NMIT1 Numerik 1	Serie 11	Zürcher Hochschule für Angewandte Wissenschaften School of Engineering
Autor	Rémi Georgiou	
Datum	20. Mai 2015	

NMIT1 – Serie 11 Rémi Georgiou

Aufgabe 2

a)

$$f_1(x_1,x_2) = 5x_1x_2 \quad , \quad f_2(x_1,x_2) = x_1^2x_2^2 + x_1 + 2x_2 \quad , \quad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\frac{\partial f_1}{\partial x_1} = x_2 \cdot \frac{d(5x_1)}{dx_1} = 5x_2$$

$$\frac{\partial f_2}{\partial x_2} = 5x_1 \frac{d(x_2)}{dx_2} = 5x_1$$

$$\frac{\partial f_2}{\partial x_1} = x_2^2 \cdot \frac{d(x_1^2)}{dx_1} + \frac{d(x_1)}{dx_1} + \frac{d(2x_2)}{dx_1} = 2x_1 \cdot x_2^2 + 1$$

$$\frac{\partial f_2}{\partial x_2} = x_1^2 \cdot \frac{d(x_2^2)}{dx_2} + \frac{d(x_1)}{dx_2} + \frac{d(2x_2)}{dx_2} = 2x_1^2x_2 + 2$$

$$Df(x_1,x_2) = \begin{pmatrix} 5x_2 & 5x_1 \\ 2x_1x_2^2+1 & 2x_1^2x_2+2 \end{pmatrix}$$

$$Df(x_1,x_2) = \begin{pmatrix} 10 & 5 \\ 9 & 6 \end{pmatrix}$$

b)

$$f_1(x_1,x_2,x_3) = \ln(x_1^2+x_2^2) + x_3^2 \quad , \quad f_2(x_1,x_2,x_3) = e^{x_2^2+x_3^2} + x_1^2$$

$$f_3(x_1,x_2,x_3) = \frac{1}{x_3^2+x_1^2} + x_2^2 \quad , \quad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$u = x_1^2 + x_2^2$

1) $$\frac{\partial f_1}{\partial x_1} = \frac{d(x_3^2)}{dx_1} + \frac{d}{dx_1}(\ln(x_1^2+x_2^2)) = 0 + \frac{d(\ln(u))}{du} \cdot \frac{du}{dx_1} = \frac{d(x_1^2+x_2^2)}{dx_1} \cdot \frac{1}{u}$$

$$= \frac{\frac{d(x_1^2)}{dx_1} + \frac{d(x_2^2)}{dx_1}}{x_1^2+x_2^2} = \frac{2x_1}{x_1^2+x_2^2}$$

2) $$\frac{\partial f_1}{\partial x_2} = \frac{d(x_3^2)}{dx_2} + \frac{d}{dx_2}(\ln(x_1^2+x_2^2)) = 0 + \frac{d(\ln(u))}{du} \cdot \frac{du}{dx_2} = \frac{d(x_1^2+x_2^2)}{dx_2} \cdot \frac{1}{u}$$

$$\frac{\frac{d(x_1^2)}{dx_2} + \frac{d(x_2^2)}{dx_2}}{x_1^2+x_2^2} = \frac{2x_2}{x_1^2+x_2^2}$$

3) $$\frac{\partial f_1}{\partial x_3} = \frac{d}{dx_3}(\ln(x_1^2+x_2^2)) + \frac{d(x_3^2)}{dx_3} = 0 + 2 \cdot x_3 = 2x_3$$

NMIT1 Numerik 1	Serie 11	Zürcher Hochschule für Angewandte Wissenschaften School of Engineering
Autor	Rémi Georgiou	
Datum	20. Mai 2015	

4) $$\frac{\partial f_2}{\partial x_1} = \frac{d}{dx_1}\left(e^{x_2^2+x_3^2}\right) + \frac{d(x_1^2)}{dx_1} = 0 + 2 \cdot x_1 = \underline{2x_1}$$

$u = x_2^2 + x_3^2$

5) $$\frac{\partial f_2}{\partial x_2} = \frac{d}{dx_2}\left(e^{x_2^2+x_3^2}\right) + \underbrace{\frac{d(x_1^2)}{dx_2}}_{\to 0} = \frac{d(e^u)}{du} \cdot \frac{du}{dx_2} + 0$$

$$= e^u \cdot \frac{d}{dx_2}\left(x_2^2 + x_3^2\right) = e^{x_2^2+x_3^2} \cdot \left(\frac{d}{dx_2}(x_2^2) + \underbrace{\frac{d}{dx_2}(x_3^2)}_{\to 0}\right)$$

$$= \underline{e^{x_2^2+x_3^2} \cdot 2x_2}$$

6) $$\frac{\partial f_2}{\partial x_3} = \frac{d}{dx_3}\left(e^{x_2^2+x_3^2}\right) + \underbrace{\frac{d(x_1^2)}{dx_3}}_{\to 0} = \frac{d(e^u)}{du} \cdot \frac{du}{dx_3} + 0$$

$$= e^u \cdot \frac{d}{dx_3}\left(x_2^2 + x_3^2\right) = e^{x_2^2+x_3^2} \cdot \left(\underbrace{\frac{d}{dx_3}(x_2^2)}_{\to 0} + \frac{d}{dx_3}(x_3^2)\right) = \underline{e^{x_2^2+x_3^2} \cdot 2x_3}$$

7) $u = x_3^2 + x_1^2$

$$\frac{\partial f_3}{\partial x_1} = \frac{d}{dx_1}\left(\frac{1}{x_3^2+x_1^2}\right) + \underbrace{\frac{d}{dx_1}(x_2^2)}_{\to 0} = \frac{d\left(\frac{1}{u}\right)}{du} \cdot \frac{du}{dx_1} + 0$$

$$= -\frac{1}{u^2} \cdot \frac{d}{dx_1}\left(x_3^2 + x_1^2\right) = -\frac{1}{(x_3^2+x_1^2)^2} \cdot \left(\underbrace{\frac{d}{dx_1}(x_3^2)}_{\to 0} + \frac{d}{dx_1}(x_1^2)\right)$$

$$= -\frac{1}{(x_3^2+x_1^2)^2} \cdot 2x_1 = \underline{-\frac{2x_1}{(x_3^2+x_1^2)^2}}$$

8) $$\frac{\partial f_3}{\partial x_2} = \underbrace{\frac{d}{dx_2}\left(\frac{1}{x_3^2+x_1^2}\right)}_{\to 0} + \frac{d}{dx_2}(x_2^2) = 0 + 2x_2 = \underline{2x_2}$$

9) $$\frac{\partial f_3}{\partial x_3} = \frac{d}{dx_3}\left(\frac{1}{x_3^2+x_1^2}\right) + \underbrace{\frac{d}{dx_3}(x_2^2)}_{\to 0} = \frac{d\left(\frac{1}{u}\right)}{du} \cdot \frac{du}{dx_3} + 0$$

$$= -\frac{1}{u^2} \cdot \frac{d}{dx_3}\left(x_3^2 + x_1^2\right) = -\frac{1}{(x_3^2+x_1^2)^2} \cdot \left(\frac{d}{dx_3}(x_3^2) + \underbrace{\frac{d}{dx_3}(x_1^2)}_{\to 0}\right)$$

$$= -\frac{1}{(x_3^2+x_1^2)^2} \cdot 2 \cdot x_3 = \underline{-\frac{2x_3}{(x_3^2+x_1^2)^2}}$$

$$Df(x_1, x_2, x_3) = \begin{pmatrix} \frac{2x_1}{x_1^2+x_2^2} & \frac{2x_2}{x_1^2+x_2^2} & 2x_3 \\ 2x_1 & e^{x_2^2+x_3^2} \cdot 2x_2 & e^{x_2^2+x_3^2} \cdot 2x_3 \\ -\frac{2x_1}{(x_3^2+x_1^2)^2} & 2x_2 & -\frac{2x_3}{(x_3^2+x_1^2)^2} \end{pmatrix} = \begin{pmatrix} \frac{2}{5} & \frac{4}{5} & 6 \\ 2 & 4e^{13} & 6e^{13} \\ -\frac{1}{50} & 4 & -\frac{3}{50} \end{pmatrix} = \begin{pmatrix} \frac{2}{5} & \frac{4}{5} & 6 \\ 2 & 1{,}77 \cdot 10^6 & 2{,}65 \cdot 10^6 \\ -\frac{1}{50} & 4 & -\frac{3}{50} \end{pmatrix}$$