Difference-in-Differences with Spatial Spillovers

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Spatial Spillovers

Researchers aim to estimate the average treatment effect on the treated:

$$\tau \equiv \mathbb{E}\left[Y_{i1}(1) - Y_{i1}(0) \mid D_i = 1\right]$$

Spillover effects are when effect of treatment extend over boundaries such as states, counties, etc.

- e.g. large employer opening/closing in a county have positive employment effects on nearby counties

Bias from Spatial Spillovers

The canonical difference-in-differences estimate is:

$$\hat{\tau} = \underbrace{\hat{\mathbb{E}}\left[Y_{i1} - Y_{i0} \mid D_i = 1\right]}_{\text{Counterfactual Trend } + \ \tau} - \underbrace{\hat{\mathbb{E}}\left[Y_{i1} - Y_{i0} \mid D_i = 0\right]}_{\text{Counterfactual Trend}}$$

Two problems in presence of spillover effects:

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- **Spillover onto Control Units:**
  Nearby "control" units fail to estimate counterfactual trends because they are affected by treatment

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Two problems in presence of spillover effects:

- **Spillover onto Control Units:**
  Nearby "control" units fail to estimate counterfactual trends because they are affected by treatment

- **Spillover onto other Treated Units:**
  Treated units are also affected by nearby units and therefore combines "direct" effects with spillover effects

Contribution

Difference-in-Differences Estimation with Spillovers

[Clarke (2017), Berg and Streitz (2019), and Verbitsky-Savitz and Raudenbush (2012)]

- I generalize the work by deriving the bias in terms of potential outcomes and show how to estimate treatment effects by removing spillover effects

Potential Outcome Framework for Spillovers

[Miguel and Kremer (2004), Vazquez-Bare (2019), Angelucci and Di Maro (2016), Angrist (2014)]

- I focus on a spatial setting whereas these papers consider spillovers in network clusters

Place-based Policies

[Kline and Moretti (2014b), Kline and Moretti (2014a), Busso, Gregory, and Kline (2013), and Neumark and Kolko (2010)]

- I highlight the need to control for general equilibrium effects to properly estimate local effects of policies

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Roadmap of Talk

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Potential Outcomes Framework

For exposition, I will label units as counties. Assume all treatment occurs at the same time (2-periods or pre-post averages).[1]

- $Y_{it}(D_i, h(\vec{D}, i))$ is the potential outcome of county $i \in \{1, \ldots, N\}$ at time t with treatment status $D_i \in \{0, 1\}$.
- $\vec{D} \in \{0, 1\}^N$ is the vector of all units treatments.
- The function $h(\vec{D}, i)$ maps the entire treatment vector into an 'exposure mapping' which can be a scalar or a vector.
- No exposure is when $h(\vec{D}, i) = \vec{0}$.

[1] I extend this into an event study framework in the paper, but the intuition is the same as in the 2×2 setting.

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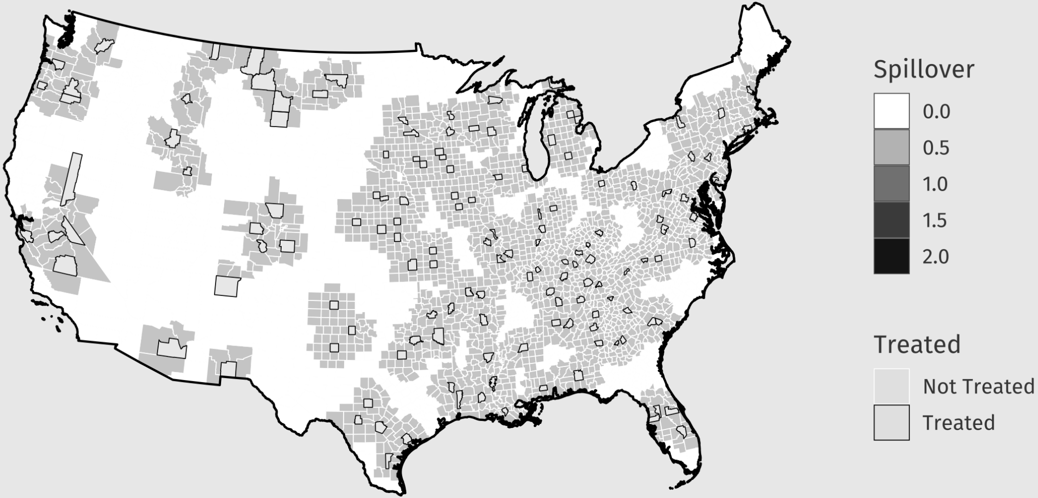
- **Treatment within x miles:**
  $h(\vec{D}, i) = max_j \; 1(d(i,j) \leq x)$ where $d(i,j)$ is the distance between counties i and j.
  - e.g. library access where x is the maximum distance people will travel
  - Spillovers are non-additive

Within 80mi.



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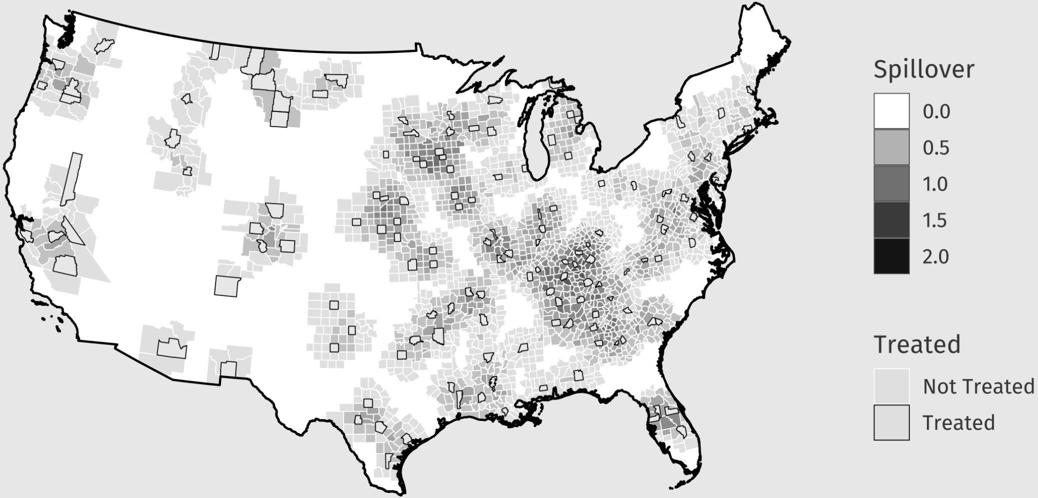
- **Number of Treated within x miles:**
  $h(\vec{D}, i) = \sum_{j=1}^{k} 1(d(i, j) \leq x)$.
  - e.g. large factories opening
  - Agglomeration economies suggest spillovers are additive

Within 80mi. (Additive)



Estimand of Interest

Estimand of Interest:

$$\tau_{\text{direct}} \equiv \mathbb{E}\left[Y_{i,1}(1,0) - Y_{i,1}(0,0) \mid D_i = 1\right]$$

This is the direct effect in the absense of exposure to spillovers.

Parallel Trends

I assume a modified version of the parallel counterfactual trends assumption:

$$\mathbb{E}\big[\underbrace{Y_{i,1}(0,\vec{0}) - Y_{i,0}(0,\vec{0})}_{\text{Counterfactual Trend}} \mid D_i = 1\big]$$
$$= \mathbb{E}\big[\underbrace{Y_{i,1}(0,\vec{0}) - Y_{i,0}(0,\vec{0})}_{\text{Counterfactual Trend}} \mid D_i = 0\big],$$

In the complete absence of treatment (not just the absence of individual i's treatment), the change in potential outcomes from period 0 to 1 would not depend on treatment status

What does Diff-in-Diff identify?

With the parallel trends assumption and random assignment of D_i, I decompose the diff-in-diff estimate as follows:

$$\mathbb{E}\left[\hat{\tau}\right] = \underbrace{\mathbb{E}\left[Y_{i1} - Y_{i0} \mid D_i = 1\right] - \mathbb{E}\left[Y_{i1} - Y_{i0} \mid D_i = 0\right]}_{\text{Difference-in-Differences}}$$

$$\begin{aligned}
&= \mathbb{E}\left[Y_{i1}(1,0) - Y_{i1}(0,0) \mid D_i = 1\right] \\
&\quad + \quad \mathbb{E}\left[Y_{i1}(1, h_i(\vec{D})) - Y_{i1}(1,0) \mid D_i = 1\right] \\
&\quad - \quad \mathbb{E}\left[Y_{i1}(0, h_i(\vec{D})) - Y_{i1}(0,0) \mid D_i = 0\right]
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$$= \tau_{\text{direct}} + \tau_{\text{spillover, treated}} - \tau_{\text{spillover, control}}$$

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Biased Estimate for τ_{direct}

$$\mathbb{E}[\hat{\tau}_{\text{diff-in-diff}}] - \tau_{\text{direct}} = \tau_{\text{spillover, treated}} - \tau_{\text{spillover, control}}$$

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Problems with ”Dropping Bad Controls”

It is common in empirical applications to drop control units near the treated units when estimating the direct effect of treatment.

This is not recommended for two reasons:

1. If not all control units that experience spillover effects are removed, then bias remains
2. There is a second source of bias, the spillover effects on treated units, that still remains

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Spillovers as estimand of interest

Until now, we assumed our estimand of interest is τ_{direct}.

However, the two other spillover effects are of interest as well:

- $\tau_{\text{spillover, control}}$: Do the benefits of a treated county come at a cost to neighbor counties?
- $\tau_{\text{spillover, treated}}$: Does the estimated effect change based on others treatment? (This is what you should consider if you are a policy maker)

To estimate the spillover effects, we have to parameterize $h(\vec{D}, i)$ function and the potential outcomes function $Y_i(D_i, h(\vec{D}, i))$.

Robustness to Misspecification

Generate data using the same data-generating process as before but with different spillover functions:

$$y = \mu_t + \mu_i + 2D_{it} + \beta_{\text{spill,control}} * (1 - D_{it})h(\vec{D}, i) + \varepsilon_{it}$$

Then, I estimate each data-generating process using (potentially) misspecified $\tilde{h}(\vec{D}, i)$ and report the average estimate bias.

Results

I find that an indicator for being Within x miles from treated area will remove all bias so long as the indicator contains all the affected units.

Estimation of Spillover Effects

In a lot of settings, estimating the spillover effects are also an estimand of interest.

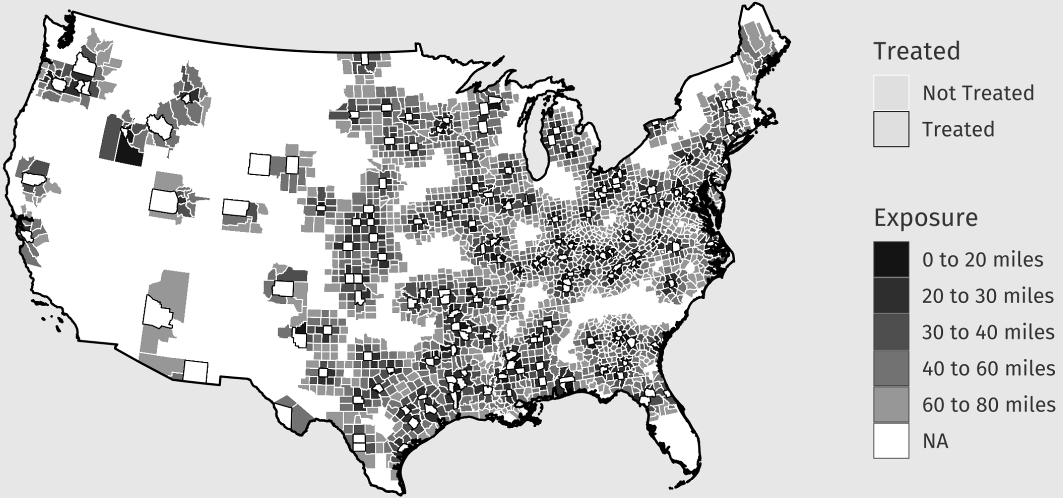
I repeat the same exercise and estimate the spillover effects for each control unit, $\hat{\beta}_{\text{spill, control}} * \tilde{h}(\vec{D}, i)$.

Then calculate

$$1 - \frac{\overbrace{\sum_{i:D_i=0} (\beta_{\text{spill, control}} h(\vec{D}, i) - \hat{\beta}_{\text{spill, control}} \tilde{h}(\vec{D}, i))^2}^{\text{Mean Square Prediction Error}}}{\underbrace{\sum_{i:D_i=0} (\beta_{\text{spill, control}} h(\vec{D}, i))^2}_{\text{Normalization}}}$$

This gives the proportion of spillovers explained by $\tilde{h}(\vec{D}, i)$

Rings (0-20, 20-30, 30-40, 40-60, 60-80)



Rings perform best at estimating spillover effects.

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- I decomposed the TWFE estimate into the direct effect and two spillover terms
- I showed that a set of concentric rings removes two spillover terms from treatment effect estimate and models spillovers well
- For place-based policies, I show the importance of considering spatial spillovers when estimating treatment effects