PHYS 521 Fall 2024
Exam #2 Review

Format: Short answer, some calculations. Calculators required!

Scope: Midterm #2 is continuing. Study from the **Exam #1 Review sheet AND this one**: concepts, application, and units. This review sheet may *not* cover every concept or equation that could be used on the exam – study from your readings, problem sets, and lecture notes.

Review your homework, class notes, and textbook!! Important formulae have an asterisk (*).

1. **Interstellar Dust**

   50-100 nm, silicates/graphite/...

   interstellar extinction = dust clouds obscuring stars (behind) scatters & absorbs blue λ $\Rightarrow$ reddening (so bluer stars are even more affected)

   - size, composition of grains
   - * optical depth: $\tau_\lambda = \sigma_\lambda n s = \kappa_\lambda \rho s$ cross-section; number density; opacity = mass absorption coefficient
   - * Extinction: $I = I_0 e^{-\tau}$, optically thin vs. thick
   - relation between τ and Δm
   - * $V - M_v = 5\ \log d - 5 + A_v$
   - $\frac{A_\lambda}{A_v} = (\frac{\lambda_v}{\lambda})^{1.8}$
   - $A_B\ A_v\ A_R\ A_I\ A_J\ A_H\ A_K$
   - Stellar spectral energy distribution (e.g., Planck function) and interstellar reddening
   - Scattering: Reflection Nebula
   - Thermal Emission and Equilibrium Temperature

2. **Interstellar Gas**

   - Gas phases of HI, H_2, HII: density and temperatures
   - Classical HII Region
   - * Stromgren Sphere $R_{ss} = (\frac{3}{4\pi\alpha})^{1/3}\ N_{uv}^{1/3}\ n_p^{-2/3}$
   - Atomic HI and 21-cm line emission
   - Molecular H_2, known molecular species
   - Molecular quantized vibrational, rotational Energy levels, dipole moment required for rotational emission lines
   - CO J = 1-0, λ = 2.7 mm, $CO/H_2 \sim 10^{-4}$
   - Very Hot Coronal Gas, galactic halo

Interstellar dust

size/composition: 50-100nm, silicates/graphite/ice mantle
dust clouds block light from stars behind → interstellar extinction
scatters & absorbs blue λ → Reddening (& bluer stars are more affected)

Optical depth: $\tau_\lambda = \sigma_\lambda n s = \kappa_\lambda \rho s$

- κ_λ: opacity = mass absorption coefficient
- ρ: mass density
- s: thickness
- n: number density

Extinction: $I_{out} = I_{in} e^{-\tau}$ (I_{in}: before extinction)

→ thick = high τ ⟹ I_{out} small
→ thin = low τ → $I_{out} \approx I_{in}$

τ & Δm: $\Delta m = -2.5 \log\left(\underbrace{\frac{I}{I_0}}_{e^{-\tau}}\right)$

$\Delta m = 1.086\,\tau_\lambda = A_\lambda$

Δm: change in apparent magnitude due to extinction; A_λ: line of sight extinction ⟹ higher A = more dust

extinction = effect of scattering by dust, λ (& ν) dependent

distance modulus w/ dust: $m_{obs} - m_{10pc} = 5\log d + A$
└ corrects for dust dimming
$m_\lambda - M_\lambda = 5\log d + A_\lambda$

$V - M_V = 5\log d + A_V$ in visible band

(V: apparent; M_V: abs; d: distance; A_V: extra factor bc of dust)

λ dep. extinction: $\frac{A_\lambda}{A_V} = \left(\frac{\lambda_V}{\lambda}\right)^{-1.8}$

Ratio w/ visible band
⟹ how extinction scales w/ λ
⟹ A is larger for smaller λ (blue)
bc λ is closer to physical size of dust so more scattering
$A_\lambda \sim \lambda^{-1.8}$

Reddening: larger A = more opaque & optically thick. $E(B-V) = A_B - A_V$
in ISM, dust scatters blue light more → makes nebula have a blue 'reflection'/aura

Thermal emission: dust absorbs & re-emits radiation at longer λ
equil. T depends on abs/emit balance → Wien's law
cold dust (20 K) emits in IR

Interstellar gas

Gas phases: HI neutral hydrogen. cooler regions. 21 cm line (rare) → penetrates dust
H_2 molecular hydrogen, denser cooler clouds, primary sites of star formation
harder to trace bc no permanent dipole
HII ionized hydrogen, active star formation, young hot stars, hot dense regions

classical HII region: hot O,B stars emit in UV → ionize surrounding gas

3. **Star Formation and Giant Molecular Clouds**
    - Star Formation Regions
    - * Gravitational Energy vs. Thermal Energy $\frac{3}{5}GM^2/R \geq \frac{3}{\cancel{2}\,4}(M/m)kT$
    - * Jeans Mass $M_J \propto (kT)^{3/2}\ n^{-1/2}$
    - * Virial Theorem $KE = -1/2\ PE$
    - Free fall time $t_{ff} \approx (G\rho)^{-1/2}$
    - Gravitational Collapse and Angular Momentum $a = GM/R^2 - \omega^2 R$ acceleration
      spins faster
      counter grav pull
      + formation of rotation disk
    - Disk Equilibrium Radius $R/R_o = R_o v_o^2/GM$ vel
    - Protostars, Accretion Disks, Jets
4. **Stellar Evolution After the Main Sequence**
    - red giant and core/envelope structure
    - evolution of core
    - evolution of envelope
    - evolutionary phases for $M_* < 8\mathrm{M}_\odot$
    - equation of state for electron degenerate gas
    - equation of state for relativistic degenerate gas
    - * Chandraskar mass limit $M < 1.4\mathrm{M}_\odot$
    - evolutionary phases for $M_* > 8\mathrm{M}_\odot$
    - pulsating variables and instability strip
    - iron catastrophe and core collapse (II) supernova
    - nucleosynthesis and cosmic abundance of the elements
    - historical supernovae and supernova remnants
5. **Black Holes & Close Binaries**
    - neutron degeneracy, neutron star, pulsar, black hole
    - cataclysmic variables and type Ia SN
    - * Schwarzschild radius: $R_\mathrm{S} = 2GM/c^2$
    - * BH energy production: $E_{phot} = \eta\ mc^2$, where $\eta \sim 0.1$
    - * Accretion luminosity: $L = \eta(dm/dt)c^2$
    - * Eddington luminosity limit: $L_{Edd} = \frac{4\pi Gc}{\bar{\kappa}}M = \frac{4\pi Gc\ m_H}{\sigma_T}M$
    - * Eddington accretion: $\dot{M}_\mathrm{Edd} = \frac{L_\mathrm{Edd}}{\eta c^2}$
    - Eddington limit, Eddington mass
    - Accretion disks (accretion rates and disk temperatures)

- * Sound speed: $c_s = \sqrt{\gamma P/\rho}$ and $\gamma = 5/3$ for monatomic gas
- * Bondi accretion: $\dot{M} \simeq q(\gamma)\frac{16\pi\rho G^2 M^2}{c_s 3} \simeq \frac{4\pi\rho G^2 M^2}{c_s 3}$
- * Radius of the innermost stable orbit (ISCO): $R_{\rm ISCO} = \frac{6GM}{c^2}$

6. **The Milky Way Galaxy & Normal Galaxies**
    - Shape, size of Milky Way from Star Counts
    - Objects in/out of plane of Milky Way
    - Pop I vs Pop II stars (+ stellar orbits), Pop II stars
    - Constituents of the Milky Way and their location in the galaxy
    - The Great Debate about the Milky Way's size
    - * Enclosed mass from centripetal and gravitational forces: $M(R) = v(R)^2 R/G$
    - Rotation curves and Dark Matter
    - Evidence for Sgr A*, the Milky Way's SMBH
    - Spiral density waves
    - Galaxy Types: Spiral vs. Elliptical galaxies, Irregulars

7. **Hubble's Law, Distances, and Structure**
    - Standard candles as distance estimators (Cepheids, RR Lyrae, SNe Ia)
    - The distance ladder
    - * Hubble's Law: $v = H \times D$
    - * Cosmological Redshift: $z = (\lambda_{obs} - \lambda_o)/\lambda_o = \Delta\lambda/\lambda_o$
    - Groups, Clusters, and Voids
    - Evidence for Dark Matter in Galaxy clusters
    - Large Scale Structure

NOT COVERED: **Active Galaxies**, **Big Bang Cosmology**, and **Early Universe** (C&O Chapters 28, 29, 30).

Exam #2 Review

1. Interstellar dust

- **Size / composition of grains:** → 0.01–0.5 μm
  → Silicates / carbon compounds (graphite)
  ice mantle in cold areas (CO, H_2O)

- **optical depth:** $\tau = \sigma_\lambda n s = \kappa_\lambda \rho s$ (cross-sectional area: σ_λ; path length: s) amount of light blocked while traveling through medium

- **Extinction:** $I = I_0 e^{-\tau}$ → optically thick: high τ, light very attenuated
  → optically thin: low τ, light passes through

- **τ vs. Δm:** extinction in magnitude ∝ optical depth. More dust = higher extinction, $A \approx 1.086$

- **Distance modulus w/ dust:** $V - M_v = 5\log d - 5 + A_v$ corrects for dust-dimming

- **Wavelength-dependent extinction:** $\frac{A_\lambda}{A_v} = \left(\frac{\lambda_v}{\lambda_\lambda}\right)^{1.8}$ shorter wavelengths experience higher extinction. shifts stellar color index across diff. bands

- **Stellar spectral energy distribution & interstellar reddening**
  → dust scatters / absorbs blue light more ⟹ net reddening
  → shifts spectral energy dist., $E(B-V) = A_B A_V$ measures reddening.

- **Scattering & reflection nebulae:** bluer "reflection" nebula from scattering, bright patch in dark sky

- **Thermal emission and equilibrium temp of dust:** → dust absorbs & re-emits @ longer λ
  eq. Temp depends on absorb-emit balance
  given by Wein's law.
  → cooler dust emits in infrared (20K ⇒ 0.15mm)

2. Interstellar gas

- **Gas phases of different regions**
  → HI: neutral hydrogen, cooler (50–100K), $\rho \sim 1\,cm^{-3}$
  → H_2: molecular hydrogen, dense cool clouds (10–50K), $\rho \sim 10\text{–}1000\,cm^{-3}$ star formation
  → HII: ionized hydrogen, hot sparse areas (>10 000K), $\rho \sim 0.1\text{–}10\,cm^{-3}$ young stars

- **Classical HII region:** → hot O, B stars emit in UV ⇒ ionize surrounding gas
  → glows in optical around star, observable in star-forming regions
  → allows us to study star formation & feedback w/ ISM.

- **Strömgren sphere:** $R_{ss} = \left(\frac{3}{4\pi\alpha}\right)^{1/3} N_{uv}^{1/3} n_p^{-2/3}$ (uv photon output: N_{uv}; recomb. rate: α) ionized volume around hot star

- **Atomic HI & 21-cm line:** spin-flip transition in neutral hydrogen emits @ 1420 MHz (21cm)
  used to map dist + motion of hydrogen gas in galaxies
  penetrates dust clouds

- Molecular H_2 & known molecular species: → H_2 most common, weak emission (no perm. dipole)
  → CO, etc. easier to detect, tracers of H_2 (10^{-4} ratio)
  → over 200 molecules in ISM including organic comp.

- Molecular vibrational & rotational energy levels: → ISM molecules have quantized vibration/rotation E-levels
  → rotation = radio/mm spectra
  → vibration = infrared spectra
  → need permanent dipole moment to detect ⇒ CO good ☺

- CO $J=1\to 0$ transition: $\lambda = 2.7$mm, $CO/H_2 \sim 10^{-4}$
  → Rotational transition in CO, used to map molecular (hydrogen) clouds → easier to study

- Very hot coronal gas & galactic halo: → hot component of ISM ($\sim 10^6$K)
  → in galactic halo + intergalactic medium
  → mainly ionized atoms (O^{6+})
  → emits in X-ray, from galaxy interaction / feedback.

3. Star formation & giant molecular clouds

- Star formation regions: cold, dense ISM w/ GMCs collapse into stars; see in radio/infrared
  10-50K
  hundreds of ly

- Gravitational vs. thermal energy: $\frac{3}{5}\frac{GM^2}{R} \geq \frac{3}{2}\frac{M}{m}kT$ determines if gas cloud will collapse into star

- Jean's mass: $M_J \propto (kT)^{3/2} n^{-1/2}$ critical mass for a gas cloud to collapse; dense + cold ⇒ collapse
  temp
  density
- Virial theorem: $KE = -\frac{1}{2}PE$ condition in stable systems (not collapsing)

- Free fall time: $t_{ff} \approx (G\rho)^{-1/2}$ timescale for gravitational collapse. Dense GMC ⇒ rapid star formatn

- Gravitational collapse and angular momentum: $a = \frac{GM}{R^2} - \omega^2 R$ conservation of momentum
  → spins faster as it collapses, rotating disk forms around protostar

- Disk equilibrium radius: $R/R_0 = R_0 V_0^2/GM$ forming disk around protostar, accretion disk size/dist

- Protostars, accretion disks, jets: → protostars: form in dense GMC regions, heat up as $E_{grav} \to E_{therm}$
  → accretion disk: how material falls onto protostar
  ↓ channels matter onto growing star ↓ enables outflows/jets
  → Jets: along rotation axis, clear out nearby material
  regulates angular momentum

4. Stellar evolution after the main sequence

- Red Giant & core/envelope structure: → exhaust H in core → expand into RG.
  → inert He (or heavier) core, H-burning shell
  → strong luminosity w/ large cool envelope (red)

- Evolution of core: contracts → temp. increase → fusion advances
  - ↳ if $M_* < 8 M_\odot$, core contracts until He burning (He flash, 1-2 $M_\odot$)
    $M_* > 8 M_\odot$, no flash, gradually build up C & O
  - ↳ exhaust all fuel, contract into qm state (e⁻ degeneracy pressure)

- Evolution of envelope: energy from core flows out → outer layers expand + cool
  - ↳ mass loss via stellar winds ⇒ enriches ISM w/ fusion elements

- Evolutionary phases for $M_* < 8 M_\odot$: red giant ⇒ helium burning $\overset{?}{\Rightarrow}$ asymptotic giant branch
  - ↳ asymptotic GB: hydrogen & helium shells burn around degenerate C-O core
    expel outer layers → planetary nebulae w/ WD core (e⁻ pressure)

- Equation of state for electron degenerate gas: Pressure depends on e⁻ density, not temperature
  - ↳ non-relativistic: $P \propto n^{5/3}$

- Equation of state for relativistic degenerate gas: extreme densities, WD near chandrasekhar lim
  - ↳ $P \propto n^{4/3}$, less density dependent

- Chandrasekhar limit: $M < 1.4 M_\odot$ max. mass for WD can resist grav. collapse
  beyond this, neutron star or black hole

- Evolutionary phases for $M > 8 M_\odot$: successive fusion C → Ne → Si ... Fe → core collapse
  doesn't produce enough energy

- Pulsating variables & instability strip: region of HR diagram, Cepheids & RR lyrae
  → periodic expansion from opacity changes w/i layers ⇒ distance indicates

- Iron catastrophe & core collapse (Type II Supernova): after iron builds up in core → grav. collapse, type II supernova → BH or NS

- Nucleosynthesis & cosmic abundance of elements: elements fused in stars, s-process (supernovae) or r-processes (neutron capture) form heavier elements ⇒ into ISM

- Historical supernovae & supernova remnants: insights into stellar death / remnants.
  → supernovae emit X-ray, ex SN 1054 → Crab Nebula

5. Black holes & close binaries

- Neutron degeneracy, neutron stars, pulsars, black holes:
  stars exhaust fuel → collapse from gravity
  if remnant core is... $< 1.4 M_\odot$ ⇒ WD
  $1.4 < \text{core} < 3 M_\odot$ ⇒ neutron star, $R_* \sim 10$ km, strong magnetic field + fast rotation → pulsar!
  $> 3 M_\odot$ ⇒ Black Hole

- Cataclysmic variables & type Ia SN: binary systems w/ WD accreting from companion star ⇒ periodic outbursts
  if $M_{WD} \to 1.4 M_\odot$, thermonuclear runaway ⇒ Type Ia supernova.

- **Schwarzschild radius**: $R_S = 2GM/c^2$ radius of event horizon, $V_{escape} > c$

- **BH energy production**: $E_{phot} = \eta mc^2$ w/ $\eta \sim 0.1$ efficiency of energy release through accretion onto BH; 10% of rest mass energy ⇒ radiation

- **Accretion luminosity**: $L = \eta (d_t m) c^2$ mass accretion rate; material spins inward ⇒ friction etc. heats → emits EM radiation

- **Eddington luminosity limit**: $L_{edd} = 4\pi G c M / \bar{\kappa} = 4\pi G c m_H M / \sigma_T$ thomson scattering crosssection; max. L where radiation pressure balances grav. force
  ↳ beyond this limit, radiation pressure will halt/expel accreting material.

- **Eddington accretion**: $\dot{M}_{edd} = L_{edd} / \eta c^2$ max. steady accretion rate for BH/NS w/i eddington limit.
  ↳ maximizes luminosity w/o significant mass loss

- **Accretion disks & disk temperature**: form around compact objects as matter spirals inward
  → grav. energy ⇒ heat + light.
  → higher temp @ inner disk → x-ray emission

- **Sound speed**: $C_s = \sqrt{\gamma P / \rho}$, $\gamma = 5/3$ monotomic gas for how dynamics work w/i region near c.o's.

- **Bondi accretion**: $\dot{M} \simeq q(\gamma) \dfrac{16\pi \rho G^2 M^2}{C_s^3} \simeq \dfrac{4\pi \rho G^2 M^2}{C_s^3}$ spherical accretion of gas onto compact object w/i static medium

- **Radius of innermost stable orbit (ISCO)**: $R_{isco} = 6GM/c^2$ for non-rotating BH, $R_{isco} = 3R_s$
  ↳ beyond this radius, material will fall into BH

6. The milky way galaxy & normal galaxies

- **Shape/size of milky way from star counts**: barred-spiral galaxy, disk diameter ~100 000ly
  ↳ estimate size & shape from star count ⇒ flat disk w/ central bulge & extended halo

- **Objects in/out of plane of milky way**: → most stars/gas/dust in spiral arms (young & forming stars)
  → above/below: halo w/ globular clusters, high velocity stars, DM
  ↳ old and metal-poor stars

  - **Pop I, II stars & stellar orbits**: → pop I: young, metal-rich, within disk, circular orbits w/i galaxy
    → pop II: old, metal-poor, in halo/bulge, elliptic orbit above/below plane

  - **constituents of milky way + location**:
    1. disk: gas, dust, pop I stars
    2. Bulge: dense, spheroidal region w/ pop I & II stars
    3. Halo: sparsely populated w/ pop II + globular clusters
    4. SMBH: galactic center

- **Great debate about milky way size**: 20^{th}, Shapely vs. Curtis debate.
  → Shapely: milky way = whole universe
  → Curtis: proposed spiral nebulae = separate galaxies
  ⇒ Hubble: resolved debate w/ cepheid variables in andromeda.

- **Enclosed mass from centripetal and gravitational forces**: $M(R) = v^2(R) R / G$ w/i galaxy, helps constrain DM

- **Rotation curves and dark matter:** orbital velocity of stars as fxn of distance from center
  ↳ surprisingly, flat @ large radii, no drop off ⟹ must be DM

- **Evidence for Sgr A*:** → strong radio source in galactic center
  → rapid orbits of nearby stars constrain mass to ~4 million $M_\odot$

- **Spiral density waves:** gravitational perturbations organize stars + gas into arms
  ↳ density waves move through disk ⟹ trigger star formation as gas clouds compress into arms.

- **Galaxy types:** 1. Spiral well-defined arms, sometimes w/ central bar
  2. Elliptical "red and dead", spherical or elongated, older pop II stars w/ little formation
  3. Irregular no distinct shape, from grav. interactions/mergers, mixed star pop.s & gas

7. Hubble's law, distances, and structure

- **Standard candles as distance estimators:** objects w/ known intrinsic can be used to estimate distances
  → Cepheid variables pulsating stars w/ known period-luminosity reln ⟹ known intrinsic brightness
  → RR lyrae variables shorter periods and lower luminosity ⟹ distances w/i galaxy & nearby galaxies
  → Type Ia supernovae WD explosions reach standard peak luminosity ⟹ far galaxies

- **Distance ladder:** series of methods for measuring cosmic distances
  → nearby stars = parallax, cepheids, then RR lyrae, then type Ia supernovae

- **Hubble law:** $v = H \times D$ speed of galaxy moving away from us wrt distance D

- **Cosmological redshift:** $z = (\lambda_{obs} - \lambda_0)/\lambda_0$ higher redshift = further object

- **Groups:** collection of galaxy (local group)
  clusters: collection of groups (Virgo cluster)
  Voids: relatively empty regions btwn structures
  } ⟹ "cosmic web", contributes to large-scale structure, infl. of DM

- **Evidence of dark matter in galaxy clusters:**
  → galaxy velocities higher than expected
  → gravitational lensing around areas w/ no normal matter
  → x-ray emissions would see more from hot gas, doesn't escape due to DM.

$I_\lambda = \sigma_\lambda n s = k_\lambda \rho s$	$M_J \propto (kT)^{3/2} n^{-1/2}$	$L = \eta (d_t m) c^2$	$R_{isco} = \frac{6GM}{c^2}$
$I = I_0 e^{-\tau}$	$KE = 1/2\, PE$	$L_{edd} = \frac{4\pi G c m_H}{\sigma_T} M$	$M(R) = v^2(R) \cdot R/G$
$V - M_V = 5\log d - 5 + A_V$	$M_{ch} < 1.4 M_\odot$	$\dot{M}_{edd} = L_{edd}/\eta c^2$	$v = H \times D$
$R_{SS} = \left(\frac{3}{4\pi\alpha}\right)^{1/3} N_{uv}^{1/3} n_p^{-2/3}$	$R_s = \frac{2GM}{c^2}$	$c_s = \sqrt{\gamma P/\rho}$	$z = (\lambda_{obs} - \lambda_0)/\lambda_0$
$\frac{3}{5}\frac{GM^2}{R} \geq \frac{3}{2}\left(\frac{M}{m}\right) kT$	$E_{phot} = \overset{0.1}{\eta} mc^2$	$\dot{M} \simeq \frac{4\pi \rho G^2 M^2}{c_s^3}$	

Review - midterm 2 *important formulae*

1. Interstellar dust

- size, composition of grains

ISM composed of dust & gas

size of dust: 50 - 100 nm (~ blue λ)

composition: silicates, graphite, icy mantles (Si, C, O, Mg, Fe)

- * optical depth: $\tau_\lambda = \sigma_\lambda n s = \kappa_\lambda \rho s$

ρ → density

κ_λ → opacity

n → nb density of scattering dust grains

σ_λ → scattering cross section

typically $\kappa \sim 1 cm^2/g$ at 500nm

$\sigma_g = \pi a^2$ cross-section that a dust particle presents to passing photon

$Q_\lambda \equiv \frac{\sigma_\lambda}{\sigma_g}$

- extinction

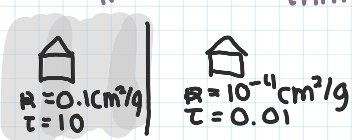
scattering & absorption of light by dust λ dependent

$I = I_0 e^{-\tau}$

↳ obs. intensity

if medium optically thick $\tau \gg 1$, light significantly scattered & abs. no correlation b/w input & output dir.

" " thin $\tau \ll 1$, light minimally " " => more light passes through



$s = 1km$

$\rho_{air} = 0.001 g/cm^3$

- relation b/w τ & Δm

$\Delta m = 2.5 \log\left(\frac{I}{I_0}\right) = 2.5 \log(e^{-\tau}) = -\frac{2.5}{\ln(10)} \ln(e^{-\tau}) = 1.086\tau$ → change in mag. due to extinction ~ τ along LOS

$\tau_V \sim \Delta V \sim A_V$

A_V: dust along LOS & effectiveness of dust absorption in V band.

- $V - M_V = 5\log d - 5 + A_V$

no dust: $V_{nodust} - M_V = 5\log d - 5 + A_V$

but $V_{obs} = V_{nodust} + A_V$

$V_{obs} - M_V = 5\log d - 5 + A_V$

- $\frac{A_\lambda}{A_V} = \left(\frac{\lambda_V}{\lambda}\right)^{1.8}$ $A_\lambda \sim \lambda^{-1.8}$ alters stellar spectral energy distributions

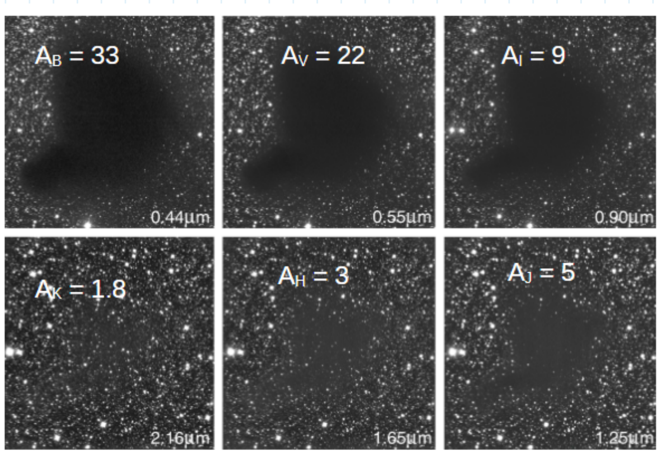
extinction more significant smaller λ, blue light scattered more than red

extinction causes reddening of starlight passing through dust

- A_B A_V A_R A_I A_J A_H A_K

reddening in blue, visible, red ... bands

B68 Dark Cloud at different wavelengths.
D = 0.08 pc
d = 125 pc



From ESO

A_K is smaller, bc biggest λ

- stellar spectral energy (e.g. Planck fct) and interstellar reddening
  extinction ↑ w/ ↓ λ
  extinction ↓ when λ becomes ↑ size of dust
  dust scatters blue light more than red → alters spectral energy distribution of stars
  also changes apparent colour
  w/o dust: $B-V = M_B - M_V$
  w/ dust: $B-V = M_B - M_V + (A_B - A_V)$
  colour excess (reddening): $E(B-V) = A_B - A_V$
  $B-V_{(obs)} = B-V_{(intr.)} + E(B-V)$
  $A_B \sim 1.32 A_V \Rightarrow A_B > A_V$
  $\Rightarrow$ reddening ratio : $R = \frac{A_V}{E(B-V)} \sim 3.1$
- scattering: reflection nebula
  dust in the ISM can scatter vis light (bluer light) producing <u>blue reflection nebulae</u> and it can absorb visible starlight to produce dark nebulae
- thermal emission and eq. temp.
  dust particles absorb stellar radiation and re-emit at longer IR λ
  the eq T° depends on balance b/w abs. and emitted radiation, often calc. using Wien's law
  Cold dust ($T \sim 20K$) emits thermal radiation at long λ (IR & radio)
  $\lambda_{max} T = 0.290$ cmK , if $T = 20K$, $\lambda = 0.15$nm in far IR

2- Interstellar Gas

- gas phases of HI, H_2, HII. density & T°
  $\frac{1}{2}$ gas by mass in ISM is H & He
  gas too cold to produce detectable emission lines in vis. & UV they're in lowest electronic state
  ↳ can only be traced through 21cm line of H

info about internal motion of gas atoms & molecules due to doppler. width & shape of lines

- HI regions: neutral hydrogen typically found in cooler regions 50-100K w/ densities of $\sim 1 cm^{-3}$
  emits 21cm photon HI abundant, 21cm em/abs rare for individual atoms
  → center of this line is optically thin over large dist.
- H_2 regions: molecular Hydrogen exists in denser cooler clouds (10-20K) w/ densities from 10^6 cm^{-3} these are primary sites of star formation
- HII regions: Ionized hydrogen in areas w/ active star formation or near young, hot stars, w/ $T \geq 10000$K and lower densities ($\sim 0.1 - 10 cm^{-3}$)

- Classical HII region
  form around hot O or B type stars emitting significant UV radiation, which ionizes surrounding hydrogen gas. Creates glowing region of HII gas around that star, visible in optical λ Classical HII regions are obs. in star forming regions across galaxies
  ↳ mostly Balmer
  → crucial for understanding star formation rates, as they trace areas w/ recent stellar births contributing to the feedback cycle within the ISM.

- Stromgren Sphere $R_{ss} = \left(\frac{3}{4\pi\alpha}\right)^{1/3} N_{uv}^{1/3} n_p^{-2/3}$

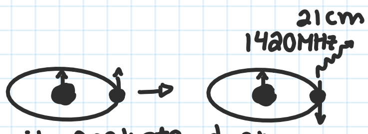
recombination rate. · UV output · n_p → density of surrounding photons

  - assume gas is pure H, other elements don't affect analysis
  - when in eq., rate of ionizing photons emitted by central source must be equal rate of removal of photons by ionization.
  - this photon removal rate must also equal to the ionic recombination rate in the region

  It describes ionized V around hot star (young)

- Atomic HI & 21 cm emission

  21cm emission line arises from spin-flip transition in neutral hydrogen HI



  this line is key for mapping the distribution & motion of Hydrogen gas in galaxies ∵ it penetrates dust clouds that obscure optical λ

  could help measure rotational curves, and estimate mass of hydrogen clouds within galaxies

- Molecular H_2, known molecular species

  H_2: most common molecular form in ISM, but doesn't strongly emit in radio ∵ lacks permanent dipole moment

  molecules like CO are easier to detect and often used as tracers for H_2. CO very common in molecular clouds where $\frac{CO}{H_2}$ ratio $\sim 10^{-4}$

  >200 molecules have been detected in ISM including complex organic compounds. Obs. these molecules helps to trace star-forming regions & study chem processes within ISM.

- Molecular quantized vibrational, rotational Energy levels, dipole moment required for rotational emission lines

  molecules in ISM have quantized vibrational & rotational energy levels. rotational transitions often occur in radio/millimeter λ & vibrational transitions in IR detecting these transitions requires a molecule w/ permanent dipole → CO effective tracer for molecular clouds

- CO $J=1-0$, $\lambda = 2.7$ mm, $\frac{CO}{H_2} \sim 10^{-4}$

  rotational transition occurs at $\lambda = 2.7$mm

  this line widely used to map molecular clouds w/ $\frac{CO}{H_2} \sim 10^{-4}$ abundance ratio providing estimates of molecular hydrogen density in clouds → aids in studying star formation & cloud dynamics

- very hot coronal gas & galactic halo.

  coronal gas is very hot ($\sim 10^6$ K) component of ISM located in galactic halo (extends beyond galactic plane) & intergalactic medium, primarly composed of ionized atoms. emits in X-ray, indicating high T° from shock heating or interactions b/w galactic outflows & halo.

  important to understand galaxy formation & feedback processes.

3 - Star formation and Giant Molecular Clouds

- Star Formation Regions

  occurs in cold, dense regions of ISM, particularly within Giant Molecular Clouds (GMCs)
  GMCs often span hundreds of ly., contain cold gas w/ H_2 as main component & are visible in radio & IR. Dense cores within GMCs eventually collapse under grav. to form stars.
  molecular clouds fragmented into clumps provide ideal conditions for star formation
  IR obs reveal protostellar objects embedded within these clouds, tracing early star formation stages.

- Gravitational Energy vs Thermal Energy $\frac{3}{5}\frac{GM^2}{R} \geq \frac{3}{2}\frac{M}{m}KT$ *

  balance b/w grav. & thermal energy determines if a gas cloud will collapse to form a star,
  for collapse to occur grav. E must exceed thermal E to ensure that gravity overcomes internal pressure to initiate contraction.

  * either typo, or ignore $\frac{1}{2}$
  $U \sim -\frac{3}{5}\frac{GM^2}{R}$, $K = \frac{3}{2}NKT = \frac{3}{2}\frac{M}{\mu m_H}$
  by virial thm $2K + U = 0$
  $\frac{3MKT}{\mu m_H} < \frac{3}{5}\frac{GM^2}{R}$

- Jeans Mass $M_J \propto (KT)^{3/2} n^{-1/2}$

  It's the critical mass above which a gas cloud will grav. collapse.
  cloud w/ $M > M_J \rightarrow$ unstable and contracts under its own grav.
  $\uparrow T^\circ \rightarrow \uparrow M_J$, $\uparrow n \rightarrow \downarrow M_J \rightarrow$ Jeans Criterion
  => denser colder regions are more prone to collapse.

- Virial theorem. $KE = -\frac{1}{2}PE \rightarrow$ grav. bound

  In stable, self-gravitating cloud in eq. $KE = -\frac{1}{2}PE$
  crucial for understanding the conditions under which a cloud remains stable vs. when it collapses to form stars

- free fall time $t_{ff} \approx (G\rho)^{-1/2} \rightarrow$ satisfies Jeans Criterion

  timescale over which a cloud collapses under grav in absence of pressure support.
  shorter $t_{ff} \rightarrow$ higher ρ => dense regions within GMCs conducive to rapid star formation

- Gravitational Collapse and Angular mom. $a = \frac{GM}{R^2} - \omega^2 R$

  as molecular cloud collapses, conservation of angular momentum causes it to spin faster, which counteracts inward pull of grav.
  the balance b/w acc. $\frac{GM}{R^2}$ & centrifugal force acc $-\omega^2 R$ due to rotation defines the dynamics during collapse, often leading to formation of a rotating disk around the nascent protostar

- disk eq. radius $\frac{R}{R_0} = \frac{R_0 v_0^2}{GM}$

  eq. radius of a forming disk around protostar can be approx. by $\frac{R}{R_0} = \frac{R_0 v_0^2}{GM}$ R_0 initial radius, v_0 initial vel.
  helps describe eventual distribution of material in the accretion disk around star

- protostars accretion disks & jets

  protostars form as dense regions within GMCs collapse & heat up as grav. E converts into thermal
  material falls into protostar through an accretion disk, which channels matter efficiently onto the growing star while enabling outflows and jets these jets emerge along the rotational axis, clearing surrounding material & influencing nearby star formation
  accretion disks are crucial in protostellar ev. & jets help regulate ang. mom., allowing additional material to accrete onto the protostar.
  ↳ obs in IR & radio, often revealing bipolar jets

4- Stellar Evolution after MS

- red giant & core/enveloppe structure

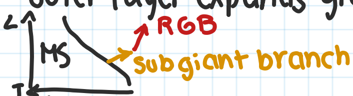
after exhausting H in the core, stars expand & become red giants.

inert He core contracts, releases GPE which heats up surrounding regions and promotes H fusion in shell above

At first, changes are slow, L↑ slightly, the star expands & cools. Star no longer in stable eq., He core contracts & heats up, ↑T°core, fusion of H→He in shell ↑↑, L increases dramatically

energy transport in star can't keep pace w/ rising L of H burning shell. new thermal energy trapped in star => outer shell expands a lot => surface cools. L, R↑, T↓ => Red giant forms

outer layer expands greatly & cool, inner core contracts & T↑. The star is a Red giant star ×100 MS size



star like Sun 10^{10} years on MS

in 10^9 years moves to tip of RGB ↗ 10^9 until becomes RGB or of being RGB??

- Evolution of the core

as core contracts, T↑ & fusion advances. In stars w/ $M_* < 8M_\odot$, the core contracts until He burning begins in a process called the helium flash for stars of 1-2 $M_\odot$. In more massive stars, the core ignite He w/o a flash, gradually building C & O nuclei

for stars tha eventually exhaust all fuel, the core may continue contracting into a degenerate state, governed by QM and supported e degeneracy pressure

- evol. of envelope

as nuclear reactions intensify in core, energy flows outward, causing outer layers of star to expand & cool. For many years, this leads to mass loss from the outer envelope through stellar winds, which contributes to enrichment of ISM w/ processed elements

- evol phases for $M_* < 8M_\odot$

after becoming Red Giant, for stars w. $M > 0.4M_\odot$, the core reaches $T > 120$ million k & He can fuse via triple alpha process to produce heavier elements: ${}^4He + {}^4He \rightarrow {}^8Be$ $E = -92$ keV ${}^8Be$ unstable

$4He + 8Be \rightarrow {}^{12}C + \gamma$ $E = 7.37$ MeV

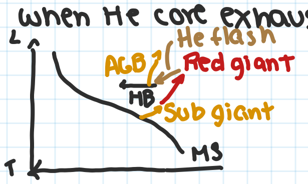
Helium fusion requires $T > 10^8 K$ net $3He \rightarrow {}^{12}C$ $E = 7.28$ MeV

He ignition occurs & now star has central source of E. core ceases to contract & star reestablishes hydrostatic eq. The new eq. for a star like Sun is now yellow giant ×100 L than when on MS

star now horizontal branch star fuse He into C in the core, and H into He in the shell. central T > 120 million k

∵ L↑ than when MS & ∵ He fusion produces 4 times less E than H fusion, HB much shorter than MS. for star like Sun, ~100 million years

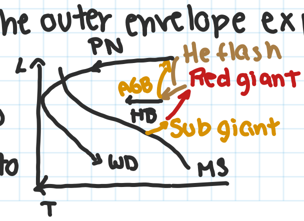
when He core exhausted, similar ev. as MS. core contracts, invigorates shell burning, outer layer expands & star undergoes 2nd red giant phase: Asymtotic giant branch (AGB)



AGB stars have collapsing C/O rich core, surrounded by shell of He fusion to C surrounded by shell of H fusing into He

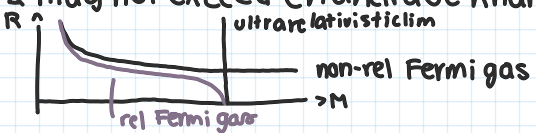
- In stars w/ $M < 4M_\odot$, the outer enveloppe expands, inert C/O core contracts e^- degeneracy pressure halts core collapse, and no further fusion reaction occurs. The outer envelope expands away becoming a planetary nebula, leaving a white dwarf remnant.

$M < 0.4M_\odot$ He rich

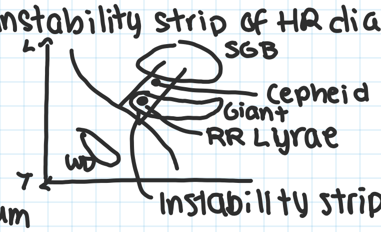


- for stars w/ $4M_\odot < M < 8M_\odot$, the core can contract and heat up to fusion by alpha capture into Ne & Mg. v. short. they end life sim. to to lower mass stars, producing planetary nebulae & leaving behind an O, Ne & Mg rich WD

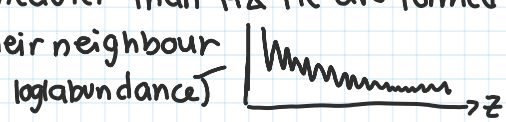
- eq. of state for e^- degenerate gas
  ↳ typically in WD, P not ∝ to T if total degeneracy. If partial, some T dep.
  $P \propto n_e^{5/3}$ → due to non-thermal motion of e^-
  M of core can > Chandrasekhar limit if additional pressure can be found to supplement ideal gas pressure
  & can occur if e^- are degenerate. when ρ of gas high enough, e^- need to occupy lowest possible E level w/o being in same quantum state. e^- stacked in progressively higher E states
  less massive stars → ↑ level of degeneracy on MS & may not exceed Chandrasekhar limit
- eq. of state for relativistic degenerate gas
  v. dense WD close to Chandrasekhar limit
  $P \propto n_e^{4/3}$ less pressure support for given density



- Chandrasekhar mass limit $M < 1.4 M_\odot$
  max mass a WD can support itself against grav. collapse through e^- degeneracy pressure alone.
  beyond this limit, the star would collapse
- Evol. phases for $M_* > 8 M_\odot$
  In stars w/ $M > 8 M_\odot$, fusion of O, Si, can result in building up heavier & heavier nuclei in the core.
  which requires higher & higher T to have reasonable prob. of tunneling, the E must ↑ as charge & mass ↑. for E generation, end at Fe, there's no further fusion that can produce E.
  BE / nucleon released in the fusion of heavy elements ↓ as products approach iron
  Near end massive star's lifetime, build up inert iron core.
  after Fe, core undergoes catastrophic collapse resulting in Type II SN leaving behind either NS or BH.
- Pulsating variables & instability strip
  pulsating stars like Cepheids & RR Lyrae, lie within the instability strip of HR diagram
  stars typically in HB phase can be unstable to pulsations
  low mass $1-2 M_\odot$ RR Lyrae
  high mass $2-15 M_\odot$ Cepheid



- Iron catastrophe & core collapse SN II → H present in spectrum
  When massive star builds up Fe core, reaches point where no further fusion can release energy ('Iron Catastrophe'). Core collapses to grav. causing Type II SN. Violent expulsion of outer layers & formation of NS or BH
- Nucleosynthesis & cosmic abundance of elements
  ↳ process that creates new atomic nuclei from existing nucleons & nuclei
  first nuclei we formed after BB in BBN
  stars are sites of nucleosynthesis where elements heavier than H & He are formed
  elements w/ even atomic nb are more common than their neighbour
  abundance of element ↓ as become heavier



- Historical SN & SN remnants
  e.g. SN 1054 (formed Crab Nebula) has provided insight into stellar death & remnants. SN remnants emit radiation across the spectrum (radio → X-ray) and are key to studying lifecycles of stars & enrichment of ISM
  Observing SN remnants helps understand the dynamics of explosions, element distribution & impact of these events on galactic evolution.

5. Close binaries & BH

- Neutron degeneracy, NS, Pulsars & BH

  When stars w/ sufficient mass exhaust their nuclear fuel, they collapse under grav

  if remnant core

  - < Chandrasekhar limit (~1.4 $M_\odot$)
  - ~1.4 & 3 $M_\odot$ → NS
  - > 3 $M_\odot$, no force can counterbalance grav. → BH

  NS v. dense remnants w/ r ~ 10 km, often obs. as pulsars due to rapid rotation & strong mag. fields, emitting beams of radiation detectable as pulses

- Cataclysmic variables & type Ia SN

  binary systems where a WD accretes material from companion star, leading to periodic outbursts. Sometimes, if WD mass approaches Chandrasekhar limit, undergoes thermonuclear runaway, producin Type Ia SN. → crucial for measuring cosmic distances as standard candle

- Schwarzschild radius: $R_S = \frac{2GM}{c^2}$

  radius of event horizon of BH, where escape vel > c

  more massive BH → larger event horizon

- BH energy production: $E_{phot} = \eta mc^2$ where $\eta \sim 0.1$

  → efficiency of E release through accretion

  ~10% of rest mass E of accreted material becomes radiation, contributing to immense L of AGN & x-ray binaries

- accretion luminosity: $L = \eta \frac{dm}{dt} c^2$

  depends on accretion rate $\frac{dm}{dt}$ & efficiency η.

  as material spirals in, friction etc. heat it ⇒ emits E accross EM spectrum.

- Eddington luminosity limit: $L_{Edd} = \frac{4\pi Gc}{\kappa} M = \frac{4\pi G c m_H}{\sigma_T} M$

  Eddington luminosity represents max L where outward radiation P balances inward grav. F, preventing further accretion. exceeding this limit causes intense radiation P that stop or expel accreting material

- Eddington accretion: $\dot{M}_{Edd} = \frac{L_{Edd}}{\eta c^2}$

  max steady accretion rate for BH or NS under accretion lim.

  accreting at this rate maximizes L w/o causing significant mass loss from radiation P

- accreting disks (accretion rates & disk T) im matter lmao

  they form around compact objects as matter spirals inwards, converting grav. E into heat & light.

  disk T ↑ closer to compact object, and for BH, inner disk regions can reach X-ray emitting T

- sound speed: $c_s = \sqrt{\frac{\gamma P}{\rho}}$ w/ $\gamma = \frac{5}{3}$ monoatomic gas

- Bondi Accretion. $\dot{M} \simeq q(\gamma) \frac{16\pi \rho G^2 M^2}{c_s^3} \simeq \frac{4\pi \rho G^2 M^2}{c_s^3}$

  spherical accretion of gas onto compact object from static medium.

- radius of innermost stable orbit: $R_{isco} = \frac{6GM}{c^2} = 3R_S$

  closest stable orbit around BH (non-rotating)

  material within this radius inevitably falls into BH, as stable orbits are no longer possible.

6 - The MW & Normal Galaxies

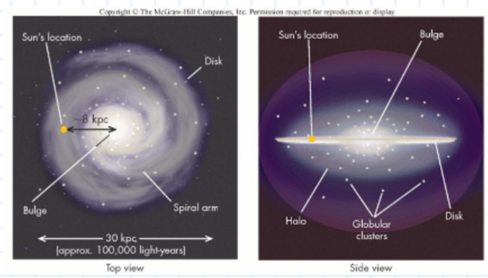
- Shape, size of MW from star counts

MW is a (barred) spiral galaxy w/ disk $d \sim 100\,000$ ly prob. looks like Andromeda.

size & shape estimates from star counts & studies of stellar distributions, revealing flat, disk-like structure w/ central bulge & extended halo. Vis. matter (by mass) 90% stars, 10% ISM (gas & dust)

- objects in/out of plane of MW

- within galactic plane, majority of stars, gas & dust, organized in spiral arms.
- above & below plane, halo contains globular clusters, high vel. stars. & DM. The galactic plane is heavily populated w/ young stars & star forming regions, while halo hosts older, metal-poor stars.



- Pop I vs Pop II stars (+ stellar orbits), Pop II

	Pop I	Pop II → spheroïdal pop
location	disc	halo & bulge
orbits	circular	elliptical
age	< 10 billion years	> 10 billion yrs
metal	metal-rich	metal poor

sun is Pop I w/ 2% of its mass being metallic. Pop II stars have 10 times lower metallic abundances.

- Pop I stars roughly circular orbit confined to the disc of the MW w/ all stars orbiting the same direction
- Pop II stars have elliptical orbits that extend into the halo. orbits random

- Constituents of the MW & their location in galaxy

- disk: gas, dust, Pop I stars
- bulge: dense, spheroïdal region around gal. center w/ Pop II stars & Pop I (on the plane)
- halo. sparsely pop. region w/ old, metal poor stars (Pop II) & globular clusters
- SMBH SgrA* at center of galaxy.

- the great debate about MW size

- William & Caroline Herschley in 1780s tried to deduce size counting stars. assume sun is center → $d \sim 2500$ pc
- In 1920, Kapteyn & van Rhijn found same thing, & that most stars within few thousand pc of sun.
- In 1920s, Shapley used RR Lyrae variable stars that have predictable L to get dist. to many globular clusters. He found that glob. clus. were distributed well offset from the sun.

where they went wrong:

- didn't consider dust effect on ISM
  - ↳ mass fraction of dust isn't high, but very effective at blocking vis. star light → distorts view of MW

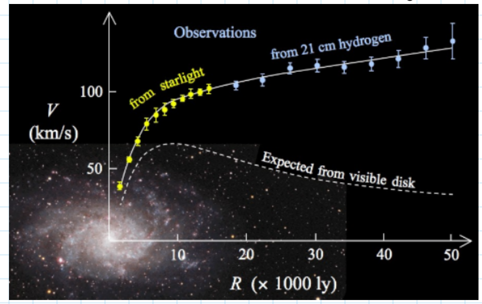
- enclosed mass from centripetal & grav. forces: $M(R) = \frac{v(R)^2 R}{G}$

determined by balancing centripetal & grav. forces. Mass estimation is key in studying DM in galaxy

$$\frac{GmM(R)}{R^2} = \frac{m\,v(R)^2}{R} \Rightarrow M(R) = \frac{R\,v(R)^2}{G}$$

- rotational curves & DM

  the rotational curve of MW showing orbital vel. as fct of dist. from center, remains flat at large radii, contrary to expected drop-off

  based on rot. vel. obs, we expect $M(R) = 10^{12} M_\odot$

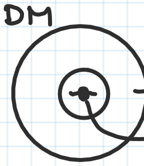
  if we statistically add all mass in stars, gas & dust, we find $M \sim 10^{10} M_\odot$

  → either our theory of grav. is wrong OR we need more mass

  the total mass could be as large as $10^{12} M_\odot$ more than what's seen

  the missing mass is DM. 90-95% of total mass of MW is DM

  thought to be distributed in large halo surrounding MW

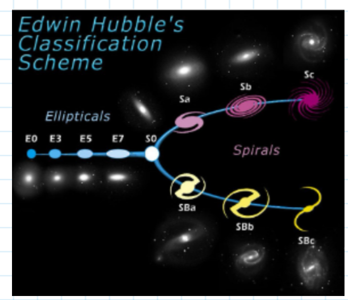
- Evidence for Sgr A*

  Sgr A* is obs. as strong radio source in MW center w/ indirect evidence from rapid orbits of nearby stars indicating an enclosed mass of ~4 million $M_\odot$ This compact mass suggests it is a BH, confirmed by detailed obs. of stellar motions & high res. imaging

- Spiral density waves

  spiral struct. of galaxies like MW is maintained by spiral density waves, which are grav. perturbations that organize stars & gas into arm patterns These density waves move through the disk, triggering star formation as gas clouds are compressed in the arms

- galaxy types. spiral vs elliptical galaxies, irregulars



20% but brightest - spiral galaxies are classified by degree of winding in the arms have both disk & spheroid component. stricking spiral arms → more dust & gas

  spheroid mostly old stars, bit of gas & dust

  disk relatively young stars & dust star formation mostly on spiral arms. relatively massive, optically bright.

60% - elliptical galaxies are classified by how flattened they are

  - mostly old stars w/ less gas & dust few new stars formed wide var. of sizes, most ~1% MW mass

20% - Irregular galaxies, like spiral, contain stars, gas & dust star formation still ongoing, but no disk or spherical component.

  smol, relatively faint w/ lots of gas & dust

  no spiral arms or bright centers

  mostly dwarf galaxies, w/ $\frac{1}{100}$ MW mass.