NMIT2 Numerik 2	Serie 5	Zürcher Hochschule für Angewandte Wissenschaften zh aw School of Engineering
Autoren	Rémi Georgiou, André Stocker	
Datum	18. Oktober 2015	

Aufgabe 2

$\frac{dy}{dx} = \frac{x^2}{y}$
$0 \leq x \leq 2.1, \quad y_0 = y(0) = 2$
Exakte Lösung der DGL : $y(x) = \sqrt{\frac{2x^3}{3} + 4}$

a) Lösen mittels Eulerverfahren, $h = 0.7$

i	x_i	y_i	abs_err
0	$x_0 = 0$	$y_0 = 2$	$\left\|\sqrt{4} - 2\right\| = 0$
1	$x_1 = x_0 + h = 0.7$	$y_1 = y_0 + h \cdot \left(\frac{x_0^2}{y_0}\right) = 2 + 0.7 \cdot \frac{0^2}{2} = 2$	$\left\|\sqrt{2 \cdot \frac{(0.7)^3}{3} + 4} - 2\right\| = 0.0564$
2	$x_2 = x_1 + h = 1.4$	$y_2 = y_1 + h \cdot \left(\frac{x_1^2}{y_1}\right) = 2 + 0.7 \cdot \frac{(0.7)^2}{2} = 2.1715$	$\left\|\sqrt{2 \cdot \frac{(1.4)^3}{3} + 4} - 2.1715\right\| = 0.2429$
3	$x_3 = x_2 + h = 2.1$	$y_3 = y_2 + h \cdot \left(\frac{x_2^2}{y_2}\right) = 2.1715 + 0.7 \cdot \frac{(1.4)^2}{2.1715} = 2.8033$	$\left\|\sqrt{2 \cdot \frac{(2.1)^3}{3} + 4} - 2.8033\right\| = 0.3863$

b) Lösen mittels Mittelpunktverfahren, $h = 0.7$

i	x_i	y_i	abs_err
0	$x_0 = 0$	$y_0 = 2$	$\left\|\sqrt{4} - 2\right\| = 0$
1	$x_1 = x_0 + h = 0.7$ $x_{\frac{h}{2}} = x_0 + \frac{h}{2} = 0 + 0.35 = 0.35$	$y_{\frac{h}{2}} = y_0 + \frac{h}{2} \cdot \left(\frac{x_0^2}{y_0}\right) = 2 + 0.35 \cdot \frac{0^2}{2} = 2$ $y_1 = y_0 + h \cdot \left(\frac{x_{\frac{h}{2}}^2}{y_{\frac{h}{2}}}\right) = 2 + 0.7 \cdot \frac{(0.35)^2}{2} = 2.0429$	$\left\|\sqrt{2 \cdot \frac{(0.7)^3}{3} + 4} - 2.0429\right\| = 0.0135$
2	$x_2 = x_1 + h = 1.4$ $x_{\frac{h}{2}} = x_1 + \frac{h}{2} = 1.05$	$y_{\frac{h}{2}} = y_1 + \frac{h}{2} \cdot \left(\frac{x_1^2}{y_1}\right) = 2.0429 + 0.35 \cdot \frac{(0.7)^2}{2.0429} = 2.1268$ $y_2 = y_1 + h \cdot \left(\frac{x_{\frac{h}{2}}^2}{y_{\frac{h}{2}}}\right) = 2.0429 + 0.7 \cdot \frac{(1.05)^2}{2.1268} = 2.4057$	$\left\|\sqrt{2 \cdot \frac{(1.4)^3}{3} + 4} - 2.4057\right\| = 0.0087$
3	$x_3 = x_2 + h = 2.1$ $x_{\frac{h}{2}} = x_3 + \frac{h}{2} = 1.75$	$y_{\frac{h}{2}} = y_2 + \frac{h}{2} \cdot \left(\frac{x_2^2}{y_2}\right) = 2.4057 + 0.35 \cdot \frac{(1.4)^2}{2.4057} = 2.6908$ $y_3 = y_2 + h \cdot \left(\frac{x_{\frac{h}{2}}^2}{y_{\frac{h}{2}}}\right) = 2.4057 + 0.7 \cdot \frac{(1.75)^2}{2.6908} = 3.2024$	$\left\|\sqrt{2 \cdot \frac{(2.1)^3}{3} + 4} - 3.2024\right\| = 0.0127$

c) Lösen mittels modifiziertem Eulerverfahren, $h = 0.7$

i	x_i	y_i	abs_err
0	$x_0 = 0$	$y_0 = 2$	$\left\|\sqrt{4} - 2\right\| = 0$
1	$x_1 = x_0 + h = 0.7$	$y_1^{Euler} = y_0 + h \cdot \left(\frac{x_0^2}{y_0}\right) = 2$ $k_1 = \frac{x_0^2}{y_0} = \frac{0^2}{2} = 0$ $k_2 = \frac{x_1^2}{y_1^{Euler}} = \frac{(0.7)^2}{2} = 0.245$ $y_1 = y_0 + h \cdot \left(\frac{k_1 + k_2}{2}\right) = 2 + 0.7 \cdot \frac{0.245}{2} = 2.0858$	$\left\|\sqrt{2 \cdot \frac{(0.7)^3}{3} + 4} - 2.0858\right\| = 0.0294$
2	$x_2 = x_1 + h = 1.4$	$y_2^{Euler} = y_1 + h \cdot \left(\frac{x_1^2}{y_1}\right) = 2.2502$ $k_1 = \frac{x_1^2}{y_1} = \frac{(0.7)^2}{2.0858} = 0.2349$ $k_2 = \frac{x_2^2}{y_2^{Euler}} = \frac{(1.4)^2}{2.2502} = 0.871$ $y_2 = y_1 + h \cdot \left(\frac{k_1 + k_2}{2}\right) = 2.0858 + 0.7 \cdot \frac{1.1059}{2} = 2.4728$	$\left\|\sqrt{2 \cdot \frac{(1.4)^3}{3} + 4} - 2.4728\right\| = 0.0584$
3	$x_3 = x_2 + h = 2.1$	$y_3^{Euler} = y_2 + h \cdot \left(\frac{x_2^2}{y_2}\right) = 3.0276$ $k_1 = \frac{x_2^2}{y_2} = \frac{(1.4)^2}{2.4728} = 0.7926$ $k_2 = \frac{x_3^2}{y_3^{Euler}} = \frac{(2.1)^2}{3.0276} = 1.4566$ $y_3 = y_2 + h \cdot \left(\frac{k_1 + k_2}{2}\right) = 2.4728 + 0.7 \cdot \frac{2.2492}{2} = 3.26$	$\left\|\sqrt{2 \cdot \frac{(2.1)^3}{3} + 4} - 3.26\right\| = 0.0703$