Problem Set 2

$$= \begin{bmatrix} ① & 2 & 3 \\ \phi & ④ & 5 \\ 4 & \phi & ⑥ \end{bmatrix} \leftarrow \text{upper triangular matrix so eigenvalues are the diagonal}$$

eigenvalues $= \lambda = 1, 4, 6$ so the

character polynomial is $(x-1)\underbrace{(x-4)}_{a}(x-6)$

$x^2 - 4x - 1x + 4 = \overline{(x^2 - 5x + 4)(x-6)} =$

$x^3 - 5x^2 + 4x - 6x^2 + 30x - 24 =$

$-11 \boxed{x^3 - 11x^2 + 34x - 24} \leftarrow \text{original equation}$

$\alpha_A = (1) \quad \alpha_A = (4) \quad \alpha_A = (6)$

Eigenspaces

$\lambda = 1 \quad A - (1)J_3 = \begin{bmatrix} 1-1 & 2 & 3 \\ \phi & 4-1 & 5 \\ \phi & \phi & 6-1 \end{bmatrix} = \begin{bmatrix} \phi & 2 & 3 \\ \phi & 3 & 5 \\ \phi & \phi & \phi \end{bmatrix} \overset{\text{RREF}}{\to}$

$\to \begin{bmatrix} \phi & 1 & \phi \\ \phi & \phi & 1 \\ \phi & \phi & \phi \end{bmatrix} \Delta \quad \mathcal{E}_A(1) = N(A - (1)I_3) = \left\langle \left\{ \begin{bmatrix} \phi \\ 1 \\ 1 \end{bmatrix} \right\} \right\rangle$

$\lambda = 4 \quad A - (4)I_4 = \begin{bmatrix} 1-4 & 2 & 3 \\ \phi & 4-4 & 5 \\ \phi & \phi & 6-4 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 3 \\ \phi & \phi & 5 \\ \phi & \phi & 2 \end{bmatrix} \overset{\text{RREF}}{\to}$

$\to \begin{bmatrix} 1 & -2/3 & \phi \\ \phi & \phi & \phi \\ \phi & \phi & \phi \end{bmatrix} \Delta \quad \mathcal{E}_A(4) = N(A - (4)I_3$

$= \left\langle \left\{ \begin{bmatrix} 2/3 \\ 1 \\ 1 \end{bmatrix} \right\} \right\rangle = \left\langle \left\{ \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix} \right\} \right\rangle$

$$\lambda = 6 \qquad A - (6)I_3 = \begin{bmatrix} 1-6 & 2 & 3 \\ 0 & 4-6 & 5 \\ 0 & 0 & 6-6 \end{bmatrix}$$

$$\begin{bmatrix} -5 & 2 & 3 \\ 0 & -2 & 5 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & -1.6 \\ 0 & 1 & -2.5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$E_A(6) = N_A(6) = N(A - (6)I_3) =$$

$$\left\langle \left\{ \begin{bmatrix} 1.6 \\ 2.5 \\ 1 \end{bmatrix} \right\} \right\rangle = \left\langle \left\{ \begin{bmatrix} 8/5 \\ 5/2 \\ 1 \end{bmatrix} \right. \right.$$