C19

$C = \begin{vmatrix} -1 & 2 \\ -6 & 6 \end{vmatrix}$ $\quad p_C(x) = \begin{vmatrix} -1-x & 2 \\ -6 & 6-x \end{vmatrix}$

$= (-1-x)(6-x) - (2)(-6)$

$\frac{2}{3} \cdot 3 = \frac{6}{3} = 2$

$\frac{2}{3} \cdot \frac{3}{1}$ or $\frac{2}{1} = 2$

$= -6 + x - 6x + x^2 + 12$

$= x^2 - 5x + 6$

$= (x-3)(x-2)$

eigenvalues of C are solutions to $p_C(x) = \emptyset$, $\lambda = 2$ and $\lambda = 3$
each appears once in characteristic polynomial
$\alpha_C(2) = 1$, $\alpha_C(3) = 1$

eigenspaces $\lambda = 2$

$C - (2)I_2 = \begin{bmatrix} -1-2 & 2 \\ -6 & 6-2 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & -\frac{2}{3} \\ \emptyset & \emptyset \end{bmatrix}$

$\mathcal{E}_C(2) = N(C - (2)I_2) = \left\langle \left\{ \begin{bmatrix} 2/3 \\ 1 \end{bmatrix} \right\} \right\rangle = \left\langle \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\} \right\rangle$

$\lambda = 3$ $\begin{bmatrix} -1-3 & 2 \\ -6 & 6-3 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ -6 & 3 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & -\frac{1}{2} \\ \emptyset & \emptyset \end{bmatrix}$

$\mathcal{E}_C(3) = N(C - (3)I_2) = \left\langle \left\{ \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} \right\} \right\rangle = \left\langle \left\{ \begin{matrix} 1 \\ 2 \end{matrix} \right\} \right\rangle$

Each eigenspace has one dimension thus geometric multiplicities $\gamma_C(2) = 1$ and $\gamma_C(3) = 1$

C2$\cancel{0}$

$B = \begin{bmatrix} -12 & 30 \\ -5 & 13 \end{bmatrix}$ $p_B(x) = \det(B - xI_2)$

$= \begin{vmatrix} -12-x & 30 \\ -5 & 13-x \end{vmatrix}$

$= (-12-x)(13-x) - (30)(-5)$

$= -156 + 12x - 13x + x^2 - (-150)$

Char poly $= x^2 - x - 6$ ✓ eigenvalues $\lambda = 3$ $\lambda = -2$

$= (x-3)(x+2)$ $\alpha_B(3) = 1$ $\alpha_B(-2) =$

algebraic multiplic

$\lambda = 3$ $B - 3I_2 = \begin{bmatrix} -12-3 & 30 \\ -5 & 13-3 \end{bmatrix} = \begin{bmatrix} -15 & 30 \\ -5 & 10 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$

$\mathcal{E}_B(3) = N(B - (3)I_2) = \left\langle \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\} \right\rangle$

$\lambda = -2$ $B - (-2)I_2 = \begin{bmatrix} -12-(-2) & 30 \\ -5 & 13-(-2) \end{bmatrix} = \begin{bmatrix} -10 & 30 \\ -5 & 15 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & -3 \\ 0 & 0 \end{bmatrix}$

$\mathcal{E}_B(-2) = N(B - (-2)I_2) = \left\langle \left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right\} \right\rangle$

Each eigenspace has one dimension thus geometric multiplicties $\gamma_B(3) = 1$ and $\gamma_B(-2) = 1$

C21 $A = \begin{bmatrix} 18 & -15 & 33 & -15 \\ -4 & 8 & -6 & 6 \\ -9 & 9 & -16 & 9 \\ 5 & -6 & 9 & -4 \end{bmatrix}$ eigenvalue $\lambda = 2$

$$A-(2)I_4 = \begin{bmatrix} (18-2) & -15 & 33 & -15 \\ -4 & (8-2) & -6 & 6 \\ -9 & 9 & (-16-2) & 9 \\ 5 & -6 & 9 & (-4-2) \end{bmatrix}$$

$$A-(2)I_4 = \begin{bmatrix} 16 & -15 & 33 & -15 \\ -4 & 6 & -6 & 6 \\ -9 & 9 & -18 & 9 \\ 5 & -6 & 9 & -6 \end{bmatrix} \rightarrow (1)$$

$$\xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

two free variables

→ basis of a Null Space will contain two vectors thus the Null Space of $A-(2)I_4$ has 2 dimensions and geometric multiplicity = 2