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HW3

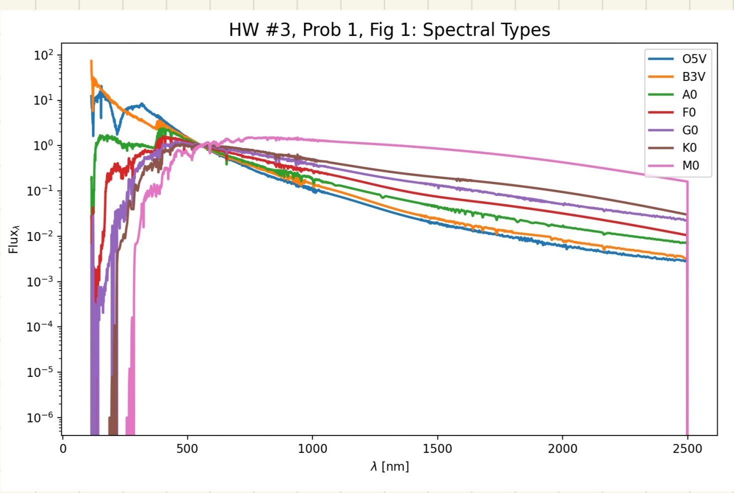
1. Selected spectra of Main Sequence stars, taken from the **Pickles Stellar Spectral Flux Library**, are on the class MyCourses site. Each is an ascii file identified by the spectral type O5, B3, A0, F0, G0, K0, M0 containing two columns of numbers, (1) wavelength and (2) the corresponding monochromatic flux, normalized to unity at 555 nm.
   - ~~a)~~ Use Table 2 on the Pickles Library website to find the effective temperature in degrees Kelvin for each of the stars. Present this in a table. Which one is most like the sun?
   - ~~b)~~ Look at and run the Python script *pickles.py*. The script reads each file with the command `open(file.txt)` and the for loop defines a wavelength vector (column 1) and flux vector (column 2) for each spectrum. Figure 1 plots the spectra normalized to unity at 555 nm and Figure 2 plots them again, but now each one is simply shifted vertically to separate them visually. Be sure you see how this was done.
   - ~~c)~~ Modify the script to add a Figure 3 that shows the spectra only between 350 and 800 nm. Make adjustments in the y-axis range and flux normalization so the differences in the spectral lines can be easily assessed. Put a vertical line at the location of Hα.
   - ~~d)~~ Describe the key differences among the main sequence spectra. Which star shows the most prominent Hydrogen lines?

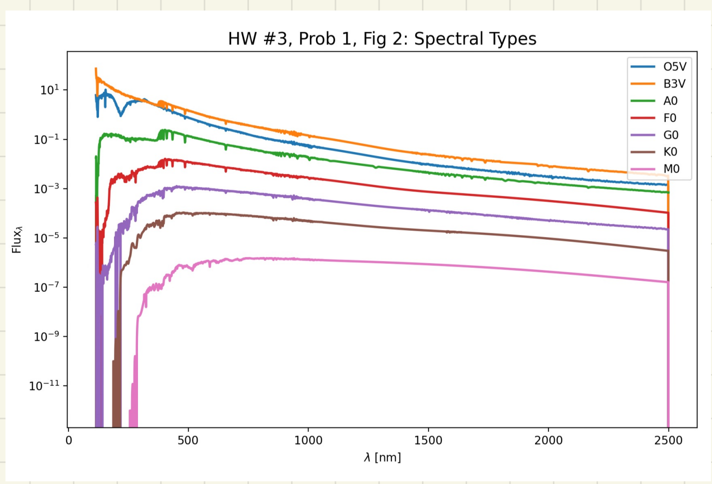
a)

Spectral Type	T_eff [K]
O5	39810.7
B3	19054.6
A0	9549.93
F0	7211.08
G0	5807.64
K0	5188.0
M0	3801.89

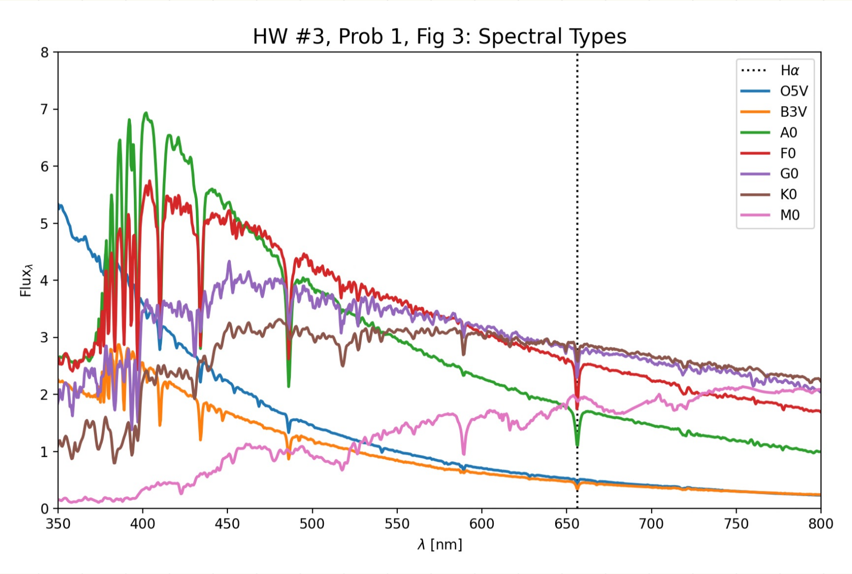
The sun has T~5800 K so the G0 is the most like the sun

b)



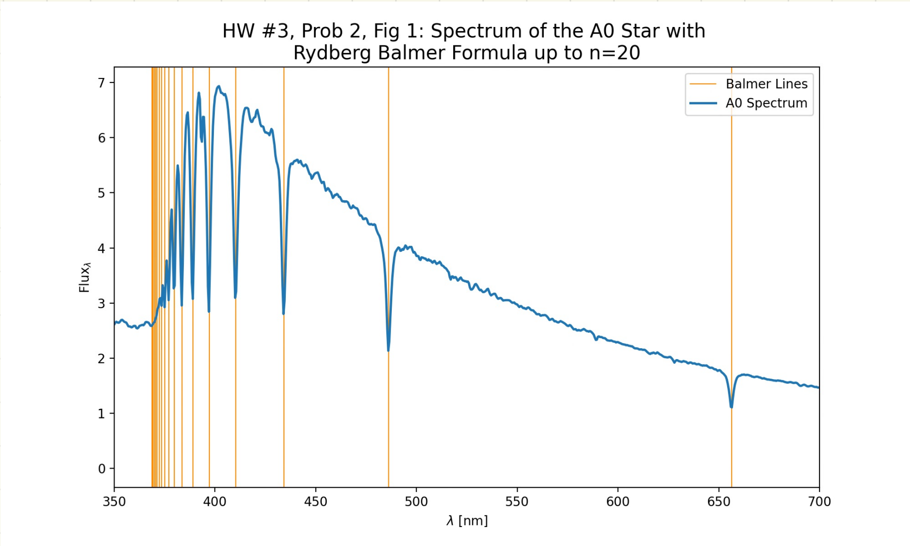


c)



d) The M0 star doesn't really follow the same trend as the other spectral types, and doesn't show the Hα line either.
The largest Hα dips are seen in the A0 & F0 types (followed by the G0 line)
We can also see that the location of the peak flux goes to lower wavelengths as temperatures increase

2. Write a new script that combines relevant parts of *balmer.py* and *pickles.py* to plot only the spectrum of the A0 star and overplot it on the same figure with the line locations from the Balmer Formula. Set the x-axis range from 350 to 700 nm.
   a) What is the temperature of the A0 star? At what wavelength in nm is it expected to have its peak brightness due to thermal radiation? Is that where it actually peaks?
   b) What is the wavelength in nm of a photon that would ionize hydrogen atoms with electrons in the n=2 level, (E_∞-E_2). Where is that relative to the Balmer lines in the figure?
   c) The wavelength of 365 nm is sometimes called the "Balmer Jump". What might be going on here in the A0 star?



a) from the table in Q1, $T = 9549.93$ K.
we can calculate the location of the peak with Wien's displacement law

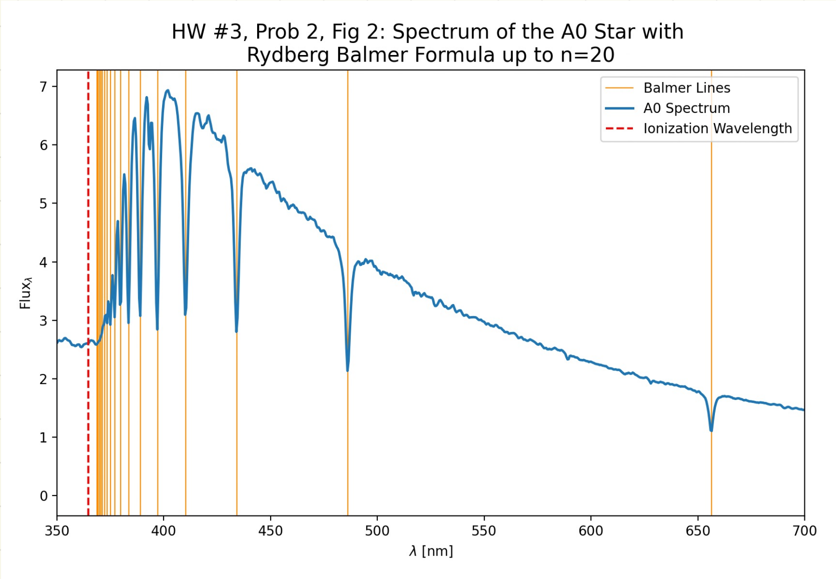
$$\lambda_{max} T = 2.9 \times 10^6 \text{ nm} \cdot \text{K}$$

$$\lambda_{max} = \frac{2.9 \times 10^6}{9549.93}$$

$$\lambda_{max} = 303.43 \text{ nm}$$

from the graph, the peak occurs around ~400 nm, so the predicted location of the peak is much lower than what we observe.

b) ionization wavelength is 364.71 nm
→ on the left end of the Balmer lines



c) This line represents the transition from $n=2$ to $n=\infty$, ie the H is getting ionized from the 2nd energy level. This occurs in A0 stars due to their high temperatures, and represents the transition between a regime dominated by state transitions (<365 nm) and a regime dominated by ionization (>365nm)

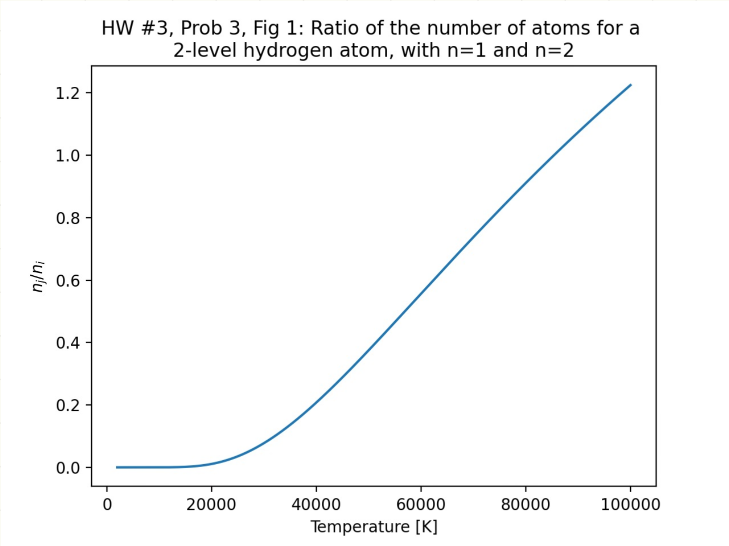
3. The ***Boltzmann Excitation*** formula gives the ratio of the number of atoms of the same element with electrons in any two energy levels, *i* (lower) and *j* (upper).

$$\frac{n_j}{n_i} = \frac{g_j}{g_i} e^{-[(E_j - E_i)/kT_k]}$$

The *g*'s are atomic parameters called statistical weights, that specify the number of sublevels in each *n* energy level. **For hydrogen they are given by $g_n = 2n^2$**. The E's are the binding energy of each level and T_k is the 'kinetic' temperature in K.

a) Write a Python script to create a plot of n_j / n_i for a 2-level hydrogen atom with $j = 2$ and $i = 1$ from T = 2000 to 100,000 K.
b) Describe the plot in words.
c) At what temperature is there an equal number of electrons in energy levels 1 and 2?
d) At what temperature is the number of atoms in n=2 equal to 10% of the number in the ground state?
e) As T approaches infinity, what is the expected ratio of electrons in the n=2 state compared to the ground state?

a)



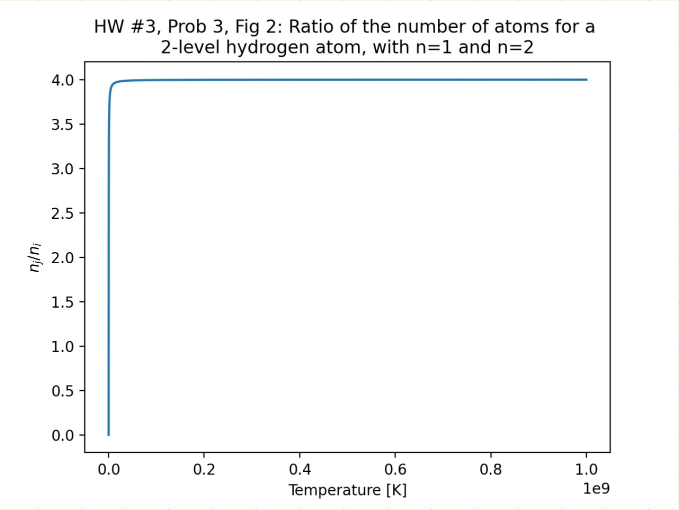
b) At low temperatures, the atoms are mainly in the ground state, and as T increases more atoms get excited to the n=2 level due to the increase in thermal energy $\rightarrow n_j > n_i$ so n_j/n_i increases

c) $n_j/n_i = 1$ at $T = 85386.74$ K

d) $n_j/n_i = 0.1$ at $T = 32089.01$ K

e) as $T \rightarrow \infty$, $e^{-\Delta E/kT} \rightarrow$ cst

so we get $\frac{n_j}{n_i} = \frac{g_j}{g_i} = 4$



4. The ***Saha Ionization*** formula gives the ratio of the number of atoms in two different states of ionization, (X_{r+1}) and (X_r).

$$\frac{N(X_{r+1})}{N(X_r)} = \frac{2kTg_{r+1}}{P_e g_r}\left(\frac{2\pi m_r kT}{h^2}\right)^{3/2} e^{-\chi_i/kT}$$

The g's are atomic parameters, m_r is the reduced mass which is almost the same as the mass of an electron, P_e is the electron pressure and χ_{ion} is the energy to ionize X_r from the ground state. For the case of hydrogen X_r and X_{r+1} correspond to H I (neutral) and H II (singly ionized) and the g's are 1 for H I and 2 for H II.

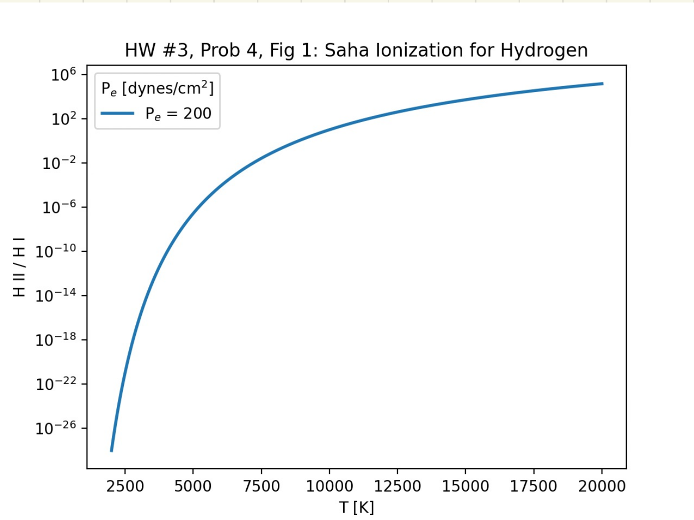
a) What is the ionization energy of H I in ev and in erg? Explain what this energy represents.

b) The Python script *saha.py* calculates and plots the ratio of the number of ionized to neutral hydrogen atoms, N_{HII}/N_{HI}, for temperatures from 2000 K to 20,000 K. Run the program and look at the plot.
- Describe the behavior of N_{HII}/N_{HI} in words.
- At T = 2000 K what is the ratio of ionized to neutral hydrogen?
- At T = 6000 K?
- At T = 10000 K?
- At what temp is there 10 times more ionized than neutral H?

c) In the script the electron pressure, P_e, is set to that in the sun's photosphere, P_e = 200 dyne cm^{-2}. Modify the script to overplot N_{HII}/N_{HI} for 3 different electron pressures, P_e = 20, 200, 2000 dyne cm^{-2}. This covers the range for main sequence stars.
- What effect does changing the electron pressure have?

a) 13.54 eV = 2.166×10^{-11} erg
This is the minimum energy required to remove an e^- from the ground state of H (ie, ionize H)

b)



At low T, the atoms are neutral

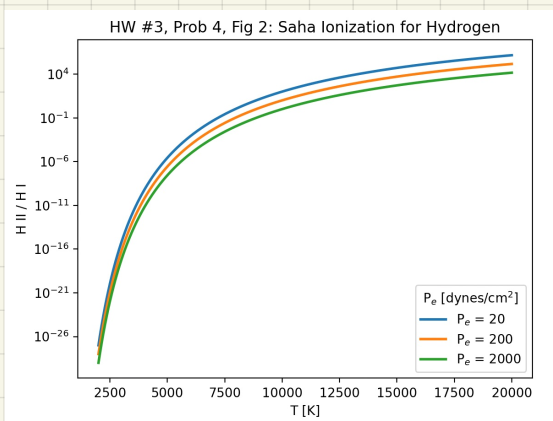
As T increases, the available thermal energy allows the atoms to go to higher energy states so we get a higher fraction of HII (ie, HII/HI increases) → get ionized

T	HII/HI
2000 K	1.009 ×10^{-28}
6000 K	8.16 ×10^{-5}
10 000 K	10.22

*from the code

HII/HI = 10 at T = 9987.94 K

c)



Increasing the electron pressure brings down the HII/HI curve. This means that at a given temperature, a higher P_e will result in a lower amt of ionized H.

5. Derive an expression for the *fraction of the total number hydrogen atoms that are ionized*, N_{HII}/N_{total}, in terms of the number of ionized N_{HII} and neutral N_{HI} atoms. Express your answer so N_{HII} is given as a function of N_{HII} / N_{HI}.

let $r = \frac{N_{HII}}{N_{HI}}$. we know $N_{total} = N_{HI} + N_{HII}$, so...

$$\frac{N_{HII}}{N_{total}} = \frac{N_{HII}}{N_{HI}+N_{HII}} = \left(\frac{N_{HI}+N_{HII}}{N_{HII}}\right)^{-1}$$

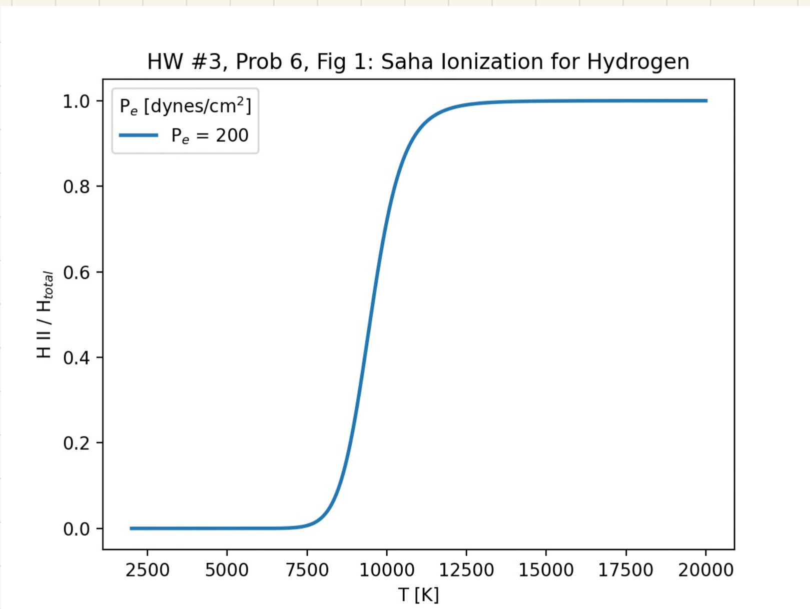
$$= \left(\frac{N_{HI}}{N_{HII}} + 1\right)^{-1}$$

$$= \left(\frac{1}{r} + 1\right)^{-1}$$

$$= \left(\frac{1+r}{r}\right)^{-1}$$

$$\frac{N_{HII}}{N_{total}} = \frac{r}{1+r}$$

6. Use your result from the previous problem to add a new figure to *saha.py* that plots N_{HII}/N_{total} for temperatures from 2000 to 20,000 K (the ratio should be from 0 to 1).
a) At T = 8000 K, what percent of hydrogen atoms are ionized?
b) At T = 12,000 K?



a) 2.81%
b) 98.22%

7. *Fusion of H to He.* Main sequence stars support themselves against gravitational collapse by releasing energy in the fusion of H to He. Whether via the proton-proton or CNO sequence the basic reaction is $4\,^1H$ morphs in ^4He, releasing nuclear binding energy in the process.

- ~~a)~~ The mass of one proton ^1H is 1.6726×10^{-24} gm and the mass of a helium nucleus ^4He is 6.6447×10^{-24} gm. What is the mass difference in gm between 4 protons and one helium nucleus?
- ~~b)~~ Calculate the ratio of the mass difference to the original mass of the 4 protons.
- ~~c)~~ Calculate the binding energy of $4\,^1H$ to ^4He corresponding the mass difference, in both erg and eV.

~~d)~~ Use the above binding energy of one fusion reaction coupled with the solar luminosity to calculate how many fusion reactions per second occur in the solar core.

*all computed in code

a) $\Delta m = 4m_H - m_{He} = 4.57 \times 10^{-26}\,g$

b) $\frac{\Delta m}{4m_H} = 6.83 \times 10^{-3}$

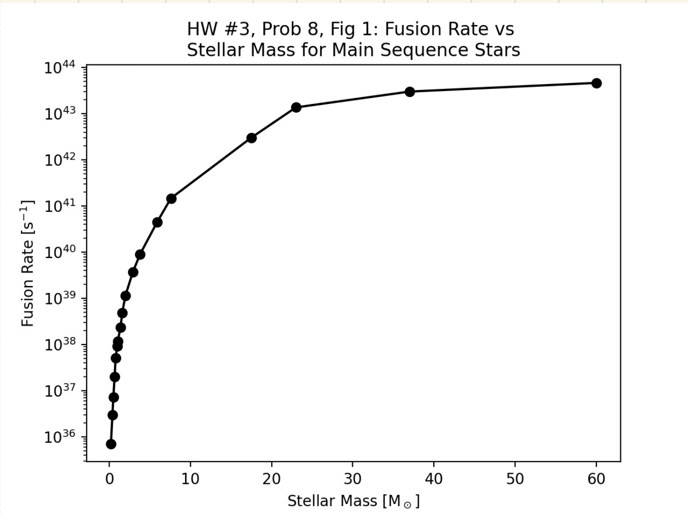
c) $E_{bind} = \Delta m c^2 = 4.11 \times 10^{-5}\,erg$
$= 2.56 \times 10^{7}\,eV$

d) $rate = \frac{L}{E} = \frac{L_\odot}{E_{bind}} = 9.36 \times 10^{37}\,s^{-1}$

8. *Main Sequence Fusion Rates & Lifetimes*

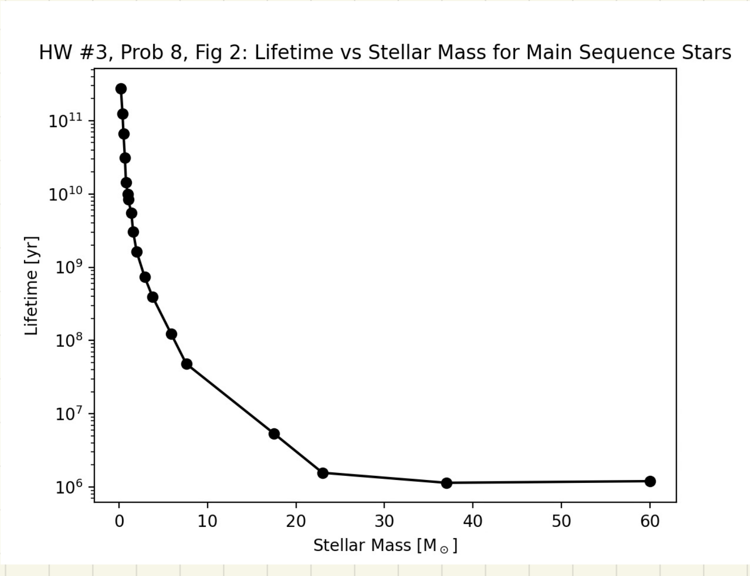
- ~~a)~~ Write a Python script that calculates the rate of fusion reactions per second for all the MS stars in Appendix G. Plot the rates as a function of Stellar Mass.
- ~~b)~~ What is the range of fusion rates across all MS stars?
- ~~c)~~ Write a Python script to calculate τ in years for all the main sequence stars in Appendix G. Plot τ against the mass in solar masses.
- ~~d)~~ What is the range in lifetimes of MS stars expressed as a ratio from max to min?

a)



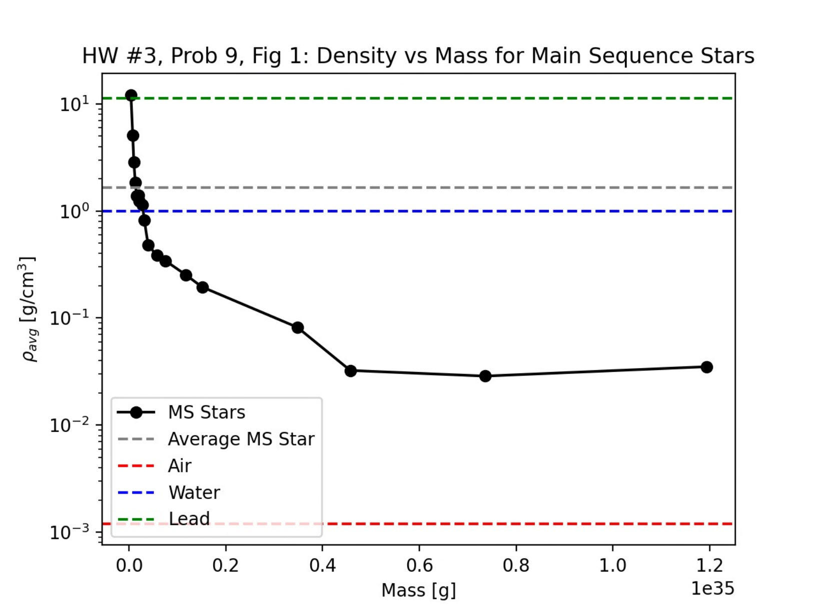
b) min fusion rate: $7.12 \times 10^{35}\,s^{-1}$
max fusion rate: $4.67 \times 10^{43}\,s^{-1}$
$\rightarrow$ Range = max - min $= 4.67 \times 10^{43}\,s^{-1}$

c) star lifetime $\approx 10^{10} \cdot \left(\frac{M}{M_\odot}\right) / (L/L_\odot)$



d) min lifetime: 1.14×10^{6} yr
max lifetime: 2.76×10^{11} yr
range $= \frac{max}{min} = 2.42 \times 10^{5}$

9. *Stellar Interiors*: *Average density ρ_{avg}* . Write a script to find the average density of the main sequence stars in Appendix G. Express your answer in gm cm^{-3}, and plot it against the mass of the star. Compare to the density of air, water, and lead (find on the web). Any surprises?



Avg ρ : 1.65 g/cm^3

In general, the main sequence stars have a density comparable to water. which is surprising!

10. *Stellar Interiors: core temperature* T_c. The equation of state for an ideal gas, $\mathbf{P = \rho\, k\, T/m}$, enables you to estimate the core temperature for main sequence stars, assuming a star is a single 'shell' with a density corresponding to the average for the entire star. (Remember that m is the 'reduced mass', about 0.6 times the mass of a proton for a fully ionized gas of H, He.)

a) Show that the expression for T_c can be expressed as $\mathbf{T_c = mG/k\ M/R}$.

b) Write a Python script to find the core temperature for all the main sequence stars in Appendix G. Plot T_c as a function of the mass of the star in solar masses and indicate the location of the sun.

c) At what spectral type/stellar mass does the T_c reach 17 x 10^6 K, where the fusion reaction transitions from primarily the proton-proton to the CNO chain?

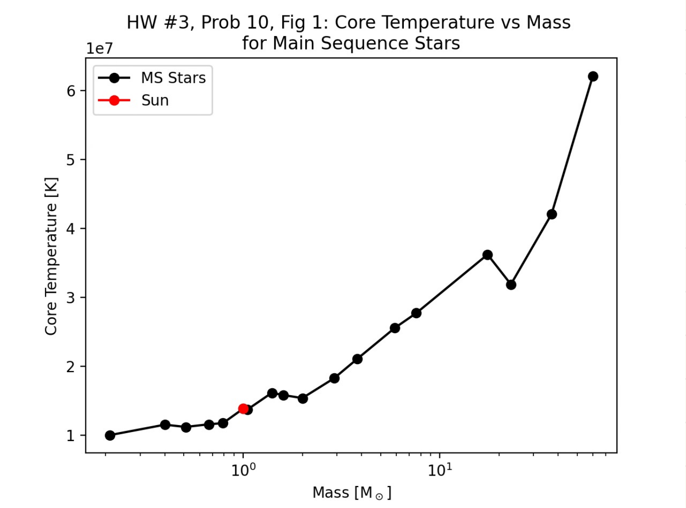
a) $P = \rho k \frac{T}{m} \rightarrow T_c = \frac{m P_c}{\rho_{avg} k}$

$P_c = G \rho_{avg} M/R$

$= \frac{m}{\cancel{\rho_{avg}} k} \cdot \frac{G \cancel{\rho_{avg}} M}{R}$

$T_c = \frac{GmM}{kR}$

b)



c) M ~ 1.40 Msun (from code)

so F5 type

11. *Stellar Interiors: core density* $\boldsymbol{\rho_c}$**.** The equation of mass continuity relates the mass in each shell to the density and volume of the shell.

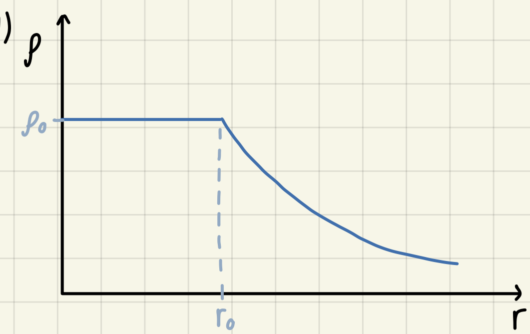
$$M(r) = \int 4\pi r^2 \rho(r) dr$$

We can approximate a value for the central density assuming the density was constant throughout the star. Now let there be two regions in the star:

- Core: $\rho(r) = \rho_0$ for $0 < r < r_0$
- Envelope: $\rho(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-2}$ for $r_0 < r < R$

~~a)~~ Make a sketch of how density varies with radius in each regime.
~~b)~~ Integrate the equation for M_r in each regime to find the expression for how mass varies with radius in range. You will need to use and solve for definite integrals.
~~c)~~ Add the masses from the two regimes to find the total M inside R.
~~d)~~ If $M = 1\ M_{sun}$, $R = 1\ R_{sun}$, and $r_o = 0.07\ R_{sun}$, find the resultant density in the core, $\boldsymbol{\rho_o}$, in gm cm^{-3}.

a)



b) ① $0 < r < r_0$

$$M(r) = \int 4\pi r^2 \rho(r) dr$$
$$= 4\pi \rho_0 \int_0^{r_0} r^2 dr$$
$$= 4\pi \rho_0 \frac{1}{3} r_0^3$$
$$M_{in} = \frac{4\pi}{3} r_0^3 \rho_0$$

② $r_0 < r < R$

$$M(r) = \int 4\pi r^2 \rho(r) dr$$
$$= 4\pi \int_{r_0}^{R} \cancel{r^2} \cdot \rho_0 \frac{\cancel{r^{-2}}}{r_0^{-2}} dr$$
$$M_{out} = 4\pi r_0^2 \rho_0 \cdot (R - r_0)$$

c) $M_{tot} = M_{in} + M_{out}$

$$= \frac{4\pi}{3} r_0^3 \rho_0 + 4\pi r_0^2 \rho_0 (R - r_0)$$
$$= 4\pi r_0^2 \rho_0 \left(\frac{r_0}{3} - r_0 + R\right)$$
$$M_{tot} = 4\pi r_0^2 \rho_0 \left(R - \frac{2}{3} r_0\right)$$

d) $M = M_{sun}$, $R = R_{sun}$, $r_0 = 0.07\ R_{sun}$

$$M_{tot} = 4\pi (0.07 R_{sun})^2 \rho_0 \left(R_{sun} - \frac{2}{3} \cdot 0.07 R_{sun}\right) = M_{sun}$$

$\rightarrow \rho_0 = 100\ g/cm^3$ (see code)