A hybrid approach of physical laws and data-driven modeling and estimation

The example of queuing networks

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May 3$^{\mathrm{rd}}$, 2013

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Queuing networks

Numerous applications:

- Communication networks
- Manufacturing and supply-chain
- Parallel computing
- Transportation networks

Goals:

- Estimation of queue lengths, waiting time, bottlenecks
- Routing, network optimization

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Numerous applications:

- Communication networks
- Manufacturing and supply-chain
- Parallel computing
- Transportation networks
  - Air traffic control
  - Freight
  - Intelligent Transportation Systems

Goals:

- Estimation of queue lengths, waiting time, bottlenecks
- Routing, network optimization

Mathematical modeling considerations

Scale:

Mathematical modeling considerations

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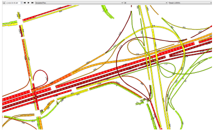
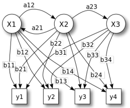
Mathematical abstraction (model) of the reality:

Microscopic
Agent-based



Macroscopic

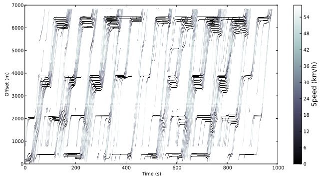


Data driven

Mathematical modeling considerations

- Input
  - Amount of data
  - Accuracy
- Output
  - Scale
  - Desired level of details
- Insights and prior information
  - First principles
  - System characteristics
- Constraints
  - Real-time computation
  - Scalability

Fleet of 500 taxis reporting their position every minute.



Detailed trajectories from high resolution video camera.

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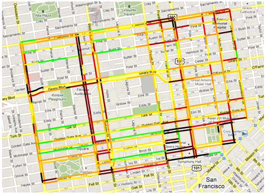
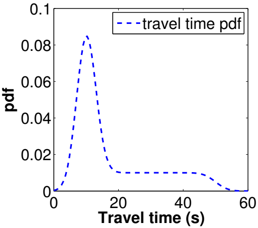


Color-map of congestion levels.



Distribution of travel time.

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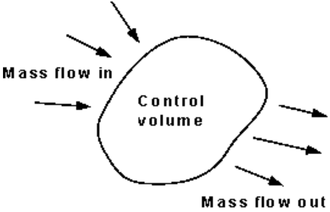

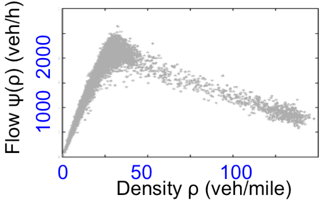

Empirical relationship between flow and density.

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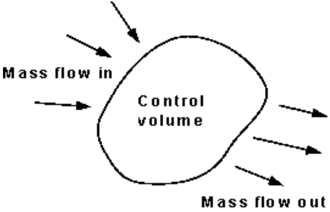

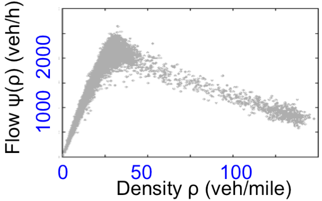

Empirical relationship between flow and density.

Modeling and estimation in queuing networks

1. Macroscopic model (Chap. 3)
   - Lighthill-Whitham-Richards model and Moskowitz equation
   - Solution of the Hamilton-Jacobi partial differential equation
   - Incorporating internal value conditions
2. Statistical model of horizontal queues (Chap. 5)
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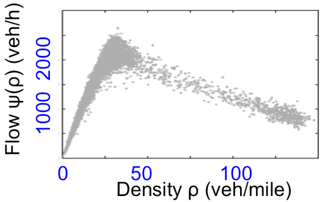
Hydrodynamic model of traffic flow

- Macroscopic variables:
  - velocity $v(x,t)$ (m/s),
  - density $\rho(x,t)$ (veh/m),
  - flow $q(x,t)$ (veh/s)
- Relations between the variables:
  - Definition of flow: $q = \rho\, v$
  - (Empirical) fundamental diagram: $q = \psi(\rho)$
- Conservation of vehicles



Fundamental diagram ψ (generally concave).

Lighthill - Whitham - Richards (LWR) model

Partial differential equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \psi(\rho)}{\partial x} = 0$$

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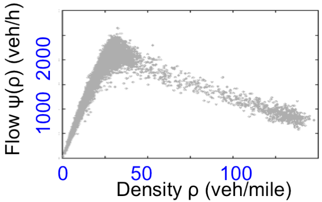

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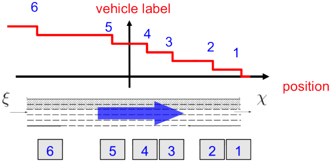
The Moskowitz function: cumulative number of vehicles

Continuous function representing labels of vehicles, assigned as they enter a road segment.

Isolines = vehicle trajectories



Moskowitz Hamilton-Jacobi partial differential equation

$$\frac{\partial \mathbf{M}}{\partial t} - \psi\left(-\frac{\partial \mathbf{M}}{\partial x}\right) = 0. \quad \psi \text{ is the Hamiltonian (concave).}$$

If $\mathbf{M}$ is differentiable in (t, x),

$$\frac{\partial \mathbf{M}(t,x)}{\partial x} = -\rho(t,x) \quad \text{and} \quad \frac{\partial \mathbf{M}(t,x)}{\partial t} = q(t,x)$$

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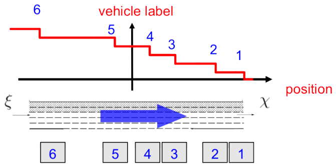

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Solution with upstream and initial value conditions

Exact solution computed at any (t, x) in the domain [Claudel].

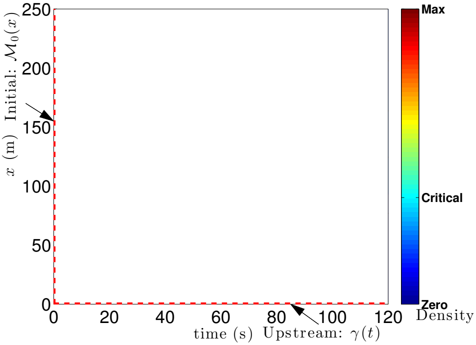
$$\frac{\partial \mathbf{M}}{\partial t} - \psi\left(-\frac{\partial \mathbf{M}}{\partial x}\right) = 0,$$

$$\text{s.t.} \quad \begin{aligned} \mathbf{M}(0, x) &= \mathcal{M}_0(x), \\ \mathbf{M}(t, x_{\min}) &= \gamma(t) \end{aligned}$$

For example:

Initial condition: aerial photo at $t = 0$.

Upstream condition: vehicle counts at $x = x_{\text{in}}$.



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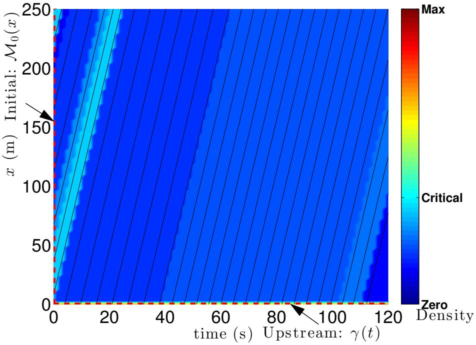

Colormap of density and isolines of the Moskowitz function (trajectories)

Solution in a signalized queue

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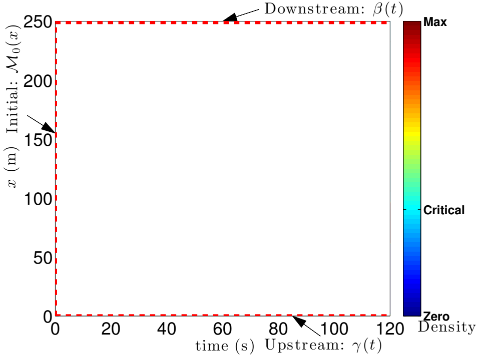
Downstream condition: signal info at $x = x_{\text{out}}$.



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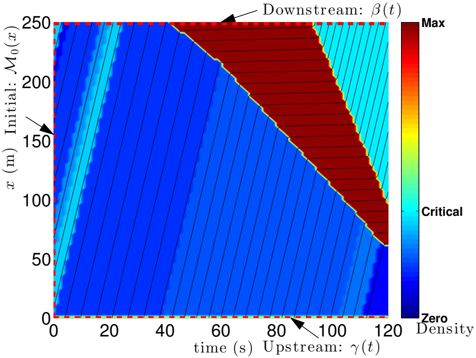

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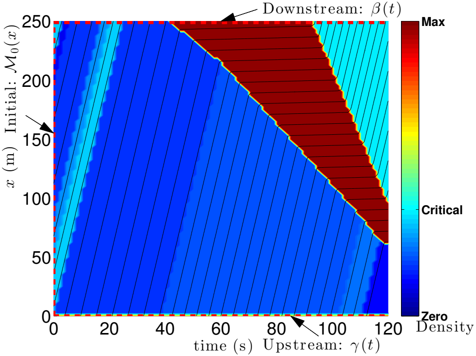
Downstream unknown

For example:
Signal timing unknown.



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Adding Lagrangian sensor measurements

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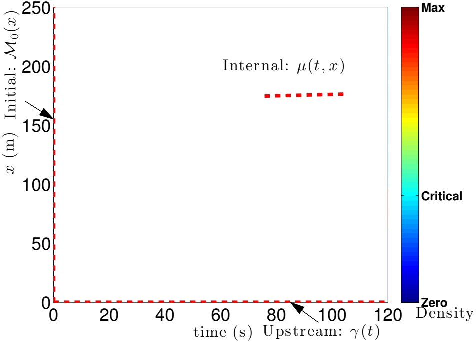
Downstream unknown

Internal measurement

For example:

Lagrangian sensor.

GPS sensor in vehicle.



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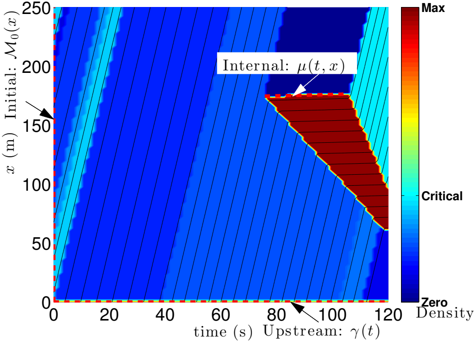

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Reconstruct downstream boundary condition

Given: an affine internal value condition μ, piecewise affine upstream (γ) and initial ($\mathcal{M}_0$) value conditions.
Reconstruct: the downstream boundary condition $\hat{\beta}$,
s.t.: the solution with the value conditions γ, $\mathcal{M}_0$ and $\hat{\beta}$ satisfies the internal condition μ.

Result

There is an algorithm consisting of solving **scalar** *equations and/or convex optimization problems to compute $\hat{\beta}$.*
The complexity is **linear** *in the number of affine pieces of γ and $\mathcal{M}_0$.*
The solution $\hat{\beta}$ is unique on an interval which corresponds to a **constant limitation of the maximum flow** *(e.g. traffic signal).*

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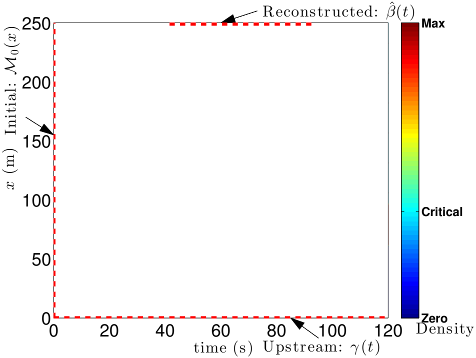
For example:

Reconstruct signal timing



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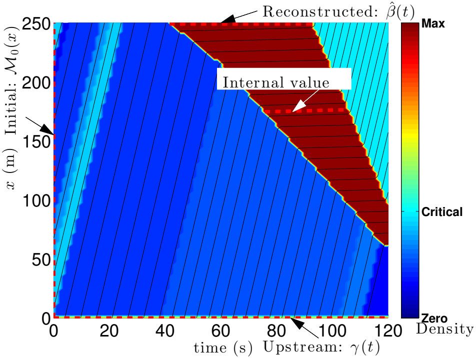
Reconstruct signal timing

The solution satisfies

$\mathbf{M}(t,x) = \mu(t,x)$



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Potentials and limitations

+ Leverages first principles to reconstruct missing information (downstream boundary condition).
+ Can be extended to incorporate random (noisy) value conditions (Chapter 4).
- Requires more detailed measurements than are usually available.
- Does not leverage the periodicity of arterial traffic.

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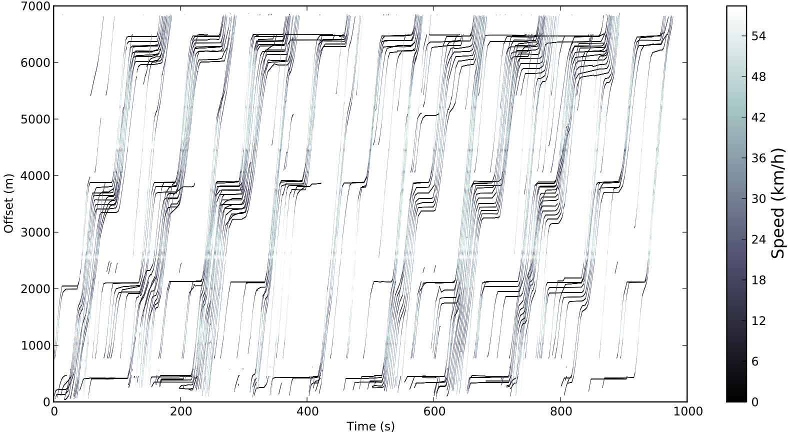
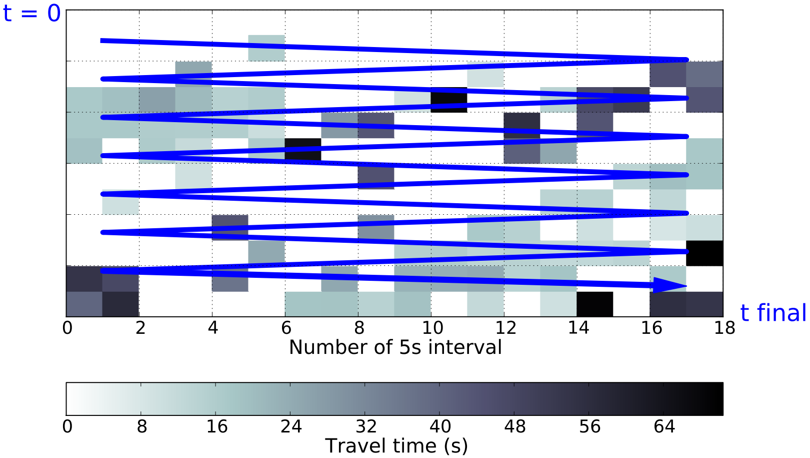
Queue formation and dissolution at signals



Detailed trajectories [NGSIM].

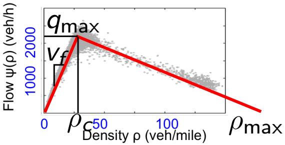
Visualization of the time series of travel times



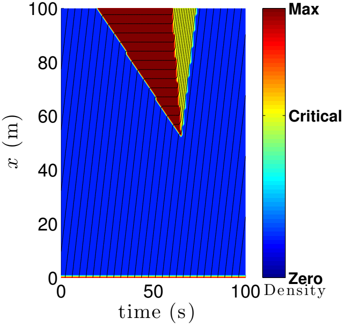
The color of the rectangles indicate the median travel time on the road segment during each 5 second interval.

Horizontal queues

- Modeling:
  - LWR model with triangular fundamental diagram ψ
  - Constant arrival rates
  - Periodicity: signal cycle C



Triangular diagram ψ.



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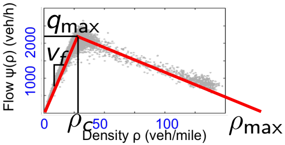

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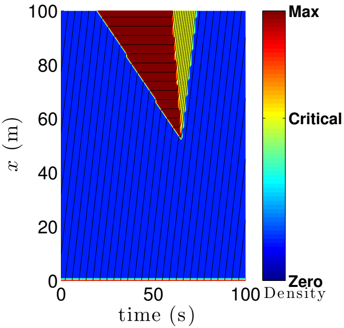

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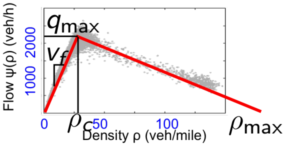
- Modeling:
    - LWR model with triangular fundamental diagram ψ
    - Constant arrival rates
    - Periodicity: signal cycle C
- Constant speed of formation and dissolution of queues (Rankine Hugoniot)
- Delay δ
    - Decreases linearly with the stop location
    - Depends on the entrance time



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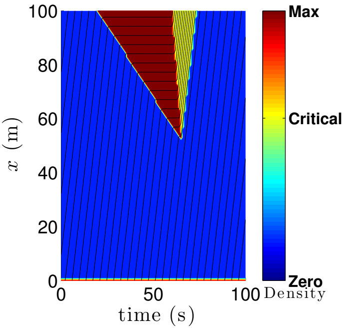
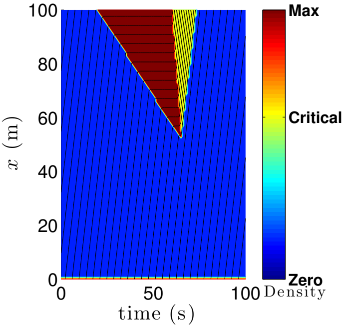
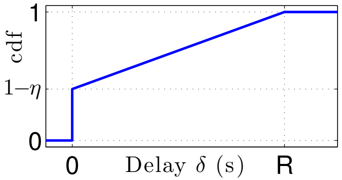


Delay probability distribution function P_δ

- Stopping vehicles ($s = 1$):
  - Delay uniformly distributed on $[0, R]$ (R: red time)
  - Probability of stopping: η
- Non-stopping vehicles ($s = 0$):
  - Delay: 0 s. (not stopping)
  - Probability of not stopping: $1 - \eta$
- Mixture distribution: condition on delay pattern (stop or no-stop)

$$\begin{aligned} P_\delta(\delta) &= P(\delta | s = 0)P(s) \\ &+ P(\delta | s = 1)P(s = 1) \end{aligned}$$





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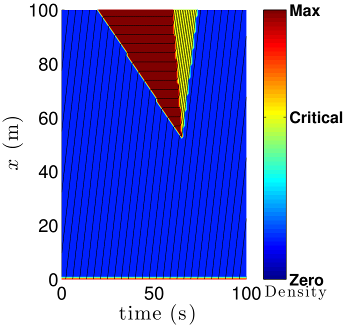
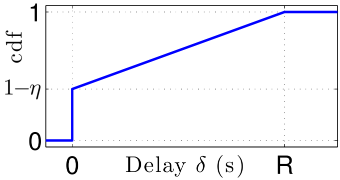
$$P_\delta(\delta) = P(\delta|s=0)P(s) + P(\delta|s=1)P(s=1)$$





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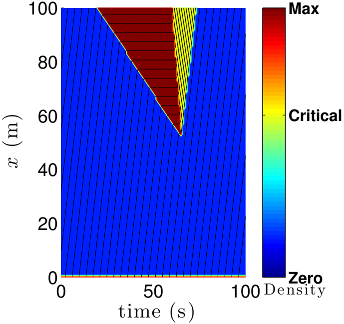

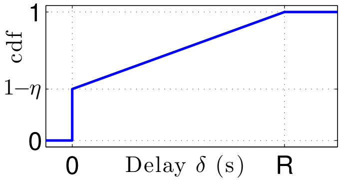

Travel time distribution function

Travel time = free flow travel time + delay

- Free flow travel time: pdf φ_f (e.g. log-concave, Normal, Gamma, ...)
- Delay: pdf P_δ (mixture distribution)
- Sum of independent variables

Pdf of travel time g

$$g(y) = (\varphi_f \star P_\delta)(y)$$

Travel time distribution function

Travel time = free flow travel time + delay

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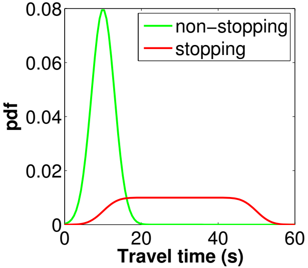
Mixture of log-concave distributions

- Convolution of each weighted component of the pdf of delay with φ_f (linearity of convolution)
- Sum the components



Proposition

The pdf of travel time is a mixture of log-concave distributions. Each component corresponds to a delay pattern

Proof.

The convolution of log-concave functions is log-concave □

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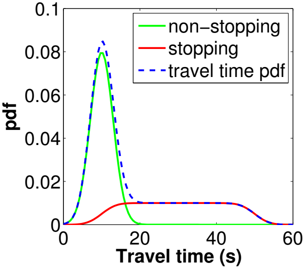

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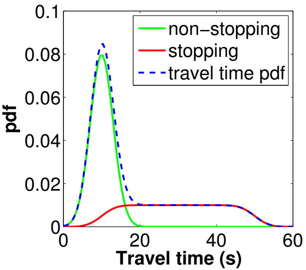

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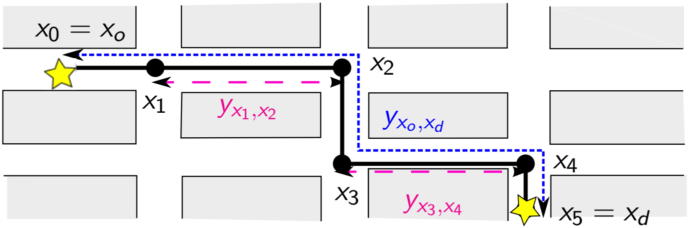
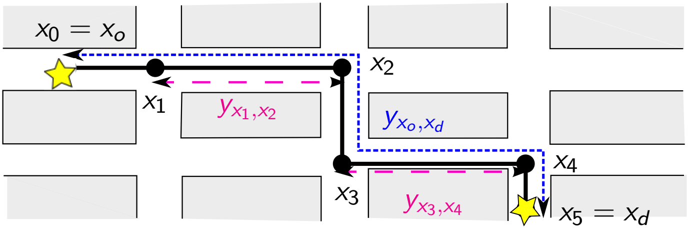
Sparse probe vehicle data characteristics

- Vehicles report their position periodically ($\sim$ once per minute).
- Map-matching and Path Inference (Tim Hunter).
- $M+1$ links traversed between successive location reports.



Sparse probe vehicle data characteristics

- Vehicles report their position periodically ($\sim$ once per minute).
- Map-matching and Path Inference (Tim Hunter).
- $M+1$ links traversed between successive location reports.
- Iterative (hard) Expectation Maximization algorithm:
  - Decompose the path travel time: $y_{x_o,x_d} = \sum_{i=0}^{M} y_{x_i,x_{i+1}}$.
  - Learn the pdf of travel time on each link using the allocated travel times.



Travel time decomposition

Maximize the log-likelihood of the travel time on each link

$$\underset{(y^{i_m}_{x_m,x_{m+1}})_{m=0\ldots M}}{\textbf{maximize}} \quad : \quad \sum_{m=0}^{M} \ln\left(g^{i_m}_{x_m,x_{m+1}}\left(y^{i_m}_{x_m,x_{m+1}}\right)\right)$$

$$\textbf{s.t.} \ : \quad y_{x_o,x_d} = \sum_{m=0}^{M} y^{i_m}_{x_m,x_{m+1}} \text{ and } \forall m \ y^{i_m}_{x_m,x_{m+1}} \geq 0$$

Notation:

$g^{i_m}_{x_m,x_{m+1}}$ Pdf of travel time on link i_m, between x_m and x_{m+1}.

$y^{i_m}_{x_m,x_{m+1}}$ Travel time on link i_m, between x_m and x_{m+1}.

Travel time decomposition

Maximize the log-likelihood of the travel time on each link

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Optimization problem is not convex.

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Maximize the log-likelihood of the travel time on each link

$$\underset{(y^{i_m}_{x_m,x_{m+1}})_{m=0\dots M}}{\textbf{maximize}} \; : \; \sum_{m=0}^{M} \ln\left(\sum_{k=0}^{K_{i_m}} \alpha^{i_m,k}_{x_m,x_{m+1}} g^{i_m,k}_{x_m,x_{m+1}}(y^{i_m}_{x_m,x_{m+1}})\right)$$

$$\textbf{s.t.} \; : \; y_{x_o,x_d} = \sum_{m=0}^{M} y^{i_m}_{x_m,x_{m+1}} \text{ and } \forall m \; y^{i_m}_{x_m,x_{m+1}} \geq 0$$

Can we exploit the concavity structure of the pdf of travel time?

Notation:

$g^{i_m,k}_{x_m,x_{m+1}}$	Pdf of travel time on link i_m for delay pattern k.
K_{i_m}	Number of delay patterns on link i_m.
$\alpha^{i_m,k}_{x_m,x_{m+1}}$	Weight of component (delay pattern) k on link i_m.
$y^{i_m}_{x_m,x_{m+1}}$	Travel time on link i_m, between x_m and x_{m+1}.

Travel time decomposition

A vehicle has one delay pattern on each link

$$\underset{\substack{(y_{x_m,x_{m+1}})_m \\ (\beta_{i_m,k})_{m,k}}}{\textbf{maximize}} : \sum_{m=0}^{M} \sum_{k=0}^{K_{i_m}} \beta_{i_m,k} \ln(g^{i_m,k}(y_{x_m,x_{m+1}}))$$

$$\textbf{s.t.} : y_{x_o,x_d} = \sum_{m=0}^{M} y_{x_m,x_{m+1}} \text{ and } \forall m \; y_{x_m,x_{m+1}} \geq 0,$$

$$\beta_{i_m,k} \in \{0,1\}, \quad \sum_{k=1}^{K_{i_m}} \beta_{i_m,k} = 1.$$

Enumerate $\prod_{m=0}^{M} K_{i_m}$ convex problems

Notation:

$g^{i_m,k}_{x_m,x_{m+1}}$	Pdf of travel time on link i_m for delay pattern k.
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K_{i_m}	Number of delay patterns on link i_m.
$\beta_{i_m,k}$	Indicates if vehicle has delay pattern k on link i_m.

Parameter estimation

Maximize the log-likelihood of the allocated travel times

$$\underset{\eta^i, R^i, \theta_p^i, l^i}{\textbf{maximize}} \sum_{j=1}^{J^i} \ln(g^i_{x_1,x_2}(y^{i,j}_{x_1,x_2})) \quad \textbf{s.t.} \quad \eta^i \in [0,1],\ R^i > 0$$

Notation:

$y^{i,j}_{x_1,x_2}$	j^{th} travel time allocated to link i
η^i	Proportion of stopping vehicles on link i
R^i	Red time of link i
l^i	Queue length of link i
θ^i_p	Parameters of the free flow travel time distribution

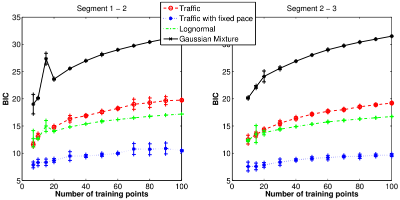
Comparison with other distributions

Minimize the Bayesian Information Criterion

- Gaussian Mixture: penalized for its complexity (number of parameters).
- Log-Normal: best fit among "classic" distributions.
- Traffic distribution: Fixing the free flow pace a-priori limits the risk of over-fitting.

$$\mathrm{BIC} = -2\ln(\Lambda) + k\ln(n)$$

Λ	Log-likelihood
n	Size of training set
k	Number of parameters



Comparison with other distributions

Kolmogorov-Smirnov statistics: measures the distance between distributions:

- Proposal distribution learned with training data
- Empirical distribution of validation data

Test hypothesis: "The validation data is drawn from the proposal distribution".

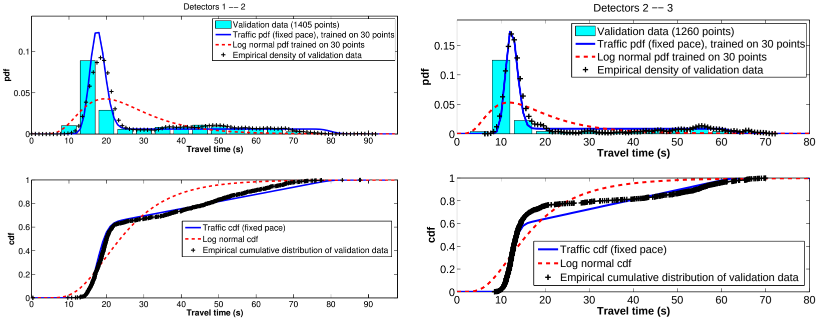
Size of training set	Traffic	Fix pace	Log-Normal	GMM
15	0.014	0.048	$4.4 \cdot 10^{-4}$	0.025
20	0.018	0.055	$2.3 \cdot 10^{-4}$	0.044
30	0.020	0.066	$4.1 \cdot 10^{-5}$	0.021
50	0.014	0.076	$1.3 \cdot 10^{-5}$	0.020

p-value

Qualitative fit

The traffic model captures the multi-modality of the travel times:

- "Peaked" distribution (non-stopping vehicles)
- "Flat" distribution (stopping vehicles)



Potentials and limitations

+ Captures specificities of the distribution
+ Performs well with little data (parameters have physical interpretation)
- Modeling assumptions may be restrictive
- Distributions have fewer *properties* than “classic” distributions

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Model extensions

- Propagation of congestion in the network (Chap. 6 & 7): Dynamic Bayesian Network (DBN)
  - Hidden variables:
    - Queue length (Chap. 6)
    - Discrete "congestion state" (Chap. 7)
  - Dynamics:
    - Conservation of vehicles (Chap. 6)
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  - Observed variables: travel times
- Release the assumption of uniform arrivals
  - Piecewise-constant arrivals
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Modeling and estimation in queuing networks

1. Macroscopic model (Chap. 3)
2. Statistical model of horizontal queues (Chap. 5)
3. Data driven model refinements and conclusion

Change detection and pattern discovery (Chap. 8 & 9)

Sparse estimation: Homotopy algorithm for a generalization of the LASSO (Chap. 8)

- Least-square estimation with l_1 regularization
- Detects spatial or temporal changes in traffic conditions
  - Choose interval of data aggregation, discard old data
  - Improve the structure of the DBN

Dimensionality reduction: Non-negative Matrix Factorization (NMF) (Chap. 9)

- Similar to Singular Value Decomposition but all entries are non-negative
- Spatial clustering: *part-based* decomposition of the network
- Temporal clustering: identification of *times of day*
- Hierarchical model: Parameters for different days of the week, parts of the network *Times of the day*

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  - Reconstruct missing information
  - Produce reliable estimates with little data
- Data driven models:
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  - Computationally efficient
  - Discover patterns in large scale systems which are hard to describe with physical laws
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Acknowledgements

- My family, in particular my sister Céline and my father Patrick
- Ryan Herring
- Alex Bayen
- Pieter Abbeel
- Laurent El Ghaoui and Tarek Rabbani
- Alex Skabardonis
- Christian Claudel (KAUST)
- Saurabh Amin (MIT)
- Fabien Moutarde, Yufei Han and Arnaud De La Fortelle (Mines - INRIA, France)
- Timothy Hunter
- Saneesh Apte (Facebook)
- Dan Work, Olli-Pekka Tossavainen, Joe Butler, Greg Merritt, Anthony Patire and all the members of the *Mobile Millennium* project and PaTH.
- Samitha Samaranayake, Jack Reilly, Leah Anderson, Walid Krichène, Jérôme Thai, Andrew Tinka, Samuel Rosat, Constant Bails and all the students, alumni and interns of the research group.
- Rebecca Miller, Dana Jantz, Rosita Alvarez-Croft, Patrick Hernan, Sam Rifkin, Shirley Salanio, and all the members of the Center for Student Affairs and the EECS administrative support.
- Ana Ferreira
- Woody Hoburg
- Kristen Parrish
- Maryam Kamgarpour, Gireeja Ranade, Floraine Berthouzoz, Erin Summers, Colleen Lewis, Todd & Heather Roman, Eric & Irma Hernandez, James Magee & Michele Mangelli and all the great friends I met during these past five years
- Sheila Humphreys

Questions

A hybrid approach of physical laws and data-driven modeling and estimation

The example of queuing networks

Aude Hofleitner
Electrical Engineering and Computer Science
UC Berkeley

May 3$^{\mathrm{rd}}$, 2013

app