ME535 Assignment 9

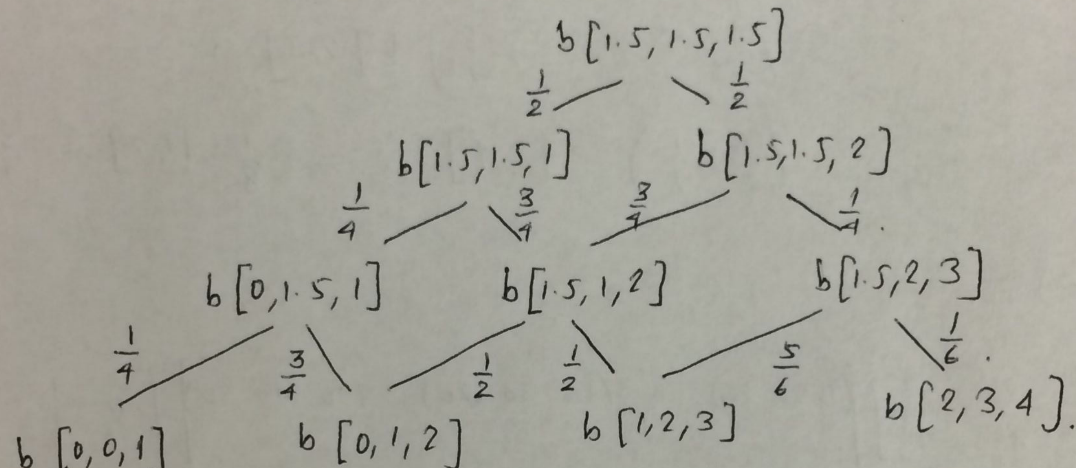
6.7.

Knot vectors in u : $[0, 0, 0, 1, 2, 3, 4, 4, 4]$

Knot vectors in v : $[0, 0, 1, 2, 3, 4, 5, 5]$

For $u \in [1, 2]$ and $v \in [2, 3]$.

For u the blossom: $[u = 1.5]$



$$b[1.5, 1.5, 1.5] = \left(\tfrac{1}{2} \times \tfrac{1}{4} \times \tfrac{1}{4}\right) b[0, 0, 1] +$$

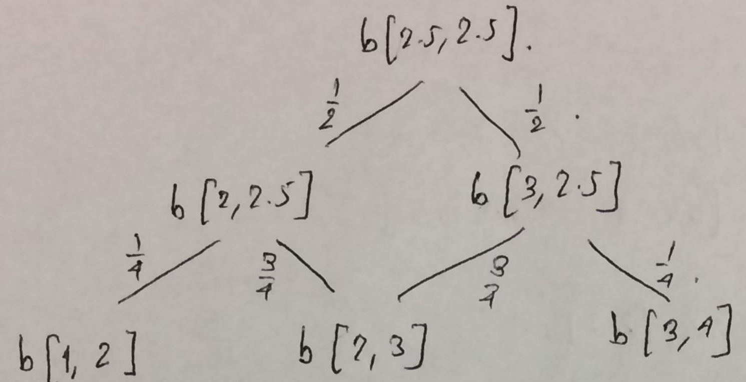
$$\left(\tfrac{1}{2} \times \tfrac{1}{4} \times \tfrac{3}{4} + \tfrac{1}{2} \times \tfrac{3}{4} \times \tfrac{1}{2} + \tfrac{1}{2} \times \tfrac{3}{4} \times \tfrac{1}{2}\right) b[0, 1, 2] +$$

$$\left(\tfrac{1}{2} \times \tfrac{3}{4} \times \tfrac{1}{2} + \tfrac{1}{2} \times \tfrac{3}{4} \times \tfrac{1}{2} + \tfrac{1}{2} \times \tfrac{1}{4} \times \tfrac{5}{6}\right) b[1, 2, 3] +$$

$$\left(\tfrac{1}{2} \times \tfrac{1}{4} \times \tfrac{1}{6}\right) b[2, 3, 4].$$

$$= \left(\tfrac{1}{32}\right) b[0, 0, 1] + \left(\tfrac{15}{32}\right) b[0, 1, 2] + \left(\tfrac{23}{48}\right) b[1, 2, 3] + \tfrac{1}{48} b[2, 3, 4$$

Blossom along v :



$$b[2.5, 2.5] = \left(\frac{1}{2} \times \frac{1}{4}\right) b[1,2] + \left(\frac{1}{2} \times \frac{3}{4} + \frac{1}{2} \times \frac{3}{4}\right) b[2,3] + \left(\frac{1}{2} \times \frac{1}{4}\right) b[3,4] = \frac{1}{8} b[1,2] + \frac{3}{4} b[2,3] + \frac{1}{8} b[3,4]$$

$\therefore \; s(1.5, 2.5) =$

$$= \begin{bmatrix} \frac{1}{32} & \frac{15}{32} & \frac{23}{48} & \frac{1}{48} \end{bmatrix} \begin{bmatrix} (10\ 20\ 20) & (10\ 30\ 20) & (10\ 40\ 30) \\ (20\ 20\ 40) & (20\ 30\ 35) & (20\ 40\ 35) \\ (30\ 20\ 45) & (30\ 30\ 40) & (30\ 40\ 35) \\ (40\ 20\ 35) & (40\ 30\ 45) & (40\ 40\ 50) \end{bmatrix} \begin{bmatrix} 1/8 \\ 3/4 \\ 1/8 \end{bmatrix}$$

$$= \begin{bmatrix} \begin{pmatrix} 24.8958 \\ 20 \\ 41.1667 \end{pmatrix} & \begin{pmatrix} 24.8958 \\ 30 \\ 37.1354 \end{pmatrix} & \begin{pmatrix} 24.8958 \\ 40 \\ 35.1562 \end{pmatrix} \end{bmatrix} \begin{bmatrix} 1/8 \\ 3/4 \\ 1/8 \end{bmatrix} = \begin{pmatrix} 24.8958 \\ 30 \\ 37.4544 \end{pmatrix} \rightarrow s(1.5, 2.5)$$

Ans.

$$\frac{\partial S}{\partial u} = 3b[1.5, 1.5, \vec{T}] = \left(-1 \times \frac{1}{4} \times \frac{1}{4}\right) b[0,0,1] +$$

$$\left(-1 \times \frac{1}{4} \times \frac{3}{4} + (-1)\frac{3}{4}\cdot\frac{1}{2} + 1\cdot\frac{3}{4}\cdot\frac{1}{2}\right) b[0,1,2] +$$

$$\left(-1 \times \frac{3}{4} \times \frac{1}{2} + (1)\frac{3}{4}\cdot\frac{1}{2} + 1\cdot\frac{1}{4}\cdot\frac{5}{6}\right) b[1,2,3] +$$

$$\left(1\cdot\frac{1}{4}\cdot\frac{1}{6}\right) b[2,3,4]$$

$$= 3\begin{bmatrix} -\frac{1}{16} & -\frac{3}{16} & \frac{5}{24} & \frac{1}{24} \end{bmatrix} \begin{bmatrix} b[0,0,1] \\ b[0,1,2] \\ b[1,2,3] \\ b[2,3,4] \end{bmatrix}$$

$$\therefore S_v(1.5, 2.5) =$$

$$= \begin{bmatrix} -\frac{3}{16} & -\frac{9}{16} & \frac{15}{24} & \frac{3}{24} \end{bmatrix} \begin{bmatrix} (10\ 20\ 20) & (10\ 30\ 20) & (10\ 40\ 30) \\ (20\ 20\ 10) & (20\ 30\ 35) & (20\ 40\ 35) \\ (30\ 20\ 45) & (30\ 30\ 40) & (30\ 40\ 35) \\ (40\ 20\ 35) & (40\ 30\ 45) & (40\ 40\ 50) \end{bmatrix} \begin{bmatrix} 1/8 \\ 3/4 \\ 1/8 \end{bmatrix}$$

$$= \left[\begin{pmatrix} 10.625 \\ 0 \\ 6.2500 \end{pmatrix} \begin{pmatrix} 10.625 \\ 0 \\ 7.1875 \end{pmatrix} \begin{pmatrix} 10.625 \\ 0 \\ 2.8125 \end{pmatrix}\right] \begin{bmatrix} 1/8 \\ 3/4 \\ 1/8 \end{bmatrix} = \begin{bmatrix} 10.6250 \\ 0 \\ 6.5234 \end{bmatrix}.$$

Ans.

$$\frac{\partial S}{\partial v} = 2\left\{\left[-1\times\frac{1}{4}\right] b[1,2] + \left[-1\times\frac{3}{4} + 1\times\frac{3}{4}\right] b[2,3] + \left[\frac{1}{4}\right] b[3,4]\right\}$$

$$2b[2.5, \bar{1}]$$

$$= 2\left\{\begin{bmatrix} -\frac{1}{4} & 0 & \frac{1}{4}\end{bmatrix}\begin{bmatrix} b[1,2] \\ b[2,3] \\ b[3,4]\end{bmatrix}\right\}$$

or,

$$S_v = \begin{bmatrix} \frac{1}{32} & \frac{15}{32} & \frac{23}{48} & \frac{1}{48}\end{bmatrix}\begin{bmatrix} (10\ 20\ 20) & (10\ 30\ 20) & (10\ 40\ 30) \\ (20\ 20\ 40) & (20\ 30\ 35) & (20\ 40\ 35) \\ (30\ 20\ 45) & (30\ 30\ 40) & (30\ 40\ 35) \\ (40\ 20\ 35) & (40\ 30\ 45) & (40\ 40\ 50)\end{bmatrix}\begin{bmatrix} -\frac{1}{2} \\ 0 \\ \frac{1}{2}\end{bmatrix}$$

$$= \begin{bmatrix}\begin{pmatrix}24.8958 \\ 20 \\ 41.667\end{pmatrix} & \begin{pmatrix}24.8958 \\ 30 \\ 37.1354\end{pmatrix} & \begin{pmatrix}24.8958 \\ 40 \\ 35.1562\end{pmatrix}\end{bmatrix}\begin{bmatrix} -\frac{1}{2} \\ 0 \\ \frac{1}{2}\end{bmatrix} = \begin{bmatrix} 0 \\ 10.0 \\ -3.2553\end{bmatrix} \rightarrow \text{Ans.}$$

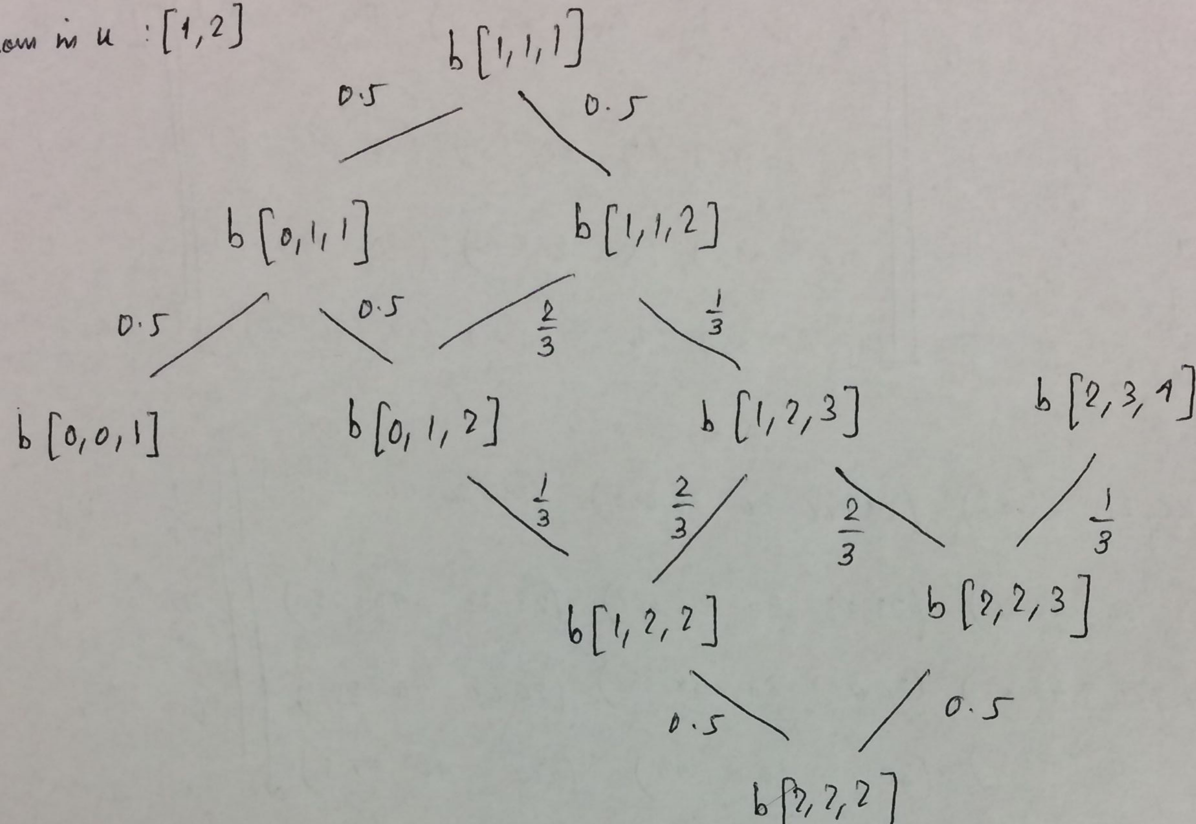
$$S_{uv} = \begin{bmatrix} -\frac{3}{16} & -\frac{9}{16} & \frac{15}{24} & \frac{1}{8}\end{bmatrix}\begin{bmatrix} (10\ 20\ 20) & (10\ 30\ 20) & (10\ 40\ 30) \\ (20\ 20\ 40) & (20\ 30\ 35) & (20\ 40\ 35) \\ (30\ 20\ 45) & (30\ 30\ 40) & (30\ 40\ 35) \\ (40\ 20\ 35) & (40\ 30\ 45) & (40\ 40\ 50)\end{bmatrix}\begin{bmatrix} -\frac{1}{2} \\ 0 \\ \frac{1}{2}\end{bmatrix}.$$

$$= \begin{bmatrix}\begin{pmatrix}10.625 \\ 0 \\ 6.2500\end{pmatrix} & \begin{pmatrix}10.625 \\ 0 \\ 7.1875\end{pmatrix} & \begin{pmatrix}10.625 \\ 0 \\ 2.8125\end{pmatrix}\end{bmatrix}\begin{bmatrix} -\frac{1}{2} \\ 0 \\ \frac{1}{2}\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1.7188\end{bmatrix}.$$

Ex 6.8)

Develop Bezier representation of the patch over domain $[1,2] \times [2,3]$.

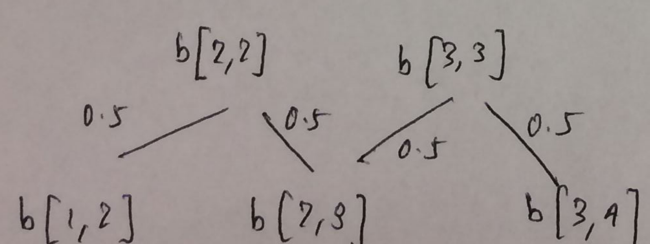
Blossom in u : $[1,2]$



$$\begin{bmatrix} b[1,1,1] \\ b[1,1,2] \\ b[1,2,2] \\ b[2,2,2] \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{7}{12} & \frac{1}{6} & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{bmatrix} \begin{bmatrix} b[0,0,1] \\ b[0,1,2] \\ b[1,2,3] \\ b[2,3,4] \end{bmatrix}$$

Basis function matrix $N(u)$

Blossom in v :



$$\begin{bmatrix} b[2,2] \\ b[2,3] \\ b[3,3] \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} b[1,2] \\ b[2,3] \\ b[3,4] \end{bmatrix}$$

Basis function matrix $N(v)$

$\therefore [1,2] \times [2,3]$ Bezier patch : $N(u)\,[P]\,N(v)^T$

$$= \begin{bmatrix} \frac{1}{4} & \frac{7}{12} & \frac{1}{6} & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{bmatrix} \begin{bmatrix} (10\ 20\ 20) & (10\ 30\ 20) & (10\ 40\ 30) \\ (20\ 20\ 40) & (20\ 30\ 35) & (20\ 40\ 35) \\ (30\ 20\ 45) & (30\ 30\ 40) & (30\ 40\ 35) \\ (40\ 20\ 35) & (40\ 30\ 45) & (40\ 40\ 50) \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}.$$

$$= \begin{bmatrix} (19.166\ 20\ 35.83) & (19.166\ 30\ 32.0833) & (19.166\ 40\ 33.75) \\ (23.33\ 20\ 41.66) & (23.33\ 30\ 36.667) & (23.33\ 40\ 35) \\ (26.67\ 20\ 43.33) & (26.67\ 30\ 38.333) & (26.67\ 40\ 35) \\ (30\ 20\ 42.5) & (30\ 30\ 40) & (30\ 40\ 37.5) \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} (19.166\ 25\ 33.956) & (19.166\ 30\ 32.0833) & (19.166\ 35\ 32.9167) \\ (23.333\ 25\ 39.165) & (23.333\ 30\ 36.667) & (23.333\ 35\ 35.8335) \\ (26.67\ 25\ 40.83) & (26.67\ 30\ 38.33) & (26.67\ 35\ 36.665) \\ (30\ 25\ 41.25) & (30\ 30\ 40) & (30\ 35\ 38.75) \end{bmatrix}$$

Bezier representation of domain.

Ex 6.9

For the Nurbs surface.

$$N(u) = \begin{bmatrix} \frac{1}{32} & \frac{15}{32} & \frac{23}{48} & \frac{1}{48} \end{bmatrix} \qquad N(v) = \begin{bmatrix} \frac{1}{8} & \frac{3}{4} & \frac{1}{8} \end{bmatrix}$$

Multiplying patch by weights and extending it to 4 dimensions:

$$S^w(1.5, 2.5) = N(u)\,[P^w]\,N(v)^T$$

$$\begin{bmatrix} \frac{1}{32} & \frac{15}{32} & \frac{23}{48} & \frac{1}{48} \end{bmatrix} \begin{bmatrix} (10\ 20\ 20\ 1) & (10\ 30\ 20\ 1) & (10\ 40\ 30\ 1) \\ (20\ 20\ 40\ 1) & (20\ 30\ 35\ 1) & (20\ 40\ 35\ 1) \\ (30\ 20\ 45\ 1) & (120\ 120\ 160\ 4) & (30\ 40\ 35\ 1) \\ (40\ 20\ 35\ 1) & (40\ 30\ 45\ 1) & (40\ 40\ 50\ 1) \end{bmatrix} \begin{bmatrix} 1/8 \\ 3/4 \\ 1/8 \end{bmatrix}$$

$$S^w(1.5, 2.5) = \underbrace{\begin{bmatrix} (24.8958 & 20.00 & 41.6667 & 1) \\ (68.0208 & 73.125 & 94.6354 & 2.4375) \\ (24.8958 & 40.00 & 35.1562 & 1) \end{bmatrix}^T}_{1\times3} \underbrace{\begin{bmatrix} 1/8 \\ 3/4 \\ 1/8 \end{bmatrix}}_{3\times1}.$$

$$S^w(1.5, 2.5) = (57.2395,\ 62.3438,\ 80.5794,\ 2.0781)$$

$$S(1.5, 2.5) = (27.5442,\ 30.0004,\ 38.7755). \leftarrow \text{Ans.}$$

ME 535 Assignment 9 - Fall 2018

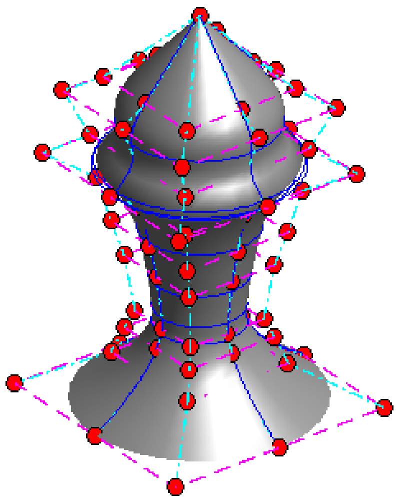
Debabrata Auddya

Ex. 6.12 — Revolution

1. Create a NURBS represented revolved surface.

```
% Copyright: Xiaoping Qian @ UW-Madison
close all; clear all;

% profile 1
P = [ 0 0 0  1;
      5 0 0  1;
      5 0 5  1;
      0 0 5  1;];  % curve in x-z plane
knots = [0 0 0 0 1 1 1 1];
order = 4;


% profile 2
 P = [0 0 0 1;
     7 0 -8 1;
     10 0 -10 1;
     8 0 -15 1;
     11.5 0 -17.5 1;
     7.5 0 -20 1;
     6.5 0 -25 1;
     5.5 0 -29 1;
     4.7 0 -35.5 1;
     5.5 0 -37.5 1;
     6.5 0 -40.5 1;
     13.5 0 -45 1;];
knots = [0 0 0 0 0.333 0.47 0.48 0.49 0.54 0.59 0.62 0.666 1 1 1 1];


%do plot of control polygon
Q = bsplineCurve(P, order, knots, 50);
plot3(P(:,1),P(:,2), P(:,3),'r-s');
hold on;
plot3(Q(:,1),Q(:,2), Q(:,3),'b','linewidth',2);
axis equal;
%print('-dpdf','-painters','revolution0.pdf')

P0 = P;  % curve CPs
P1 = P0(:,1:4)+P(:,1)*[0 1 0 0]; P1(:,4) = P(:,4).*sqrt(2)/2; % offset CPs
P2 = P1(:,1:4)+P(:,1)*[-1 0 0 0];P2(:,4) = P(:,4);  % offset CPs
P3 = P2(:,1:4)+P(:,1)*[-1 0 0 0]; P3(:,4) = P(:,4)*sqrt(2)/2;
P4 = P3(:,1:4)+P(:,1)*[0  -1 0 0]; P4(:,4) = P(:,4); % offset CPs
P5 = P4(:,1:4)+P(:,1)*[ 0 -1 0 0]; P5(:,4) = P(:,4)*sqrt(2)/2;
P6 = P5(:,1:4)+P(:,1)*[1 0 0 0]; P6(:,4) = P(:,4); % offset CPs
P7 = P6(:,1:4)+P(:,1)*[1 0 0 0]; P7(:,4) = P(:,4)*sqrt(2)/2;
P8 = P7(:,1:4)+P(:,1)*[0  1 0 0]; P8(:,4) = P(:,4); % offset CPs

CPs = [P0; P1; P2; P3; P4; P5; P6; P7;P8];  % v first
knots_u = [0 0 0 1 1 2 2 3 3 4 4 4]/4.;
knots_v = knots;
k1 = 3;
k2 = order;
ncp_u = 9;
ncp_v = size(P,1);

cph=[CPs(:,1:3).*CPs(:,4), CPs(:,4)]; % turn into homogeneous coorinates
tCP = reshape(cph',[4,ncp_v,ncp_u]);
```

```
CPs_d3 = permute(tCP,[1,3,2]); % change it to u first by switching 3rd dimension w/ 2nd dimension

nrbs.type   = 'Surface';
nrbs.number = [ncp_u, ncp_v];
nrbs.coefs = CPs_d3;  % CPs in homogeneous coordinates
nrbs.knots = {knots_u knots_v};
nrbs.order = [k1 k2];

plotNrbs(nrbs);
```



```
%print('-dpdf','-painters','revolution1.pdf')

nrbs_Spink.form='B-NURBS';
nrbs_Spink.dim = 4
```

```
nrbs_Spink = struct with fields:

    form: 'B-NURBS'
     dim: 4
```

```
nrbs_Spink.number = [ncp_u, ncp_v];
tCP = reshape(CPs',[4,ncp_v,ncp_u]);
CPs_d3 = permute(tCP,[1,3,2]); % change it to u first by switching 3rd dimension w/ 2nd dimension
nrbs_Spink.coefs = CPs_d3;  % CPs in Euclidean coordinates
nrbs_Spink.knots = {knots_u knots_v};
nrbs_Spink.order = [k1 k2];
NrbsSrf2IGES(nrbs_Spink,'revolutionChess.igs','./');
```

```
ans = 'Finished Export to IGES'
```

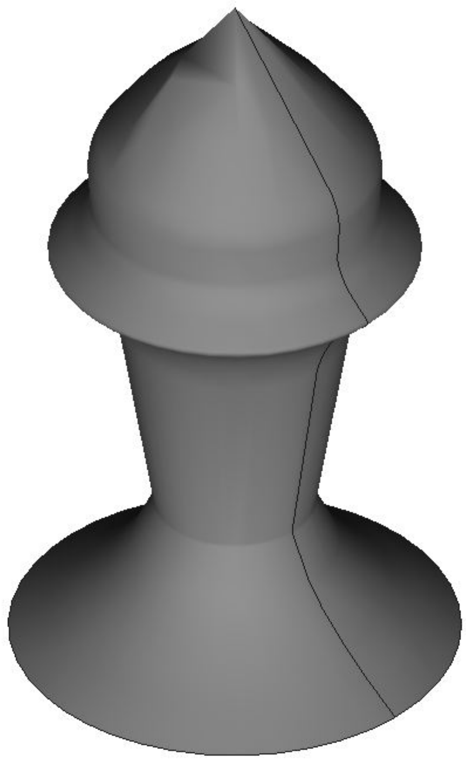
2. Export the surface into an IGES file.

```
disp('Igs file exported and named as revolutionChess.igs')
```

```
Igs file exported and named as revolutionChess.igs
```

3. Import the IGES file into a CAD software and render the surface.

```
figure(2)
image1 = imread('image1.png');
imshow(image1)
```

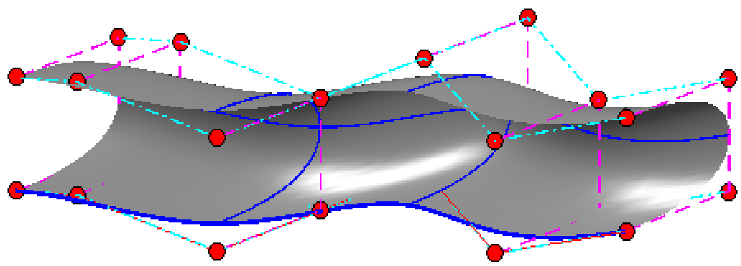


Ex. 6.13 — Sweeping

1. Create a NURBS represented translationally sweeping surface.

```
% Copyright: Xiaoping Qian @ UW-Madison
close all; clear;

% profile 1
P = [ 0 0 0  1;
      5 0 0  1;
      5 0 5  1;
      0 0 5  1;];  % curve in x-z plane
knots = [0 0 0 0 1 1 1 1];
order = 4;



% profile 2
P = [25 5 0 1;
     10 10 0 1;
     15 30 0 1;
     -15 30 0 1;
     -20 50 0 1;
     -25 55 0 1];
```

```
knots = [0 0 0 0 0.333 0.666 1 1 1 1];

P2= [0 0 0 1;
    15 0 0 1;
    15 0 15 1;
    0 0 15 1];


%do plot of control polygon
Q = bsplineCurve(P, order, knots, 50);
plot3(P(:,1),P(:,2), P(:,3),'r-s');
hold on;
plot3(Q(:,1),Q(:,2), Q(:,3),'b','linewidth',2);
axis equal;
%print('-dpdf','-painters','revolution0.pdf')

P0 = P;  % curve CPs
CPs=[];
for i=1:size(P2,1)
    tCP=P(:,1:3)+P2(i,1:3);
    tCP(:,4)= P(:,4)*P2(i,4);
    CPs=[CPs;tCP];
end
%
%     P1()=P0(:,1:3)+P2(i,1:3)
%     P1
%
%
% end



% P1 = P0(:,1:4)+P(:,1)*[0 1 0 0]; P1(:,4) = P(:,4).*sqrt(2)/2; % offset CPs
% P2 = P1(:,1:4)+P(:,1)*[-1 0 0 0];P2(:,4) = P(:,4);  % offset CPs
% P3 = P2(:,1:4)+P(:,1)*[-1 0 0 0]; P3(:,4) = P(:,4)*sqrt(2)/2;
% P4 = P3(:,1:4)+P(:,1)*[0  -1 0 0]; P4(:,4) = P(:,4); % offset CPs
% P5 = P4(:,1:4)+P(:,1)*[ 0 -1 0 0]; P5(:,4) = P(:,4)*sqrt(2)/2;
% P6 = P5(:,1:4)+P(:,1)*[1 0 0 0]; P6(:,4) = P(:,4); % offset CPs
% P7 = P6(:,1:4)+P(:,1)*[1 0 0 0]; P7(:,4) = P(:,4)*sqrt(2)/2;
% P8 = P7(:,1:4)+P(:,1)*[0  1 0 0]; P8(:,4) = P(:,4); % offset CPs

% CPs = [P0; P1; P2; P3; P4; P5; P6; P7;P8];  % v first
knots_u = [0 0 0 4 8 8 8]/8.;
knots_v = knots;
k1 = 3;
k2 = order;
ncp_u = size(P2,1);
ncp_v = size(P,1);

cph=[CPs(:,1:3).*CPs(:,4), CPs(:,4)]; % turn into homogneous coorinates
tCP = reshape(cph',[4,ncp_v,ncp_u]);
CPs_d3 = permute(tCP,[1,3,2]); % change it to u first by switching 3rd dimension w/ 2nd dimension

nrbs.type   = 'Surface';
nrbs.number = [ncp_u, ncp_v];
nrbs.coefs = CPs_d3;  % CPs in homogeneous coordinates
nrbs.knots = {knots_u knots_v};
nrbs.order = [k1 k2];

plotNrbs(nrbs);
```



```
%print('-dpdf','-painters','revolution1.pdf')

nrbs_Spink.form='B-NURBS';
nrbs_Spink.dim = 4
```

```
nrbs_Spink = struct with fields:

    form: 'B-NURBS'
     dim: 4
```

```
nrbs_Spink.number = [ncp_u, ncp_v];
tCP = reshape(CPs',[4,ncp_v,ncp_u]);
CPs_d3 = permute(tCP,[1,3,2]); % change it to u first by switching 3rd dimension w/ 2nd dimension
nrbs_Spink.coefs = CPs_d3;  % CPs in Euclidean coordinates
nrbs_Spink.knots = {knots_u knots_v};
nrbs_Spink.order = [k1 k2];
NrbsSrf2IGES(nrbs_Spink,'sweep.igs','./')
```

```
ans = 'Finished Export to IGES'
```

2. Export the surface into an IGES file.

```
disp('Igs file exported and named as sweep.igs')
```

```
Igs file exported and named as sweep.igs
```

3. Import the IGES file into a CAD software and render the surface.

```
figure(3)
image2 = imread('image2.png');
imshow(image2)
```