Genetic Algorithm
Step-by-Step Calculation

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Problem Identification

Suppose we have an equation

$$3X_1 + 2X_2 + 4X_3 + 2X_4 + 5X_5 = 100$$

To find the variables $(X_1, X_2, \dots, X_5)$, we can use the Genetic Algorithm in which trying to minimize the objective function

$$f(x) = 3X_1 + 2X_2 + 4X_3 + 2X_4 + 5X_5 - 100$$

Five variables $(X_1, X_2, \dots, X_5)$ help us construct the chromosomes, as follows

X_1	X_2	X_3	X_4	X_5
Gene 1	Gene 2	Gene 3	Gene 4	Gene 5

Initialization

Generation 0

We draw 5 chromosomes in which each genes contains integer from 0 to 10

	Gene 1	Gene 2	Gene 3	Gene 4	Gene 5
Chromosome 1	0	3	2	9	3
Chromosome 2	7	5	0	0	9
Chromosome 3	6	3	2	1	2
Chromosome 4	4	1	1	6	3
Chromosome 5	7	4	1	2	8

For the selection, we need to calculate the fitness function $\boldsymbol{F(c)}$

$$F(c) = \frac{1}{|Error|}$$

Where $Error = f(x)$

Selection

Generation 0

For example

	Gene 1	Gene 2	Gene 3	Gene 4	Gene 5
Chromosome 1	0	3	2	9	3

$$f(x) = |3X_1 + 2X_2 + 4X_3 + 2X_4 + 5X_5 - 100|$$
$$f(x) = |3(0) + 2(3) + 4(2) + 2(9) + 5(3) - 100|$$
$$f(x) = \mathbf{53}$$

So, the fitness value of chromosome 1

$$F(c) = \frac{1}{|1 + f(x)|} = \frac{1}{|1 + 53|}$$
$$F(c) = \mathbf{0.0185}$$

Note

To avoid the zero problem, we add 1 in the fitness function

Selection

Generation 0

Fittest chromosomes have higher probability to be selected in next generation

	Gene 1	Gene 2	Gene 3	Gene 4	Gene 5	$\lvert f(x) \rvert$	$F(c)$
Chromosome 1	0	3	2	9	3	53	0.0185
Chromosome 2	7	5	0	0	9	24	0.0400
Chromosome 3	6	3	2	1	2	56	0.0178
Chromosome 4	4	1	1	6	3	55	0.0181
Chromosome 5	7	4	1	2	8	23	0.0434

Selection

Generation 0

From previous calculation, we have the total value of fitness function

$$Total = 0.0185 + 0.0400 + 0.0178 + 0.0181 + 0.0434$$
$$Total = \mathbf{0.1378}$$

So, the probability of chromosomes is formulated as follows

$$\boxed{P = \frac{F(c)}{Total}}$$

For example, for chromosome 1, the probability to be selected is as follows

$$P_1 = \frac{F(c)_1}{Total} = \frac{0.0185}{0.1378}$$

$$P_1 = \mathbf{0.1342}$$

Selection

Generation 0

For each chromosomes, calculate its probability using previous formula

	Gene 1	Gene 2	Gene 3	Gene 4	Gene 5	$\lvert f(x) \rvert$	$F(c)$	P
Chromosome 1	0	3	2	9	3	53	0.0185	0.1342
Chromosome 2	7	5	0	0	9	24	0.0400	0.2902
Chromosome 3	6	3	2	1	2	56	0.0178	0.1291
Chromosome 4	4	1	1	6	3	55	0.0181	0.1313
Chromosome 5	7	4	1	2	8	23	0.0434	0.3149

Selection

Generation 0

For the roulette method, we should calculate its cumulative probability

	Gene 1	Gene 2	Gene 3	Gene 4	Gene 5	P	C
Chromosome 1	0	3	2	9	3	0.1342	0.1342
Chromosome 2	7	5	0	0	9	0.2902	0.4244
Chromosome 3	6	3	2	1	2	0.1291	0.5535
Chromosome 4	4	1	1	6	3	0.1313	0.6848
Chromosome 5	7	4	1	2	8	0.3149	1.0000

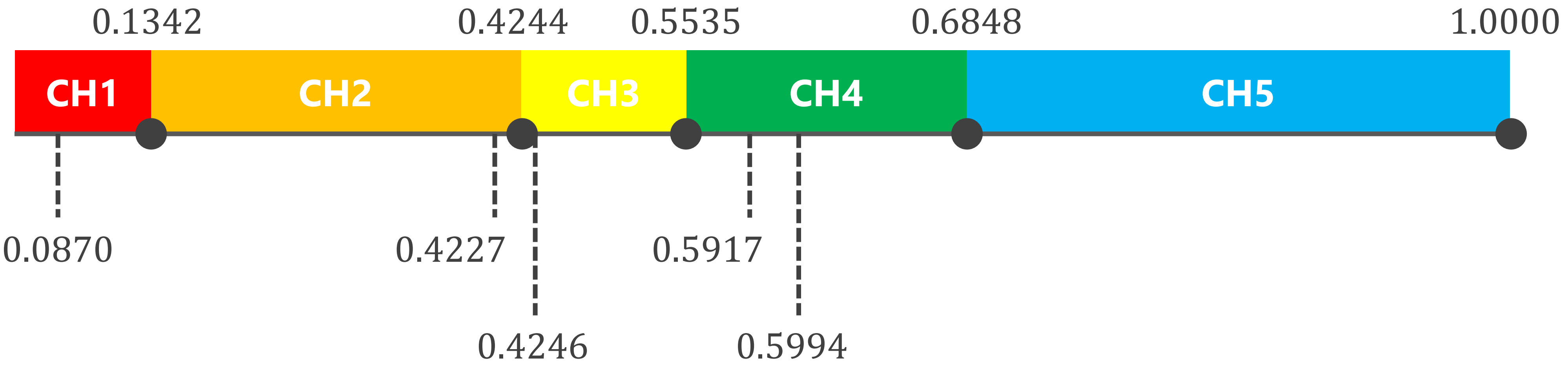
Selection

On the previous slide, each chromosomes has its own cumulative probability. To select the chromosomes, we will generate 5 random number using **Uniform(0, 1)**

$$R_1 = 0.0870$$
$$R_2 = 0.5917$$
$$R_3 = 0.4227$$
$$R_4 = 0.4246$$
$$R_5 = 0.5994$$

For example

When the $R_1 = 0.0870$ is less than $CH_1 = 0.1342$ and less than $CH_2 = 0.4224$, so new chromosome will be CH_1



Selection

Generation 0

New chromosomes are selected as follows

	Gene 1	Gene 2	Gene 3	Gene 4	Gene 5
New Chromosome 1	0	3	2	9	3
New Chromosome 2	4	1	1	6	3
New Chromosome 3	7	5	0	0	9
New Chromosome 4	6	3	2	1	2
New Chromosome 5	4	1	1	6	3

Note

Genes in new chromosomes are adjusted by the roulette method in previous slide

Cross Over

Generation 0

To select the chromosomes for cross over, we will generate 5 random number using **Uniform(0, 1)**. Chromosome ***k*** will be selected if the random number is less than cross over rate (Pc)

$$
\begin{aligned}
R_1 &= 0.1066 \\
R_2 &= 0.3917 \\
R_3 &= 0.1929 \\
R_4 &= 0.5626 \\
R_5 &= 0.2408
\end{aligned}
$$

For example

We set the cross over rate 25%, so the chromosomes that has the random number less than 0.25 will be selected for cross over

The selected chromosomes are CH_1, CH_3, and CH_5. These combinations are as follows

$\boxed{CH_1}$ **X** $\boxed{CH_3}$ $\boxed{CH_1}$ **X** $\boxed{CH_5}$ $\boxed{CH_3}$ **X** $\boxed{CH_5}$

Cross Over

Generation 0

To determine the position of cross over, we generate random number between 1 to *n* where n is length of chromosome – 1. So, we generate between 1 and 4

$$CO_1 = 2$$
$$CO_2 = 1$$
$$CO_3 = 3$$

For example

The cross over between CH_1 and CH_3 (known as CO_1) is as follows

	Gene 1	Gene 2	Gene 3	Gene 4	Gene 5
New Chromosome 1	0	3	2	9	3
New Chromosome 3	7	5	0	0	9

0	3	0	0	9

Cross Over

Generation 0

The cross over between CH_1 and CH_5 (known as CO_2) is as follows

	Gene 1	Gene 2	Gene 3	Gene 4	Gene 5
New Chromosome 1	0	3	2	9	3
New Chromosome 5	4	1	1	6	3
➡	0	3	1	6	3

Cross Over

Generation 0

The cross over between CH_3 and CH_5 (known as CO_3) is as follows

	Gene 1	Gene 2	Gene 3	Gene 4	Gene 5
New Chromosome 3	7	5	0	0	9
New Chromosome 5	4	1	1	6	3
	7	5	1	6	3

Cross Over

Generation 0

The population after performing cross over

	Gene 1	Gene 2	Gene 3	Gene 4	Gene 5
Chromosome 1	4	1	1	6	3
Chromosome 2	6	3	2	1	2
Chromosome 3	0	3	0	0	9
Chromosome 4	0	3	1	6	3
Chromosome 5	7	5	1	6	3

Old chromosomes   New chromosomes

Note

Chromosome 1 and 2 are coming from the new chromosomes 2 and 4. They are not selected for cross over. Meanwhile, the chromosome 3, 4, and 5 are from the cross over

Mutation

Generation 0

Mutation is a process in which we assign new value to any genes. Number of genes that have mutations is determined by the mutation rate (Pm). Firstly, count the number of genes in population

$$\#genes = \#chromosome \ * \#gen\ in\ chromosome$$

Next, number of genes that have mutation is as follows

$$\#genes\ mutation = \#genes \ * Pm$$

So, number of genes in population

$\#genes = 5 \ * 6$
$\#genes = \mathbf{30}$

Number of mutated genes ($Pm = 10\%$)

$\#genes\ mutation = 30 * 0.1$
$\#genes\ mutation = \mathbf{3}$

Mutation

After mutation, we have 6 new chromosomes in generation 0

	Gene 1	Gene 2	Gene 3	Gene 4	Gene 5
Chromosome 1	4	1	1	6	3
Chromosome 2	6	8	2	1	2
Chromosome 3	0	3	0	0	9
Chromosome 4	0	3	1	5	3
Chromosome 5	7	5	2	6	3

Note

Firstly, we generate random number from 1 to 30. The result is 7, 19, and 23. They are mutated genes

Next, for each selected genes, generate random number from 0 to 9 for replacing the old values

Evaluation

Generation 1

Evaluate the chromosomes to the objective function

	Gene 1	Gene 2	Gene 3	Gene 4	Gene 5	$\lvert f(x) \rvert$	$F(c)$
Chromosome 1	4	1	1	6	3	55	0.0178
Chromosome 2	6	8	2	1	2	46	0.0212
Chromosome 3	0	3	0	0	9	49	0.0200
Chromosome 4	0	3	1	5	3	65	0.0151
Chromosome 5	7	5	2	6	3	34	0.0285

Next Process

The process is repeated using the new generation to get the best solution for $X_1, X_2, \dots, X_5$

For example

After several generations, the best chromosome is obtained as follows

	Gene 1	Gene 2	Gene 3	Gene 4	Gene 5
Best Chromosome	5	7	9	5	5

$$f(x) = 3X_1 + 2X_2 + 4X_3 + 2X_4 + 5X_5 - 100$$

$$f(x) = 3(5) + 2(7) + 4(9) + 2(5) + 5(5) - 100$$

$$\boldsymbol{f(x) = 0}$$

Thank You

Let's get connected

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