Aufgabe 1

$$I(a) = 2\int_1^a x \cdot \ln(x^2)\,dx$$

i	0	1	2	3
a	$e-\frac{1}{2}$	$e-\frac{1}{4}$	$e+\frac{1}{4}$	$e+\frac{1}{2}$
$I(a)$	3.9203	5.9169	11.3611	14.8550

a) Lagrange-Interpolation:
$P_n(x_i) = y_i \qquad i = 0,1,2,3$

i	***Lagrange Polynome***
0	$l_0 = \prod_{\substack{j=0\\j\neq 0}}^{3} \frac{x-x_j}{x_0-x_j} = \frac{e-\left(e-\frac{1}{4}\right)}{\left(e-\frac{1}{2}\right)-\left(e-\frac{1}{4}\right)} \cdot \frac{e-\left(e+\frac{1}{4}\right)}{\left(e-\frac{1}{2}\right)-\left(e+\frac{1}{4}\right)} \cdot \frac{e-\left(e+\frac{1}{2}\right)}{\left(e-\frac{1}{2}\right)-\left(e+\frac{1}{2}\right)}$ $= (-1)\cdot\left(-\frac{1}{3}\right)\cdot\left(-\frac{1}{2}\right) = -\frac{1}{6}$
1	$l_1 = \prod_{\substack{j=0\\j\neq 1}}^{3} \frac{x-x_j}{x_1-x_j} = \frac{e-\left(e-\frac{1}{2}\right)}{\left(e-\frac{1}{4}\right)-\left(e-\frac{1}{2}\right)} \cdot \frac{e-\left(e+\frac{1}{4}\right)}{\left(e-\frac{1}{4}\right)-\left(e+\frac{1}{4}\right)} \cdot \frac{e-\left(e+\frac{1}{2}\right)}{\left(e-\frac{1}{4}\right)-\left(e+\frac{1}{2}\right)}$ $= 2\cdot\frac{1}{2}\cdot\frac{2}{3} = \frac{2}{3}$
2	$l_2 = \prod_{\substack{j=0\\j\neq 2}}^{3} \frac{x-x_j}{x_2-x_j} = \frac{e-\left(e-\frac{1}{2}\right)}{\left(e+\frac{1}{4}\right)-\left(e-\frac{1}{2}\right)} \cdot \frac{e-\left(e-\frac{1}{4}\right)}{\left(e+\frac{1}{4}\right)-\left(e-\frac{1}{4}\right)} \cdot \frac{e-\left(e+\frac{1}{2}\right)}{\left(e+\frac{1}{4}\right)-\left(e+\frac{1}{2}\right)}$ $= \frac{2}{3}\cdot\frac{1}{2}\cdot 2 = \frac{2}{3}$
3	$l_3 = \prod_{\substack{j=0\\j\neq 3}}^{3} \frac{x-x_j}{x_3-x_j} = \frac{e-\left(e-\frac{1}{2}\right)}{\left(e+\frac{1}{2}\right)-\left(e-\frac{1}{2}\right)} \cdot \frac{e-\left(e-\frac{1}{4}\right)}{\left(e+\frac{1}{2}\right)-\left(e-\frac{1}{4}\right)} \cdot \frac{e-\left(e+\frac{1}{4}\right)}{\left(e+\frac{1}{2}\right)-\left(e+\frac{1}{4}\right)}$ $= \frac{1}{2}\cdot\frac{1}{3}\cdot(-1) = -\frac{1}{6}$

$$P_3(x) = \sum_{i=0}^{3}\left(l_i(x)\cdot y_i\right) = 3.9203\cdot\left(-\frac{1}{6}\right) + 5.9169\cdot\frac{2}{3} + 11.3611\cdot\frac{2}{3} + 14.8550\cdot\left(-\frac{1}{6}\right)$$
$$= (-0.653383) + 3.9446 + 7.57407 + (-2.47583) = 8.38945$$

b) Analytische Berechnung des exakten Wertes:

$$I(e) = 2\int_1^e x \cdot \ln(x^2)\,dx = |x^2 \cdot (\ln(x^2) - 1)|_1^e$$
$$= \left(e^2 \cdot (\ln(\mathrm{e}^2) - 1)\right) - \left(1^2 \cdot (\ln(1^2) - 1)\right) = (e^2 \cdot 1) - (1^2 \cdot (-1))$$
$$= e^2 + 1 \approx 8.3890561$$

absoluter Fehler: $|8.38945 - 8.38906| = 0.39 \cdot 10^{-3}$

relativer Fehler: $\frac{|8.38945-8.38906|}{8.38906} = 0.46 \cdot 10^{-4}$

c) Berechnung mittels Romberg-Extrapolation:

$$Näherung_{Romberg} = 8.3890576$$

absoluter Fehler: $|8.3890576 - 8.3890561| = 0.15 \cdot 10^{-5}$

relativer Fehler: $\frac{|8.3890576-8.3890561|}{8.3890561} = 0.18 \cdot 10^{-6}$

Die Romberg-Extrapolation schneidet in diesem Beispiel besser ab als die Lagrange-Interpolation.

Aufgabe 2

Höhe über Meer [m]	0	1250	2500	3750	5000	10000
Atmosphärendruck [hPa]	1013	NaN	747	NaN	540	226

```
1013,       0,                  0,                                  0

            133 x
 747, 1013 - -----,             0,                                  0
             1250

                          2
            207 x    59 x       591 x
 540,  954 - -----, -------- - ----- + 1013,                        0
             2500  12500000     5000

                        2                             3             2
            157 x      x       257 x   2962          77 x        313 x      3623 x
 226,  854 - -----,  ------ - ----- + ----,  -  ------------ + -------- - ------ + 1013
             2500    375000    2500     3       375000000000   50000000    30000
```

Berechnung für $x = 1250$

x	y			
x_0	$y_0 = p_{00} = 1013$			
x_1	$y_1 = p_{10} = 747$	$p_{11} = 880$		
x_2	$y_2 = p_{20} = 540$	$p_{21} = 880.5$	$p_{22} = 872.625$	
x_3	$y_3 = p_{30} = 226$	$p_{31} = 775.5$	$p_{32} = 863$	$p_{33} = 871.421875$

Berechnung für $x = 3750$

x	y			
x_0	$y_0 = p_{00} = 1013$			
x_1	$y_1 = p_{10} = 747$	$p_{11} = 614$		
x_2	$y_2 = p_{20} = 540$	$p_{21} = 643.5$	$p_{22} = 636.125$	
x_3	$y_3 = p_{30} = 226$	$p_{31} = 618.5$	$p_{32} = 639.333$	$p_{33} = 637.328125$