Modeling Durotaxis using Mechanical Bidomain model

October 4, 2018

Analytical treatment - 10/03/2018

The equations formulated below consider a gradient in the extracellular stiffness in a 1-d strand of tissue using the mechanical bidomain model.

Consider u_x and w_x as the intra- and extra-cellular displacements of the bidomain layer. x being the principal direction of the one-dimensional tissue strand, T being the tension, p and q being the intra- and extra-cellular pressure contributions arising mostly from hydrostatic forces.

The standard equations of the bidomain model are as follows [1]:

$$-\frac{\partial p}{\partial x} + \nu(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2}) + \gamma\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial T}{\partial x} = K(u_x - w_x) \tag{1}$$

$$-\frac{\partial q}{\partial x} + \mu(\frac{\partial^2 w_x}{\partial x^2} + \frac{\partial^2 w_x}{\partial y^2}) = -K(u_x - w_x) \tag{2}$$

For this problem we consider:

- $\mu = \mu_0 + gx$
- No contributions along the y axis
- p,q and T are constant and hence their gradients are zero along the x axis
- Slope of μ is a constant
- K is same for intra- and extra-cellular displacements

$$\nu\frac{\partial^2 u_x}{\partial x^2} + \gamma\frac{\partial^2 u_x}{\partial x^2} = K(u_x - w_x) \tag{3}$$

$$\mu_0\frac{\partial^2 w_x}{\partial x^2} + gx\frac{\partial^2 w_x}{\partial x^2} = -K(u_x - w_x) \tag{4}$$

Simplifying the above expression, replacing u_x in equation (2) from (1):

$$u_x = w_x + \frac{\nu}{K}\frac{\partial^2 u_x}{\partial x^2} + \frac{\gamma}{K}\frac{\partial^2 u_x}{\partial x^2} + \gamma\frac{\partial^2 u_x}{\partial x^2} \tag{5}$$

$$\therefore \frac{\mu_0}{K}\frac{\partial^2 w_x}{\partial x^2} + x\frac{g}{k}\frac{\partial^2 w_x}{\partial x^2} = -(\frac{\nu}{K}\frac{\partial^2 u_x}{\partial x^2} + \frac{\gamma}{K}\frac{\partial^2 u_x}{\partial x^2}) \tag{6}$$

Final form the equation can be written as:

$$\frac{\partial^2 w_x}{\partial x^2}(\frac{\mu_0}{K} + \frac{gx}{K}) + \frac{\partial^2 u_x}{\partial x^2}(\frac{\nu}{K} + \frac{\gamma}{K}) = 0 \tag{7}$$

Analytical treatment - 10/04/2018

The equations formulated below consider a gradient in the extracellular stiffness in a 1-d strand of tissue using the mechanical bidomain model.

Consider u_x and w_x as the intra- and extra-cellular displacements of the bidomain layer. x being the principal direction of the one-dimensional tissue strand, T being the tension, p and q being the intra- and extra-cellular pressure contributions arising mostly from hydrostatic forces.

The intra and extra-cellular stresses arising in the 1-d strand of tissue can be expressed as [2]:

$$\tau_{ix} = -p + 2\nu\epsilon_{ix} + T \tag{8}$$

$$\tau_{ex} = -q + 2\mu(x)\epsilon_{ex} \tag{9}$$

Relationship between the strains and displacement can be written as:

$$\frac{\partial \tau_{ix}}{\partial x} = K(u_x - w_x) \tag{10}$$

$$\frac{\partial \tau_{ex}}{\partial x} = -K(u_x - w_x) \tag{11}$$

Using equation (8) and (9) in (10) and (11) the resulting intra- and extra-cellular equations are:

$$\frac{\partial}{\partial x}(-p + 2\nu\frac{\partial u_x}{\partial x}) = K(u_x - w_x) \tag{12}$$

$$\frac{\partial}{\partial x}(-q + 2\mu(x)\frac{\partial w_x}{\partial x}) = -K(u_x - w_x) \tag{13}$$

For the problem we have assumed $\mu = \mu_0 + gx$, where g is a constant. Rewriting (12) and (13) we have:

$$-\frac{\partial p}{\partial x} + 2\nu\frac{\partial^2 u_x}{\partial x^2} = K(u_x - w_x) \tag{14}$$

$$-\frac{\partial q}{\partial x} + 2(g\frac{\partial w_x}{\partial x} + \frac{\partial^2 w_x}{\partial x^2}\mu(x)) = -K(u_x - w_x) \tag{15}$$

Bibliography

[1] Bradley J Roth *The Mechanical Bidomain Model: A Review* ISRN Tissue Eng. 2013

[2] Sharma, Kharananda; Al-Asuoad, Nofe; Shillor, Meir; Roth, Bradley J. *Intracellular, extracellular, and membrane forces in remodeling and mechanotransduction: The mechanical bidomain model* Journal of Coupled Systems and Multiscale Dynamics, Volume 3, Number 3, September 2015, pp. 200-207(8)