PHYS 357 Pset 7. Due 11:59 PM Thursday Nov. 7

1. Townsend 4.9. I suggest you start with Townsend eqn. 4.41 and throw away the $\omega + \omega_0$ terms. Proceed as we did in class, and you should end up with a second-order differential equation for $d(t)$ with constant coefficients. If you still have an explicit $t-$dependence, something has gone wrong. Once you have the form for $d(t)$, you can go back to eqn. 4.41 and use the fact that the starting wave function is $(1, 0)$ to get the full initial conditions.

2. Verify Rabi's formula by solving the magnetic resonance equations numerically for a range of values for ω. You can start with the magnetic resonance solver we used in class, which is on github in codes/magres_class.py. For $\omega_0 = 1, \omega_1 = 0.1$, plot the max probability to be in the spin-down state as a function of ω for $0.7 \leq \omega \leq 1.3$, and overplot Rabi's approximation. It's pretty good!

3. Townsend 4.16. The wavelength of this transition (called Lyman-α) is 121.6 nm, and as a reminder, the energy of a photon is $E = h\nu$.

4. Repeat the previous problem, but this time do it for the electron-proton spin-flip in hydrogen (we'll see more of this in Chapter 5). This is a forbidden transition, and so it has a lifetime of 11 million *years*. This transition has an emission frequency of 1420 MHz.

   The extreme narrowness of this transition means that atomic clocks based on hydrogen are some of the most accurate in the world, especially on timescales of 1 day. Hydrogen clocks are used on the EU's Galileo satellites, which keep time to about half a nanosecond per day.

5. Townsend 4.15. Feel free to do this on a computer. Note - Townsend doesn't specify, but the starting state $|3/2, 3/2\rangle$ is in the $z-$basis.

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Townsend 4.9: Derive Rabi's formula

$$|\langle -\mathbf{z}|\psi(t)\rangle|^2 = \frac{\omega_1^2/4}{(\omega_0-\omega)^2+\omega_1^2/4}\sin^2\frac{\sqrt{(\omega_0-\omega)^2+\omega_1^2/4}}{2}t$$

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