Problem 7: (Stewart & Day 8.6) Show that for any arbitrary nonsingular matrix A, its inverse A^{-1} is unique. *Hint: Show that if there are two inverses, they must be equal.*

My Attempt: | Solution:

Challenge Problem: (Stewart & Day 8.4) Suppose that A is an $n \times n$ diagonal matrix with entries d_{ii}. Show that A^{-1} is an $n \times n$ diagonal matrix with entries $1/d_{ii}$.