4.1

(a)

$$N_{i,p} = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1} + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}$$

$u = \{0, 1, 2, 3, 3, 4, 5, 6\}.$
$u_0\ u_1\ u_2\ u_3\ u_4\ u_5\ u_6\ u_7.$

$p = 2$

Solution: $(p=0)$

$i=0$ $N_{0,0} = \begin{cases} 1 & u \in [u_0, u_1) \\ 0 \end{cases}$

$i=1$ $N_{1,0} = \begin{cases} 1 & u \in [1, 2) \\ 0 \end{cases}$

$i=2$ $N_{2,0} = \begin{cases} 1 & u \in [2,3) \\ 0 \end{cases}$

$i=3$ $N_{3,0} = \begin{cases} 0 & u \in [3, 3) \\ 0 \end{cases}$

$i=4$ $N_{4,0} = \begin{cases} 1 & u \in [3, 4) \\ 0 \end{cases}$

$i=5$ $N_{5,0} = \begin{cases} 1 & u \in [4,5) \\ 0 \end{cases}$

$i=6$ $N_{6,0} = \begin{cases} 1 & u \in [5,6) \\ 0 \end{cases}$

$(p=1)$

$i=0$ $N_{0,1} = \frac{u - 0}{u_1 - u_0} N_{0,0}(u) + \frac{u_2 - u}{u_2 - u_1} N_{1,0}(u)$

$= \frac{u}{1 - 0} N_{0,0}(u) + \frac{2 - u}{2 - 1} N_{1,0}(u)$

$= u N_{0,0}(u) + (2 - u) N_{1,0}(u)$

$i=1$ $N_{1,1} = \frac{u - u_1}{u_2 - u_1} N_{1,0}(u) + \frac{u_3 - u}{u_3 - u_2} N_{2,0}(u)$

$= \frac{u - 1}{2 - 1} N_{1,0}(u) + \frac{3 - u}{3 - 2} N_{2,0}(u)$

$= (u-1) N_{1,0}(u) + (3-u) N_{2,0}(u)$

$i=2$ $N_{2,1} = \frac{u - u_2}{u_3 - u_2} N_{2,0}(u) + \frac{u_4 - u}{u_4 - u_3} N_{3,0}(u)$

$= \frac{u - 2}{3 - 2} N_{2,0}(u)$

$= (u-2) N_{2,0}(u).$

$i=3$ $$N_{3,1} = \frac{u-u_3}{u_4-u_3} N_{3,0}(u) + \frac{u_5-u}{u_5-u_4} N_{4,0}(u)$$

$$= (4-u)\, N_{4,0}(u).$$

$i=4$ $$N_{4,1} = \frac{u-u_4}{u_5-u_4} N_{4,0}(u) + \frac{u_6-u}{u_6-u_5} N_{5,0}(u)$$

$$= \frac{u-3}{4-3} N_{4,0}(u) + \frac{5-u}{5-4} N_{5,0}(u)$$

$$= (u-3)\, N_{4,0}(u) + (5-u)\, N_{5,0}(u).$$

$i=5$ $$N_{5,1} = \frac{u-u_5}{u_6-u_5} N_{5,0}(u) + \frac{u_7-u}{u_7-u_6} N_{6,0}(u)$$

$$= \frac{u-4}{5-4} N_{5,0}(u) + \frac{6-u}{6-5} N_{6,0}(u)$$

$$= (u-4)\, N_{5,0}(u) + (6-u)\, N_{6,0}(u).$$

$p=2$

$i=0$
$$N_{0,2} = \frac{u-u_0}{u_2-u_0} N_{0,1}(u) + \frac{u_3-u}{u_3-u_1} N_{1,1}(u).$$

$$= \frac{u-0}{2-1}\Big(u\, N_{0,0}(u) + (2-u)\, N_{1,0}(u)\Big) + \frac{3-u}{3-1}\Big((u-1)N_{1,0}(u) + (3-u)N_{2,0}(u)\Big)$$

$$= \frac{u}{2}\Big(u\, N_{0,0}(u) + (2-u)\, N_{1,0}(u)\Big) + \frac{3-u}{2}\Big((u-1)\, N_{1,0}(u) + (3-u)N_{2,0}(u)\Big)$$

$$= \underbrace{\frac{u^2}{2} N_{0,0}(u)}_{[0,1)} + \underbrace{\frac{u}{2}(2-u)N_{1,0}(u) + \frac{(3-u)(u-1)}{2} N_{1,0}(u)}_{[1,2)} + \underbrace{\frac{(3-u)^2}{2} N_{2,0}(u)}_{[2,3)}.$$

$i = 1$

$$N_{1,2} = \frac{u - u_1}{u_3 - u_1} N_{1,1}(u) + \frac{u_4 - u}{u_4 - u_2} N_{2,1}(u).$$

$$= \frac{u-1}{3-1}\left((u-1)N_{1,0}(u) + (3-u)N_{2,0}(u)\right) + \frac{3-u}{3-2}\left((u-2)N_{2,0}(u)\right)$$

$$= \left(\frac{u-1}{2}\right)\left((u-1)N_{1,0}(u) + (3-u)N_{2,0}(u)\right) + (3-u)(u-2)N_{2,0}(u).$$

$$= \underbrace{\frac{(u-1)^2}{2} N_{1,0}(u)}_{[1,2)} + \underbrace{\frac{(3-u)(u-1)}{2} N_{2,0}(u) + (3-u)(u-2)N_{2,0}(u)}_{[2,3)}.$$

$i = 2$

$$N_{2,2} = \frac{u - u_2}{u_4 - u_2} N_{2,1}(u) + \frac{u_5 - u}{u_5 - u_3} N_{3,1}(u).$$

$$= \frac{u-2}{3-2} N_{2,1}(u) + \frac{4-u}{4-3.} N_{3,1}(u).$$

$$= \frac{u-2}{3-2}\left((u-2)N_{2,0}(u)\right) + \frac{4-u}{4-3}\left((4-u)N_{4,0}(u)\right)$$

$$= \underbrace{(u-2)^2 N_{2,0}(u)}_{[2,3)} + \underbrace{(4-u)^2 N_{4,0}(u)}_{[4,5)}.$$

$i = 3$

$$N_{3,2} = \frac{u - u_3}{u_5 - u_3} N_{3,1}(u) + \frac{u_6 - u}{u_6 - u_4} N_{4,1}(u)$$

$$= \frac{u-3}{4-3} N_{3,1}(u). + \frac{5-u}{5-3} N_{4,1}(u)$$

$$= (u-3)\left((4-u)N_{4,0}(u)\right) + \frac{5-u}{2}\left((u-3)N_{4,0}(u) + (5-u)N_{5,0}(u)\right)$$

$$= \underbrace{\left((u-3)(4-u) + \frac{(5-u)(u-3)}{2}\right) N_{4,0}(u)}_{[4,5)} + \underbrace{\frac{(5-u)^2}{2} N_{5,0}(u)}_{[5,6)}.$$

$i=4$

$$N_{4,2} = \frac{u-u_4}{u_6-u_4} N_{4,1}(u) + \frac{u_7-u}{u_7-u_5} N_{5,1}(u).$$

$$= \frac{u-3}{5-3}\left((u-3)N_{4,0}(u) + (5-u)N_{5,0}(u)\right) + \frac{6-u}{6-4}\left((u-4)N_{5,0}(u) + (6-u)N_{6,0}(u)\right)$$

$$= \frac{(u-3)^2}{2} N_{4,0}(u) + \frac{(u-3)(5-u)}{2} N_{5,0}(u) + \frac{(6-u)(u-4)}{2} N_{5,0}(u) + \frac{(6-u)^2}{2} N_{6,0}(u)$$

$$= \underbrace{\frac{(u-3)^2}{2} N_{4,0}(u)}_{[4,5)} + \underbrace{\left(\frac{(u-3)(5-u)+(6-u)(u-4)}{2}\right)}_{[5,6)} N_{5,0}(u) + \underbrace{\frac{(6-u)^2}{2}}_{[6,7)} N_{6,0}(u)$$

$\boxed{4.1.b}$

Values at $u = 2.5$:

Since the region of influence lies in $[2,3)$ we consider specific contributions:

from $N_{0,2} = \frac{(3-u)^2}{2} N_{2,0}(u)$

from $N_{1,2} = \frac{(3-u)(u-1)}{2} N_{2,0}(u) + (3-u)(u-2)\, N_{2,0}(u)$

from $N_{2,2} = (u-2)^2 N_{2,0}(u)$.

Summing them: $N_{0,2} + N_{1,2} + N_{2,2}$ within $[2,3)$

$$= \frac{(3-2.5)^2}{2} + (3-u)\left(\frac{u-1}{2} + u-2\right) + (2.5-2)^2$$

$$= (0.5)^2 \frac{3}{2} + (0.5)\left(\frac{7.5-5}{2}\right) = 0.5\left(\frac{1.5}{2} + \frac{2.5}{2}\right) = 1$$

Satisfying the "partition of unity" condition.

$\boxed{4.1.d}$

$$c(u) = \sum_{i=0}^{m} p_i N_{i,p}(u). \qquad \left[p_0 = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \quad p_1 = \begin{Bmatrix} 1.5 \\ 0.7 \end{Bmatrix} \quad p_2 = \begin{Bmatrix} 3 \\ 0.85 \end{Bmatrix} \quad p_3 = \begin{Bmatrix} 3.5 \\ 1.2 \end{Bmatrix} \quad p_4 = \begin{Bmatrix} 5 \\ 1.1 \end{Bmatrix} \right]$$

$$= \left(\frac{u^2}{2} N_{0,0}(u) + \frac{u}{2}(2-u) N_{1,0}(u) + \frac{(3-u)(u-1)}{2} N_{1,0}(u) + \frac{(3-u)^2}{2} N_{2,0}(u) \right) p_0$$

$$+ \left(\frac{(u-1)^2}{2} N_{1,0}(u) + \left[\frac{(3-u)(u-1)}{2} + (3-u)(u-2) \right] N_{2,0}(u) \right) p_1$$

$$+ \left((u-2)^2 N_{2,0}(u) + (4-u)^2 N_{4,0}(u) \right) p_2$$

$$+ \left(\left[(u-3)(4-u) + \frac{(5-u)(u-3)}{2} \right] N_{4,0}(u) + \frac{(5-u)^2}{2} N_{5,0}(u) \right) p_3$$

$$+ \left(\frac{(u-3)^2}{2} N_{4,0}(u) + \frac{(u-3)(5-u) + (6-u)(u-4)}{2} N_{5,0}(u) + \frac{(6-u)^2}{2} N_{6,0}(u) \right) p_4$$

FOR $u = 2.5$ Domain : $[2,3)$

$$c(2.5) = \frac{(3-u)^2}{2} p_0 + \left[\frac{(3-u)(u-1)}{2} + (3-u)(u-2) \right] p_1 + (u-2)^2 p_2$$

$$= \frac{(3-2.5)^2}{2} \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} + (0.5)\left(\frac{2.5}{2}\right) \begin{Bmatrix} 1.5 \\ 0.7 \end{Bmatrix} + (2.5-2)^2 \begin{Bmatrix} 3 \\ 0.85 \end{Bmatrix}$$

$$= \frac{(0.5)^2}{2} \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} + \left(\frac{1.25}{2}\right) \begin{Bmatrix} 1.5 \\ 0.7 \end{Bmatrix} + (0.5)^2 \begin{Bmatrix} 3 \\ 0.85 \end{Bmatrix} = \begin{Bmatrix} 1.6875 \\ 0.775 \end{Bmatrix} \rightarrow ①$$

FOR $u = 3.5$ Domain : $[3,4)$

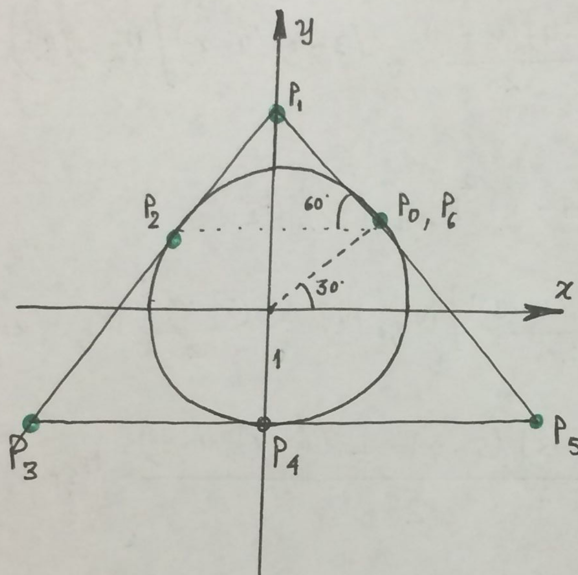
$$c(3.5) = (4-u)^2 p_2 + \left[(u-3)(4-u) + \frac{(5-u)(u-3)}{2} \right] p_3 + \frac{(u-3)^2}{2} p_4$$

$$= (0.5)^2 \begin{Bmatrix} 3 \\ 0.85 \end{Bmatrix} + \left(\frac{1.25}{2}\right) \begin{Bmatrix} 3.5 \\ 1.2 \end{Bmatrix} + \frac{(0.5)^2}{2} \begin{Bmatrix} 5 \\ 1.1 \end{Bmatrix} = \begin{Bmatrix} 3.5625 \\ 1.1 \end{Bmatrix} \rightarrow ②$$

$$c(2.5) = \begin{pmatrix} 1.6875 \\ 0.775 \end{pmatrix}$$

$$c(3.5) = \begin{pmatrix} 3.5625 \\ 1.1 \end{pmatrix}$$

4.2



Given:
- Radius of circle : 1
- Equilateral triangle

a. Co-ordinates of control points:

$P_0 : (1\cos 30, 1\sin 30) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$P_1 : (0, 1\operatorname{cosec} 30) = (0, 2)$

$P_2 : (-1\cos 30, 1\sin 30) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$P_3 : (-3\cot 60, -1) = (-\sqrt{3}, -1)$

$P_4 : (0, -1) = (0, -1)$

$P_5 : (3\cot 60, -1) = (\sqrt{3}, -1)$

$P_6 : (1\cos 30, 1\sin 30) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

b. Weights

$w_0 : 1$

$w_1 : \cos 60 = \frac{1}{2}$

$w_2 : 1$

$w_3 : \frac{1}{2}$

$w_4 : 1$

$w_5 : \frac{1}{2}$

$w_6 : 1$

$\boxed{4.2\ (c)}$:

$$U = \{0, 0, 0, 1, 1, 2, 2, 3, 3, 3\}.$$

$$N_{i,p} = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u).$$

$\boxed{p=0}$

$N_{0,0} = 0$

$N_{1,0} = 0$

$N_{2,0} = 1 \quad u \in [u_2, u_3)$

$N_{3,0} = 0$

$N_{4,0} = 1 \quad u \in [u_4, u_5)$

$N_{5,0} = 0$

$N_{6,0} = 1 \quad u \in [u_6, u_7)$

$N_{7,0} = 0 \quad u \in [u_7, u_8)$

$N_{8,0} = 0 \quad u \in [u_8, u_9)$

$\boxed{p=1}$

$N_{0,1} = 0$

$N_{1,1} = \frac{u_3 - u}{u_3 - u_2} N_{2,0} = (1-u) N_{2,0}$

$N_{2,1} = \frac{u}{1} N_{2,0} + \frac{u_4 - u}{u_4 - u_3} N_{3,0} = u N_{2,0}$

$N_{3,1} = \frac{u_5 - u}{u_5 - u_4} N_{4,0} = \frac{2-u}{2-1} N_{4,0} = (2-u) N_{4,0}$

$N_{4,1} = \frac{u - u_4}{u_5 - u_4} N_{4,0}. = (u-1) N_{4,0}$

$N_{5,1} = \frac{u_7 - u}{u_7 - u_6} N_{6,0} = (3-u) N_{6,0}$

$N_{6,1} = \frac{u - u_6}{u_7 - u_6} N_{6,0} + \frac{u_8 - u}{u_8 - u_7} N_{7,0} = (u-2) N_{6,0}$

$N_{7,1} = 0$

$\boxed{p=2}$

$N_{0,2} = (1-u)((1-u) N_{2,0}) = (1-u)^2 N_{2,0}$

$N_{1,2} = u(1-u) N_{2,0} + (1-u) u N_{2,0} = 2u(1-u) N_{2,0}$

$N_{2,2} = u(u N_{2,0}) + (2-u)((2-u)) N_{4,0} = u^2 N_{2,0} + (2-u)^2 N_{4,0}$

$N_{3,2} = (u-1)((2-u) N_{4,0}) + (2-u)((u-1) N_{4,0}) = 2(u-1)(2-u) N_{4,0}$

$N_{4,2} = (u-1)^2 N_{4,0} + (3-u)^2 N_{6,0}$

$N_{5,2} = 2(u-2)(3-u) N_{6,0}$; $N_{6,2} = (u-2)^2 N_{6,0}$

Computing curve points at $u = 0.25$; $u = 0.5$; $u = 0.75$

It has a common domain $u \in [0, 1)$

$$C(u) = \frac{(1-u)^2 N_{2,0} \begin{Bmatrix} \sqrt{3}/2 \\ 1/2 \end{Bmatrix} 1 + 2u(1-u) N_{2,0} \begin{Bmatrix} 0 \\ 2 \end{Bmatrix} \frac{1}{2} + u^2 N_{2,0} \begin{Bmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{Bmatrix} \cdot 1}{(1-u)^2 N_{2,0} + 2u(1-u) N_{2,0} + u^2 N_{2,0}}$$

For $u = 0.25$: $\dfrac{\begin{pmatrix} 0.433 \\ 0.6875 \end{pmatrix}}{(0.8125)} = \begin{pmatrix} 0.5329 \\ 0.8461 \end{pmatrix}$

For $u = 0.5$ $\dfrac{1}{0.8125} \begin{pmatrix} 0 \\ 0.8125 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

For $u = 0.75$ $\dfrac{1}{0.8125} \begin{pmatrix} -0.433 \\ 0.6875 \end{pmatrix} = \begin{pmatrix} -0.5329 \\ 0.8461 \end{pmatrix}$

Curve point :

$C(0.25) = (0.5329, 0.8461)$

$C(0.50) = (0, 1)$

$C(0.75) = (-0.5329, 0.8461)$