ME 535 Assignment 7 - Fall 2018

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Exercise 6.2(e) :

```
dsdu = [0.399 -0.399 -1.518];
dsdv = [-1.991 -1.991 0];
disp('The normal vector is given by')
```

```
The normal vector is given by
```

```
x = cross(dsdu,dsdv)
```

```
x =
   -3.0223    3.0223   -1.5888
```

```
suv = [1.309 -1.309 1.621];
y = dot(suv,x)
```

```
y = -10.4880
```

```
disp('The equation of the tangent plane is given as')
```

```
The equation of the tangent plane is given as
```

```
disp('-3.0223x + 3.0223y -1.5888z = -10.4880')
```

```
-3.0223x + 3.0223y -1.5888z = -10.4880
```

$-3.0223x + 3.0223y - 1.5888z = -10.4880$

Exercise 6.2 (f) :

Render this patch and its control net.

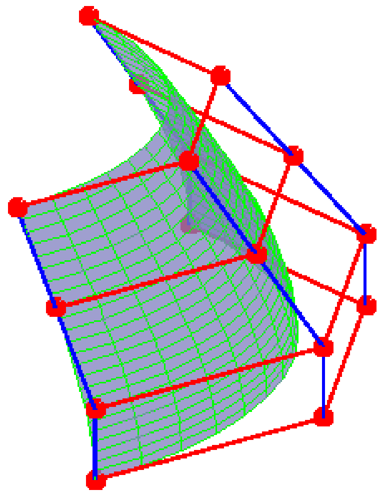
```
clear all, close all;

% control points A for a degree 3 * 3 patch
 P =  [1.5 0 2.4; 1.5 -0.84 2.4; 0.84 -1.5 2.4; 0 -1.5 2.4;
       1.75 0 1.875; 1.75 -0.98 1.875; 0.98 -1.75 1.875; 0 -1.75 1.875;
       2 0 1.35; 2 -1.12 1.35; 1.12 -2 1.35; 0 -2 1.35;
       2 0 0.9; 2 -1.12 0.9; 1.12 -2 0.9; 0 -2 0.90];  % Example problem
 nr = 4;
 nc = 4;

% surface evaluation
u = 1/2.; v = 1/2.;
tQ = deCasteljauSurf(P,nr, nc, u, v);


%number of sampled points
snr = 25; % number of sampled points in row (in u direction)
snc = 15; % number of sampled points in col (in v direction)
hold on;
Q = bezierSurf(P, nr, nc, snr, snc);
%plot the surface
bezierSurfPlot(P, Q, nr, nc, snr, snc);
view(3)

axis off;
```



Exercise 6.3

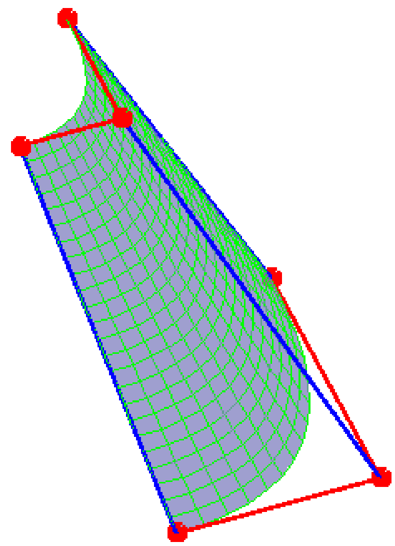
Render the B´ezier patch and its control net

```
clear all;
close all;
%close all;
figure(1)
% Control poits for a degree 2 * 1 patch
P1 = [0 -4 0; 1 -4 0; 2 0 0;
    0 -2 2; 0.5 -2 2; 1 0 2;];
 nr = 2;
 nc = 3;

% surface evaluation
u = 1/2.; v = 1/2.;
tQ = deCasteljauSurf(P1,nr, nc, u, v);


%number of sampled points
snr = 25; % number of sampled points in row (in u direction)
snc = 15; % number of sampled points in col (in v direction)
%hold on;
Q1 = bezierSurf(P1, nr, nc, snr, snc);
%plot the surface
bezierSurfPlot(P1, Q1, nr, nc, snr, snc);
view(3)

axis off;
```



Functions added:

deCasteljau

```
function [Q] = deCasteljau(P, u)
% computer point with paremeter value u on bezier curve defined by control points P
% P control points, in matrix format: {size of P} * {dimension of P}
% dimension of P is 2 or 3
% u parameter with value [0 1]
% Q point lying on the bezier curve
% input: P: control points P; u:  parameter value
% output: Q: the Bezier curve point at u
% example: >>
%       P = [0 0; 1 2; 3 5; 4,4; 5 0];
% call:  Q=deCasteljau(P, 0.5)
% output:Q =
%   2.6875    3.3750

% m: # of control points; m = the degree of the curve +1
[m, n] = size(P);
% if m <= 2
%     err('please specify more than 2 control points');
% end

if u < 0 | u > 1
    err('u must be in range from 0 to 1');
end

for i=1:(m-1)
    for j=1:(m-i) % the array index in Matlab starts with 1, not 0.
        P(j,:) = (1-u)*P(j,:) + u*P(j+1,:);
        % ':' meaning for all the columns: x, y, z
    end
end

Q=P(1,:);
end
```

Bezier Surf Plot:

```
function bezierSurfPlot(P, Q, nl, nc, snl, snc)
```

```
% plot the given control points P and points Q on beizer curve
% P control points
% Q points on bezier curve
% nr number of rows of control points
% nc number of columns of control points
% snl number of sampled points in u-dir
% snc number of sampled points in v-dir

hold on;
%plot control points
for i=0:(nl-1)
     PL = P((i*nc+1):(i+1)*nc,:);
     plot3(PL(:,1),PL(:,2),PL(:,3),'-ro', 'linewidth',2,'MarkerFaceColor', 'r', 'MarkerSize',8);
     hold on;
end

for i=0:(nc-1)
     k = (i+1):nc:(nc*nl);
     PL = P(k,:);
     plot3(PL(:,1),PL(:,2),PL(:,3),'-b', 'linewidth',2);
end

%plot surface points
% for i=0:(snl-1)
%      QL = Q((i*snc+1):(i+1)*snc,:);
%      plot3(QL(:,1),QL(:,2),QL(:,3),'-g');
%      hold on;
% end
%
% for i=0:(snc-1)
%      k = (i+1):snc:(snc*snl);
%      QL = Q(k,:);
%      plot3(QL(:,1),QL(:,2),QL(:,3),'-g');
% end

%figure(2);

[r,c] = size(Q);
out   = (reshape(Q,[snc,snl, c]));
 p = surf(out(:,:,1),out(:,:,2), out(:,:,3));

% style 1
%  p.EdgeColor=0.1*[1 1 1 ];
%  p.FaceAlpha=.75;
%  p.FaceColor='green';

 % style 2
p.FaceColor = [0.5 0.5 0.75];
p.FaceAlpha=.75; %0.75 %.75;
p.EdgeColor = 'green'; %'interp';
p.LineStyle = '-'; %'none';'



hold off;
end
```

deCasteljau surf function

```
function [Q] = deCasteljauSurf(P, nr, nc, u, v)
%P bezier control points with format [p0; p1; p2;....p(nr*nc)]
%every p(i) is of form (x, y, z) or (x, y)
%nr number of rows
%nc number of columns
%u  u-parameter value
```

```
%v  v-parameter value

for i=0:(nr-1)  %do decasteljau on every row in v-dir
    PR = P((i*nc+1):(i+1)*nc,:);  % i-th row CPs
    QR(i+1,:) = deCasteljau(PR, v);
end
Q = deCasteljau(QR, u);  %do decastlejau in u-dir
end
```

Bezier surf function

```
function [Q] = bezierSurf(P, nr, nc, snr, snc)
% return points on the surface.
% compute points on bezier surface defined by control points P
% P control points, in matrix format: {size(nr*nc) of P} * {dimension of P}
% nr number of rows of control points
% nc number of colums of control points
% snr the number of points need to be computed in row
% snc the number of points need to be computed in column
% Q points lying on the bezier surface

i = 1;
for u=0:(1/(snr-1)):1
    for v=0:(1/(snc-1)):1
    Q(i,:) = deCasteljauSurf(P,nr, nc, u, v);
    i = i + 1;
    end
end
end
```

6.2

(a) Bernstein functions can be written as:

$$B_{0,3}(u)\,B_{3,3}(v) = (1-u)^3 v^3 = (0.5)^3(0.5)^3 = \frac{1}{64}$$

$$B_{1,3}(u)\,B_{3,3}(v) = 3(1-u)^2 u v^3 = 3(0.5)^2(0.5)(0.5)^3 = \frac{3}{64}$$

$$B_{2,3}(u)\,B_{3,3}(v) = 3(1-u)u^2 v^3 = 3(0.5)(0.5)^2(0.5)^3 = \frac{3}{64}$$

$$B_{3,3}(u)\,B_{3,3}(v) = u^3 v^3 = (0.5)^3(0.5)^3 = \frac{1}{64}$$

at $u = v = 0.5$

$$\sum_{i=0}^{3} B_{i,3}(u)\,B_{3,3}(v) = \frac{1}{64} + \frac{3}{64} + \frac{3}{64} + \frac{1}{64} = \frac{1}{8}$$

(b) Surface point $S(u,v)$ =

$$[P] = \begin{bmatrix} (1.5\ \ 0\ \ 2.4) & (1.5\ \ -0.84\ \ 2.4) & (0.84\ \ -1.5\ \ 2.4) & (0\ \ -1.5\ \ 2.4) \\ (1.75\ \ 0\ \ 1.875) & (1.75\ \ -0.98\ \ 1.875) & (0.98\ \ -1.75\ \ 1.875) & (0\ \ -1.75\ \ 1.875) \\ (2\ \ 0\ \ 1.35) & (2\ \ -1.12\ \ 1.35) & (1.12\ \ -2\ \ 1.35) & (0\ \ -2\ \ 1.35) \\ (2\ \ 0\ \ 0.9) & (2\ \ -1.12\ \ 0.9) & (1.12\ \ -2\ \ 0.9) & (0\ \ -2\ \ 0.90) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} (1-u)^3 & 3(1-u)^2 u & 3(1-u)u^2 & u^3 \end{bmatrix} [P] \begin{bmatrix} (1-v)^3 \\ 3(1-v)^2 v \\ 3(1-v)v^2 \\ v^3 \end{bmatrix}$$

at $u = v = 0.5$

$$\begin{bmatrix} 1/8 & 3/8 & 3/8 & 1/8 \end{bmatrix} [P] \begin{bmatrix} 1/8 \\ 3/8 \\ 3/8 \\ 1/8 \end{bmatrix}$$

$$\Rightarrow \begin{array}{cccc} P_{i0} & P_{i1} & P_{i2} & P_{i3} \end{array}$$

$$\begin{bmatrix} \frac{1}{8}(1.5\ \ 0\ \ 2.4) & \frac{1}{8}(1.5\ \ -0.84\ \ 2.4) & \frac{1}{8}\begin{pmatrix} 0.84 \\ -1.5 \\ 2.4 \end{pmatrix} & \frac{1}{8}\begin{pmatrix} 0 \\ -1.5 \\ 2.4 \end{pmatrix} \\ +\frac{3}{8}(1.75\ \ 0\ \ 1.875) & +\frac{3}{8}(1.75\ \ -0.98\ \ 1.875) & +\frac{3}{8}(0.98\ \ -1.75\ \ 1.875) & +\frac{3}{8}\begin{pmatrix} 0 \\ -1.75 \\ 1.875 \end{pmatrix} \\ +\frac{3}{8}(2\ \ 0\ \ 1.35) & +\frac{3}{8}(2\ \ -1.12\ \ 1.35) & +\frac{3}{8}(1.12\ \ -2\ \ 1.35) & +\frac{3}{8}\begin{pmatrix} 0 \\ -2 \\ 1.85 \end{pmatrix} \\ +\frac{1}{8}(2\ \ 0\ \ 0.9) & +\frac{1}{8}(2\ \ -1.12\ \ 0.9) & +\frac{1}{8}\begin{pmatrix} 1.12 \\ -2 \\ 0.9 \end{pmatrix} & +\frac{1}{8}\begin{pmatrix} 0 \\ -2 \\ 0.9 \end{pmatrix} \end{bmatrix} \begin{bmatrix} 1/8 \\ 3/8 \\ 3/8 \\ 1/8 \end{bmatrix}$$

$$= \left[\begin{pmatrix} 1.84375 \\ 0 \\ 1.621875 \end{pmatrix} \begin{pmatrix} 1.84375 \\ -1.0325 \\ 1.621875 \end{pmatrix} \begin{pmatrix} 1.0325 \\ -1.84375 \\ 1.621875 \end{pmatrix} \begin{pmatrix} 0 \\ -1.84375 \\ 1.621875 \end{pmatrix} \right] \begin{bmatrix} 0.125 \\ 0.375 \\ 0.375 \\ 0.125 \end{bmatrix}$$

$$= \begin{pmatrix} 1.3090625 \\ -1.3090625 \\ 1.621875 \end{pmatrix} = S\left(\tfrac{1}{2}, \tfrac{1}{2}\right) \longrightarrow \text{Ans.}$$

(c)(i) $$\frac{\partial S(1/2, 1/2)}{\partial u} = \left[-3(1-u)^2 \quad 3(3u^2-4u+1) \quad 3(2u-3u^2) \quad 3u^2 \right]$$: Derivative of blending functions w.r.t. u

at $u = 0.5$ and $v = 0.5$

$$B_{i,m}(u) = \left[-\frac{3}{4} \quad -\frac{3}{4} \quad \frac{3}{4} \quad \frac{3}{4} \right] \qquad B_{i,m}(v) = \left[\frac{1}{8} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{1}{8} \right]^T$$

$$\Rightarrow \left[-\frac{3}{4} \quad -\frac{3}{4} \quad \frac{3}{4} \quad \frac{3}{4} \right] \begin{bmatrix} (1.5 \; 0 \; 2.4) & (1.5 \; -0.84 \; 2.4) & (0.84 \; -1.5 \; 2.4) & (0 \; -1.5 \; 2.4) \\ (1.75 \; 0 \; 1.875) & (1.75 \; -0.98 \; 1.875) & (0.98 \; -1.75 \; 1.875) & (0 \; -1.75 \; 1.875) \\ (2 \; 0 \; 1.35) & (2 \; -1.12 \; 1.35) & (1.12 \; -2 \; 1.35) & (0 \; -2 \; 1.35) \\ (2 \; 0 \; 0.9) & (2 \; -1.12 \; 0.9) & (1.12 \; -2 \; 0.9) & (0 \; -2 \; 0.90) \end{bmatrix} \begin{bmatrix} 1/8 \\ 3/8 \\ 3/8 \\ 1/8 \end{bmatrix}$$

$$\Rightarrow \left[\begin{pmatrix} 0.5625 \\ 0 \\ -1.51875 \end{pmatrix} \begin{pmatrix} 0.5625 \\ -0.315 \\ -1.51575 \end{pmatrix} \begin{pmatrix} 0.315 \\ -0.5625 \\ -1.51875 \end{pmatrix} \begin{pmatrix} 0 \\ -0.5625 \\ -1.5187 \end{pmatrix} \right] \begin{bmatrix} 1/8 \\ 3/8 \\ 3/8 \\ 1/8 \end{bmatrix} = \begin{pmatrix} 0.399375 \\ -0.399375 \\ -1.51875 \end{pmatrix} \rightarrow \text{Ans}$$

(d)(ii) $\frac{\partial s}{\partial v}(\frac{1}{2},\frac{1}{2}) = \left[-3(1-v)^2 \quad 3(3v^2-4v+1) \quad 3(2v-3v^2) \quad 3v^2\right]$.

at $u = 0.5$ and $v = 0.5$

$B_{i,m}(u) = \left[\frac{1}{8} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{1}{8}\right]$ $B_{i,m}(v) = \left[-\frac{3}{4} \quad -\frac{3}{4} \quad \frac{3}{4} \quad \frac{3}{4}\right]$

$$\Rightarrow \left[\frac{1}{8} \quad \frac{3}{8} \quad \frac{3}{8} \quad \frac{1}{8}\right] \begin{bmatrix} (1.5\ 0\ 2.1) & (1.5\ -0.84\ 2.1) & (0.84\ -1.5\ 2.1) & (0\ -1.5\ 2.1) \\ (1.75\ 0\ 1.875) & (1.75\ -0.98\ 1.875) & (0.98\ -1.75\ 1.875) & (0\ -1.25\ 1.875) \\ (2\ 0\ 1.35) & (2\ -1.12\ 1.35) & (1.12\ -2\ 1.35) & (0\ -2\ 1.35) \\ (2\ 0\ 0.9) & (2\ -1.12\ 0.9) & (1.12\ -2\ 0.9) & (0\ -2\ 0.90) \end{bmatrix} \begin{bmatrix} -\frac{3}{4} \\ -\frac{3}{4} \\ \frac{3}{4} \\ \frac{3}{4} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \begin{pmatrix} 1.84375 \\ 0 \\ 1.621875 \end{pmatrix} & \begin{pmatrix} 1.84375 \\ -1.0325 \\ 1.621875 \end{pmatrix} & \begin{pmatrix} 1.0325 \\ -1.84375 \\ 1.621875 \end{pmatrix} & \begin{pmatrix} \\ -1.84375 \\ 1.621875 \end{pmatrix} \end{bmatrix} \begin{bmatrix} -3/4 \\ -3/4 \\ 3/4 \\ 3/4 \end{bmatrix} = \begin{pmatrix} -1.99125 \\ -1.99125 \\ 0 \end{pmatrix} \rightarrow \text{Ans}$$

(d) $\frac{\partial^2 s}{\partial u \partial v}(\frac{1}{2},\frac{1}{2})$ at $u = 0.5$ & $v = 0.5$

$B_{i,m}(v) = \left[-\frac{3}{4} \quad -\frac{3}{4} \quad \frac{3}{4} \quad \frac{3}{4}\right]$.

$B_{i,m}(u) = \left[-\frac{3}{4} \quad -\frac{3}{4} \quad \frac{3}{4} \quad \frac{3}{4}\right]$

$$\Rightarrow \left[-\frac{3}{4} \quad -\frac{3}{4} \quad \frac{3}{4} \quad \frac{3}{4}\right] \begin{bmatrix} (1.5\ 0\ 2.1) & (1.5\ -0.84\ 2.4) & (0.84\ -1.5\ 2.4) & (0\ -1.5\ 2.4) \\ (1.75\ 0\ 1.875) & (1.75\ -0.98\ 1.875) & (0.98\ -1.75\ 1.875) & (0\ -1.25\ 1.5) \\ (2\ 0\ 1.35) & (2\ -1.12\ 1.35) & (1.12\ -2\ 1.35) & (0\ -2\ 1.35) \\ (2\ 0\ 0.9) & (2\ -1.12\ 0.9) & (1.12\ -2\ 0.9) & (0\ -2\ 0.9) \end{bmatrix} \begin{bmatrix} -\frac{3}{4} \\ -\frac{3}{4} \\ \frac{3}{4} \\ \frac{3}{4} \end{bmatrix}$$

$$\Rightarrow \left[\begin{pmatrix} 0.5625 \\ 0 \\ -1.51875 \end{pmatrix} \begin{pmatrix} 0.5625 \\ -0.315 \\ -1.51875 \end{pmatrix} \begin{pmatrix} 0.315 \\ -0.5625 \\ -1.51875 \end{pmatrix} \begin{pmatrix} 0 \\ -0.5625 \\ -1.51875 \end{pmatrix} \right] \begin{bmatrix} -3/4 \\ -3/4 \\ 3/4 \\ 3/4 \end{bmatrix} = \begin{pmatrix} -0.6075 \\ -0.6075 \\ 0 \end{pmatrix}$$

Ans.

(e) Tangent plane and surface normal at point $s(1/2, 1/2)$

Normal vector given as :

$$\frac{ds}{du} \times \frac{ds}{dv} = [-3.0223 \quad 3.0223 \quad -1.5888]$$

Tangent vector given as :

$$\{s(u,v)\} \cdot \left\{\frac{ds}{du} \times \frac{ds}{dv}\right\} = -3.0223x + 3.0223y - 1.5888z = -10.488$$

Ans.

[CALCULATION IN CODE].

6.3

- Parametric representation of a parabolic cone

$$S(u,v) = (u(2-v),\ 2(u^2-1)(2-v),\ 2v)$$

* With tensor product blossom form, blossoming in u or v can be represented as:

$$S(u,v) = (u(2-v),\ 2(u^2-1)(2-v),\ 2v).$$

$$b[u_1,u_2|v] = \left(\frac{u_1+u_2}{2} \times (2-v),\ 2(u_1u_2-1)(2-v),\ 2v\right)$$

* The control points can be obtained as

$$P_{0v} = b[0,0|v] = (0,-2(2-v),2v) = (0,2v-4,2v)$$

$\rightarrow P_{00} = (0,-4,0)$

$\rightarrow P_{01} = (0,-2,2)$

$$P_{1v} = b[0,1|v] = \left(\tfrac{1}{2}(2-v),-2(2-v),2v\right) = \left(\frac{2-v}{2},2v-4,2v\right)$$

$\rightarrow P_{10} = (1,-4,0)$

$\rightarrow P_{11} = (0.5,-2,2)$

$$P_{2v} = b[1,1|v] = (2-v,0,2v) = (2-v,0,2v)$$

$\rightarrow P_{20} = (2,0,0)$

$\rightarrow P_{21} = (1,0,2)$

* The Bezier representation:

$$\rightarrow \begin{bmatrix} -u^2 & 2u(1-u) & u^2 \end{bmatrix} \begin{bmatrix} (0\ -4\ 0) & (0\ -2\ 2) \\ (1\ -4\ 0) & (0.5\ -2\ 2) \\ (2\ 0\ 0) & (1\ 0\ 2) \end{bmatrix} \begin{bmatrix} (1-v) \\ (v) \end{bmatrix}$$