ME535 Assignment #6

6.1 Surface can be represented as : $x = \cos\left(\frac{\pi}{2}u\right)$, $y : \sin\left(\frac{\pi}{2}u\right)$; $z = 4v$

$u \in [0,1]$; $v \in [0,1]$

a) (i)

$$S(\bar{u}, \bar{v}) = \left(\cos\left(\frac{\pi}{2}u\right), \sin\left(\frac{\pi}{2}u\right), 4v\right)$$

$$\Rightarrow S(0.5, 0.5) = \left(\cos\left(\frac{\pi}{4}\right), \sin\left(\frac{\pi}{4}\right), 4\times 0.5\right)$$

$$= \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 2\right) = \boxed{(0.7071, 0.7071, 2)} \leftarrow S(\bar{u},\bar{v})$$

(ii) $$\frac{\partial S}{\partial u}(\bar{u},\bar{v}) = \left(-\frac{\pi}{2}\sin\left(\frac{\pi}{2}u\right), \frac{\pi}{2}\cos\left(\frac{\pi}{2}u\right), 0\right)$$

$$\Rightarrow \frac{\partial S}{\partial u}(0.5, 0.5) = \left(-\frac{\pi}{2}\cdot\frac{1}{\sqrt{2}}, \frac{\pi}{2}\cdot\frac{1}{\sqrt{2}}, 0\right)$$

$$= \boxed{(-1.1107, 1.1107, 0)} \leftarrow \frac{\partial S}{\partial u}$$

(iii) $$\frac{\partial S}{\partial v}(\bar{u},\bar{v}) = \boxed{(0,0,4)} \leftarrow \frac{\partial S}{\partial v}$$

(iv) $$\frac{\partial^2 S}{\partial u \partial v}(\bar{u},\bar{v}) = \boxed{(0,0,0)} \leftarrow \frac{\partial^2 S}{\partial u \partial v}$$

b) $$S(u,v) = (1-v)C_1(u) + vC_2(u) + (1-u)C_3(v) + uC_4(v) - (1-u)(1-v)P_{0,0} - u(1-v)P_{1,0} - v(1-u)P_{0,1} - uvP_{1,1}$$

$$= (1-v)\begin{pmatrix}\cos\frac{\pi}{2}u \\ \sin\frac{\pi}{2}u \\ 0\end{pmatrix} + v\begin{pmatrix}\cos\frac{\pi}{2}u \\ \sin\frac{\pi}{2}u \\ 4\end{pmatrix} + (1-u)\begin{pmatrix}1 \\ 0 \\ 4v\end{pmatrix} + u\begin{pmatrix}0 \\ 1 \\ 4v\end{pmatrix}$$

$$- (1-u)(1-v)\begin{pmatrix}1 \\ 0 \\ 0\end{pmatrix} - u(1-v)\begin{pmatrix}0 \\ 1 \\ 0\end{pmatrix} - v(1-u)\begin{pmatrix}1 \\ 0 \\ 4\end{pmatrix} - uv\begin{pmatrix}0 \\ 1 \\ 4\end{pmatrix}$$

where :

$P_{0,0} : (1,0,0)$ $\qquad C_1(u) = \begin{pmatrix} \cos \frac{\pi}{2} u \\ \sin \frac{\pi}{2} u \\ 0 \end{pmatrix}$ $\qquad C_3(v) = \begin{pmatrix} 1 \\ 0 \\ 4v \end{pmatrix}$

$P_{1,0} = (0,1,0)$

$P_{0,1} = (1,0,4)$ $\qquad C_2(u) = \begin{pmatrix} \cos \frac{\pi}{2} u \\ \sin \frac{\pi}{2} u \\ 4 \end{pmatrix}$ $\qquad C_4(v) = \begin{pmatrix} 0 \\ 1 \\ 4v \end{pmatrix}$

$P_{1,1} = (0,1,4)$

$$S(u,v) = \begin{pmatrix} (1-v)\cos \frac{\pi}{2} u + v\cos \frac{\pi}{2} u + (1-u) \\ (1-v)\sin \frac{\pi}{2} u + v \sin \frac{\pi}{2} u + u \\ 4v + 4v(1-u) + 4uv \end{pmatrix} - \begin{pmatrix} (1-u)(1-v) + v(1-u) \\ u(1-v) + uv \\ 4(v(1-u)) + 4uv \end{pmatrix}$$

$$= \begin{pmatrix} \cos \frac{\pi}{2} u + 1 - u \\ \sin \frac{\pi}{2} u + u \\ 8v \end{pmatrix} - \begin{pmatrix} 1-u \\ u \\ 4v \end{pmatrix}$$

$$= \begin{pmatrix} \cos \frac{\pi}{2} u \\ \sin \frac{\pi}{2} u \\ 4v \end{pmatrix} \longleftarrow \text{Coons patch}$$

(c) $S(u,v) = \left(\cos\left(\frac{\pi}{2} u\right), \sin\left(\frac{\pi}{2} u\right), 4v\right)$

$S(0,0) = (1,0,0)$ $\quad S(1,0) = (0,1,0)$ $\quad S_v(u,v) = (0,0,4)$

$S(0,1) = (1,0,4)$ $\quad S(1,1) = (0,1,4)$ $\quad S_v(0,0) = (0,0,4)$ $\quad S_v(0,1) = (0,0,4)$

$S_u(0,0) = (0, \frac{\pi}{2}, 0)$ $\quad S_u(1,0) = (-\frac{\pi}{2}, 0, 0)$ $\quad S_v(1,0) = (0,0,4)$ $\quad S_v(1,1) = (0,0,4)$

$S_u(0,1) = (0, \frac{\pi}{2}, 0)$ $\quad S_u(1,1) = (-\frac{\pi}{2}, 0, 0)$ $\quad S_{uv}(u,v) = (0,0,0)$

$S_{uv}(0,0) = (0,0,0)$ $\quad S_{uv}(0,1) = (0,0,0)$

$S_{uv}(1,0) = (0,0,0)$ $\quad S_{uv}(1,1) = (0,0,0)$

Bicubic patch :

$$S(u,v) = \begin{bmatrix} F_1(u) & F_2(u) & F_3(u) & F_4(u) \end{bmatrix} \begin{bmatrix} (1\ 0\ 0) & (1\ 0\ 4) & (0\ 0\ 4) & (0\ 0\ 4) \\ (0\ 1\ 0) & (0\ 1\ 4) & (0\ 0\ 4) & (0\ 0\ 4) \\ (0\ \frac{\pi}{2}\ 0) & (0\ \frac{\pi}{2}\ 0) & (0\ 0\ 0) & (0\ 0\ 0) \\ (-\frac{\pi}{2}\ 0\ 0) & (-\frac{\pi}{2}\ 0\ 0) & (0\ 0\ 0) & (0\ 0\ 0) \end{bmatrix} \begin{bmatrix} F_1(v) \\ F_2(v) \\ F_3(v) \\ F_4(v) \end{bmatrix}$$

~~or~~ Now : $F_1(u) : 1 - 3u^2 + 2u^3$

$F_2(u) : 3u^2 - 2u^3.$

$F_3(u) : u - 2u^2 + u^3$

$F_4(u) : -u^2 + u^3$

or, $$S(u,v) = \begin{bmatrix} 1-3u^2+2u^3 & 3u^2-2u^3 & u-2u^2+u^3 & -u^2+u^3 \end{bmatrix} \begin{bmatrix} (1\ 0\ 0) & (1\ 0\ 4) & (0\ 0\ 4) & (0\ 0\ 4) \\ (0\ 1\ 0) & (0\ 1\ 4) & (0\ 0\ 4) & (0\ 0\ 4) \\ (0\ \frac{\pi}{2}\ 0) & (0\ \frac{\pi}{2}\ 0) & (0\ 0\ 0) & (0\ 0\ 0) \\ (-\frac{\pi}{2}\ 0\ 0) & (-\frac{\pi}{2}\ 0\ 0) & (0\ 0\ 0) & (0\ 0\ 0) \end{bmatrix} \begin{bmatrix} 1-3v^2+2v^3 \\ 3v^2-2v^3 \\ v-2u^2+v^3 \\ -v^2+v^3 \end{bmatrix}$$

Approximation of surface with a bicubic patch.

(d) For a Fergusons patch the constraint is $\frac{\partial^2 S}{\partial u \partial v}$ is equal to zero., which is exactly the same as that of Bicubic patch for this problem.

$$\therefore S[u,v] = \begin{bmatrix} 1-3u^2+2u^3 & 3u^2-2u^3 & u-2u^2+u^3 & -u^2+u^3 \end{bmatrix} \begin{bmatrix} (1\ 0\ 0) & (1\ 0\ 4) & (0\ 0\ 4) & (0\ 0\ 4) \\ (0\ 1\ 0) & (0\ 1\ 4) & (0\ 0\ 4) & (0\ 0\ 4) \\ (0\ \frac{\pi}{2}\ 0) & (0\ \frac{\pi}{2}\ 0) & (0\ 0\ 0) & (0\ 0\ 0) \\ (-\frac{\pi}{2}\ 0\ 0) & (-\frac{\pi}{2}\ 0\ 0) & (0\ 0\ 0) & (0\ 0\ 0) \end{bmatrix} \begin{bmatrix} 1-3v^2+2v^3 \\ 3v^2-2v^3 \\ v-2v^2+v^3 \\ -v^2+v^3 \end{bmatrix}$$

(e) Exact surface (at parametric midpoint) can be written as:

$S(0.5, 0.5) = (0.7071, 0.7071, 2)$ ... from problem (a)

$\boxed{\text{Comparing (a) with (b)}}$

(b) can be written as: $\begin{pmatrix} \cos\frac{\pi}{2}u \\ \sin\frac{\pi}{2}u \\ 4v \end{pmatrix}$

using $u = V = 0.5$

$$\begin{pmatrix} \cos\frac{\pi}{2}u \\ \sin\frac{\pi}{2}u \\ 4V \end{pmatrix} = \begin{pmatrix} \cos\frac{\pi}{4} \\ \sin\frac{\pi}{4} \\ 4\cdot\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 2 \end{pmatrix} = \begin{pmatrix} 0.7071 \\ 0.7071 \\ 2 \end{pmatrix}$$

We see that the result from Coons patch matches with that of exact surface.

$\boxed{\text{Comparing (a) with (c) \& (d)}}$

Using the patch from (c), inputting $u = v = 0.5$ we have.

$$\rightarrow \begin{bmatrix} 0.5 & 0.5 & 0.125 & -0.175 \end{bmatrix} \begin{bmatrix} (1\ 0\ 0) & (1\ 0\ 4) & (0\ 0\ 4) & (0\ 0\ 4) \\ (0\ 1\ 0) & (0\ 1\ 4) & (0\ 0\ 4) & (0\ 0\ 4) \\ (0\ \frac{\pi}{2}\ 0) & (0\ \frac{\pi}{2}\ 0) & (0\ 0\ 0) & (0\ 0\ 0) \\ (-\frac{\pi}{2}\ 0\ 0) & (-\frac{\pi}{2}\ 0\ 0) & (0\ 0\ 0) & (0\ 0\ 0) \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \\ 0.125 \\ -0.175 \end{bmatrix}$$

$$\text{or, } \Bigg[\overbrace{\left\{ (0.5)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 0.5\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 0.125\begin{pmatrix} 0 \\ \pi/2 \\ 0 \end{pmatrix} - 0.125\begin{pmatrix} -\pi/2 \\ 0 \\ 0 \end{pmatrix} \right\}}^{F_1(u)} \overbrace{\left\{ 0.5\begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} + 0.5\begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix} + 0.125\begin{pmatrix} 0 \\ \pi/2 \\ 0 \end{pmatrix} - 0.125\begin{pmatrix} -\pi/2 \\ 0 \\ 0 \end{pmatrix} \right\}}^{F_2(u)}$$

$$\underbrace{\left\{ 1.5\begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} + 0.5\begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} + 0.125\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + 0.125\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}}_{F_3(u)} \underbrace{\left\{ 0.5\begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} + 0.5\begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}}_{F_4(u)} \Bigg]$$

Multiplying (scalar) with blending functions of v

$$\left[\begin{pmatrix} 0.5 + 0.125\frac{\pi}{2} \\ 0.5 + 0.125\cdot\frac{\pi}{2} \\ 0 \end{pmatrix} \begin{pmatrix} 0.5 + 0.125\cdot\frac{\pi}{2} \\ 0.5 + 0.125\frac{\pi}{2} \\ 0.5\times 4 + 0.5\times 4 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0.5\times 4\times 2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0.5\times 4\times 2 \end{pmatrix}\right] \begin{bmatrix} 0.5 \\ 0.5 \\ 0.125 \\ -0.125 \end{bmatrix} \begin{array}{l} \rightarrow F_1(0.5) \\ \rightarrow F_2(0.5) \\ \rightarrow F_3(0.5) \\ \rightarrow F_4(0.5) \end{array}$$

$\uparrow$ $F_1(0.5)P$ $\quad$ $\uparrow$ $F_2(0.5)P$ $\quad$ $\uparrow$ $F_3(0.5)P$ $\quad$ $\uparrow$ $F_4(0.5)$ $\quad$ (4×1)

$(1\times 4) \leftarrow$ matrix

$$= \left[0.5\begin{pmatrix} 0.5 + 0.125\frac{\pi}{2} \\ 0.5 + 0.125\frac{\pi}{2} \\ 0 \end{pmatrix} + 0.5\begin{pmatrix} 0.5 + 0.125\cdot\frac{\pi}{2} \\ 0.5 + 0.125\frac{\pi}{2} \\ 0.5 * 4\times 2 \end{pmatrix} + 0.125\begin{pmatrix} 0 \\ 0 \\ 0.5\times 8 \end{pmatrix} - 0.125\begin{pmatrix} 0 \\ 0 \\ 0.5\times 8 \end{pmatrix}\right]$$

$$= \begin{bmatrix} 0.6963 \\ 0.6963 \\ 2 \end{bmatrix} \longrightarrow \text{Fergusson and Bicubic patch.}$$

Comparing with exact result we see that it i nearly equal to the original value which i $\begin{bmatrix} 0.7071 \\ 0.7071 \\ 2 \end{bmatrix}$