HW Assignment 2

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```r
# Sys.setenv(RGL_USE_NULL=TRUE)
# library(tidyverse)
library(matlib)
library(pracma)
library(tinytex)
```

ASSIGNMENT 2

IS 605 FUNDAMENTALS OF COMPUTATIONAL MATHEMATICS - 2015

Problem set 1

$$(1)\ Show\ A^tA \neq AA^t\ in\ general.\ (Proof\ and\ demonstration.)$$

$$A \ = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}\ A^t \ = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

$$AA^t = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 35 & 44 \\ 44 & 56 \end{bmatrix}$$

Matrix multiplication results in a 2x2 matrix

$$A^tA = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 11 & 17 \\ 11 & 25 & 39 \\ 17 & 39 & 61 \end{bmatrix}$$

Multiplication results in a 3x3 matrix

$$(2)\ For\ a\ special\ type\ of\ square\ matrix\ A,\ we\ get\ A^tA \ = \ AA^t. Under\ what\ conditions\ could\ this\ be\ true?$$

$$(Hint:\ The\ Identity\ matrix\ I\ is\ an\ example\ of\ such\ a\ matrix).$$

$$A \ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\ A^t \ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AA^t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^tA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem set 2

Matrix factorization is a very important problem. There are supercomputers built just to do matrix factorizations. Every second you are on an airplane, matrices are being factorized. Radars that track flights use a technique called Kalman filtering. At the heart of Kalman Filtering is a Matrix Factorization operation. Kalman Filters are solving linear systems of equations when they track your flight using radars. Write an R function to factorize a square matrix A into LU or LDU, whichever you prefer. Please submit your response in an R Markdown document using our class naming convention, e.g. LFulton_Assignment2_PS2.png

You don't have to worry about permuting rows of A and you can assume that A is less than 5x5, if you need to hard-code any variables in your code. If you doing the entire assignment in R, then please submit only one markdown document for both the problems.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$$

Calculate the Upper Triangular Matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix} \xrightarrow{-2R1 + R2 = R2} \begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & -4 \\ 3 & 1 & 1 \end{bmatrix} \xrightarrow{-3R1 + R3 = R3} \begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & -4 \\ 0 & -2 & 5 \end{bmatrix} \xrightarrow{-2R2 + R3 = R3} \begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & -4 \\ 0 & 0 & 3 \end{bmatrix} = U$$

Take the inverse of the coefficients (-2,-3,-2) to calculate the Upper Truangular Matrix to get the Lower Triangular Matrix (L)

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

Prove A = LU

$$LU = A \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & -4 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$$