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Homework 2

Q1 Write a Python function that superposes plots of the Planck function I_λ in erg cm^{-2} s^{-1} λ^{-1} sr^{-1} for a range of temperatures from wavelengths from 300 to 5000 nm. *Describe the key similarities and differences in the plots of different temperatures.*

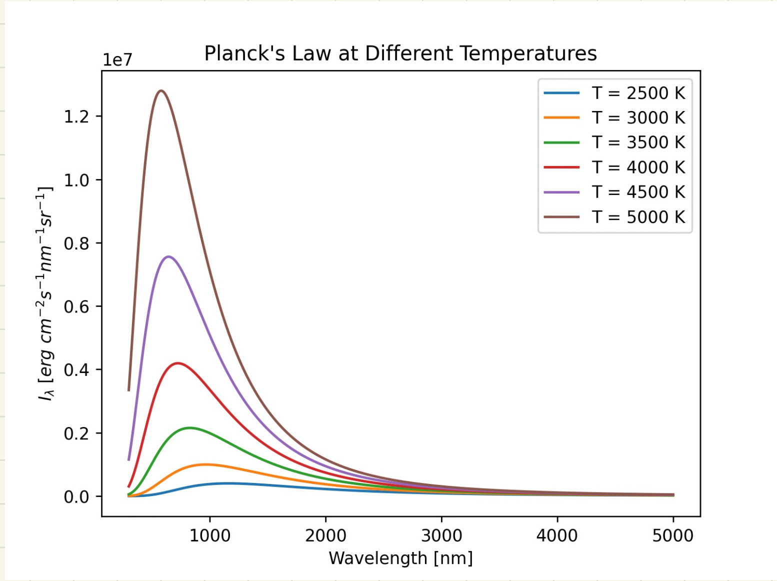
code in planck_function.py

$$I_\lambda = \frac{2hc^2}{\lambda^5}\cdot\frac{1}{e^{hc/\lambda kT}-1}$$

SI units. $(Js)\cdot(m/s)^2\cdot(m)^{-5}sr^{-1}$
$= W\cdot m^{-3} sr^{-1}$

to cgs $\Downarrow$ $W = 10^7$ erg/s, $m = 10^2$ cm, $m = 10^9$ nm

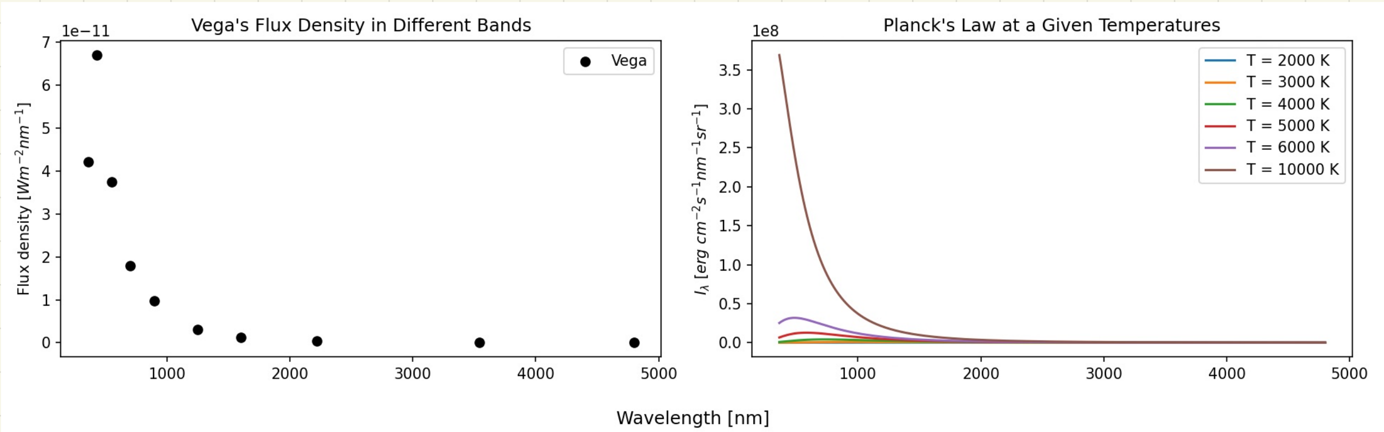
$= 10^7\, erg\cdot s^{-1}\cdot(10^2 cm)^{-2}\cdot(10^9 nm)^{-1}\cdot sr^{-1}$
$= 10^7\cdot 10^{-4}\cdot 10^{-9}\, erg\, s^{-1} cm^{-2} nm^{-1} sr^{-1}$
$= \underbrace{10^{-6}}_{\text{conversion factor}}\, erg\, s^{-1} cm^{-2} nm^{-1} sr^{-1}$



The peak of I_λ is higher and more towards lower λ for higher T. As T $\uparrow$, the peak gets lower & shifts to higher λ

Q2 Go back to the flux density plot you made for Vega from the first homework assignment. Experiment with the Planck function to find a spectral energy distribution that seems similar to Vega in its shape (not its intensity values). Use the subplot command to plot Vega and the most similar thermal radiator in two plots on the same page. What do you conclude from this comparison?

code in flux_density.py



$\Rightarrow$ The curve for T = 10 000 K is similar to the flux density plot of Vega

Since flux scales with intensity, we can conclude that Vega's surface temperature is ~10 000 K

Q3 Write a Python function that integrates the Planck function at a temperature of 6000 K, expressed as a monochromatic flux, i.e $\int \pi I_\lambda d\lambda$ erg cm^{-2} s^{-1}, over all wavelengths (use 10 to 100,000 nm). Compare your answer for the total bolometric flux to the Stefan Boltzmann law. What is your percent error?

code in planck_function.py

```
Integrated Planck function at 6000 K: 7.348800e+10 erg cm^-2 s^-1
Stefan-Boltzmann Law at 6000 K: 7.348805e+10 erg cm^-2 s^-1
Percent error: 7.014e-05%
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Q4 What is the ***Balmer Formula*** and why is it important?

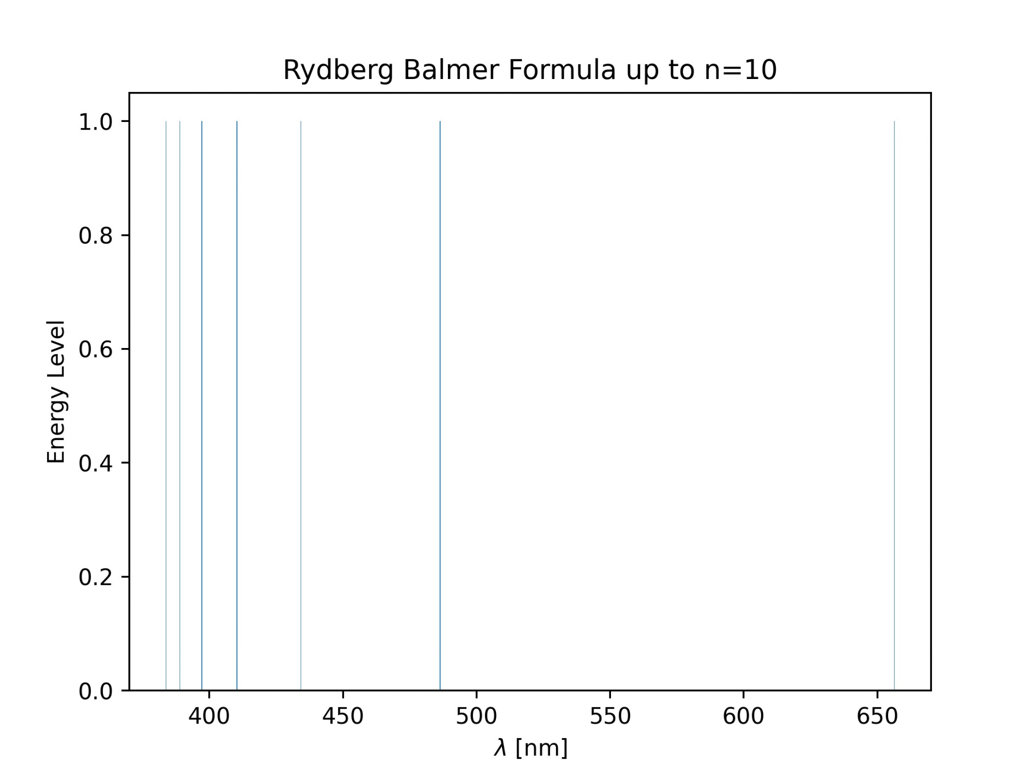
Run the Python script *balmer.py* that illustrates the wavelengths of hydrogen lines identified by the Balmer Formula for transitions up to n = 10. Note the use of a histogram in making the figure: pyplot.hist(lam,500) Read the help on the command 'pyplot.hist' and experiment with this command, for example: pyplot.hist(lam), pyplot.hist(lam,1000), etc.

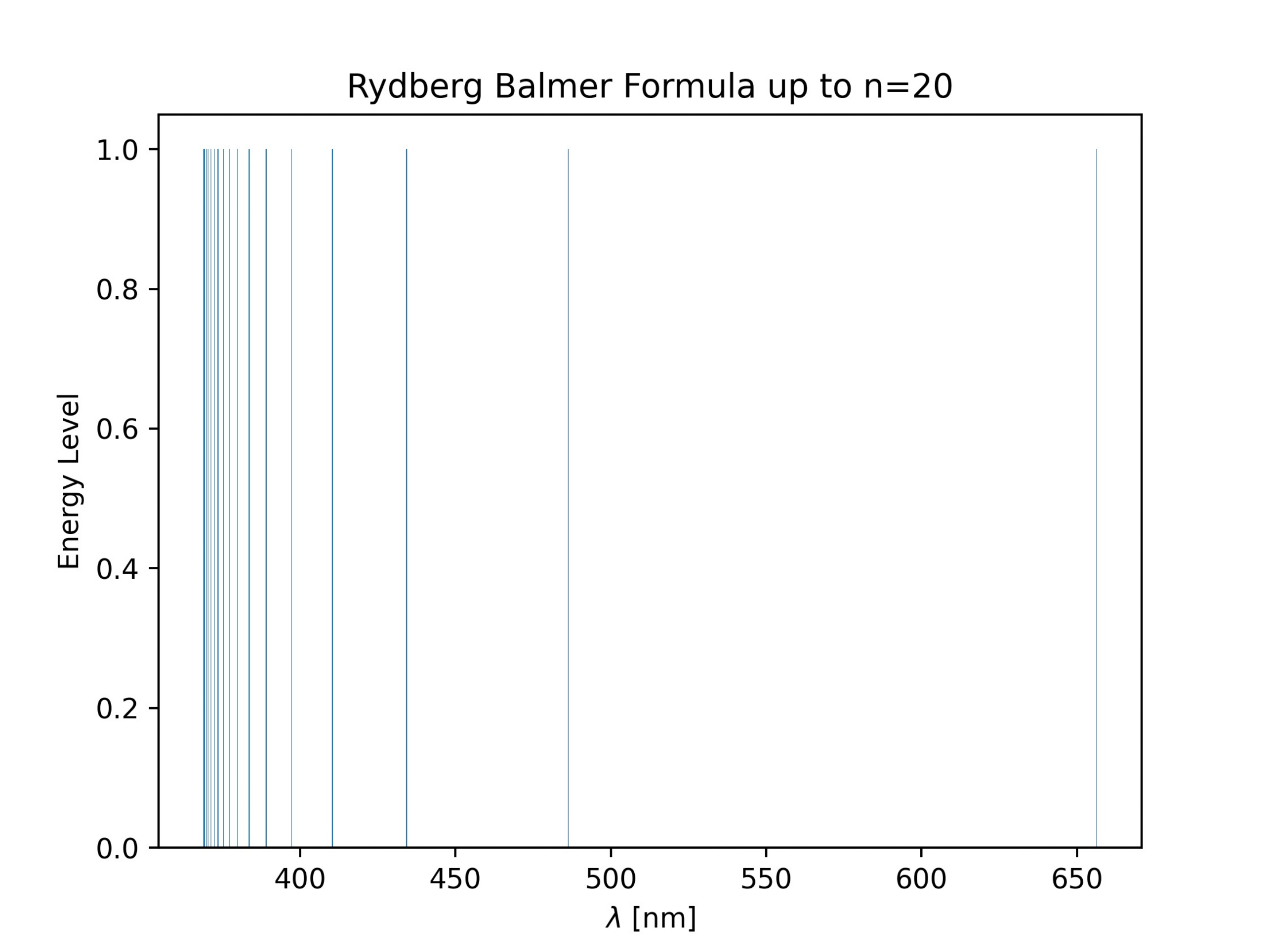
a. Modify the script to show all Balmer lines up to n=20.
b. Write a few sentences that describe the pattern of the Balmer Hydrogen lines.

① Balmer formula : $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$

This formula is important because it gives us the spectral lines of Hydrogen.

②



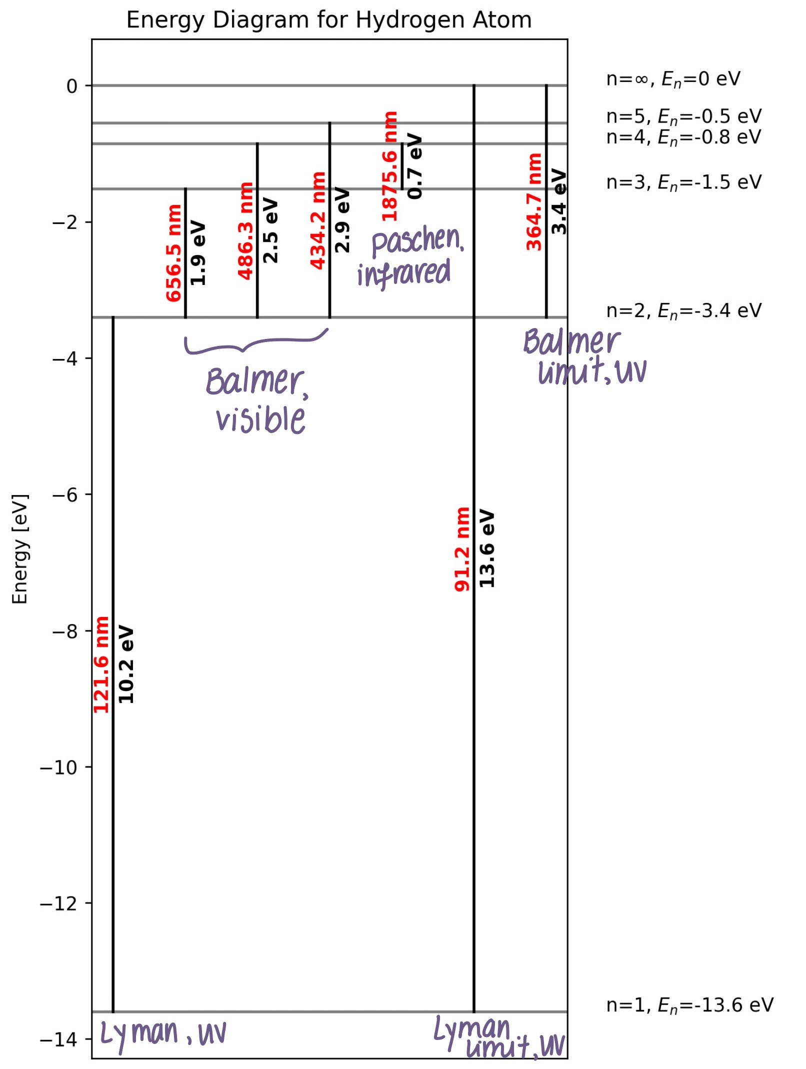


③ As n increases, we get more lines around 300-400 nm, as expected from the formula.
The high λ region still has only a few lines since these come from low n levels.

Q5 Make a sketch of the ***Energy Level Diagram*** for the Hydrogen atom that shows 6 levels: n = 1 , 2, 3, 4, 5 and ∞.

a. Compute the binding energy (eV) *for each of the 6 levels* using Equation 5.14 in your text. Label each on the diagram. What does n = ∞ mean?
b. Compute the *energy (eV)* for an electron to *make the following 7 transitions* and indicate them in the diagram.
   - 2 – 1
   - 3 - 2
   - 4 - 2
   - 5 – 2
   - 4 - 3
   - ∞ - 1
   - ∞ - 2
c. Compute the *wavelength in **nm*** that would be emitted by an atom with an electron making each of the above 7 transitions and specify in which region of the electromagnetic spectrum each falls. Show all unit conversions. Identify any lines that have recognized names.

code in energy_diagram.py



n = ∞ means the atom is gaining/losing an electron

Q6 You have a stellar binary, in circular orbits, each with mass M = $1M_\odot$. Assume the orbital period is P_{orb} = 1 day.

a. Derive an equation for the binary separation a (the distance between the two stars) in units of solar radii $R_\odot$. It should be of the form

$$a = N_0 f\left(\frac{M}{M_\odot}, \frac{P_{orb}}{1\ day}\right) \times R_\odot$$

where N_0 is some number, to allow substitution for different masses or periods.

b. Use the relation for R(M) for main sequence stars, $R \propto M^{\frac{n-1}{n+3}}$, with n=4 for low mass stars that burn hydrogen on the p-p chain, and n=19 for high mass stars that burn hydrogen on the CNO cycle. In your relation derived in (a), set the separation a = 2 R(M): this corresponds to a separation where the stars would just touch. Solve this equation for $P_{orb}(M)$ in both cases (low mass and high mass). Note that P_{orb} will be a function of M only. This is then an estimate for the limiting orbital period for a binary system of mass M.

c. Plot the relationship derived in part (b), with P_{orb} in units of days, and M in units of solar masses, for M=0.1-200 $M_\odot$. Label all axes accordingly.

d. If you were to observe a binary with period P_{orb} 15 hours, what mass range for the binary components is possible, based on your derivation?

a) Kepler's law: $P^2 = \frac{4\pi^2}{G}\frac{a^3}{m_1+m_2}$

so we have $a = \left(\frac{GM}{4\pi^2}\cdot P^2\right)^{1/3}$

$$= \underbrace{\left(\frac{GM_\odot}{4\pi^2}\cdot\frac{(1\,day)^2}{R_\odot^3}\right)^{1/3}}_{N_0 = 4.2}\left(\frac{M}{M_\odot}\right)^{1/3}\left(\frac{P}{(1\,day)}\right)^{2/3} R_\odot$$

$$a = 4.2\left(\frac{M}{M_\odot}\right)^{1/3}\left(\frac{P}{1\,day}\right)^{2/3} R_\odot$$

Plugging in values for our system, $M = 2M_\odot$, $P = 1$ day $\Rightarrow$ $a = 5.3 R_\odot$
(wolfram)

b) $R(M) \propto M^{\frac{n-1}{n+3}}$, n=4 for low mass stars, n=19 for high mass stars
$a = 2R(M)$, solve for $P_{orb}(M)$

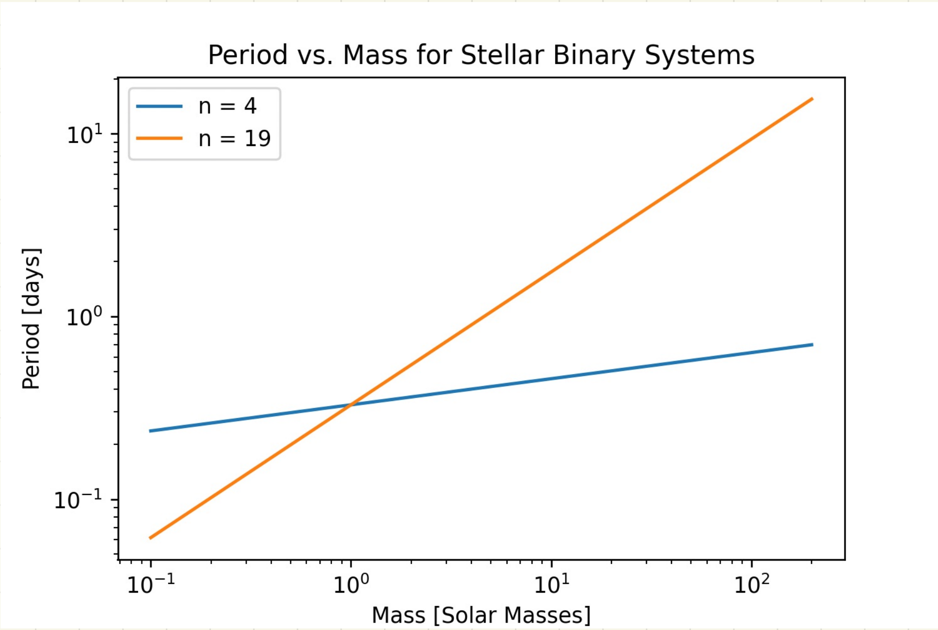
we have $2R(M) = 4.2\left(\frac{M}{M_\odot}\right)^{1/3}\cdot\left(\frac{P_{orb}(M)}{1\,day}\right)^{2/3} R_\odot$

$$\cancel{R_\odot}\left(\frac{M}{M_\odot}\right)^{\frac{n-1}{n+3}} = 2.1\left(\frac{M}{M_\odot}\right)^{1/3}\left(\frac{P_{orb}(M)}{1\,day}\right)^{2/3}\cancel{R_\odot}$$

$$P_{orb}(M) = 2.1^{-3/2}\left(M^{\frac{n-1}{n+3}-\frac{1}{3}}\right)^{3/2}$$

for n=4, we get $P_{orb}(M) \propto M^{1/7}$
for n=19, we get $P_{orb}(M) \propto M^{8/11}$

c)



code in stellar_binary.py

d)

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Mass range: [90.04564559  2.42054031] solar masses
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Q7

A light curve is a plot of flux as a function of time for an astronomical object. The plot below is the light curve of a star, where relative brightness is measured flux, normalized by the maximum stellar flux. The periodic decreases are caused by a transiting planet, that is, a planet which passes between the observer and the star. The star has a radius of 1.84 solar radii, a mass of 1.47 solar masses and a temperature of 6441 K.

a. The planet's transit depth is the decrease in relative brightness during transit. Using the fact that flux is proportional to area, find the radius of the planet. Express it in Jupiter radii .

b. Use Kepler's laws to calculate the planet's semi-major axis in AU.

c. What is the planet's equilibrium temperature, assuming the planet absorbs all of the light it receives? (Hint: light received is proportional to cross-sectional area, and light emitted is proportional to surface area.)

d. This type of planet is called a "hot Jupiter." Why are hot Jupiters the easiest planets to detect and characterize from their light curves?

e. The same planet may have a different transit depth when it is observed at another wavelength. Why does a planet's apparent radius depend on the wavelength at which it is observed?

$R_s = 1.84\ R_\odot$
$M_s = 1.47\ M_\odot$
$T_s = 6441\ K$

a) depth = ↓ relative brightness , $F \propto A \propto R^2$

depth!

$$\frac{F_P}{F_S} = \left(\frac{R_P}{R_S}\right)^2 \longrightarrow R_P = R_S\left(\frac{F_P}{\cancel{F_S}}\right)^{1/2}$$

$$= (1.84\ R_\odot)\cdot(1-0.9935)^{1/2}$$

$R_P = 1.44\ R_{jupiter}$ wolfram

b) ① $P^2 = \frac{4\pi^2}{G}\frac{a^3}{m_1+m_2} \longrightarrow M_s >> M_p$ so approx $M_{Hot} \approx M_s$

$\downarrow$

$P = 2.2$ days (from graph)

$a = \left(\frac{G \cdot 1.47 M_\odot (2.2\,\text{days})^2}{4\pi^2}\right)^{1/3}$

$\downarrow$ wolfram

$a = 0.03763$ AU

c) ① we know $\underbrace{L_s}_{\text{intrinsic to star}} = \underbrace{f_s(a)}_{\text{flux from star at panet's position (a)}} \cdot 4\pi a^2 = \underbrace{f_s(R_s)}_{\text{emitted } = \sigma T_s^4} \cdot 4\pi R_s^2 \implies f_s(a) = \frac{\sigma T_s^4 \cdot \cancel{4\pi} R_s^2}{\cancel{4\pi} a^2}$

② also $f_{received} = L_p$ (all energy absorbed)

$\underbrace{\pi R_p^2}_{\text{cross-section area}} f_s(a) = 4\pi R_p^2 \cdot \sigma T_p^4$

$\implies T_p = \left(\frac{\cancel{\pi R_p^2} f_s(a)}{4\cancel{\pi R_p^2}\sigma}\right)^{1/4}$

$= \left(\frac{1}{4\cancel{\sigma}} \cdot \frac{\cancel{\sigma} T_s^4 R_s^2}{a^2}\right)^{1/4}$

$T_p = \left(\frac{T_s^4 R_s^2}{4a^2}\right)^{1/4}$

$\downarrow$ plug in values

$T_p = 2172$ K

d) Hot Jupiters have large radii so they block more light while passing in front of stars

e) The light that reaches us first has to go through the hot Jupiter's atmosphere. While it does so, different wavelengths interact/get absorbed differently, so the planet's apparent size changes.