Q1. $T.C = O(n^2)$

$\hookrightarrow$ It takes 5 sec for $n = 10$

$$K(n^2) = 5$$
$$K(100) = 5$$
$$K = \frac{5}{100}$$

for $n = 50$

$$\text{Time} = \frac{5 \times 50}{100} \times 50$$
$$\Rightarrow \underline{\underline{125 \text{sec}}}$$

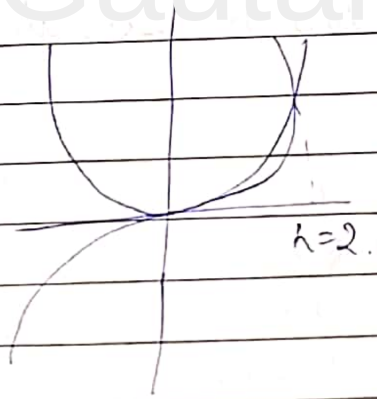
Q2. $T(A(n)) = n^3$

$T(B(n)) = 2n^2$

so $n^3 = 2n^2$

$n^2(n-2) = 0$

$n = 0$ & $n = 2$



after ($n = 2$) they will start to deviate.

Q3. $$\lim_{n\to\infty} \frac{4^n}{n2^n}$$

$$\Rightarrow \lim_{n\to\infty} \frac{2^n}{n}$$

$\rightarrow$ Applying L'Hospital Rule.

$$\Rightarrow \lim_{n\to\infty} n2^{n-1} \Rightarrow \infty.$$

$\therefore n2^n$ is in bounds of 4^n

Q4. By differentiating them we can see $\log x$ has the least growth rate i.e. $1/x$.

Q5. a) $n^4 + \log n + 17$.

$$\lim_{n \to \infty} n^4 + \log n + 17$$

$$\lim_{n \to \infty} n^4 \left(1 + \cancel{\frac{\log n}{n^4}}^{\,0} + \cancel{\frac{17}{n^4}}\right)^{\to 0} ; \quad \lim_{n \to \infty} \frac{\log n}{n^4}$$

$\therefore$ Applying L'Hospital

$$\lim_{n \to \infty} \frac{1}{4n^4} \Rightarrow 0.$$

$$\lim_{n \to \infty} n^4.$$

so, for large n f(n) behaves like n^4 so, it has $O(n^4)$.

Q7. a)
```
K=1
while K <= n
    K=K+1.
End while.
```
T.C. $= O(n)$

b)
```
for i=1 to n-1 do
    for j=1 to n do
        Swap
    End for
End for
```
$n + (n-1) + (n-2) \ldots\ldots 2+1$ — (1)

$$\Rightarrow \frac{n(n+1)}{2}$$

$$\Rightarrow O(n^2).$$

Q8. algorithm T.C $\Rightarrow O(n^2)$ it takes t_1 time for n input

for
$2n$ input $(2n)^2 \Rightarrow 4n^2$
Kn inputs $(Kn)^2 = K^2n^2$

for $K = \sqrt{2}$

if input size increase by 1.42 times then that the time for running algorithm will become twice i.e. $2t_1$.

Q9. $T_A = 100^n$
$T_B = n^4$

$$\lim_{n\to\infty} \frac{100^n}{n^4}$$

Applying L'Hospital rule

$$\lim_{n\to\infty} \frac{n100^{n-1}}{4n^3} \Rightarrow \lim_{n\to\infty} \frac{100^{n-2}}{\cancel{24}\, 8\left(\frac{1}{1-\frac{1}{n}} \to 0\right)} \Rightarrow \infty$$

$\therefore T_A$ will take more time than T_B.

Q10. $$\lim_{n\to\infty} \frac{\log(n!)}{n\log n} \to \frac{\log n}{n\log n} + \frac{\log n-1}{n\log n} \dots + \frac{\log 1}{n\log n} \to 0.$$

$\therefore \Rightarrow 0.$

Hence proved $n\log n > \log(n!)$

Q11 a) $2^{n-1} + 4^{n+1}$

$\frac{2^n}{2} + 4 \cdot 2^{2n+2}$

$\frac{2^n}{2} + 4 \cdot 2^{2n}$

$\underbrace{2^n\left(\frac{1}{2} + 4 \cdot 2^n\right)}$

$\simeq 2^n$ (for $n \to \infty$)

$\therefore 2^{2n} = 4^n$

$\Rightarrow O(4^n)$

b) $(n^2+6)^8$

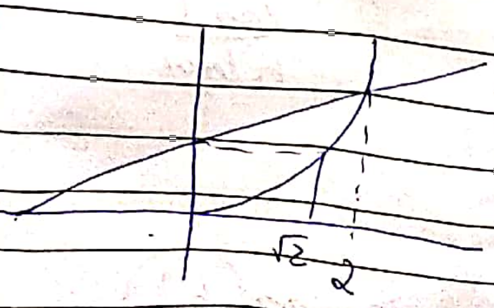
$\lim_{n \to \infty} (n^2)^8 \left(1 + \cancel{\frac{6}{n^2}}\right)^{8} \to 0$

$\Rightarrow n^{16}$

$\Rightarrow O(n^{16})$

Q12 $T_A = n^2$

$T_A = n+2$



breaking point is $n=2$ after $n=2$

n^2 grows much faster that $n+2$.

Q.14 ~~for(i=n; i≤n; i~~

for (i = n ; i ≥ 1 ; i /= 2)
{
}

$i = \cancel{2k}$

$$\frac{n}{2^k} = 1$$

$$2^k = n$$

$$k = \log_2 n$$

$\therefore O(\log_2 n)$

```
for (i = 1; i < n; i++)            -> O(n)
{ for (j = 0; j < n; j++)          -> O(n)
  {
  }
}
```


$\therefore$ T.C. $= O(n^2)$

```
for (i = 0; i < n; i++)            -> O(n)
    for (i = n-1; i >= 1; i = i/2) -> O(log2
```

T.C. $\Rightarrow O(n \log_2 n)$