NMIT2 Numerik 2	Serie 9	Zürcher Hochschule für Angewandte Wissenschaften zh aw School of Engineering
Autoren	Rémi Georgiou, André Stocker	
Datum	15. November 2015	

Aufgabe 1

$$I(a) = 2\int_1^a x \cdot \ln(x^2)\,dx$$

i	0	1	2	3
a	$e - \frac{1}{2}$	$e - \frac{1}{4}$	$e + \frac{1}{4}$	$e + \frac{1}{2}$
$I(a)$	3.9203	5.9169	11.3611	14.8550

a) Lagrange-Interpolation:
$P_n(x_i) = y_i \qquad i = 0,1,2,3$

i	***Lagrange Polynome***
0	$l_0 = \prod_{\substack{j=0 \\ j\neq 0}}^{3} \frac{x - x_j}{x_0 - x_j} = \frac{e-\left(e-\frac{1}{4}\right)}{\left(e-\frac{1}{2}\right)-\left(e-\frac{1}{4}\right)} \cdot \frac{e-\left(e+\frac{1}{4}\right)}{\left(e-\frac{1}{2}\right)-\left(e+\frac{1}{4}\right)} \cdot \frac{e-\left(e+\frac{1}{2}\right)}{\left(e-\frac{1}{2}\right)-\left(e+\frac{1}{2}\right)}$ $= (-1)\cdot\left(-\frac{1}{3}\right)\cdot\left(-\frac{1}{2}\right) = -\frac{1}{6}$
1	$l_1 = \prod_{\substack{j=0 \\ j\neq 1}}^{3} \frac{x - x_j}{x_1 - x_j} = \frac{e-\left(e-\frac{1}{2}\right)}{\left(e-\frac{1}{4}\right)-\left(e-\frac{1}{2}\right)} \cdot \frac{e-\left(e+\frac{1}{4}\right)}{\left(e-\frac{1}{4}\right)-\left(e+\frac{1}{4}\right)} \cdot \frac{e-\left(e+\frac{1}{2}\right)}{\left(e-\frac{1}{4}\right)-\left(e+\frac{1}{2}\right)}$ $= 2\cdot\frac{1}{2}\cdot\frac{2}{3} = \frac{2}{3}$
2	$l_2 = \prod_{\substack{j=0 \\ j\neq 2}}^{3} \frac{x - x_j}{x_2 - x_j} = \frac{e-\left(e-\frac{1}{2}\right)}{\left(e+\frac{1}{4}\right)-\left(e-\frac{1}{2}\right)} \cdot \frac{e-\left(e-\frac{1}{4}\right)}{\left(e+\frac{1}{4}\right)-\left(e-\frac{1}{4}\right)} \cdot \frac{e-\left(e+\frac{1}{2}\right)}{\left(e+\frac{1}{4}\right)-\left(e+\frac{1}{2}\right)}$ $= \frac{2}{3}\cdot\frac{1}{2}\cdot 2 = \frac{2}{3}$
3	$l_3 = \prod_{\substack{j=0 \\ j\neq 3}}^{3} \frac{x - x_j}{x_3 - x_j} = \frac{e-\left(e-\frac{1}{2}\right)}{\left(e+\frac{1}{2}\right)-\left(e-\frac{1}{2}\right)} \cdot \frac{e-\left(e-\frac{1}{4}\right)}{\left(e+\frac{1}{2}\right)-\left(e-\frac{1}{4}\right)} \cdot \frac{e-\left(e+\frac{1}{4}\right)}{\left(e+\frac{1}{2}\right)-\left(e+\frac{1}{4}\right)}$ $= \frac{1}{2}\cdot\frac{1}{3}\cdot(-1) = -\frac{1}{6}$

$$P_3(x) = \sum_{i=0}^{3}(l_i(x)\cdot y_i) = 3.9203\cdot\left(-\frac{1}{6}\right) + 5.9169\cdot\frac{2}{3} + 11.3611\cdot\frac{2}{3} + 14.8550\cdot\left(-\frac{1}{6}\right)$$
$$= (-0.653383) + 3.9446 + 7.57407 + (-2.47583) = 8.38945$$

b) Analytische Berechnung des exakten Wertes:

$$I(e) = 2\int_1^e x \cdot \ln(x^2)\,dx = |x^2 \cdot (\ln(x^2) - 1)|_1^e$$
$$= \left(e^2 \cdot (\ln(\mathrm{e}^2) - 1)\right) - \left(1^2 \cdot (\ln(1^2) - 1)\right) = (e^2 \cdot 1) - (1^2 \cdot (-1))$$
$$= e^2 + 1 \approx 8.3890561$$

absoluter Fehler: $|8.38945 - 8.38906| = 0.39 \cdot 10^{-3}$

relativer Fehler: $\frac{|8.38945 - 8.38906|}{8.38906} = 0.46 \cdot 10^{-4}$

c) Berechnung mittels Romberg-Extrapolation:

$$Näherung_{Romberg} = 8.3890576$$

absoluter Fehler: $|8.3890576 - 8.3890561| = 0.15 \cdot 10^{-5}$

relativer Fehler: $\frac{|8.3890576 - 8.3890561|}{8.3890561} = 0.18 \cdot 10^{-6}$

Die Romberg-Extrapolation schneidet in diesem Beispiel besser ab als die Lagrange-Interpolation.