Difference-in-Differences with Spatial Spillovers

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March 17, 2021

Spatial Spillovers

Researchers aim to estimate the **average treatment effect on the treated**:

$$\tau \equiv \mathbb{E}\left[Y_{i1}(1) - Y_{i1}(0) \mid D_i = 1\right]$$

Estimation is complicated by **Spillover Effects**

Spillover effects are when effect of treatment extend over the treatment boundaries (states, counties, etc.)

- E.g. a large employer opening/closing in a **treated** county have positive employment effects on *nearby counties*
- Having nearby counties with factories raises wages and reduces effect of **treated** counties

Bias from Spatial Spillovers

The canonical difference-in-differences estimate is:

$$\hat{\tau} = \underbrace{\hat{\mathbb{E}}\left[Y_{i1} - Y_{i0} \mid D_i = 1\right]}_{\text{Counterfactual Trend} + \tau} - \underbrace{\hat{\mathbb{E}}\left[Y_{i1} - Y_{i0} \mid D_i = 0\right]}_{\text{Counterfactual Trend}}$$

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- **Spillover onto Control Units:**
  Nearby "control" units fail to estimate counterfactual trends because they are affected by treatment

2

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Two problems in presence of spillover effects:

- **Spillover onto Control Units:**
  Nearby "control" units fail to estimate counterfactual trends because they are affected by treatment

- **Spillover onto other Treated Units:**
  Treated units are also affected by nearby units and therefore combines "direct" effects with spillover effects

2

Econometric Contribution

I formalize spillovers into a potential outcomes framework:

[Clarke (2017), Berg and Streitz (2019), and Verbitsky-Savitz and Raudenbush (2012)]

- I decompose the difference-in-differences estimator into three parts: Direct Effect of Treatment, Spillover onto Treated Units, Spillover onto Control Units

- Show that an indicator for being close to treated units remove *all bias* so long as the indicator contains all units affected by spillovers

- 'Rings' are able to estimate spillover effects non-parametrically while still removing all bias

Theory

Estimation with Spillovers

Application in Urban Economics

Conclusion

Potential Outcomes Framework

$Y_{it}(D_i, h(\vec{D}, i))$ is the potential outcome of county $i \in \{1, \dots, N\}$ at time t with treatment status $D_i \in \{0, 1\}$.

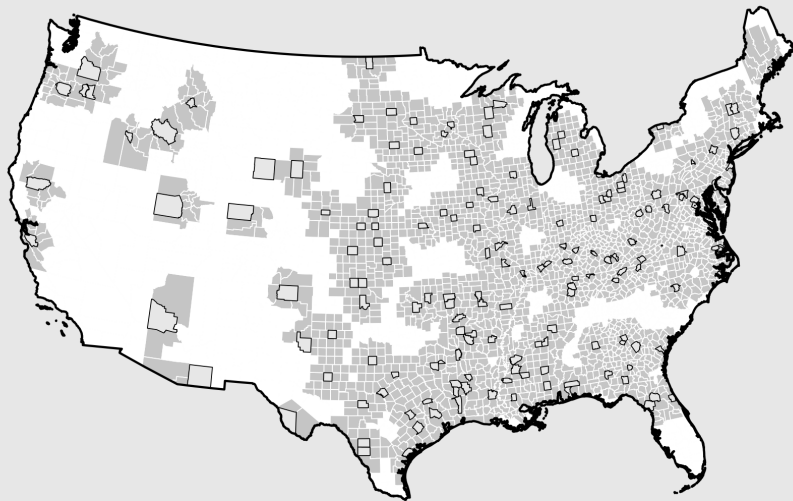The function $h(\vec{D}, i)$ maps the entire treatment vector into an 'exposure mapping'

Examples of $h_i(\vec{D})$

Treatment within x miles:

$h(\vec{D}, i) = max_j \ 1(d(i, j) \leq x)$ where $d(i, j)$ is the distance between counties i and j.

- e.g. library access where x is the maximum distance people will travel
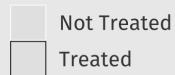- Spillovers are non-additive

Within 80mi.

Spillover
- 0.0
- 0.5
- 1.0
- 1.5
- 2.0

Treated
- Not Treated
- Treated

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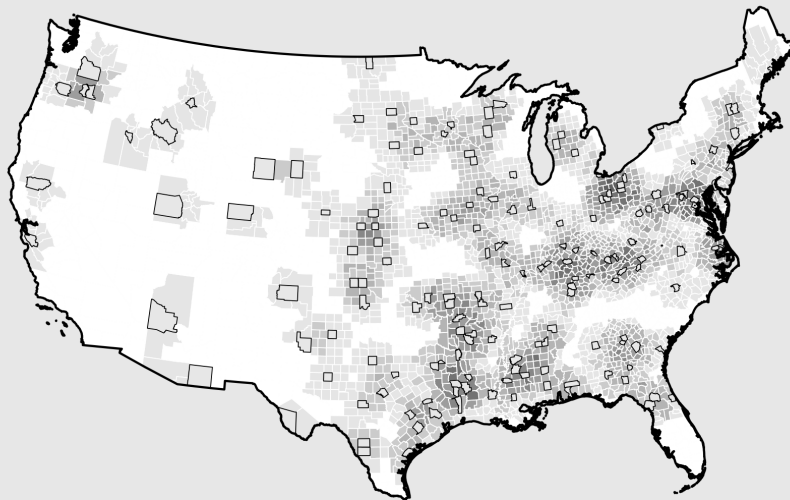
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Number of Treated within x miles:

$h(\vec{D}, i) = \sum_{j=1}^{k} 1(d(i, j) \leq x)$.

- e.g. large factories opening
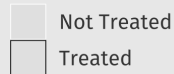- Agglomeration economies suggest spillovers are additive

Within 80mi. (Additive)

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What does Diff-in-Diff identify?

With the parallel trends assumption and a random sample, I decompose the difference-in-differences estimate as follows:

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$$= \mathbb{E}\left[Y_{i1}(1,0) - Y_{i1}(0,0) \mid D_i = 1\right]$$
$$+ \quad \mathbb{E}\left[Y_{i1}(1, h_i(\vec{D})) - Y_{i1}(1,0) \mid D_i = 1\right]$$
$$- \quad \mathbb{E}\left[Y_{i1}(0, h_i(\vec{D})) - Y_{i1}(0,0) \mid D_i = 0\right]$$

$$= \tau_{\text{direct}} + \tau_{\text{spillover, treated}} - \tau_{\text{spillover, control}}$$

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Spillovers as estimand of interest

Until now, we assumed our estimand of interest is τ_{direct}.

However, the two other spillover effects are of interest as well:

- $\tau_{\text{spillover, control}}$: Do the benefits of a treated county come at a cost to neighbor counties?

- $\tau_{\text{spillover, treated}}$: Does the estimated effect change based on others treatment? (This is what you should consider if you are a policy maker)

To estimate the spillover effects, we have to parameterize $h(\vec{D}, i)$ function and the potential outcomes function $Y_i(D_i, h(\vec{D}, i))$.

Robustness to Misspecification

I find that an indicator for being Within x miles from treated area interacted with treatment status will remove **all bias** so long as the indicator contains all the affected units.
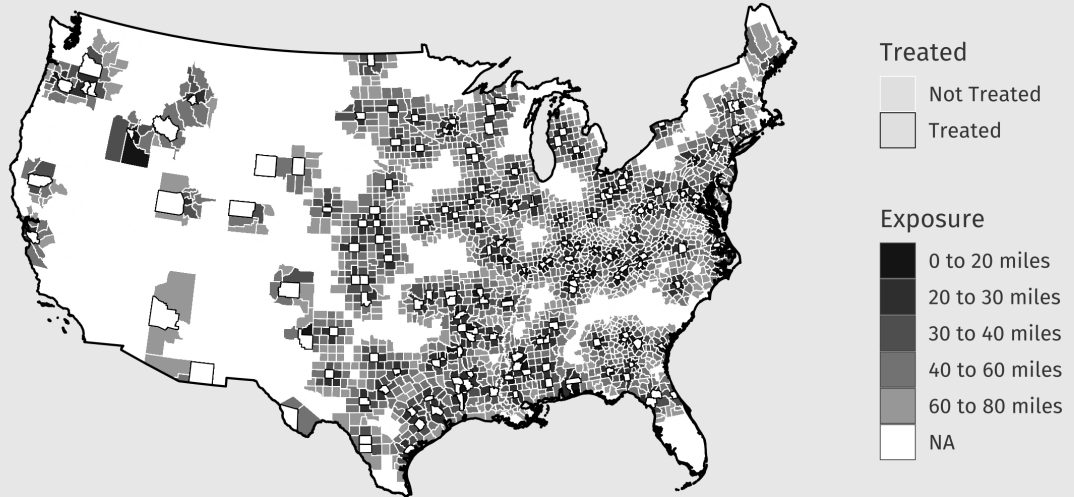
- Indicator will estimate the average spillover effect on treated and control units and remove these from estimate, $\hat{\tau}$

Estimation of Spillover Effects

In a lot of settings, estimating the spillover effects are also an estimand of interest.

A set of concentring 'rings' around treatment perform best for estimating spillover effects

Rings (0-20, 20-30, 30-40, 40-60, 60-80)

Benefits of Rings

- Still remove all bias from the treatment effect estimate
- Can trace out how spillovers spread over distance
- If spillovers are additive in the number of nearby treated units, then an additive version of rings should be used (but this loses the bias-removal property)

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Urban Economics

I apply framework to place-based analyses in Urban Economics

- Revisit Kline and Moretti (2014a) analysis of the Tennessee Valley Authority

  - The local effect estimate is contaminated by spillover effects to neighboring counties (Kline and Moretti, 2014b)

  - Large scale manufacturing investment creates an 'urban shadow' (Cuberes, Desmet, and Rappaport, 2021; Fujita, Krugman, and Venables, 2001)

- Discuss how framework can reconcile conflicting findings on effect of federal Empowerment Zones

  - Identification Strategy of using far-away rejected applicants (Busso, Gregory, and Kline, 2013) vs. census tracts within 1000 feet of empowerment zone (Neumark and Kolko, 2010)

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- I decomposed the TWFE estimate into the direct effect and two spillover terms
- I showed that a set of concentric rings removes two spillover terms from treatment effect estimate and models spillovers well
- For place-based policies, I show the importance of considering spatial spillovers when estimating treatment effects