PHYS 521 Fall 2024
Exam #1 Review

Format: Short answer, some calculations required; bring your calculator.
Scope: Class notes, Homework, and Carroll & Ostlie (2nd ed.) Chapters 1-10.

Be familiar with the following material and concepts, including units.

Important formulae to memorize/know how to derive quickly are indicated with an asterisk.*

1. **General Astronomy**
   - * small angle formula: $\theta = D/d$
   - * parallax: $d = 1/p$ (d in pc, p in arcsec)
   - * magnitude: $m_1 - m_2 = -2.5\log(\frac{f_1}{f_2})$
   - * absolute magnitude: $M \equiv m$ at $d = 10$pc
   - distance Modulus: $\equiv m - M = -2.5\log(10/d)^2 = 5$ log d - 5 (d in pc)
   - astrometry, photometry, spectroscopy
2. **Continuous (Thermal) Radiation**
   - * $\lambda\nu = c$, $E = hc/\lambda = h\nu$
   - Planck function: $B(\lambda, T) = \frac{2hc^2\lambda^{-5}}{e^{hc/\lambda kT} - 1}$ erg s^{-1}cm$^{-2}\lambda^{-1}$st^{-1}
   - Wien's Law: $\lambda_{\max}T = 0.290$ cm K
   - $\int B(\lambda, T)\ d\lambda = \sigma T^4/\pi$
   - $L = A\ \sigma T^4 = 4\pi R^2\ \sigma T^4$
   - $f = \frac{L}{4\pi d^2}$
   - effective Temperature
   - equilibrium Temperature
3. **Special Relativity**
   - Lorentz Transformations
   - time Dilation
   - length Contraction
   - * redshift: $z = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} = \frac{\Delta\lambda}{\lambda_{rest}}$
   - * rest energy: $E_{rest} = mc^2$

4. **Spectral Lines**

- spectral lines: absorption vs. emission
- Balmer formula: $1/\lambda = R(1/2^2 - 1/n^2)$
- electron angular momentum: $mvr = nh/2\pi$
- * allowed H energy levels: $E_n = 13.6/n^2$ (ev)
- binding energy of electrons, excitation, ionization
- * electron transition: $\Delta E = hc/\lambda$
- thermodynamic equilibrium, temperatures: kinetic, excitation, ionization
- Boltzmann distribution: $\frac{n_j}{n_i} = \frac{g_j}{g_i} e^{[(E_j - E_i)/kT]}$
- Saha equation: $\frac{n(X_{r+1})}{n(X_r)} = \frac{2g_{r+1}}{g_r}(kT/P_e)(\frac{2\pi m_r kT}{h^2})^{3/2} e^{-[\chi_{ion}/kT]}$
- * stellar spectral classes: OBAFGKM
- composition of stars: abundance of the elements
- Compton Effect

5. **Scaling Relations & Hertzprung-Russell diagram**

- L, T_{eff}
- bolometric correction, bolometric magnitude
- $M_{bol} - M_{bol\odot} = -2.5 \log{(L_*/L_\odot)}$
- Main Sequence, Red Giant Branch, White Dwarfs
- Stellar radii

6. **Telescopes**

- basic telescope designs (reflector, refractor)
- Airy disk
- Rayleigh criterion: $\theta_{min} = 1.22 \frac{\lambda}{D}$
- important wavelengths for astrophysics

7. **Binary Stars and Stellar Mass**

- Kepler's laws
- * doppler shift: $\Delta\lambda = \lambda_{obs} - \lambda_{lab}$
- * radial velocity: $\frac{\Delta\lambda}{\lambda} = \frac{v_r}{c}$
- * center of mass: $m_1 a_1 = m_2 a_2$
- $a = a_1 + a_2$; $v = v_1 + v_2$
- circular orbits: $F = mv^2/r$ and $v = 2\pi a/P$ for each star
- ratio of masses: $m_1/m_2 = v_2/v_1$

- sum of masses: $m_1 + m_2 = \frac{4\pi^2}{GP^2}a^3 = \frac{P}{2\pi G}(v_1 + v_2)^3$
- radial velocity curve, e = 0: $v_r = v\ \cos(\theta)\sin(i)$, with $= \theta = \omega t$
- eclipsing binary: $i = 90°$
- Main Sequence: $L/\mathrm{L}_\odot = (\mathrm{M}/\mathrm{M}_\odot)^\alpha$, with
  $\alpha = 5.0$ for low mass stars,
  $\alpha = 3.0$ for intermediate mass stars,
  $\alpha = 1.0$ for very massive stars, and
  $\alpha = 4.0$ "on average".

8. **Exoplanets**

- stars are named A B (like Alpha Centauri A) etc
- planets are a b c etc (in order of discovery)
- albedo: how reflective (or shiny) a planet or other object is
- definition of "Hot Jupiter"
- definition of the "Habitable Zone"
- What can we learn from transits, eclipses, and phase variations?
- What are some of the main planet detection methods?
- *Don't forget that all of the orbital stuff for binaries works for planets too!!*

9. **Stellar Energy, Nuclear Fusion, and Main Sequence Lifetimes**

- Luminosity of sun: 4×10^{33} erg/s
- *Gravitational potential energy of sphere: $U = -(\frac{3}{5})\frac{GM^2}{R}$
- Chemical potential energy of Sun: $E_{\mathrm{chem}} \sim N \times 1\mathrm{eV}$
- Potential mass energy of Sun: $E = \mathrm{M}_\odot \mathrm{c}^2 = 2 \times 10^{54}$ erg
- *Sun's lifetime = $E/\mathrm{L}_\odot$
- *Binding energy of nucleus: $M_{\mathrm{nucleus}}c^2 + BE = (\text{mass of nucleons})c^2$
- Binding energy per nucleon vs. atomic number, realms of fusion, fission energy release
- proton-proton chain, BE = 27 MeV
- CNO cycle, BE = 27 MeV
- Triple-alpha sequence, BE = 7 Mev
- Fusion reaction rate $R_{\mathrm{f}} = \mathrm{L}_\odot/(\text{E per H atom}) = 3.8 \times 10^{38}$ H atoms/sec for Sun
- MS Lifetime of Sun: $\tau = \frac{0.1 \times 0.007 \times \mathrm{M}_\odot \mathrm{c}^2}{\mathrm{L}_\odot}$
- Main Sequence lifetime: $\tau \propto M/L = M/M^{3.5} = M^{-2.5}$

10. **Stellar Atmospheres**

- Opacity: $\kappa_\nu = 1/\rho l_\nu$, l_ν is the mean free path

- Optical Depth: $\tau_\nu = \int_0^{s'} \rho\kappa_\nu ds$ (optically thin vs. optically thick)
- Intensity w/ τ: $I_\nu = I_\nu^0 e^{-\tau_\nu}$
- Emission Coefficient: ϵ_ν
- Radiative Transfer Function: $\frac{dI_\nu}{ds} = -\kappa_\nu \rho I_\nu + \epsilon_\nu$
(*see also Radiative Energy Transport below...*)
- Source Function: $S_\nu = \epsilon_\nu / \rho\kappa_\nu$
- Rad. Transfer w/ Source Function: $\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu(\mu, \tau) - S_\nu(\tau_\nu)$
- Equivalent Width
- Voigt Profile
- Curve of Growth

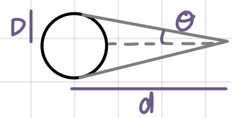
11. Stellar Interiors

- * Mass Continuity: $\frac{dM}{dr} = \rho_r 4\pi r^2$
- * Hydrostatic Equilibrium: $\frac{dP}{dr} = -G\rho_r M_r / r^2 = -\rho_r g_r$
- * Equation of State for perfect gas: $P = \rho kT/m$
- Energy Generation: $\frac{dL}{dr} = \epsilon 4\pi r^2 \rho_r$
- Radiative Energy Transport: $\frac{dT}{dr} = -(3\kappa\rho_r L_r)/(4ac\ T_r^3\ 4\pi r^2)$
can also be written: $L_r = \frac{dT}{dr}(16\pi r^2 \sigma T_r^3)/\kappa\rho_r$ since $\sigma = ac/4$
- Convective Energy Transport: $\frac{dT}{dr} = -(1 - \frac{1}{\gamma})\mu m_H G M_r / k r^2$
- stellar mass and radius
- timescales for radiative and convective energy transport
- estimate central temperature, pressure, density

1. General Astronomy

- * small angle formula: $\theta = D/d$
- * parallax: $d = 1/p$ (d in pc, p in arcsec)
- * magnitude: $m_1 - m_2 = -2.5\log(\frac{f_1}{f_2})$
- * absolute magnitude: $M \equiv m$ at $d = 10$pc
- distance Modulus: $\equiv m - M = -2.5\log(10/d)^2 = 5 \log d - 5$ (d in pc)
- astrometry, photometry, spectroscopy

$*\ \log = \log_{10}$

small angle formula: 

$\theta = \frac{D}{d}$ — size; d — distance to

parallax: parsec = distance to get $\theta = 1''$

$d = \frac{1\,AU}{\theta} = 1$ if $\theta = 1''$ — d in pc, θ in arcsec ($1° = 60'$, $1' = 60''$)

$(1pc = \frac{1}{''})$

apparent magnitude: $m_B = -2.5\log\left(\frac{f_{\nu,\lambda}(B)}{C_{\nu,\lambda}(B)}\right)$ — $f_{\nu,\lambda}(B)$: flux of star in band B; $C_{\nu,\lambda}(B)$: flux of a star w/ $m=0$ in band B

$f(r) = \frac{L}{A} = \frac{L}{4\pi d^2}$ — L: intrinsic; d: obs sphere radius

$[f] = \frac{W}{m^2} \cdot \underbrace{\frac{1}{stuff}}_{}$

eg. $\frac{1}{m}$ for f per λ or $\frac{1}{Hz}$ for f per ν

$m_1 - m_2 = -2.5\log\left(\frac{f_1}{f_2}\right)$

absolute magnitude: $M = m$ at $d = 10\,pc$

distance modulus: $f_{obs} = \frac{L}{4\pi r^2}$, $f_{10pc} = \frac{L}{4\pi(10pc)^2}$

$\rightarrow m_{obs} - m_{10pc} = -2.5\log\left(\frac{f_{obs}}{f_{10pc}}\right)$

$\vdots \qquad = -2.5\log\left(\frac{10}{d}\right)^2$

$m - M = 5\log(d) - 5$

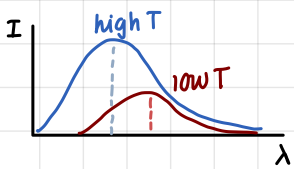
astrometry: measurement of motions & parallax
photometry: measurement of brightness as a fxn of λ or ν
spectroscopy: spectral lines

2. Continuous (Thermal) Radiation

- * $\lambda\nu = c$, $E = hc/\lambda = h\nu$
- Planck function: $B(\lambda, T) = \frac{2hc^2\lambda^{-5}}{e^{hc/\lambda kT}-1}$ erg $\mathrm{s^{-1}cm^{-2}\lambda^{-1}st^{-1}}$
- Wien's Law: $\lambda_{\max}T = 0.290$ cm K
- $\int B(\lambda, T)\, d\lambda = \sigma T^4/\pi$
- $L = A\ \sigma T^4 = 4\pi R^2\ \sigma T^4$
- $f = \frac{L}{4\pi d^2}$
- effective Temperature
- equilibrium Temperature

photons: $E = h\nu$ $\lambda\nu = c$

planck's fxn: $I_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$ valid for all λ



blackbody radiation!

Wien's law: $\lambda_{peak} T = 2.9 \times 10^3$ mK

flux & intensity...: $\overset{[W]\text{ per }\lambda}{L_\lambda} = \underbrace{A}_{\text{at distance } r} \overset{[\frac{W}{m^2}]\text{ per }\lambda}{f_\lambda} = \underbrace{4\pi R^2}_{\text{star Area}} \cdot \pi I_\lambda$ so $f(R) = \pi I$

total E/s.A. $\int_{\text{all }\lambda} I_\lambda d\lambda = \sigma T_e^4/\pi \rightarrow \underbrace{f(R) = \sigma T^4}_{\text{bolometric! } [\frac{W}{m^2}]}$ and $L = 4\pi R^2 \cdot \sigma T^4$

eg. HW2 Q7c

① $L_s = f_s(a) \cdot 4\pi a^2 = \underbrace{f_s(R)}_{\sigma T_s^4} \cdot 4\pi R_s^2 \rightarrow f_s(a) = \cdots$

② $f_{received} = L_{planet}$ (E all absorbed)

(cross-sec. area of planet) $\times f_s(a) = 4\pi R_p^2 \cdot \sigma T_p^4$

$T_p = \cdots$

effective T: T of a blackbody that emits same amt of energy, from σT^4

equilibrium T: T when $E_{emit} = E_{abs}$ (usually planets)

3. **Special Relativity**

- Lorentz Transformations
- time Dilation
- length Contraction
- * redshift: $z = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} = \frac{\Delta\lambda}{\lambda_{rest}}$
- * rest energy: $E_{rest} = mc^2$

Lorentz transformations: $S = (x, y, z, t)$

$S' = (x', y', z', t')$ moving at u along x'

for the same event measured in S and S':

$$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}}$$

$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$ = Lorentz factor

Time dilation: $\Delta t_{moving} = \frac{\Delta t_{rest}}{\sqrt{1 - u^2/c^2}}$ Relative to events!

proper time = shortest time, is measured by a clock at rest relative to the two events

length contraction: $L_{moving} = L_{rest}\sqrt{1 - u^2/c^2}$

proper length = longest length, measured in object's rest frame

*only lengths/distances // to motion are affected by length contraction

Redshift: $z = \frac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}} = \frac{\Delta\lambda}{\lambda_{emit}} = \frac{V}{C_s}$

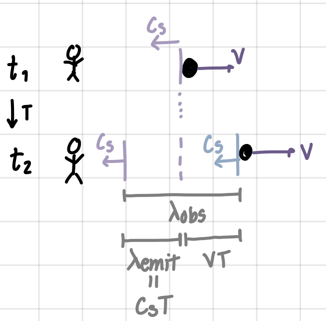
in rest frame

V — source speed

C_s — wave speed

away = ∿∿ ⟹ ∿ so $\nu\downarrow, \lambda\uparrow$

towards = ∿ ⟹ ∿∿ so $\nu\uparrow, \lambda\downarrow$



$\lambda_{obs} - \lambda_{emit} = VT = V\frac{\lambda_{emit}}{C_s}$ = Doppler shift

Rest energy: ① 2nd law: $F = \frac{dP}{dt}$ for mass m initially at rest, force F along x

② $K = W_{tot} = \int_{x_i}^{x_f} F dx = \int_{x_i}^{x_f} \frac{dP}{dt} dx = \int_{P_i = 0}^{P_f} v\,dp$) IBP

$= P_f V_f - \int_0^{V_f} P\,dv$) $P = \gamma m v$

$= \gamma m V_f^2 - \int_0^{V_f} \gamma m v^2\,dv$

: drop f subscript

$K = mc^2(\gamma - 1) \longrightarrow E_{rest} = mc^2$

4. **Spectral Lines**

- spectral lines: absorption vs. emission
- Balmer formula: $1/\lambda = R(1/2^2 - 1/n^2)$
- electron angular momentum: $mvr = nh/2\pi$
- * allowed H energy levels: $E_n = 13.6/n^2$ (ev)
- binding energy of electrons, excitation, ionization
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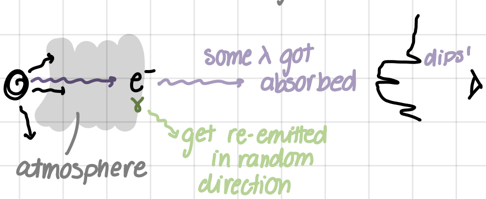
- * stellar spectral classes: OBAFGKM
- composition of stars: abundance of the elements
- Compton Effect

Spectral lines: absorption → absorbed in stellar atmosphere, e^- gets excited by γ & absorbs ⇒ dips in spectrum
emission → e^- go to lower E levels and emit γ



Balmer formula: Hydrogen, $n \to n=2$

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

Angular momentum: $L = mvr = n\hbar$ $\quad L=S$

$\hbar = \frac{h}{2\pi}$

Allowed energies: $E_n = \frac{-13.6\text{ eV}}{n^2}$ binding energy of lvl n

binding energy: energy required to keep e^- bound, ie from $n=\infty$ to n
ionization energy: energy required to remove e^-, ie from n to $n=\infty$

} same value, diff. sign
$= 13.6$ eV for H in $n=1 \leftrightarrow n=\infty$

e^- transitions: $\Delta E = E_i - E_f = \frac{hc}{\lambda}$ for $n_i \to n_f$

λ: wavelength of light emitted during transition

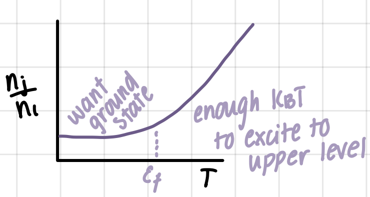
thermo. T: related to thermal equilibrium, any quantity that changes with heat can be used to measure T
Kinetic T: measure of mean kinetic energy of a gas, higher T = more motion
excitation T: Boltzmann distribution

$$\frac{n_j}{n_i} = \frac{g_j}{g_i} e^{-(E_j - E_i)/k_BT}$$

n: # atoms in level

g: # of degenerecies of lvl, $g_n = 2n^2$ for HI

E: binding energy, $E_n = \frac{-13.6\text{ eV}}{n^2}$



ionization T: ionization dominated by collisions, follows Saha eqn.

$$\frac{n(X_{r+1})}{n(X_r)} = \frac{2kTg_{r+1}}{P_e\, g_r}\left(\frac{2\pi m_r kT}{h^2}\right)^{3/2} e^{-\chi_i/kT}$$

Reduced mass

atoms in X_r state

X_r, X_{r+1} are two diff. states of ionization

e^- pressure

statistical weights, # of sub lvls

energy to excite X_r from ground state

Stellar spectral classes: hot, blue O B A F G K M cool, red

composition of stars: Hydrogen + Helium + trace metals

compton effect: γ hitting free e^- ⟶ scattering ⟶ λ_γ increases (redshifted)

5. Scaling Relations & Hertzprung-Russell diagram

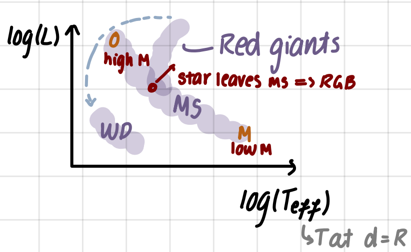
- L, T_{eff}
- bolometric correction, bolometric magnitude
- $M_{bol} - M_{bol\odot} = -2.5 \log (L_*/L_\odot)$
- Main Sequence, Red Giant Branch, White Dwarfs
- Stellar radii

luminosity: intrinsic, flow of energy, $L = \mathcal{F} A$ $\quad [W] = \frac{\text{energy}}{\text{time}}$

$= \sigma T_{eff}^4 \; 4\pi R^2$

surface T

HR diagram:



Main Sequence: majority of stellar lifetime is here, H fusion, higher mass stars = shorter lifetimes so go to Red Giant branch first

$r \sim T^2$ so $L \sim T^4 \cdot r^2 = T^8$

Red Giants: No more H to fuse in core ⟹ He shell burning ⟹ He core ⟹ He flash...

L & R ↑ but T_* ↓ $\quad T^4 = \frac{L}{R^2}$ ⟶ dominates

White Dwarfs: core can't contract after a point due to quantum pressure ⟹ inert core held up by e^- degen. pressure, ~same T as M, lower T & L

stellar radii: $L = 4\pi R^2 \cdot \sigma T^4 \implies \frac{L}{L_\odot} = \left(\frac{R}{R_\odot}\right)^2 \left(\frac{T}{T_\odot}\right)^4$ get rid of constants!

isolate $R/R_\odot$

Bolometric magnitude: apparent & absolute magnitudes measured over all wavelengths

$M_{bol} - M_{bol\odot} = -2.5 \log\left(\frac{L}{L_\odot}\right)$

Bolometric correction: correction to compare to visual magnitude, $BC = M_{bol} - M_V$

6. **Telescopes**

- basic telescope designs (reflector, refractor)
- Airy disk
- Rayleigh criterion: $\theta_{min} = 1.22\frac{\lambda}{D}$
- important wavelengths for astrophysics

Telescope designs

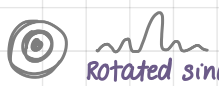
Refractor: bend light to focus to point (sensor) w/ lenses

chromatic aberrations → focal length of a lens is λ-dependent, different λ are refracted by diff amts

Reflecting: uses mirrors

Airy disk:

circular aperture →

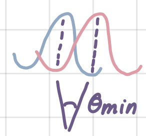


Two light sources with angular separation < angular resolution ⇒ Airy beams get too close, hard to distinguish the 2 sources

Rayleigh's criterion:

$\theta_{min} = 1.22\frac{\lambda}{D}$ — λ: operating λ, D: telescope diameter

⇒ This occurs when peak of one matches with 1st min of other



Important wavelengths:

- Radio: cold interstellar gas, CMB
- Infrared: cool dust clouds, young stars
- visible: 400-700 nm, stars & galaxies
- UV: 10-400 nm, hot young stars
- xrays: 0.01-10 nm, high energy processes (BH, NS, novae...)
- gamma rays: < 0.01 nm, most energetic processes

space-based required

7. Binary Stars and Stellar Mass

- Kepler's laws
- * doppler shift: $\Delta\lambda = \lambda_{obs} - \lambda_{lab}$
- * radial velocity: $\frac{\Delta\lambda}{\lambda} = \frac{v_r}{c}$
- * center of mass: $m_1 a_1 = m_2 a_2$
- $a = a_1 + a_2$; $v = v_1 + v_2$
- circular orbits: $F = mv^2/r$ and $v = 2\pi a/P$ for each star
- ratio of masses: $m_1/m_2 = v_2/v_1$
- sum of masses: $m_1 + m_2 = \frac{4\pi^2}{GP^2}a^3 = \frac{P}{2\pi G}(v_1 + v_2)^3$
- radial velocity curve, e = 0: $v_r = v\ \cos(\theta)\sin(i)$, with $= \theta = \omega t$
- eclipsing binary: $i = 90°$
- Main Sequence: $L/L_\odot = (M/M_\odot)^\alpha$, with
  $\alpha = 5.0$ for low mass stars,
  $\alpha = 3.0$ for intermediate mass stars,
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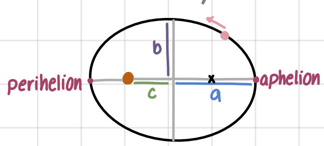
  $\alpha = 4.0$ "on average".

Kepler's laws: ① orbits are elliptical



② equal areas are swept in equal times

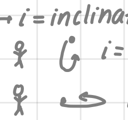
③ $T = 2\pi\sqrt{\frac{a^3}{GM}}$ → semi-major axis; → mass of sun

$T = a^3$ if in years & AU

Doppler shift: see ch3, $\Delta\lambda = \lambda_{obs} - \lambda_{emit}$

Radial velocity: $\frac{\Delta\lambda}{\lambda} = \frac{V_r}{c}$ — observed radial velocity ⇓ projected along line of sight

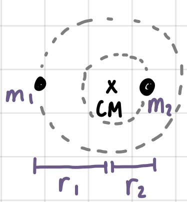
$|V_{obs}| = |V_r| \sin i$ → i = inclination



$i = 0°$

$i = 90°$

center of mass:



$m_1 r_1 = m_2 r_2$

total separation: $a = r_1 + r_2$

total velocity: $V = V_1 + V_2$

circular orbits: $F_i = \frac{m_i V_i^2}{r_i}$ and $V_i = \frac{2\pi r_i}{T}$ → same orbit period for both m!

true

$\Rightarrow \underbrace{\frac{V_1}{V_2} = \frac{r_1}{r_2}}_{\text{from above}} \underbrace{= \frac{m_2}{m_1}}_{\text{from CM}}$

both

Kepler for binaries: $a = r_1 + r_2 = \frac{\tau}{2\pi}(v_1 + v_2)$ ← observed

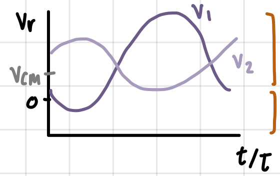
↓ plug in Kepler's 3rd
for $i = 90°$, ie, $\sin i = 1$ & $|v_{obs}| = v_r$

$M = \frac{\tau a^3}{2\pi G}$

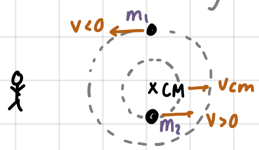
$m_1 + m_2 = \frac{\tau}{2\pi G}(v_1 + v_2)^3$ ← observed

Radial velocity curve: $v_r = v\cos\theta\sin i$ (e = 0 so circle)

$\theta = \omega t$, i = inclination



smaller mass = larger velocity amplitude



eclipsing binary ⇒ $i = 90°$

main sequence: $\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^\alpha$

$$\alpha = \begin{cases} 5, & \text{low mass} \\ 3, & \text{intermediate} \\ 1, & \text{very massive} \end{cases}$$

$\alpha_{avg} = 4$

8. Exoplanets

- stars are named A B (like Alpha Centauri A) etc
- planets are a b c etc (in order of discovery)
- albedo: how reflective (or shiny) a planet or other object is
- definition of "Hot Jupiter"
- definition of the "Habitable Zone"
- What can we learn from transits, eclipses, and phase variations?
- What are some of the main planet detection methods?
- *Don't forget that all of the orbital stuff for binaries works for planets too!!*

Naming conventions: stars → A B ... eg. Alpha Centauri A
planets → a b ... (in order of discovery)

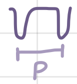
Albedo: amount of light reflected by a planet

$\begin{cases} 1 = \text{reflects all light (shiny)} \\ 0 = \text{absorbs all light (dark)} \end{cases}$ ⇒ affects atmosphere

Hot Jupiter: planets w/ similar size & mass as Jupiter but orbit close to star (shorter P, higher T)

Habitable zone: distance from star s.t. liquid water exists on planet surface

Transits: planets pass in front of stars → lightcurves ⇒ size, period, atmospheric composition



eg. $\frac{F_P}{F_S} = \left(\frac{R_P}{R_S}\right)^2 \rightarrow$ get R_P

└ from lightcurve

Kepler's 3rd for a

Eclipses: planet passing behind. dip = amt of brightness change, can tell us the T of dayside
also used to measure albedo

phase variation:  vs  → brightness changes ⇒ albedo & T

Main detection methods:
- → transit method: lightcurves
- → radial velocity: small doppler shifts in stellar spectrum
- → direct imaging (rare)
- → gravitational microlensing: planet passes in front of background star
- → astrometry: movements of stars from planet's grav. influence

9. Stellar Energy, Nuclear Fusion, and Main Sequence Lifetimes

- Luminosity of sun: 4×10^{33} erg/s
- *Gravitational potential energy of sphere: $U = -(\frac{3}{5})\frac{GM^2}{R}$
- Chemical potential energy of Sun: $E_{\text{chem}} \sim N \times 1\text{eV}$
- Potential mass energy of Sun: $E = \text{M}_\odot \text{c}^2 = 2 \times 10^{54}$ erg
- *Sun's lifetime $= E/\text{L}_\odot$
- *Binding energy of nucleus: $M_{\text{nucleus}}c^2 + BE = (\text{mass of nucleons})c^2$
- Binding energy per nucleon vs. atomic number, realms of fusion, fission energy release
- proton-proton chain, BE = 27 MeV
- CNO cycle, BE = 27 MeV
- Triple-alpha sequence, BE = 7 Mev
- Fusion reaction rate $R_{\text{f}} = \text{L}_\odot/(\text{E per H atom}) = 3.8 \times 10^{38}$ H atoms/sec for Sun
- MS Lifetime of Sun: $\tau = \frac{0.1 \times 0.007 \times \text{M}_\odot \text{c}^2}{\text{L}_\odot}$
- Main Sequence lifetime: $\tau \propto M/L = M/M^{3.5} = M^{-2.5}$

L sun: $L_\odot = 4 \times 10^{33}$ erg/s

Grav. potential: uniform $\rho = \frac{M}{\frac{4}{3}\pi R^3}$

$$U = -\int \frac{G M_{enc}}{r}\,dm = -\int_0^R \frac{G M_{enc}}{r} 4\pi r^2 \rho(r)\,dr = -\int_0^R G 4\pi r \cdot \rho \cdot \rho \frac{4}{3}\pi r^3\,dr = \cdots = \frac{-3}{5}\frac{GM^2}{R}$$

$dm = \underbrace{4\pi r^2}_{\text{S.A.}} \rho(r) \underbrace{dr}_{\text{thickness}}$ $\qquad M_{enc} = \rho \cdot \frac{4}{3}\pi r^3$

potential E chem.: $E_{chem,sun} = N \cdot 1\text{eV}$. energy stored in chemical bonds (H) → not very significant for sun.

└ # particles

E mostly comes from nuclear fusion

Potential mass E: $E = M_\odot c^2 = 2\times10^{54}$ erg, if all mass was used as energy

solar lifetime: how long sun can shine at $L_\odot$ before running out of fuel

$$\tau_{sun} = \frac{E_{fus}}{L_\odot} = \frac{0.1 \times 0.007 \times M_\odot c^2}{L_\odot} \qquad \frac{\text{amt of fuel total}}{\text{burning rate}}$$

Binding energy of nucleus: $\overbrace{E_{bind}}^{\text{E released when binding}} = E_{nucleons} - E_{nucleus}$

$E_{bind} = \Delta mc^2$

$\Delta m = m_{nucleons} - m_{nucleus}$

BE per nucleon vs atomic num: increases until Fe, then decreases (Fe = most tightly bound, cost E to break or to make heavier elements)

lighter → Fe ← heavier

release E when binding — fusion; " — fission

Fe: highest E_{bind}/nucleon

p-p chain: $4H \rightarrow He + \underbrace{E_{bind}}_{=\,27\text{ MeV}}$

CNO cycle: in more massive stars, at higher T, BE = 27 MeV

Triple α: $\underbrace{3\alpha}_{\text{He nuclei}} \rightarrow C + \underbrace{BE}_{=\,7\text{ MeV}}$

fusion rate: $\text{rate} = \dfrac{L}{E \text{ released per } H}$ $\left[\dfrac{\#\text{fusion}}{s}\right]$

$= 3.8\times10^{38}\ \dfrac{H\text{ atom}}{s}$ for sun

lifetime: $\tau \propto \dfrac{M}{L} = \dfrac{M}{M^{3.5}} = M^{-2.5}$

10. Stellar Atmospheres

- Opacity: $\kappa_\nu = 1/\rho l_\nu$, l_ν is the mean free path
- Optical Depth: $\tau_\nu = \int_0^{s'} \rho\kappa_\nu ds$ (optically thin vs. optically thick)
- Intensity w/ τ: $I_\nu = I_\nu^0 e^{-\tau_\nu}$
- Emission Coefficient: ϵ_ν
- Radiative Transfer Function: $\frac{dI_\nu}{ds} = -\kappa_\nu \rho I_\nu + \epsilon_\nu$
  (*see also Radiative Energy Transport below...*)
- Source Function: $S_\nu = \epsilon_\nu / \rho\kappa_\nu$
- Rad. Transfer w/ Source Function: $\mu\frac{dI_\nu}{d\tau_\nu} = I_\nu(\mu, \tau) - S_\nu(\tau_\nu)$
- Equivalent Width
- Voigt Profile
- Curve of Growth

Opacity: amt of light scattered/absorbed

$$\kappa_\nu = \frac{1}{\rho l_\nu}$$

ρ = density, l_ν = mfp

Optical depth: Total probability to interact ⇒ transparency at ν

$$\tau_\nu = \int_0^{s'} \rho\kappa_\nu ds$$ (path)

optically thin = $\tau_\nu < 1$
optically thick = $\tau_\nu > 1$ ie high prob.

Intensity: $I_\nu = I_\nu^0 e^{-\tau_\nu}$ (over path)
⇒ intensity decreases as it travels

Radiation emission: ϵ_ν = emission coeff. [energy per unit (time, freq. OR vol, solid angle)]

$$\frac{dI_\nu}{ds} = -\kappa_\nu \rho I_\nu + \epsilon_\nu$$

Radiative transfer eqn
→ How radiation propagates through stellar matter

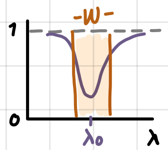
Source fxn: $S_\nu \equiv \frac{\epsilon_\nu}{\rho\kappa_\nu}$

$$\Rightarrow \mu\frac{dI_\nu}{d\tau_\nu} = I_\nu(\mu, \tau_\nu) - S_\nu(\tau_\nu)$$

($dx = \cos\theta ds = \mu ds$)

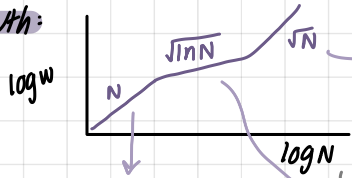
approx. that atm. is thin enough that photon doesn't see sphere

Equivalent width: strength of a spectral line is the area of the spectral line in an intensity vs λ plot



Voigt profile: convolution of Gaussian (Doppler broadening from thermal motion) with Lorentzian (broadening from qm effects / collisions)

curve of growth:



collision broadening takes over
$W \propto \sqrt{N}$

abundance of absorbing elements

- few absorbers
- optical depth small
- $W \propto N$

- line saturates
- Doppler = only way for width to grow
- $W \propto \sqrt{\ln N}$

11. Stellar Interiors

- * Mass Continuity: $\frac{dM}{dr} = \rho_r 4\pi r^2$
- * Hydrostatic Equilibrium: $\frac{dP}{dr} = -G\rho_r M_r/r^2 = -\rho_r g_r$
- * Equation of State for perfect gas: $P = \rho kT/m$
- Energy Generation: $\frac{dL}{dr} = \epsilon 4\pi r^2 \rho_r$
- Radiative Energy Transport: $\frac{dT}{dr} = -(3\kappa\rho_r L_r)/(4ac\ T_r^3\ 4\pi r^2)$
  can also be written: $L_r = \frac{dT}{dr}(16\pi r^2 \sigma T_r^3)/\kappa\rho_r$ since $\sigma = ac/4$
- Convective Energy Transport: $\frac{dT}{dr} = -(1-\frac{1}{\gamma})\mu m_H G M_r/kr^2$
- stellar mass and radius
- timescales for radiative and convective energy transport
- estimate central temperature, pressure, density

Mass continuity: $dM = \underbrace{\rho \cdot 4\pi r^2}_{\text{surface area}}\, dr$ — shell thickness

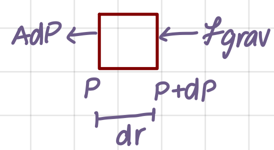
mass ↑ w/ radius

HSE: equilibrium b/w inward gravity & outward pressure

$f_{tot} = f_{pressure} + f_{grav}$

$ma \quad -AdP\,\hat{r} \quad -\frac{GM_{enc}\,dm}{r^2}\,\hat{r}$

outward pressure — towards center

* 

so $f_{P,out} = -AdP$

$$dm\frac{d^2r}{dt^2} = -AdP - \frac{GM_{enc}dm}{r^2}$$

$dm = \rho \cdot Adr$

$$\frac{d^2r}{dt^2} = \frac{-1}{\rho}\frac{dP}{dr} - \frac{GM_{enc}}{r^2} = 0$$

$$\frac{dP}{dr} = \frac{-GM_{enc}}{r^2}\rho(r) = -\rho(r)\,g(r)$$

Eqn of state: $PV = NkT \Rightarrow P = \frac{\rho kT}{m}$ — avg particle mass $m = \mu m_H$ (assume perfect gas)

Energy generation: $dL = 4\pi r^2 \epsilon(r)\rho(r)\,dr$

ϵ — energy generated per unit mass per second

Radiative energy transport: flow of energy carried by radiation through shells

$$\frac{dT}{dr} = \frac{-3\kappa \rho_r L_r}{4ac T_r^3 4\pi r^2} \rightarrow ac = 4\sigma$$

radiation is always present

mean molecular weight

convective energy transport: $\frac{dT}{dr} = -\left(1-\frac{1}{\gamma}\right)\mu m_H G M_r \frac{1}{kr^2}$

adiabatic index

stellar mass & radius: determined by HSE & energy transport
larger star (more massive) => higher internal pressure + T => faster rxns => shorter life

timescales: Radiative → time for energy to radiate away if nuclear rxn stopped, $\propto \frac{E_{tot}}{L}$
Convective → time for convective cell to transport energy from core to surface

Reduced mass

center: $P_c \sim \frac{G \rho_{avg} M}{R}$ $\xrightarrow{\text{eqn of state}}$ $T_c \sim \frac{mGM}{Rk}$