Chapter 1

$|\psi\rangle = C_+|+z\rangle + C_-|-z\rangle = \begin{pmatrix} C_+ \\ C_- \end{pmatrix}$

↳ state = superposition of basis states $|+z\rangle$, $|-z\rangle$

$\langle\psi| = |\psi\rangle^\dagger = C_+^* \langle +z| + C_-^* \langle -z| = (C_+^* \; C_-^*)$

$\langle\varphi|\psi\rangle$ = amplitude for $|\psi\rangle$ to be in $|\varphi\rangle$

$\langle\psi|\psi\rangle = 1$

$|\langle\varphi|\psi\rangle|^2$ = probability that you measure $|\varphi\rangle$ for a particle in $|\psi\rangle$

$|\psi\rangle = |+z\rangle\underbrace{\langle +z|\psi\rangle}_{C_+} + |-z\rangle\underbrace{\langle -z|\psi\rangle}_{C_-}$ $\Rightarrow$ $\langle\psi|\varphi\rangle = \langle\varphi|\psi\rangle^*$

$\langle\psi| = \langle\psi|+z\rangle\langle +z| + \langle\psi|-z\rangle\langle -z|$

p.7 $|+x\rangle = \frac{1}{\sqrt{2}}(|+z\rangle + |-z\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad |-x\rangle = \frac{1}{\sqrt{2}}(|+z\rangle - |-z\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Expectation value: $\langle S_z\rangle = P(\hbar/2)\cdot\frac{\hbar}{2} + P(-\hbar/2)\cdot(-\hbar/2)$

$= |\langle +z|\psi\rangle|^2\left(\frac{\hbar}{2}\right) + |\langle -z|\psi\rangle|^2\left(-\frac{\hbar}{2}\right)$

uncertainty: $\sigma(f) = [\langle f^2\rangle - \langle f\rangle^2]^{1/2}$ → this but w/ the $(\pm\hbar/2)^2$

eg. $\langle S_z^2\rangle - \langle S_z\rangle^2$ p.7

$|+y\rangle = \frac{1}{\sqrt{2}}(|+z\rangle + i|-z\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$ $\quad |-y\rangle = \frac{1}{\sqrt{2}}(|+z\rangle - i|-z\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$

Chapter 2

$|\psi\rangle = \begin{pmatrix} \langle +z|\psi\rangle \\ \langle -z|\psi\rangle \end{pmatrix} = \begin{pmatrix} C_+ \\ C_- \end{pmatrix}$

In z basis: $|+z\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\; |-z\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\; |+x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\; |-x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $\; |+y\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$ $\; |-y\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$

change of basis: $|\psi\rangle_x = R_{z\to x}|\psi\rangle_z$

$R_{z\to x} = \begin{pmatrix} |+x\rangle_x\langle +x|_z \\ +|-x\rangle_x\langle -x|_z \end{pmatrix}$

$R_{x\to z} = R_{z\to x}^{-1}$

Rotations: $\hat{R}_n(\theta) = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix} \xrightarrow{\theta \text{ small}} = I - \frac{i\theta}{\hbar}\hat{J}_n = e^{-i\hat{J}_n\theta/\hbar}$ ← $J_n^\dagger = J_n$

(true no matter the basis) unitary!

$\langle S_z\rangle = \langle\psi_z|J_{zz}|\psi_z\rangle$ ← chosen basis (any)

Raise z

$\langle A\rangle = \langle\psi|\hat{A}|\psi\rangle$

if A observable, $\hat{A}$ Hermitian $(A^\dagger = A)$

$\hat{J}_{xz} = \underset{\text{kets}}{R_{z\to x}^\dagger} J_{xx} \underset{\text{bras}}{R_{z\to x}}$

$J_{zz} = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\quad J_{xz} = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad J_{yz} = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

eigenvals: $A = V\Lambda V^{-1} \xrightarrow[V^{-1}=V^\dagger]{A \text{ herm.}} = V\Lambda V^\dagger$

eigenstates ⟹ uncert = 0

$V^\dagger V = I$ → unitary

p.19 A hermitian: $A = V\Lambda V^\dagger \rightarrow U = e^{iA} = Ve^{i\Lambda}V^\dagger$

chapter 3

Rotating a vector → p. 23

$[J_a, J_b] = i\hbar\,\varepsilon_{abc} J_c$

$J^2 = J_x^2 + J_y^2 + J_z^2$

$J^2|\ell,m\rangle = \ell\hbar^2|\ell,m\rangle$ $\quad J_z|\ell,m\rangle = m\hbar|\ell,m\rangle$ $\quad$ OR

$J_z^2 \rightarrow m^2\hbar^2$

p.27 $J_{\pm,z} = J_x \pm iJ_y$ $\quad$ Raise/lower along J_z (all in same basis)

$J_+^\dagger = J_-$ $\quad$ ↳ permute to get other ones

to raise the state by a unit of $\hbar$ along the J_x momentum

kets @ $J_{+,n}$ n @ bras @ state ↳ expressed in z basis

- kets: x to z basis; $x$kets in z basis
- $J_{+,n}$: so $J_{+,x}$ in x basis = $J_{+,z}$ in z basis; Raise in x basis
- bras: z to x basis; $x$bras in z basis

→ J^2 commutes w/ the J_n & J_n^2

$J_\pm$ don't commute w/ J_n

$J_z\underbrace{(J_+|\ell,m\rangle)}_{\text{eigenstate of } J_z} = \hbar\underbrace{(m+1)}_{\text{changed eigenval by 1 (raised, units of } \hbar)}\underbrace{(J_+|\ell,m\rangle)}_{\text{still an eigenstate}}$ $\qquad J_z(J_-|\ell,m\rangle) = \hbar(m-1)(J_-|\ell,m\rangle)$

max $m = j$ → p. 29-30 $\quad J^2|j,m\rangle = j(j+1)\hbar^2|j,m\rangle$ $\quad \ell = j(j+m)$

$J_z|j,m\rangle = m\hbar|j,m\rangle$ $\quad m_{max} = j$, $m_{min} = -j$

$J_+|j,m\rangle = \sqrt{j(j+1) - m(m+1)}\,\hbar\,|j,m+1\rangle$ $\quad = 0$ if $m = j$

$J_-|j,m\rangle = \sqrt{j(j+1) - m(m-1)}\,\hbar\,|j,m-1\rangle$ $\quad = 0$ if $m = -j$

$J_x = \frac{J_+ + J_-}{2}$ $\quad J_y = \frac{J_+ - J_-}{2}$ $\quad$ for $J_\pm$ of J_z

$\langle\alpha|\alpha\rangle\langle\beta,\beta\rangle \geq |\langle\alpha|\beta\rangle|^2 \Rightarrow \sigma(A)\sigma(B) \geq \frac{1}{2}|\langle C\rangle|$ for $[A,B] = iC$ $\quad$ so $\sigma(J_x)\sigma(J_y) \geq \frac{\hbar}{2}|\langle J_z\rangle|$