ME535 : ASSIGNMENT NO. 4

DEBABRATA AUDDYA

5.1

Given control points of the problem:

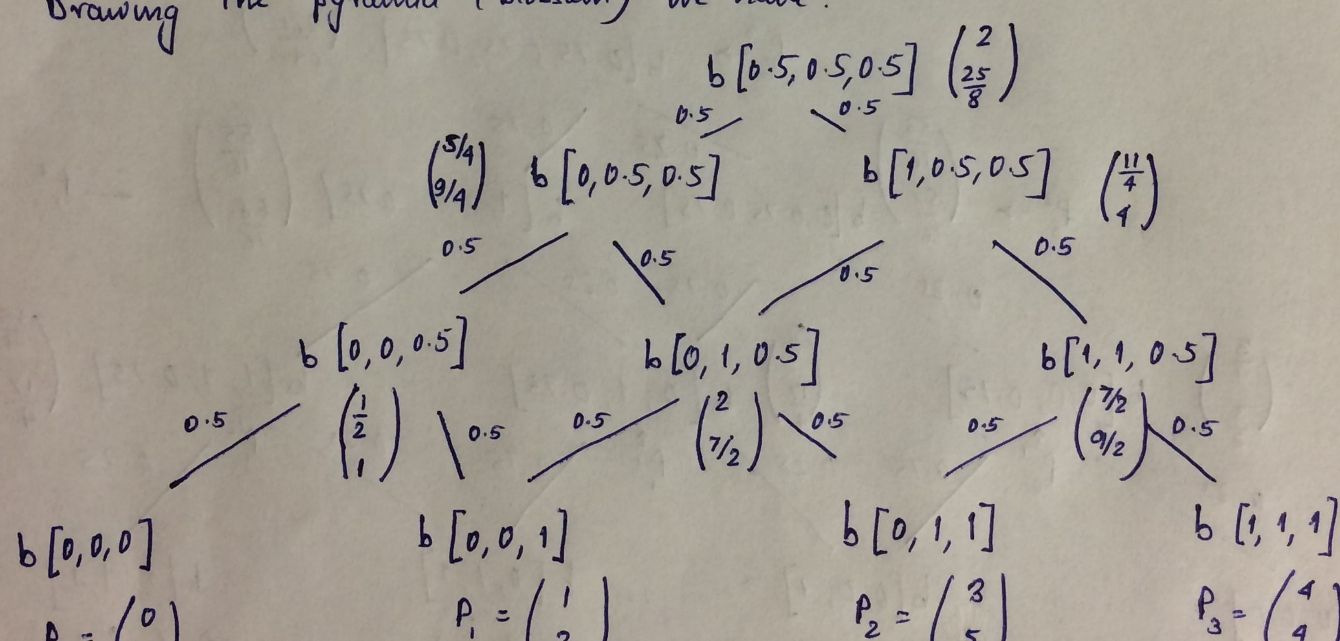
$$P = \begin{bmatrix} 0 & 0 \\ 1 & 2 \\ 3 & 5 \\ 4 & 4 \end{bmatrix} \quad \text{or,} \quad (0,0)\ ;\ (1,2)\ ;\ (3,5)\ ;\ (4,4)$$

(a) According to the formula : (at $u = 0.5$)

$$c'(u) = db\left[\left(\tfrac{1}{2}\right)^{\langle 2 \rangle}, \vec{1}\right]$$

$$= 3\left[b\left[\left(\tfrac{1}{2}\right)^{\langle 2 \rangle}, 1\right] - b\left[\left(\tfrac{1}{2}\right)^{\langle 2 \rangle}, 0\right]\right]$$

Drawing the pyramid (blossam) we have:

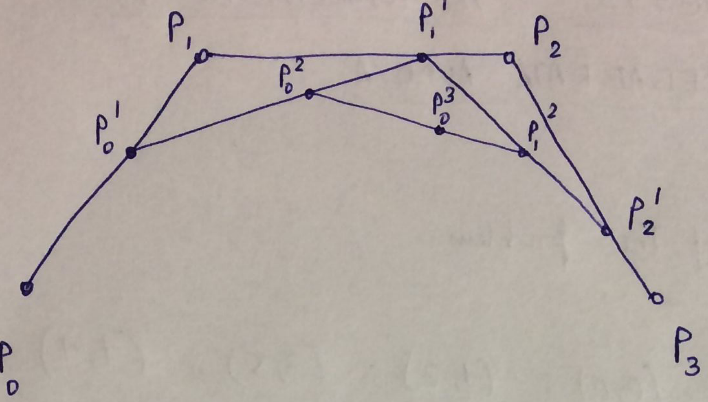


From the figure we have:

$$c'\left(\tfrac{1}{2}\right) = 3\left[\begin{pmatrix} \frac{11}{4} \\ 4 \end{pmatrix} - \begin{pmatrix} \frac{5}{4} \\ \frac{9}{4} \end{pmatrix}\right] = 3\begin{pmatrix} \frac{6}{4} \\ \frac{7}{4} \end{pmatrix} = \begin{pmatrix} \frac{9}{2} \\ \frac{21}{4} \end{pmatrix}$$

Ans: $c'(0.5) = \left(\frac{9}{2}, \frac{21}{4}\right) = \boxed{(4.5,\ 5.25)}$

5.1 (c)



We have: $P_0^1 : b[0, 0, 0.75]$

$P_1^1 : b[0, 1, 0.75]$ $\quad P_0^2 : b[0, 0.75, 0.75]$

$P_2^1 : b[1, 1, 0.75]$ $\quad P_1^2 : b[1, 0.75, 0.75]$ $\quad P_0^3 : [0.75\ 0.75\ 0.75]$

Using Pyramid representation.

For $u = 0.75$

$P_0^3 \rightarrow b[0.75, 0.75, 0.75] \begin{pmatrix} \frac{198}{64} \\ \frac{261}{64} \end{pmatrix}$

0.25   0.75

$P_0^2 \rightarrow \begin{pmatrix} \frac{33}{16} \\ \frac{57}{16} \end{pmatrix} b[0, 0.75, 0.75]$ $\quad b[1, 0.75, 0.75] \begin{pmatrix} \frac{55}{16} \\ \frac{68}{16} \end{pmatrix} \leftarrow P_1^2$

0.25   0.75   0.25   0.75

$\begin{pmatrix} \frac{3}{4} \\ \frac{3}{2} \end{pmatrix} b[0, 0, 0.75]$ $\quad \begin{pmatrix} \frac{5}{2} \\ \frac{17}{4} \end{pmatrix} b[0, 1, 0.75]$ $\quad b[1, 1, 0.75] \begin{pmatrix} \frac{15}{4} \\ \frac{17}{4} \end{pmatrix}$

$\uparrow P_0^1$ $\quad \uparrow P_1^1$ $\quad \uparrow P_2^1$

0.25   0.75   0.25   0.75   0.25   0.75

$b[0, 0, 0]$ $\quad b[0, 0, 1]$ $\quad b[0, 1, 1]$ $\quad b[1, 1, 1]$

$P_0 \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\quad P_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\quad P_2 \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ $\quad P_3 \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

The ~~a~~pyramid is shown above gives control points and the interpolated points.

From the first figure we see that the point P_0^3 divides the curve into two distinct regions:

$$P_0,\ P_0^1,\ P_0^2,\ P_0^3 \quad \& \quad P_0^3,\ P_1^2,\ P_2^1,\ P_3.$$

Whose co-ordinates are given below:

~~Co ar~~

Control points for first Bezier curve	Control Points for Second Bezier curve
$P_0 : \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$P_0^3 : \begin{pmatrix} 198/64 \\ 261/64 \end{pmatrix}$
$P_0^1 : \begin{pmatrix} 3/4 \\ 3/2 \end{pmatrix}$	$P_1^2 : \begin{pmatrix} 55/16 \\ 68/16 \end{pmatrix}$
$P_0^2 : \begin{pmatrix} 33/16 \\ 57/16 \end{pmatrix}$	$P_2^1 : \begin{pmatrix} 15/4 \\ 17/4 \end{pmatrix}$
$P_0^3 : \begin{pmatrix} 198/64 \\ 261/64 \end{pmatrix}$	$P_3 : \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

5.1 (d)

Raising the degree of curve by 1 we have:

$$b[0,0,0,0] = \frac{1}{4}\cdot 4b[0,0,0] = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$b[0,0,0,1] = \frac{1}{3+1}\left[3b[0,0,\cancel{0}] + b[0,0,0]\right] = \frac{1}{4}\left[3\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right] = \begin{pmatrix} \frac{3}{4} \\ \frac{6}{4} \end{pmatrix}$$

$$b[0,0,1,1] = \frac{1}{3+1}\left[2b[0,1,1] + 2b[0,0,1]\right] = \frac{1}{4}\left[2\begin{pmatrix} 3 \\ 5 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 2 \end{pmatrix}\right] = \begin{pmatrix} \frac{8}{4} \\ \frac{14}{4} \end{pmatrix}$$

$$b[0,1,1,1] = \frac{1}{3+1}\left[b[1,1,1] + 3b[0,1,1]\right] = \frac{1}{4}\left[\begin{pmatrix} 4 \\ 4 \end{pmatrix} + 3\begin{pmatrix} 3 \\ 5 \end{pmatrix}\right] = \begin{pmatrix} \frac{13}{4} \\ 19/4 \end{pmatrix}$$

$$b[1,1,1,1] = \frac{1}{3+1}\left[4\left(b[1,1,1]\right)\right] = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

We have points: $(0,0)$; $\left(\frac{3}{4}, \frac{6}{4}\right)$; $\left(2, \frac{7}{2}\right)$; $\left(\frac{13}{4}, \frac{19}{4}\right)$; $(4,4)$

ME 535 Assignment 4, Fall 2018

Debabrata Auddya

Q 5.1(d) Raise the degree of the B´ezier curve by 1 and give the control points for this curve. Draw the original and new control points and the two curves in the same plot.

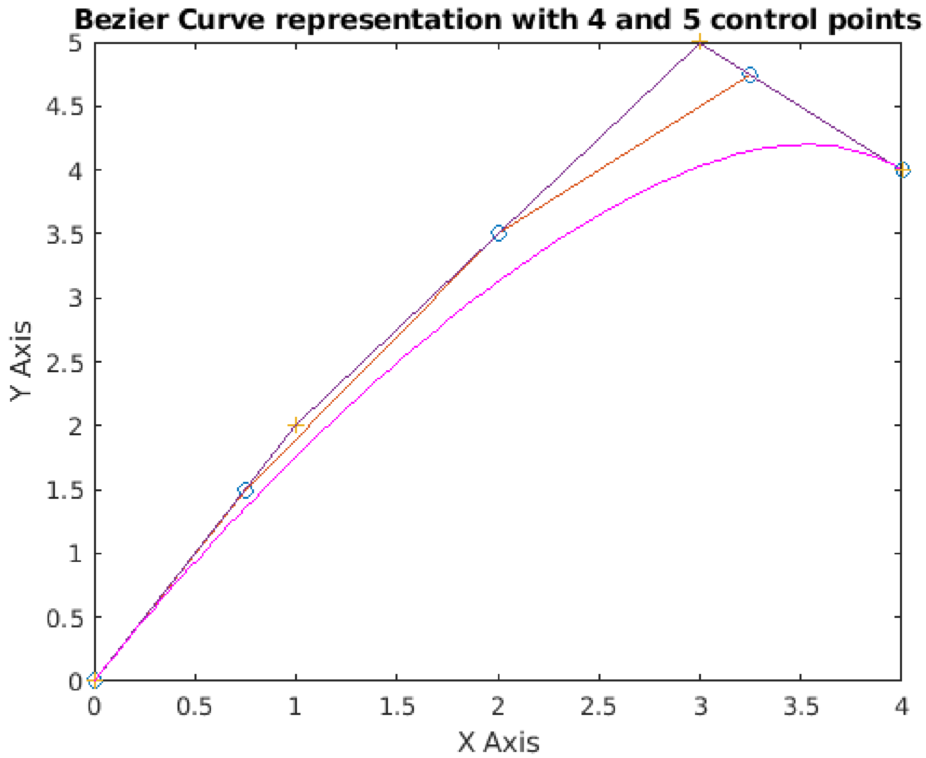
```
%Question 5.1 (d)
% Bezier curve with 5 control points
% Bezier curve with 4 control points
% Magenta line represents curve

P = [0 0; 3/4 6/4; 2 7/2; 13/4 19/4; 4 4]
```

```
P =

         0         0
    0.7500    1.5000
    2.0000    3.5000
    3.2500    4.7500
    4.0000    4.0000
```

```
plot(P(:,1),P(:,2),'o');
hold on
plot(P(:,1),P(:,2));
n = 100;
i = 1;
for u=0:(1/(n-1)):1
    Q(i,:) = deCasteljau(P, u);
    i = i + 1;
    plot(Q(:,1),Q(:,2),'b')
    hold on
end

P = [0 0; 1 2; 3 5; 4 4]
```

```
P =

     0     0
     1     2
     3     5
     4     4
```

```
plot(P(:,1),P(:,2),'+');
hold on
plot(P(:,1),P(:,2));
n = 100;
i = 1;
for u=0:(1/(n-1)):1
    Q(i,:) = deCasteljau(P, u);
    i = i + 1;
    plot(Q(:,1),Q(:,2),'m')
    hold on
end

title("Bezier Curve representation with 4 and 5 control points");
xlabel("X Axis");
ylabel("Y Axis");
```



Using deCasteljau function (mentioned below)

```
function [Q] = deCasteljau(P, u)
% computer point with parameter value u on bezier curve defined by control points P
% P control points, in matrix format: {size of P} * {dimension of P}
% dimension of P is 2 or 3
% u parameter with value [0 1]
% Q point lying on the bezier curve

% input: P: control points P; u:  parameter value
% output: Q: the Bezier curve point at u

% example: >> P = [0 0; 1 2; 3 5; 4,4; 5 0];
% >> Q=deCasteljau(P, 0.5)
% output:Q =
%   2.6875    3.3750

% m: # of control points; m = the degree of the curve +1
[m, n] = size(P);
if m <= 1
    err('please specify at least 2 control points');
end

if u < 0 | u > 1
    err('u must be in range from 0 to 1');
end

d = m-1; % degree

for r=1:d
    for i=1:(d+1-r) % the array index in Matlab starts with 1, not 0.
        P(i,:) = (1-u)*P(i,:) + u*P(i+1,:);
        % ':' operator on all columns: x, y, z
```

```
    end
end
Q=P(1,:);
end
```

5.2

Equation is : $f(t) = 5t^3 - 4t^2 - 4t + 5$

(a) Blossom in four variables can be written as :

$$f(t_1, t_2, t_3, t_4) =$$

$$\frac{5(t_1 t_2 t_3 + t_2 t_3 t_4 + t_3 t_4 t_1 + t_4 t_1 t_2)}{4}$$

$$- \frac{4(t_1 t_2 + t_2 t_3 + t_3 t_4 + t_4 t_1 + t_2 t_4 + t_1 t_3)}{6}$$

$$- \frac{4(t_1 + t_2 + t_3 + t_4)}{4}$$

$$+ 5$$

b) Using the blossom above we can write Bezier ordinates for a quartic Bezier representation of function :

$f_0 = f(0,0,0,0) = 5 \quad [t_1 = t_2 = t_3 = t_4 = 0]$

$f_1 = f(0,0,0,1) = 5 - \frac{4}{4} = 5 - 1 = 4 \quad [t_1 = t_2 = t_3 = 0 \; ; t_4 = 1]$

$f_2 = f(0,0,1,1) = 5 - \frac{4}{2} - \frac{4}{6} = 3 - \frac{2}{3} = \frac{7}{3} \quad [t_1 = t_2 = 0 \; ; t_3 = t_4 = 1]$

$f_3 = f(0,1,1,1) = \frac{5}{4} - \frac{4}{2} - 3 + 5 = \frac{5}{4} \quad [t_1 = 0 \; ; t_2 = t_3 = t_4 = 1]$

$f_4 = f(1,1,1,1) = 5 - 4 - 4 + 5 = 2 \quad [t_1 = t_2 = t_3 = t_4 = 1]$

Hence Bezier form: $f_0 (1-t)^4 + f_1 \times 4(1-t)^3 t + f_2 \times 6(1-t)^2 t^2 + f_3 \times 4(1-t) t^3 + f_4 t^4$

$f(0,0,0) = 5$

$f(0,0,1) = 5 - \frac{4}{3} = \frac{11}{3}$

$f(0,1,1) = -\frac{4}{3} - \frac{8}{3} + 5 = 1$

$f(1,1,1) = 5 - 4 - 4 + 5 = 2$

$\Rightarrow f_0(1-t)^3 + f_1 \cdot 3(1-t)^2 t + f_2 \cdot 3(1-t)t^2 + f_3 t^3$

In Bezier form: Expression

$$5(1-t)^3 + 11(1-t)^2 t + 3(1-t)t^2 + 2t^3.$$

Raising the degree:

$f[0,0,0,0] = 5$

$f[0,0,0,1] = \frac{1}{3+1}\left[3f(0,0,1) + f(0,0,0)\right] = \frac{1}{4}(11+5) = 4$

$f[0,0,1,1] = \frac{1}{3+1}\left[2f(0,1,1) + 2f(0,0,1)\right] = \frac{1}{4}\left(2 + \frac{22}{3}\right) = \frac{28}{12} = \frac{7}{3}$

$f[0,1,1,1] = \frac{1}{3+1}\left[f(1,1,1) + 3f(0,1,1)\right] = \frac{1}{4}(2 + 3 \cdot 1) = \frac{5}{4}$

$f[1,1,1,1] = f[1,1,1] = 2$

Raising the degree from 3 to 4 we have:

$$5(1-t)^4 + 16(1-t)^3 t + 14(1-t)^2 t^2 + 5(1-t)t^3 + 2t^3.$$

(4th degree: Quartic Bezier Curve)

From the first figure we see that the point P_0^3 divides the curve into two distinct regions:

$$P_0,\ P_0^1,\ P_0^2,\ P_0^3 \quad \& \quad P_0^3,\ P_1^2,\ P_2^1,\ P_3.$$

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