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HW Buddy: David Wang

Lab 07 : Combinatorial Digital Logic Design

You may collaborate on this homework with at most one person, an optional "homework buddy". You must turn in your own version of this lab, but you may discuss answers and approaches in detail. Projects should be turned in using Gradescope. Due Wednesday (March 2) at 5pm.

1. (10 pts) In lecture, we went through the example of splitting a binary addition into single bit additions. This is simplest in terms of design time, but it does not lead to the fastest implementation. Instead, let's create a 2-bit adder as the smallest unit. In this case, there are four bits of input from the two numbers, in addition to the single input carry. There are two result bits and a single output carry. A0 is the least significant bit (the one on the right). inputs: A1, A0, B1, B0, C_{in} outputs: R0, R1, C_{out}. For example, if we are performing an addition, it looks like this:

```
          A1   A0
+         B1   B0
+              Cin
  ----------------
Cout      R1   R0
```

Fill in this truth table for the function described above:

C_{in}	A1	B1	A0	B0	Cout	R1	R0
0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	1
0	0	0	1	0	0	0	1
0	0	0	1	1	0	1	0
0	0	1	0	0	0	1	0
0	0	1	0	1	0	1	1
0	0	1	1	0	0	1	1
0	0	1	1	1	1	0	0
0	1	0	0	0	0	1	0
0	1	0	0	1	0	1	1
0	1	0	1	0	0	1	1
0	1	0	1	1	1	0	0
0	1	1	0	0	1	0	0
0	1	1	0	1	1	0	1
0	1	1	1	0	1	0	1
0	1	1	1	1	1	1	0
1	0	0	0	0	0	0	1
1	0	0	0	1	0	1	0
1	0	0	1	0	0	1	0
1	0	0	1	1	0	1	1
1	0	1	0	0	0	1	1
1	0	1	0	1	1	0	0
1	0	1	1	0	1	0	0
1	0	1	1	1	1	0	1
1	1	0	0	0	0	1	1
1	1	0	0	1	1	0	0
1	1	0	1	0	1	0	0
1	1	0	1	1	1	0	1
1	1	1	0	0	1	0	1
1	1	1	0	1	1	1	0
1	1	1	1	0	1	1	0
1	1	1	1	1	1	1	1

2. (10 pts) Given this truth table

A	B	C	D	O
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

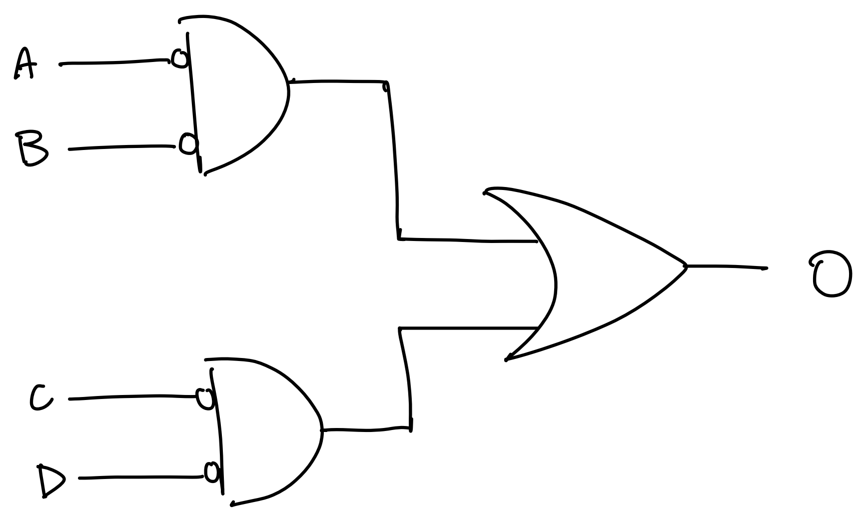
a. (5 pts) Draw the Karnaugh Map (K-Map), mark it clearly (cleanly and group optimally), and draw the optimal groupings.

AB \ CD	00	01	11	10
00	1	1	1	1
01	1	0	0	0
11	1	0	0	0
10	1	0	0	0

b. (2 pts) Write the **optimized** “sum of products” style equation for the output **O**, based on your K-Map findings.

$O = \bar{A}\bar{B} + \bar{C}\bar{D}$

c. (3 pts) Draw the optimized digital logic design for this function.



3. (10 pts) Given this truth table

A	B	C	D	O
0	0	0	0	**1**
0	0	0	1	**0**
0	0	1	0	**0**
0	0	1	1	**0**
0	1	0	0	**1**
0	1	0	1	**1**
0	1	1	0	**0**
0	1	1	1	**0**
1	0	0	0	**0**
1	0	0	1	**1**
1	0	1	0	**0**
1	0	1	1	**0**
1	1	0	0	**1**
1	1	0	1	**0**
1	1	1	0	**0**
1	1	1	1	**1**

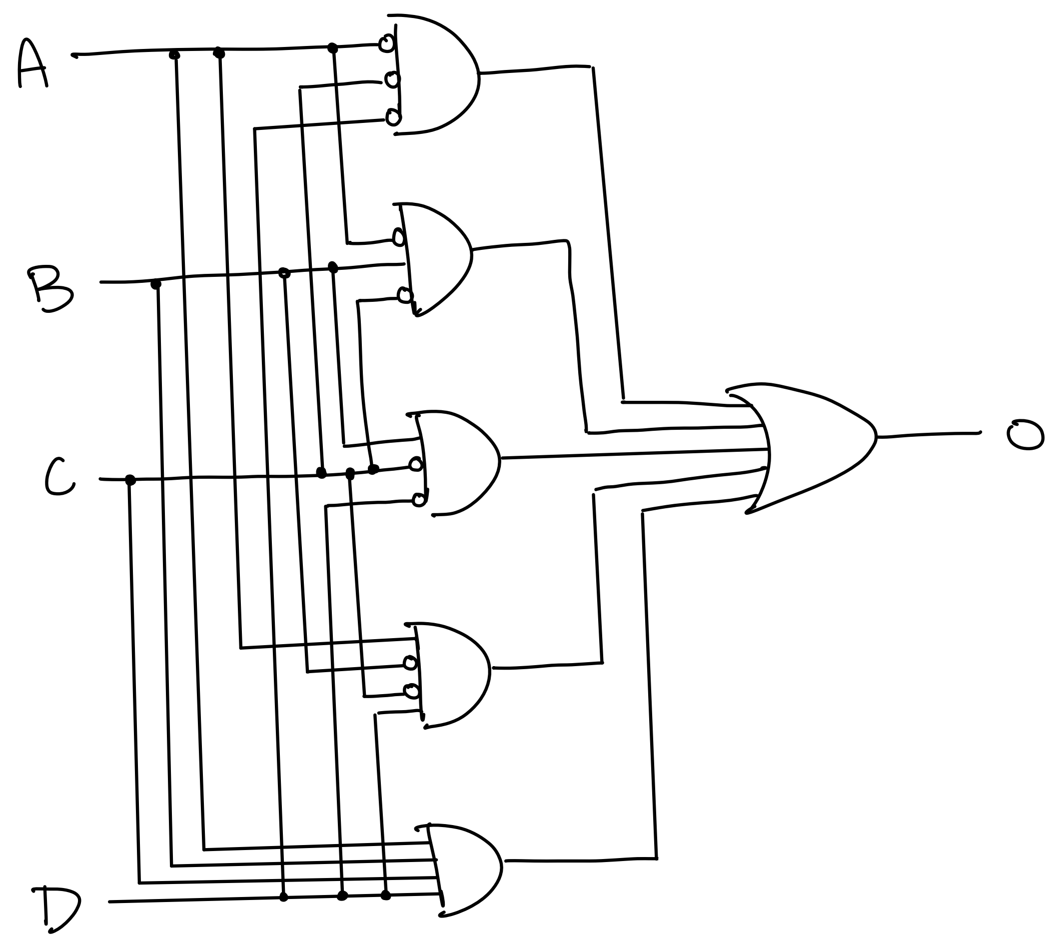
a. (5 pts) Draw the Karnaugh Map (K-Map), mark it clearly (cleanly and group optimally), and draw the optimal groupings.

AB \ CD	00	01	11	10
00	1	0	0	0
01	1	1	0	0
11	1	0	1	0
10	0	1	0	0

b. (2 pts) Write the **optimized** “sum of products” style equation for the output **O**, based on your K-Map findings.

$$O = \bar{A}\bar{C}\bar{D} + \bar{A}B\bar{C} + B\bar{C}\bar{D} + A\bar{B}\bar{C}D + ABCD$$

c. (3 pts) Draw the optimized digital logic design for this function.



4. (15 pts) Given this truth table (**and note the use of don't-cares**, represented by **X**):

A	B	C	D	O
0	0	0	0	**0**
0	0	0	1	**X**
0	0	1	0	**0**
0	0	1	1	**1**
0	1	0	0	**X**
0	1	0	1	**0**
0	1	1	0	**X**
0	1	1	1	**1**
1	0	0	0	**X**
1	0	0	1	**0**
1	0	1	0	**1**
1	0	1	1	**0**
1	1	0	0	**1**
1	1	0	1	**0**
1	1	1	0	**X**
1	1	1	1	**0**

a. (2 pts) Write the **unoptimized** "sum of products" style equation for the output **O.**

$$O = \overline{AB}CD + \bar{A}BCD + A\bar{B}C\bar{D} + AB\bar{C}\bar{D}$$

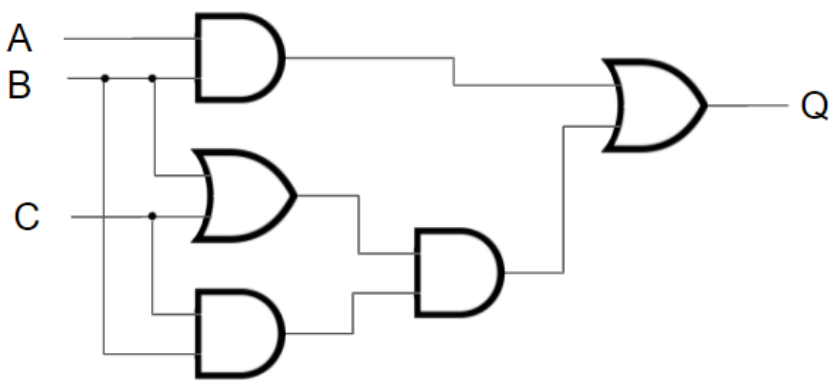
b. (10 pts) Draw the K-Map, mark it clearly (cleanly and group optimally) and **MAKE USE OF THE "DON'T-CARES".**

AB \ CD	00	01	11	10
00	0	X	1	0
01	X	0	1	X
11	1	0	0	X
10	X	0	0	1

c. (3 pts) Write the best **optimized** "sum of products" style equation for the output **O**.

$O = A\bar{D} + \bar{A}CD$

5. (10 pts) Given the following digital logic design:



a. (10 pts) Write an expression of the output **Q** – it can be unoptimized. Draw the Karnaugh Map (K-Map), mark it clearly (cleanly and group optimally), and draw the optimal groupings.

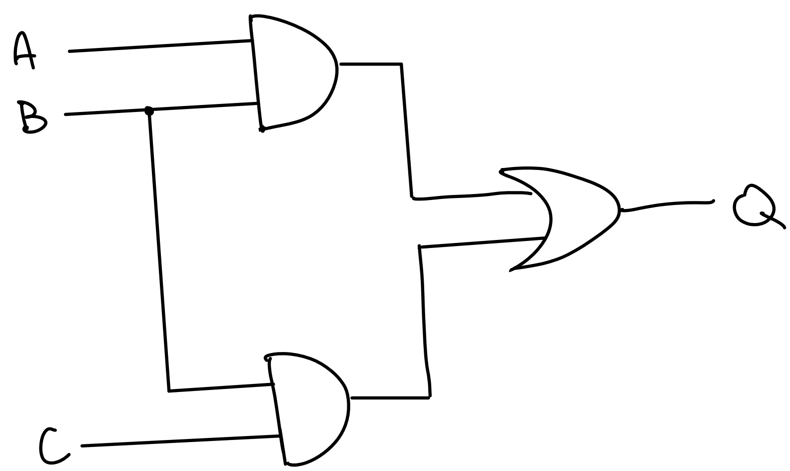
$Q = AB + (B+C)(BC)$

AB \ C	0	1
00	0	0
01	0	1
11	1	1
10	0	0

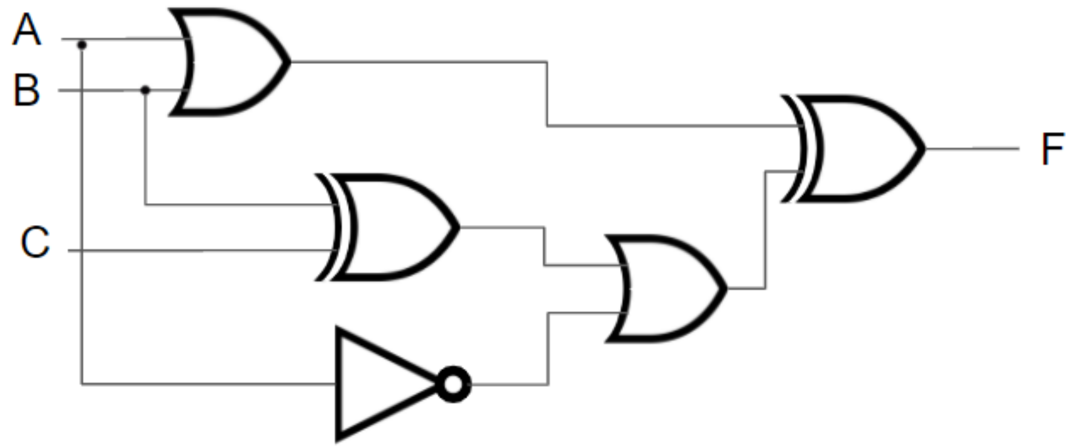
b. (2 pts) Write the best **optimized** “sum of products” style equation for the output **Q**.

$Q = AB + BC$

c. (3 pts) Re-draw the circuit based on your answers in part (b).



6. (10 pts) Given the following digital logic design:



a. (3 pts) Write the Boolean expression for the output.

$F = (A+B) \oplus ((B \oplus C) + \bar{A})$

b. (7 pts) Draw the truth table for this circuit.

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

7. (10 pts) For each problem draw the K-Maps, mark them clearly (cleanly and group optimally), and use them to optimally simplify the expressions.

a. (5 pts) f(A,B,C) = !A!B!C + A!B!C + !ABC + !AB!C + A!BC

$f(A,B,C) = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}C$

BC \ A	0	1
00	1	1
01	0	1
11	1	0
10	1	0

$\boxed{f(A,B,C) = \bar{B}\bar{C} + A\bar{B} + \bar{A}B}$

b. (5 pts) f(A,B,C,D) = (!A!C + A!D)(B(D + !BC))

$$f(A,B,C,D) = (\bar{A}\bar{C} + A\bar{D})(B \cdot (D + \bar{B}C))$$

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	1	0	0
11	0	0	0	0
10	0	0	0	0

$$\boxed{f(A,B,C,D) = \bar{A}B\bar{C}D}$$