

1 Supplementary Methods: MBITES-de

Let the behavioral state variable name from MBITES denote the expected number of mosquitoes of a given chronological age (a), that are in each behavioral state. Similarly, let the waiting times to events be modeled as a rate that is dependent on the behavioral state (x), age, and time (t): $\xi_x(a, t)$. The proportion of mosquitoes that transition from state x to state y at the end of a bout is denoted P_{xy} . Death rates can be age dependent (*i.e.*, due to senescence), which affects the proportions transitioning to other states, so we write $P_{xy}(a)$. To deal with the event-driven nature of these bouts and the possibility some bouts may be repeated many times before transitioning to another state, we index mosquitoes by the i^{th} attempt to repeat the same event as a way of computing waiting times properly; for example, a mosquito repeating a blood feeding attempt bout transitions from $B_n(a)$ to $B_{n+1}(a)$. Finally, we let $\Lambda(t)$ represent the rate of emergence of adult female mosquitoes. The following system of coupled PDEs is homologous to MBITES:

$$F_1(0, t) = \Lambda(t)$$

$$\begin{aligned} \frac{\partial F_1(a, t)}{\partial t} + \frac{\partial F_1(a, t)}{\partial a} = & \xi_O(a, t)P_{OF}(a) \sum_i O_i(a, t) + \xi_B(a, t)P_{BF}(a) \sum_i B_i(a, t) \\ & + \xi_R(a, t)P_{RF}(a)R(a, t) - \xi_F(a, t)F_1(a, t) \end{aligned}$$

$$\frac{\partial F_i(a, t)}{\partial t} + \frac{\partial F_i(a, t)}{\partial a} = \xi_F(a, t)P_{FF}(a)F_{i-1}(a, t) - \xi_F(a, t)F_i(a, t)$$

$$\begin{aligned} \frac{\partial B_1(a, t)}{\partial t} + \frac{\partial B_1(a, t)}{\partial a} = & \xi_O(a, t)P_{OB}(a) \sum_i O_i(a, t) + \xi_F(a, t)P_{FB}(a) \sum_i F_i(a, t) \\ & + \xi_R(a, t)P_{RB}(a)R(a, t) \\ & - \xi_B(a, t)B_1(a, t) \end{aligned}$$

$$\frac{\partial B_i(a, t)}{\partial t} + \frac{\partial B_i(a, t)}{\partial a} = \xi_B(a, t)P_{BB}(a)B_{i-1}(a, t) - \xi_B(a, t)B_i(a, t)$$

$$\frac{\partial R(a, t)}{\partial t} + \frac{\partial R(a, t)}{\partial a} = \xi_B(a, t)P_{BR}(a) \sum_i B_i(a, t) - \xi_R(a, t)R(a, t)$$

$$\begin{aligned} \frac{\partial L_1(a, t)}{\partial t} + \frac{\partial L_1(a, t)}{\partial a} = & \xi_R(a, t)P_{RL}(a)R(a, t) + \xi_O(a, t)P_{OL}(a) \sum_i O_i(a, t) \\ & - \xi_L(a, t)L_1(a, t) \end{aligned}$$

$$\frac{\partial L_i(a, t)}{\partial t} + \frac{\partial L_i(a, t)}{\partial a} = \xi_L(a, t)P_{LL}(a)L_{i-1}(a, t) - \xi_L(a, t)L_i(a, t)$$

$$\begin{aligned} \frac{\partial O_1(a, t)}{\partial t} + \frac{\partial O_1(a, t)}{\partial a} = & \xi_L(a, t)P_{LO}(a) \sum_i L_i(a, t) + \xi_R(a, t)P_{RO}(a)R(a, t) \\ & - \xi_O(a, t)O_1(a, t) \end{aligned}$$

$$\frac{\partial O_i(a, t)}{\partial t} + \frac{\partial O_i(a, t)}{\partial a} = \xi_O(a, t)P_{OO}(a)O_{i-1}(a, t) - \xi_O(a, t)O_i(a, t) \quad (1)$$

It is a nuisance to deal with an infinite set of equations, but if the state transitions are Markovian, then a change of variables to lump the the n^{th} states together: $x = \sum_i x_i$, with a rescaled rate variable, $\gamma_x(a, t) = \xi(a, t)(1 - P_{xx})$

<Insert John's cool proof>.

This means we can rewrite the infinite system of equations as a set of 5

differential equations:

$$\begin{aligned}
F_o(0, t) &= \Lambda(t) \\
\frac{\partial F(a, t)}{\partial t} + \frac{\partial F(a, t)}{\partial a} &= \gamma_O(a, t)P_{OF}(a)O(a, t) + \gamma_B(a, t)P_{BF}(a)B(a, t) \\
&\quad + \gamma_R(a, t)P_{RF}(a)R(a, t) + \gamma_F(a, t)P_{FF}(a)F_e(a, t) \\
&\quad - \gamma_F(a, t)F(a, t) \\
\frac{\partial B(a, t)}{\partial t} + \frac{\partial B_o(a, t)}{\partial a} &= \gamma_O(a, t)P_{OB}(a)O(a, t) + \gamma_F(a, t)P_{FB}(a)F(a, t) \\
&\quad + \gamma_R(a, t)P_{RB}(a)R(a, t) + \gamma_B(a, t)P_{BB}(a)B_e(a, t) \\
&\quad - \gamma_B(a, t)B(a, t) \\
\frac{\partial R(a, t)}{\partial t} + \frac{\partial R(a, t)}{\partial a} &= \gamma_B(a, t)P_{BR}(a)B(a, t) - \gamma_R(a, t)R(a, t) \\
\frac{\partial L(a, t)}{\partial t} + \frac{\partial L(a, t)}{\partial a} &= \gamma_R(a, t)P_{RL}(a)R(a, t) + \gamma_O(a, t)P_{OL}(a)O(a, t) \\
&\quad + \gamma_L(a, t)P_{LL}(a)L_e(a, t) - \gamma_L(a, t)L(a, t) \\
\frac{\partial O(a, t)}{\partial t} + \frac{\partial O(a, t)}{\partial a} &= \gamma_L(a, t)P_{LO}(a)L(a, t) + \gamma_R(a, t)P_{RO}(a)R(a, t) \\
&\quad + \gamma_O(a, t)P_{OO}(a)O_e(a, t) - \gamma_O(a, t)O_o(a, t)
\end{aligned} \tag{2}$$

1.1 The MBITES-de for Cohorts

Finally, we want a version of these equations to model changes in a cohort of individuals with respect to age (assuming all the mosquitoes emerge from aquatic habitats at the same time of day):

$$\begin{aligned}
F_o(0) &= 1 \\
\dot{F}_o &= \gamma_O(a)P_{OF}(a)O(a) + \gamma_B(a)P_{BF}(a)B(a) + \gamma_R(a)P_{RF}(a)R(a) \\
&\quad + \gamma_F(a)P_{FF}(a)F_e(a) - \gamma_F(a)F_o(a) \\
\dot{F}_e &= \gamma_F(a)P_{FF}(a)F_o(a) - \gamma_F(a)F_e(a) \\
\dot{B}_o &= \gamma_O(a)P_{OB}(a)O(a) + \gamma_F(a)P_{FB}(a)F(a) + \gamma_R(a)P_{RB}(a)R(a) \\
&\quad + \gamma_B(a)P_{BB}(a)B_e(a) - \gamma_B(a)B_o(a) \\
\dot{B}_e &= \gamma_B(a)P_{BB}(a)B_o(a) - \gamma_B(a)B_e(a) \\
\dot{R} &= \gamma_B(a)P_{BR}(a)\sum_i B_i(a) - \gamma_R(a)R(a) \\
\dot{L}_o &= \gamma_R(a)P_{RL}(a)R(a) + \gamma_O(a)P_{OL}(a)O(a) \\
&\quad + \gamma_L(a)P_{LL}(a)L_e(a) - \gamma_L(a)L_o(a) \\
\dot{L}_e &= \gamma_L(a)P_{LL}(a)L_o(a) - \gamma_L(a)L_e(a) \\
\dot{O}_o &= \gamma_L(a)P_{LO}(a)L(a) + \gamma_R(a)P_{RO}(a)R(a) \\
&\quad + \gamma_O(a)P_{OO}(a)O_e(a) - \gamma_O(a)O_o(a) \\
\dot{O}_e &= \gamma_O(a)P_{OO}(a)O_o(a) - \gamma_O(a)O_e(a)
\end{aligned} \tag{3}$$

1.2 Infection Dynamics in the MBITES-de Equations

To simulate infection dynamics in MBITES-de, we subdivide each variable X into new variables X_x , $x \in \{U, Y, Z\}$, to represent the fraction of mosquitoes in behavioral state X that are uninfected, U , infected, Y , or infected and infectious Z . These lead to the following systems of coupled differential equations that remain unchanged, but for the equation describing resting mosquitoes. We let $Q\kappa(t)$ the proportion of mosquitoes becoming infected after blood feeding at time t .

$$\begin{aligned} \frac{\partial R_U(a,t)}{\partial t} + \frac{\partial R_U(a,t)}{\partial a} &= (1 - Q\kappa(t)) \gamma_B(a,t) P_{BR}(a) B_U(a,t) - \gamma_R(a,t) R_U(a,t) \\ \frac{\partial R_Y(a,t)}{\partial t} + \frac{\partial R_Y(a,t)}{\partial a} &= Q\kappa(t) \gamma_B(a,t) P_{BR}(a) B_U(a,t) \\ &\quad + \gamma_B(a,t) P_{BR}(a) B_Y(a,t) - \gamma_R(a,t) R_Y(a,t) \end{aligned} \quad (4)$$

We let $\tau(t)$ denote the (possibly time-dependent) extrinsic incubation period. Because $\tau(t)$ is time dependent, we let \hat{t} denote that point in the past when the mosquito became infected in order to become infectious at time t : *i.e.*, $t = \hat{t} + \tau(\hat{t})$. Let $\rho(t)$ the proportion of mosquitoes surviving through the extrinsic incubation period (*i.e.*, from \hat{t} to $t = \hat{t} + \tau(\hat{t})$). An equation describing the proportion of infectious mosquitoes is:

$$\begin{aligned} \frac{\partial R_Z(a,t)}{\partial t} + \frac{\partial R_Z(a,t)}{\partial a} &= \rho(t) Q\kappa(\hat{t}) \gamma_B(a,t) P_{BR}(a) B_U(a,t) \\ &\quad + \gamma_B(a,t) P_{BR}(a) B_Z(a,t) - \gamma_R(a,t) R_Z(a,t) \end{aligned} \quad (5)$$

The remaining equations remain as follows:

$$\begin{aligned} F_{o,x}(0,t) &= \Lambda(t) \\ \frac{\partial F_{o,x}(a,t)}{\partial t} + \frac{\partial F_{o,x}(a,t)}{\partial a} &= \gamma_O(a,t) P_{OF}(a) O_x(a,t) + \gamma_B(a,t) P_{BF}(a) B_x(a,t) \\ &\quad + \gamma_R(a,t) P_{RF}(a) R_x(a,t) + \gamma_F(a,t) P_{FF}(a) F_{e,x}(a,t) \\ &\quad - \gamma_F(a,t) F_{o,x}(a,t) \\ \frac{\partial F_{e,x}(a,t)}{\partial t} + \frac{\partial F_{e,x}(a,t)}{\partial a} &= \gamma_F(a,t) P_{FF}(a) F_{o,x}(a,t) - \gamma_F(a,t) F_{e,x}(a,t) \\ \frac{\partial B_{o,x}(a,t)}{\partial t} + \frac{\partial B_{o,x}(a,t)}{\partial a} &= \gamma_O(a,t) P_{OB}(a) O_x(a,t) + \gamma_F(a,t) P_{FB}(a) F(a,t) \\ &\quad + \gamma_R(a,t) P_{RB}(a) R_x(a,t) + \gamma_B(a,t) P_{BB}(a) B_{e,x}(a,t) \\ &\quad - \gamma_B(a,t) B_{o,x}(a,t) \\ \frac{\partial B_{e,x}(a,t)}{\partial t} + \frac{\partial B_{e,x}(a,t)}{\partial a} &= \gamma_B(a,t) P_{BB}(a) B_{o,x}(a,t) - \gamma_B(a,t) B_{e,x}(a,t) \\ \frac{\partial L_{o,x}(a,t)}{\partial t} + \frac{\partial L_{o,x}(a,t)}{\partial a} &= \gamma_R(a,t) P_{RL}(a) R_x(a,t) + \gamma_O(a,t) P_{OL}(a) O_x(a,t) \\ &\quad + \gamma_L(a,t) P_{LL}(a) L_{e,x}(a,t) - \gamma_L(a,t) L_{o,x}(a,t) \\ \frac{\partial L_{e,x}(a,t)}{\partial t} + \frac{\partial L_{e,x}(a,t)}{\partial a} &= \gamma_L(a,t) P_{LL}(a) L_{o,x}(a,t) - \gamma_L(a,t) L_{e,x}(a,t) \\ \frac{\partial O_{o,x}(a,t)}{\partial t} + \frac{\partial O_{o,x}(a,t)}{\partial a} &= \gamma_L(a,t) P_{LO}(a) L(a,t) + \gamma_R(a,t) P_{RO}(a) R_x(a,t) \\ &\quad + \gamma_O(a,t) P_{OO}(a) O_{e,x}(a,t) - \gamma_O(a,t) O_{o,x}(a,t) \\ \frac{\partial O_{e,x}(a,t)}{\partial t} + \frac{\partial O_{e,x}(a,t)}{\partial a} &= \gamma_O(a,t) P_{OO}(a) O_{o,x}(a,t) - \gamma_O(a,t) O_{e,x}(a,t) \end{aligned} \quad (6)$$