# Difference-in-Differences with Spatial Spillovers

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# Spatial Spillovers

Researchers aim to estimate the **average treatment effect on the treated**:

$$\tau \equiv \mathbb{E}\left[Y_{i1}(1) - Y_{i1}(0) \mid D_i = 1\right]$$

Estimation is complicated by **Spillover Effects**

**Spillover effects** are when effect of treatment extend over the treatment boundaries (states, counties, etc.). Example:

- Aa large employer opening/closing in a **treated** county have positive employment effects on *nearby counties*
- Having nearby counties with factories raises wages and reduces effect of **treated** counties

# Bias from Spatial Spillovers

The canonical difference-in-differences estimate is:

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Two problems in presence of spillover effects:

- **Spillover onto Control Units:**
  Nearby "control" units fail to estimate counterfactual trends because they are affected by treatment

- **Spillover onto other Treated Units:**
  Treated units are also affected by nearby units and therefore combines "direct" effects with spillover effects

# Econometric Contribution

I formalize spillovers into a potential outcomes framework:

[Clarke (2017), Berg and Streitz (2019), and Verbitsky-Savitz and Raudenbush (2012)]

- I decompose the difference-in-differences estimator into three parts: Direct Effect of Treatment, Spillover onto Treated Units, Spillover onto Control Units
- Show that an indicator for being close to treated units remove *all bias* so long as the indicator contains all units affected by spillovers
- ‘Rings’ are able to estimate spillover effects non-parametrically while still removing all bias

# Roadmap of Talk

**Theory**

Estimation with Spillovers

Application in Urban Economics

Conclusion

# Potential Outcomes Framework

$Y_{it}(D_i, h(\vec{D}, i))$ is the potential outcome of county $i \in \{1, \ldots, N\}$ at time $t$ with treatment status $D_i \in \{0, 1\}$.

The function $h(\vec{D}, i)$ maps the entire treatment vector into an 'exposure mapping'

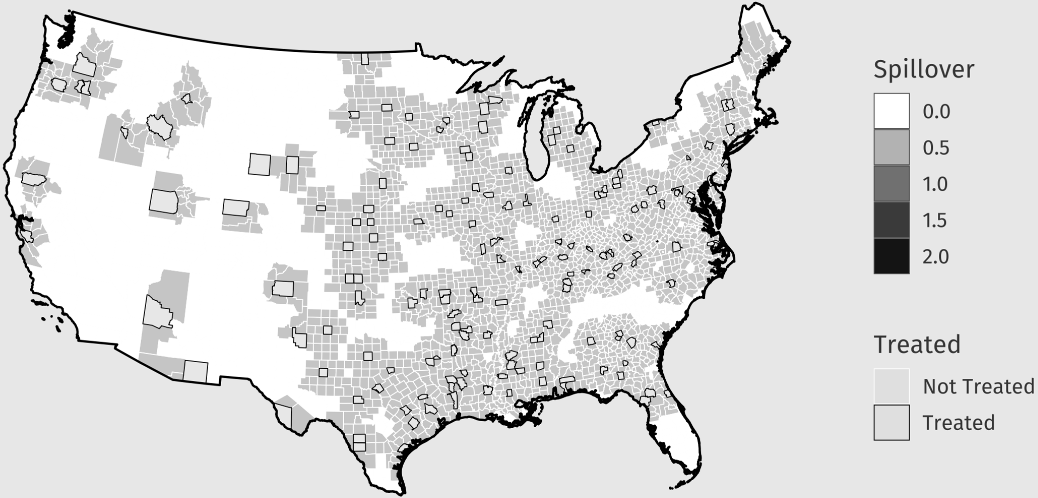
# Examples of $h_i(\vec{D})$

**Treatment within $x$ miles:**

$h(\vec{D}, i) = max_j \; 1(d(i,j) \leq x)$ where $d(i,j)$ is the distance between counties $i$ and $j$.

- e.g. library access where $x$ is the maximum distance people will travel
- Spillovers are non-additive

# Within 80mi.



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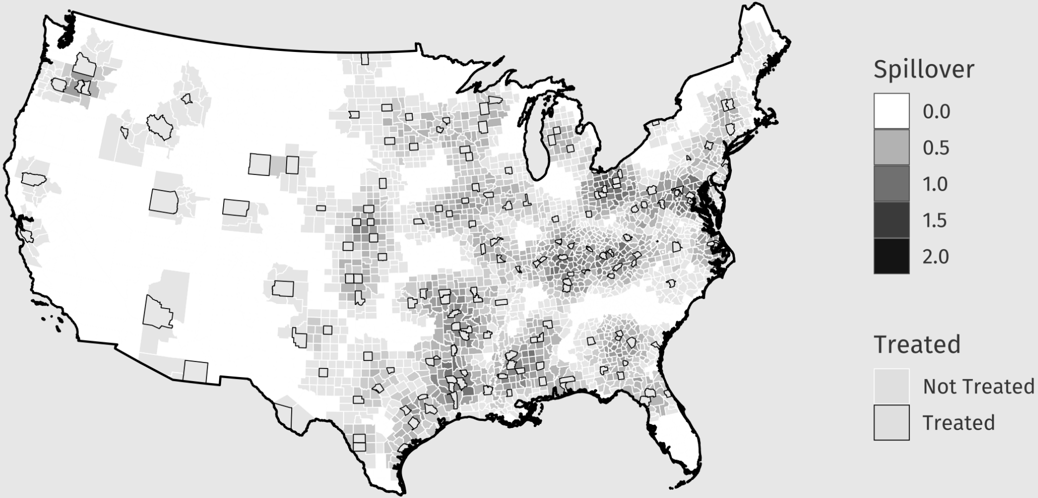
**Number of Treated within $x$ miles:**

$h(\vec{D}, i) = \sum_{j=1}^{k} 1(d(i,j) \leq x)$.

- e.g. large factories opening
- Agglomeration economies suggest spillovers are additive

# Within 80mi. (Additive)



# What does Diff-in-Diff identify?

With the parallel trends assumption and a random sample, I decompose the difference-in-differences estimate as follows:

$$\mathbb{E}\left[\hat{\tau}\right] = \underbrace{\mathbb{E}\left[Y_{i1} - Y_{i0} \mid D_i = 1\right] - \mathbb{E}\left[Y_{i1} - Y_{i0} \mid D_i = 0\right]}_{\text{Difference-in-Differences}}$$

$$\begin{aligned} &= \mathbb{E}\left[Y_{i1}(1,0) - Y_{i1}(0,0) \mid D_i = 1\right] \\ &\quad + \quad \mathbb{E}\left[Y_{i1}(1, h_i(\vec{D})) - Y_{i1}(1,0) \mid D_i = 1\right] \\ &\quad - \quad \mathbb{E}\left[Y_{i1}(0, h_i(\vec{D})) - Y_{i1}(0,0) \mid D_i = 0\right] \end{aligned}$$

$$= \tau_{\text{direct}} + \tau_{\text{spillover, treated}} - \tau_{\text{spillover, control}}$$

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# Spillovers as estimand of interest

Until now, we assumed our estimand of interest is $\tau_{\text{direct}}$.

However, the two other spillover effects are of interest as well:

- $\tau_{\text{spillover, control}}$: Do the benefits of a treated county come at a cost to neighbor counties?
- $\tau_{\text{spillover, treated}}$: Does the estimated effect change based on others treatment? (This is what you should consider if you are a policy maker)

To estimate the spillover effects, we have to parameterize $h(\vec{D}, i)$ function and the potential outcomes function $Y_i(D_i, h(\vec{D}, i))$.

# Robustness to Misspecification

I find that an indicator for being Within $x$ miles from treated area interacted with treatment status will remove **all bias** so long as the indicator contains all the affected units.

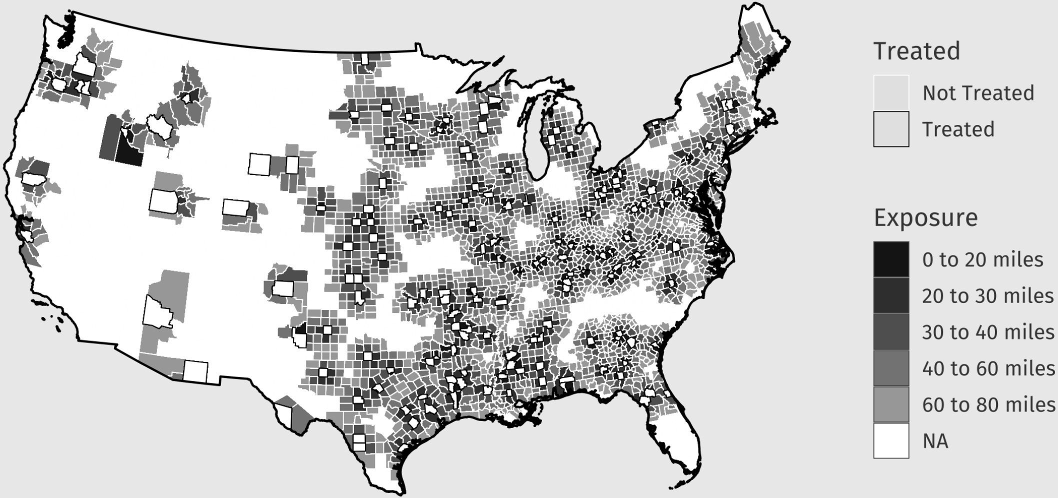
- Indicator will estimate the average spillover effect on treated and control units and remove these from estimate, $\hat{\tau}$

# Estimation of Spillover Effects

In a lot of settings, estimating the spillover effects are also an estimand of interest.

A set of concentring ‘rings’ around treatment perform best for estimating spillover effects

# Rings (0-20, 20-30, 30-40, 40-60, 60-80)



# Benefits of Rings

- Still remove all bias from the treatment effect estimate
- Can trace out how spillovers spread over distance
- If spillovers are additive in the number of nearby treated units, then an additive version of rings should be used (but this loses the bias-removal property)

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## Urban Economics

I apply framework to place-based analyses in Urban Economics

- Revisit Kline and Moretti (2014a) analysis of the Tennessee Valley Authority
    - The local effect estimate is contaminated by spillover effects to neighboring counties (Kline and Moretti, 2014b)
    - Large scale manufacturing investment creates an 'urban shadow' (Cuberes, Desmet, and Rappaport, 2021; Fujita, Krugman, and Venables, 2001)
- Discuss how framework can reconcile conflicting findings on effect of federal Empowerment Zones
    - Identification Strategy of using far-away rejected applicants (Busso, Gregory, and Kline, 2013) vs. census tracts within 1000 feet of empowerment zone (Neumark and Kolko, 2010)

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# Conclusion

- I decomposed the TWFE estimate into the direct effect and two spillover terms
- I showed that a set of concentric rings removes two spillover terms from treatment effect estimate and models spillovers well
- For place-based policies, I show the importance of considering spatial spillovers when estimating treatment effects