ASTRO

Chapter 1 : TOOLS OF ASTRONOMY

UNITS & COORDINATES

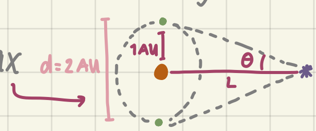
· **Units & dimensions**: (cgs, SI)

[L]: **Length**: cm, 100cm = 1m
[M]: **Mass**: g, 1000g = 1kg
[T]: **Time:** s
[LT^{-1}]: **Speed:** cms^{-1}, 100 cms^{-1} = 1ms^{-1}
[MLT^{-2}]: **Force**: dyne = $gcms^{-2}$, 10^5 dyne = 1N
[ML^2T^{-2}]: **Energy:** erg = gcm^2s^{-2}, 10^7 erg = 1J
[ML^2T^{-3}]: **Power:** $ergs^{-1}$ = gcm^2s^{-3}, 10^7 $ergs^{-1}$ = 1W
[IT],[$M^{1/2}L^{3/2}T^{-1}$]: **Charge**: esu = $g^{1/2}cm^{3/2}s^{-1}$, 3×10^9 esu = 1C !?!

· **Relative units**:
Astronomical units (AU) is the average distance from Earth to sun   1AU = 1.50×10^8 km
Light year (ly) is the distance light travels in a year   1 ly = 9.46×10^{12} km
Parsec (pc) is ~3.2 ly

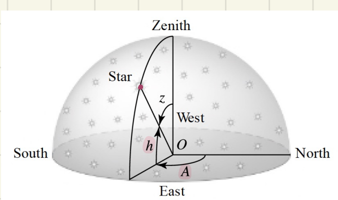
· **Measurements**: we get direct information from the stars & other objects, from their light (EM spectrum), matter (cosmic rays), and gravitational waves

photometry is the measurement of brightness as a fxn of wavelength/freq.
spectroscopy is the study of spectral lines
astrometry is the measurement of motions & parallax



$\boxed{\theta = \frac{d/2}{L}}$

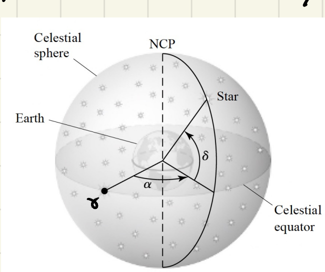
· **Altitude/Azimuth coordinate system**: viewing objects in the night sky requires only directions to them, not their distances



altitude (h) = angle measured from horizon
zenith distance (z) = angle from zenith, $z+h=90°$
azimuth (A) = angle measured along horizon eastward from north

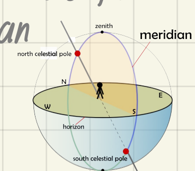
→ Difficult to use in practice, specific to local latitude & longitude

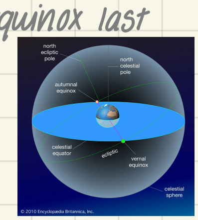
· **Equatorial coordinate system**: similar to latitude/longitude but on celestial sphere



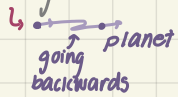
declination (δ) = equiv. to latitude, measured in degrees north/south of celestial equator
Right ascension (α) = equiv. to longitude, measured eastward along celestial equator from the **vernal equinox (♈)**
LST (of observer) = amount of time that has elapsed since the vernal equinox last traversed the meridian

equiv. to hour angle (H) of the vernal equinox



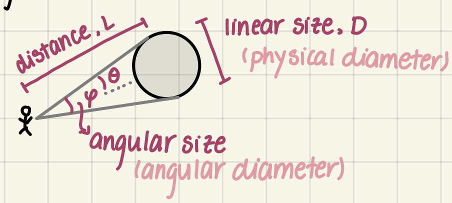


· **Celestial motion**: lunar & solar motions (wrt Earth)
planetary motions (planets moving on the fixed star background. Retrograde motion)
stellar motions (stars are 'fixed' to the sky)



· **direct proof of Earth's motion**: stellar parallax
Earth's rotation → Foucault's pendulum

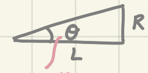
· **Angular size/distance**:



$\tan\left(\frac{\varphi}{2}\right) = \frac{D}{2L}$ (ang. diam. — diameter — distance)

⇓ small angle

$\boxed{\theta = \frac{D/2}{L}}$



$\theta = \varphi/2$

$1° = 60'$ (arcmin.), $1' = 60''$ (arcsec.)

→ can do this with Earth orbit & far away objects

· **parsec**: The distance something has to be to yield $\theta = 1''$ (ie. parallax of $1''$)

$\tan\theta = \frac{R_\oplus}{d}$

↓ small angle

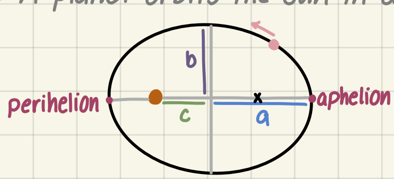
$\boxed{d = \frac{1\,AU}{\theta}}$

d: in pc

θ: in rad (convert arcsec ($''$) to rad first)

KEPLER'S LAWS

· **1st law**: A planet orbits the sun in an **ellipse**, with the sun at one focus of the ellipse



· **2nd law**: A line connecting a planet to the sun sweeps out **equal areas** in **equal times**

· **3rd law**: $\boxed{T = 2\pi\sqrt{\frac{a^3}{GM}}}$ (general)

a: semi-major axis

M: mass of sun

* or $T^2 = a^3$ if in years & AU

BRIGHTNESS MEASURES

- **Flux** (F): Total amount of light energy (of all wavelengths) received per unit time per unit area, Wm^{-2}
  Depends on intrinsic luminosity & distance to observer (same star but farther away = less bright)

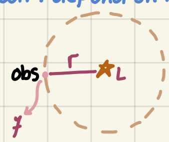
inverse square law of light!

$$\boxed{F = \frac{L}{A} = \frac{L}{4\pi r^2}}$$

*doesn't depend on r, intrinsic to source

flux at r

to obs



- **flux density** (F_ν): flux at a specific frequency (or wavelength), $Wm^{-2} \cdot m^{-1}$ (for F_λ) & $Wm^{-2} Hz^{-1}$ (for F_ν)

$$F_\nu = \frac{L_\nu}{4\pi r^2}$$

- **Photometric bands**: some standard wavelength bands...

U	365 nm
B	440 nm
V	550 nm
R	680 nm
I	800 nm
J	1220 nm
H	1630 nm
K	2190 nm

- Magnitudes are the old way to refer to luminous flux & luminosity

- **Apparent magnitude**, m: How bright a star appears in the sky, lower = brighter

$$\boxed{m_B = -2.5 \log\left(\frac{F_{\nu,\lambda}(B)}{C_{\nu,\lambda}(B)}\right)}$$

flux density of star in band B

apparent magnitude in band B

flux density in band B for a star of magnitude 0

equiv. $\frac{F_{\nu,\lambda}(B)}{C_{\nu,\lambda}(B)} = 10^{-0.4 m_B}$

- Traditionally the zero point calibration was set by the star Vega → $m_u = m_v = m_B = \cdots = 0.0$
  flux density of Vega varies w/ wavelength so C_ν, C_λ depend on band...

$$C_\nu(V) = 3.64 \times 10^{-20} \, \mathrm{erg} \cdot \mathrm{s}^{-1} \cdot \mathrm{cm}^{-2} \cdot \mathrm{Hz}^{-1},$$
$$C_\lambda(V) = 0.361 \, \mathrm{erg} \cdot \mathrm{s}^{-1} \cdot \mathrm{cm}^{-3},$$
$$C_\nu(B) = 4.26 \times 10^{-20} \, \mathrm{erg} \cdot \mathrm{s}^{-1} \cdot \mathrm{cm}^{-2} \cdot \mathrm{Hz}^{-1},$$
$$C_\lambda(B) = 0.660 \, \mathrm{erg} \cdot \mathrm{s}^{-1} \cdot \mathrm{cm}^{-3}$$

- Most photometric measurements are made relative to "standard stars" of known apparent magnitudes
  eg. comparing the B-band flux densities of 2 stars

$$m_{B,1} - m_{B,2} = -2.5\left[\log\left(\frac{F_{\nu,1}(B)}{C_{\nu,1}(B)}\right) - \log\left(\frac{F_{\nu,2}(B)}{C_{\nu,2}(B)}\right)\right]$$

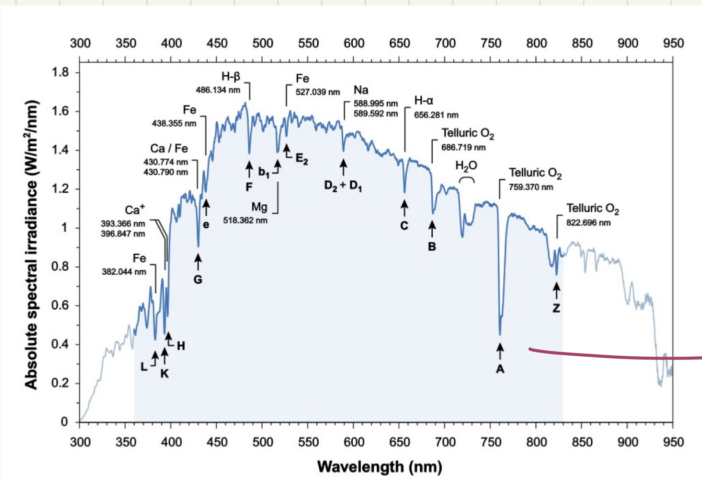
$$\boxed{m_{B,1} - m_{B,2} = -2.5 \log\left(\frac{F_{\nu,1}(B)}{F_{\nu,2}(B)}\right)}$$

- **Absolute magnitude**, M: apparent magnitude a star would have *if* it were located at a distance of 10 pc
- **Flux ratio**: $\boxed{\frac{F_2}{F_1} = 100^{(m_1 - m_2)/5}}$
- **Distance modulus**: $\boxed{\underbrace{m - M}_{\text{a measure of distance!}} = 5\log_{10}(\underbrace{d}_{\text{star's distance in pc}}) - 5}$

SPECTROSCOPY

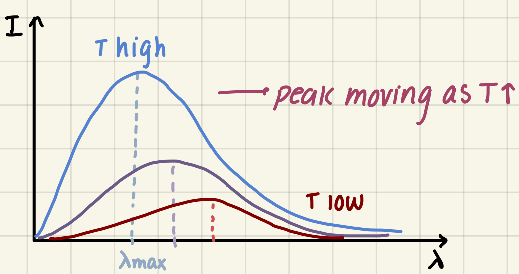
- **photons**: $\boxed{E = h\nu}$, $h = 6.6\times10^{-34}\,Js$ $\quad 1\,eV = 1.60\times10^{-19}\,J$

- Star spectrum → close to a perfect blackbody



→ Absorption spectrum

- **Blackbody**:



Wien's displacement law:

$\lambda_{max} T = cst = 2.9\times10^{6}\,nm\cdot K$

$= 2.9\times10^{3}\,mK$

R-J law: $I = \frac{2cK_BT}{\lambda^4}$ $\quad$ λ big

Wien's law: $I = \frac{2hc^2}{\lambda^5} e^{-hc/\lambda K_B T}$ $\quad$ λ small

} per wavelength

uv catastrophe!

↓ planck combined both

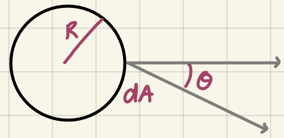
Planck: $\boxed{I_\lambda = \frac{2hc^2}{\lambda^5}\cdot\frac{1}{e^{hc/\lambda kT}\underbrace{-1}_{!}}}$ $\quad$ all λ

so when we taylor expand, we get the right thing for large & small λ perceived outside blackbody

Power per area: $\frac{P}{A} = L_\lambda d\lambda = \int I \, d\Omega \, dA \, d\lambda \cos\theta$ $d\Omega = \sin\theta \, d\theta \, d\varphi$ solid angle

per unit λ

$$= \underbrace{\int_A}_{4\pi R^2} I(\lambda) d\lambda \underbrace{\int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi/2} \sin\theta\cos\theta \, d\theta \, d\varphi}_{\pi} \, dA$$



monochromatic luminosity $\boxed{L_\lambda d\lambda = 4\pi^2 R^2 I(\lambda) d\lambda}$ $\xRightarrow{\text{integ. over } \lambda}$ $\boxed{L = \underbrace{4\pi R^2}_{\text{S.A.}} \underbrace{\sigma T_e^4}_{F_{bol.}}}$ effective temp. of star's surface

flux at a given distance per unit λ: $F_\lambda d\lambda = \frac{L_\lambda}{4\pi r^2} d\lambda$

$$= \frac{4\pi^2 R^2 I_\lambda d\lambda}{4\pi r^2}$$

monochromatic flux $\boxed{F_\lambda d\lambda = \frac{\pi R^2}{r^2} I_\lambda d\lambda}$ $\Rightarrow$ $\boxed{F_{surface} = \sigma T_e^4}$ *r=R

flux at r

Total flux: $\int_{\lambda=0}^{\infty} I(\lambda) d\lambda = \int_0^\infty \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1} d\lambda$

π?

→ Integ. I_λ over all λ

sub $\lambda = \frac{hc}{xkT}$ & $d\lambda = \frac{hc}{x^2 kT}$

$$= 2hc^2 \left(\frac{kT}{hc}\right)^5 \int_0^\infty \left(\frac{hc}{kT}\right)^5 \frac{x^5}{x^2} \frac{hc}{kT} \frac{1}{e^x - 1} dx$$

$$= \frac{k^4 T^4 \pi^4}{15 h^3 c^2}$$

do $F = \int \pi I_\lambda d\lambda$ to get F [W/m²]

Summary

*using r = R

$L_\lambda = \pi dA \cdot I_\lambda \rightarrow L = dA \cdot F$

$F_\lambda = \pi I_\lambda \rightarrow F = \sigma T^4$

* $I = \frac{F}{\Omega} = \frac{F}{\pi}$ for R=r

bolometric flux $\boxed{F = \sigma T^4}$ Stefan Boltzmann law   W/m²

by freq.: knowing $c = \lambda\nu$ $\Rightarrow \int I_\nu d\nu = \int I_\lambda d\lambda$

$$I_\nu d\nu = I_\lambda d\lambda$$

$$I_\nu = I_\lambda \frac{d\lambda}{d\nu}$$

$$\boxed{I_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}}$$

· **Balmer's lines**: $\boxed{\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)}$ $n = 3, 4, 5, \ldots$

(Hydrogen)

R = Rydberg cst, $\frac{1}{R} = 91.18$ nm

· **Why do spectral lines exist?**

semi-classical: $L = n\hbar = mvr$ (quantize L)

ang. mom. of e^- around nucleus

$$\Rightarrow v = \frac{n\hbar}{mr}$$

energy of e^- in orbit around nucleus — $E = \frac{1}{2}mv^2 - \frac{e^2}{r}$

force keeping e^- bound — $F = m\frac{v^2}{r} = \frac{e^2}{r^2}$

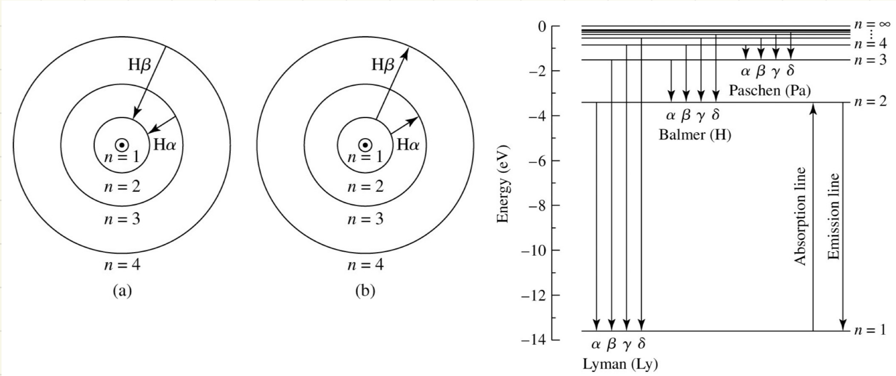
$$\frac{m}{r}\left(\frac{n\hbar}{mr}\right)^2 = \frac{e^2}{r^2}$$

$$\Rightarrow r = \frac{n^2\hbar^2}{me^2}$$

$E_n = \frac{-13.6\,eV}{n^2}$ energy of n^{th} shell/orbital level

- **Energy level diagrams:**

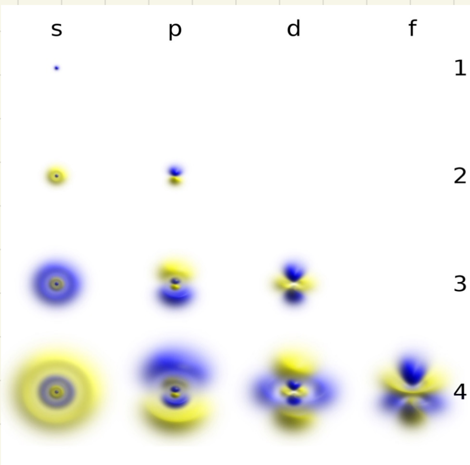
$$\boxed{\Delta E = E_i - E_f = h\nu = \frac{hc}{\lambda}}$$



- **General formula:** $\boxed{\frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)}$

Series Name	Value of m
Lyman	1
Balmer	2
Paschen	3
Brackett	4

- **Quantum:**



$$\Psi_{nlm_l}(r,\theta,\phi) = R_{nl}(r)Y_l^{m_l}(\theta,\phi)$$

$$R_{nl}(r) = -\left[\frac{(n-l-1)!}{2n[(n+l)!]^3}\right]^{\frac{1}{2}}\left(\frac{Z}{na_0}\right)^{l+\frac{3}{2}} r^l L_{n+l}^{2l+1}\left(\frac{2Zr}{na_0}\right)\mathrm{e}^{-\frac{r}{na_0}}$$

$$Y_l^{m_l}(\theta,\phi) = \left[\frac{(2l+1)(l-|m_l|)}{4\pi(l+|m_l|)}\right]^{\frac{1}{2}} P_l^{|m_l|}(\cos\theta)\mathrm{e}^{im_l\phi}$$

$$L_{n+l}^{2l+1} = \sum_{k=0}^{n-l-1}(-1)^{k+1}\frac{[(n+l)!]^2}{(n-l-k-1)!(2l+1+k)!k!}x^k$$

$$P_l^m(x) = (-1)^m \cdot 2^l(1-x^2)^{m/2}\sum_{k=m}^{l}\frac{k!}{(k-m)!}x^{k-m}\binom{l}{k}\binom{\frac{l+k-1}{2}}{l}$$

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{e^2m_e},$$

$$n = 1,2,3,...$$

$$l = 0,1,2,...n-1$$

$$m_l = -l,-l+1,...0,...l-1,l$$

- **Stellar spectra:**

Observed spectral lines → composition of gas (fingerprint)
Excitation ⟹ lines
Ionization ⟹ Opacity

THERMO

- **Thermodynamic temperature**: If two thermo systems are both in thermal equilibrium with a 3rd system, then the two systems are in thermal equilibrium with each other
  → Any physical quantity that changes with the application of heat can be used to measure temperature
  only thing that can change in thermal equil.

- **Kinetic temperature**: the temperature of a gas is a measure of the mean kinetic energy of the gas
  higher temp. = greater motion

Collisions can result in transfer from thermal (kinetic) energy to orbital (electron) energy

Average thermal energy of particles: $\boxed{E = \frac{3}{2} k_B T}$

Average thermal velocity: $E = \frac{3}{2} k_B T = \frac{1}{2} m v_{avg}^2$
(per particle)

$$\Rightarrow \boxed{v_{avg} = \sqrt{\frac{3 k_B T}{m}}}$$

- **Atomic excitation**: In Local Thermodynamic Equilibrium, this corresponds to the classical Boltzmann distribution

  The probability of finding an atom with an electron in an excited state with energy E_n above the ground state is st.
  - it ↓ exponentially w/ E_n
  - it ↑ exponentially with temp. T_K

$$\boxed{\frac{n_j}{n_i} = \frac{g_j}{g_i} e^{-(E_j - E_i)/k_B T}}$$

$i, j \Rightarrow$ upper & lower energy states ; $g_i, g_j \Rightarrow$ num. of degeneracies in each level

eg. H I has $g_n = 2n^2$

- **Atomic ionization**: In LTE, ionization is dominated by collisions
  Ionization rate = recombination rate (equilibrium)

$$\boxed{n_e \frac{N(X_{r+1})}{N(X_r)} = 2 \frac{Q_{r+1}}{Q_r} \left(\frac{2\pi m_e k_B T}{h^2}\right)^{3/2} e^{-\chi_i / k_B T}}$$

m_e: e^- mass

n_e: e^- density

$N(X_r)$: density of species X_r in ionized state r

Q_r: partition fxn of state i

$\chi = E_{r+1} - E_r$: energy diff between lvls

- Ex: ① $\frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-10.2/kT}$ → E_{Balmer}

$g_1 = 2(1)^2$, $g_2 = 2(2)^2$

$= \frac{2(2)^2}{2(1)^2} e^{-10.2/kT}$

② $Q_{II} = 1$ & $Q_I = \sum_{n=1}^{\infty} g_n e^{-(E_f - E_i)/kT}$

$= 2(1)^2 e^{-13.6(\cancel{1-1})/kT} + 2(2)^2 e^{-13.6(\cancel{1-1/2^2})/kT} + \cdots$

1   0 (tiny)

$= 2$

SO $\underbrace{\frac{P}{kT}}_{n_e} \frac{N_{II}}{N_I} = \frac{2}{2}(\cdots)^{3/2} e^{-13.6/kT}$

ionization energy

looking for this

- **Doppler shift**: allows us to calculate radial velocities (along line of sight)

$$\boxed{\frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} = \frac{\Delta\lambda}{\lambda_{rest}} = \frac{v_r}{c}}$$

TELESCOPES

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Chapter 2 : NATURE OF STARS

Chapter 3: INTERSTELLAR MEDIUM & STAR FORMATION

INTERSTELLAR DUST

- **Interstellar medium**: ISM, gas & dust between stars. During a star's lifetime, much of the material may be returned to the ISM through stellar winds & explosive events