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import numpy as np
import pandas as pd
from collections import deque
```

Problem 1.

A.

The packet will require $T + L/R$ time to finish transmitting

B.

Each packet requires $4\text{kb}/1000\text{kb} = 4\text{ms}$ of processing time at *A*. propagation delay is fixed 10ms. Thus, the packets can be modelled as follows:

Packet	arrival time	send complete time	receipt time
1	0 ms	4 ms	14ms
2	1 ms	8 ms	18 ms
3	1.5 ms	12 ms	22 ms

C.

```
res_df = pd.DataFrame(columns= ["Avg. InterArrival Time", "Avg. Packet Length", "Avg. Tran:
```

```
def simulate(lambd, mu, res_df = res_df, num_packets = 10000):
    arrival_t = np.random.exponential(1/lambd, [num_packets])
    avg_arrival_interval = np.mean(arrival_t)
    length_t = np.random.exponential(1/mu, [num_packets])
    avg_packet_length = np.mean(length_t)
    service_t = np.zeros(num_packets)
    q = deque()
    # q.append((length_t[0], 0)) # length, entrance time
    next_service = 0
    finished_service = 0
    cur_time = 0
    # print(arrival_t, length_t)
    while q or finished_service < num_packets:
        # print(q, next_service, finished_service, cur_time)
        if (q and next_service == num_packets) or (q and q[0][0] < arrival_t[next_service]):
            if next_service < num_packets:
                arrival_t[next_service] -= q[0][0]
                cur_time += q[0][0]
                service_t[finished_service] = cur_time - q[0][1]
                finished_service += 1
                q.popleft()
            elif (q and q[0][0] == arrival_t[next_service]):
                next_service += 1
                cur_time += q[0][0]
                service_t[finished_service] = cur_time - q[0][1]
                finished_service += 1
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q.popleft()
q.append(length_t[next_service])
elif (not q) or (q and q[0][0] > arrival_t[next_service]):
    if q:
        q[0][0] -= arrival_t[next_service]
        cur_time += arrival_t[next_service]
        q.append([length_t[next_service], cur_time])
        next_service += 1
res_df.loc[f"lambda: {lambda}, mu: {mu}"] = [avg_arrival_interval, avg_packet_length, n]
simulate(1, 0.8, res_df)
simulate(1, 1, res_df)
simulate(1, 1.2, res_df)
display(res_df)

```

	Avg. InterArrival Time	Avg. Packet Length	Avg. Transport Time
lambda: 1, mu: 0.8	1.006014	1.238507	1253.221346
lambda: 1, mu: 1	0.998661	0.976398	35.339267
lambda: 1, mu: 1.2	1.012268	0.825468	4.225015

Problem 2.

(Diagrams below)

A.

The state space is $\{0, 1\}$. All are communicating and recurrent because $E[A(t)] < M$

B.

The State space is non-negative integers, and has $|R| = \infty$. However, it is recurrent because $E[A(t)] < M$

C.

The State space is non-negative integers, and has $|R| = \infty$. However, it is recurrent because $E[A(t)] < M$

D.

The State space is non-negative integers, and has $|R| = \infty$. However, it is recurrent because $E[A(t)] < M$

E.

The State space is non-negative integers, and has $|R| = \infty$. However, it is transient because $E[A(t)] > M$

Problem 3.

As from class, to evaluate the stability of the system, we apply the Foster-Lyapunov theorem with $V(x) = x$. With this, we have:

$$\begin{aligned}
 E[V(t+1) - V(t) | X(t) = x_0] &= E[(X(t+1) - M(t+1))^+ + A(t+1) - X(t) | X(t) = x_0] \\
 &= E[(X(t+1) - M(t+1))^+ + A(t+1) | X(t) = x_0] - x_0 \\
 &= x_0 - E[M(X)] + E[A(t)] - x_0 \quad x_0 > 0 \qquad \qquad \qquad = E[A(t)] - E[M(X)]
 \end{aligned}$$

We know that $E[M(x)] = \frac{K}{2}$. It remains to evaluate $A(t)$, which we are told is conditional on a binary markov chain itself.

$$\begin{aligned} E[A(t)] &= E[A_1(t)(1 - S(t)) + A_2(t)S(t)] \\ &= E[p_1(1 - S(t)) + p_2S(t)] \end{aligned}$$

from the given definitions of i.i.d $A_1(t)$ and $A_2(t)$. Thus, it suffices to evaluate the EV of $S(t)$. We aren't given the values of this binary chain, so assume that the first state has value k and the second one j . Given that it is binary, we know that it is stable, and it suffices to evaluate the EV of the chain.

The transition matrix of this chain is $\begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix}$, which we want to find what it converges to.

$$\pi_0(1-p) + \pi_1(q) = \pi_0$$

$$\pi_0(p) + \pi_1(1-q) = \pi_1$$

for $\pi(0) + \pi(1) = 1$ results in $\pi = \begin{bmatrix} \frac{q}{p+q} \\ \frac{p}{p+q} \end{bmatrix}$. This implies that the EV of this system is $\frac{jq+kp}{p+q}$, so $E[A(t)] = \frac{(jq+kp)(1-p_1+p_2)}{p+q}$. This needs to be less than $\frac{K}{2}$, which is a necessary and sufficient condition for stability.

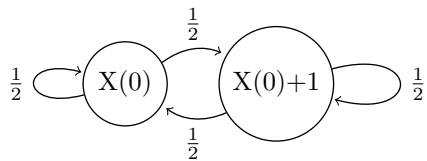


Figure 1: A.

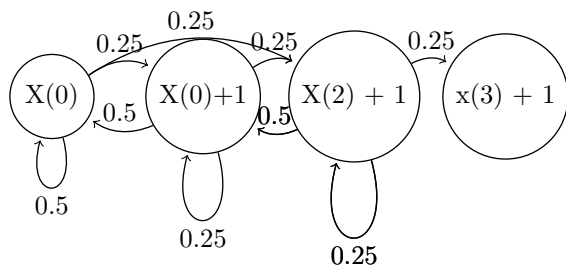


Figure 2: B.

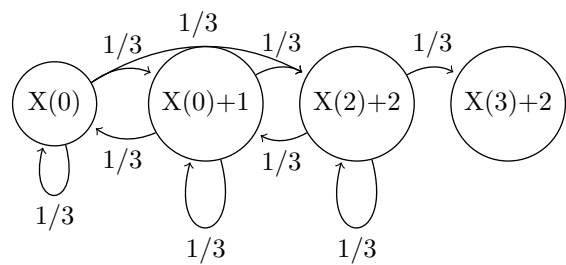


Figure 3: C.

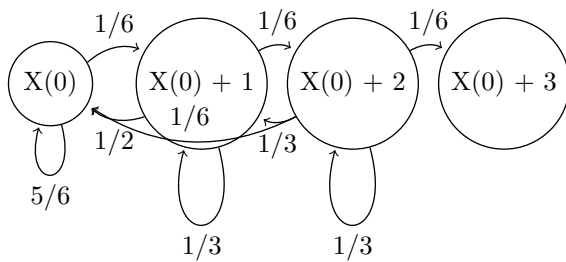


Figure 4: D.

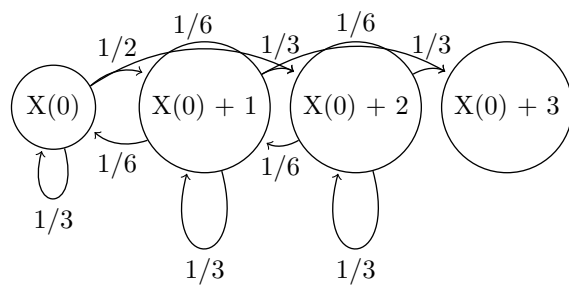


Figure 5: E.