CG Method based 3D CT Image Reconstruction

Problems

The project aims to solve the CT reconstruction problem. The algorithm applys L2 least norm and adds a huber regularization term to get better performance. The main problem consists of the following points:

- choose the suitable optimization method to solve the regularization problem.
- choose the suitable hyper parameter of the model to get better performance.

Meanwhile, we need to apply the method to 2D phantom image as well as 3D realistic CT image, respectively. After that, a comparison between the existing method and the FBP algorithm will be made.
The source code of this project can be found on github

Methods

We apply the L2 norm of error plus with Huber regularization term as our objective function:

$$\min_x \frac{1}{2}\|Ax - b\|^2 + \lambda \mathcal{H}_\gamma(\nabla x) \tag{1}$$

The Huber regularization term is able to retain the edge and avoid overfitting. The optimization of the object function is done based on the conjugated gradient nolinear function.
It can convergent faster compared to the traditional steepest gradient descend because it searches for the conjugated gradient direction rather than the gradient direction.

Experiments

phantom

The selection of γ

We first set λ to 0.01, then 10 γ values are selected in a proportional ratio from $1e^{-5}$ to 1. After iteration for 50 times, we calculate their mean error in the last 20 iterations to represent their performance. The bar chart is showed below:

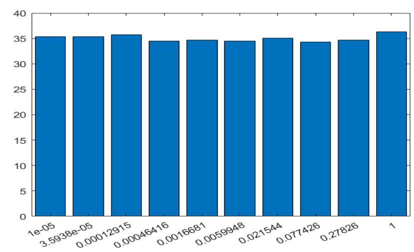

Figure 1: MeanError

It can be seen that the value of γ doesn't heavily influence the result. Thus we select 0.01 as our γ.

The selection of λ

We searched in the slide and found the corresponding method. However, it requires the whole projection matrix data to calculate the largest singular value. We are not quite familiar with the odl and not sure if it can extract the projection data from the *rayTrafo* operator.
So we manually tried several values and found that the smaller λ value contribute to the better performance of the result image. So finally we set λ to 0.01.

The selection of step length

We apply the ***BacktrackingLineSearch*** method to automatically search the maximum allowable step length of each iteration.

Results

We set the iteration time to 100, and choose several λ and white noises:

γ=0.01,λ=0.01,noise=0.1

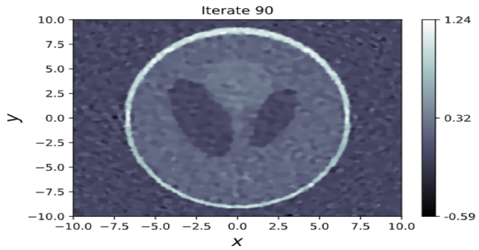

Figure 2: Phantom

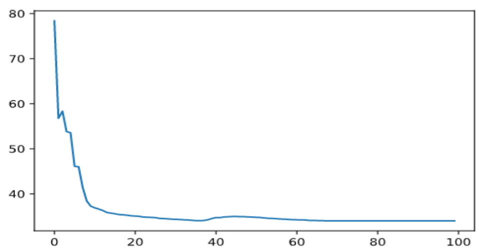

Figure 3: Error

γ=0.01,λ=0.05,noise=0.3

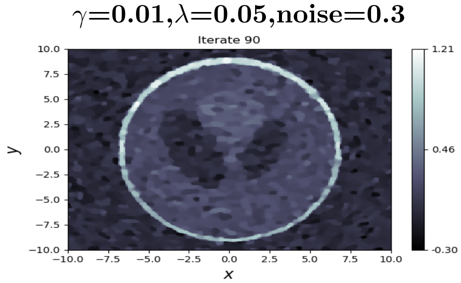

Figure 4: Phantom

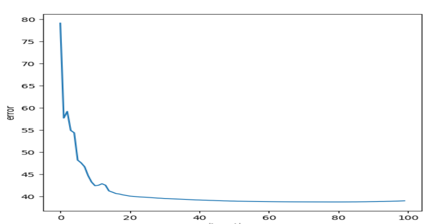

Figure 5: Error

realistic

Then we convert to realistic data. We choose the different λ to check the performance. The iteration time was constrained to 20 due to computational limitation.

λ=0.01

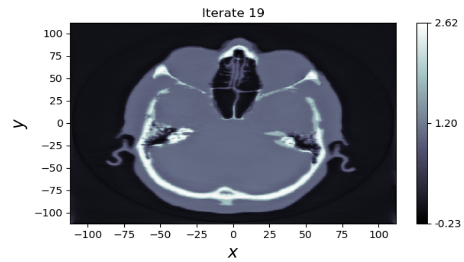

Figure 6: Iterative transverse

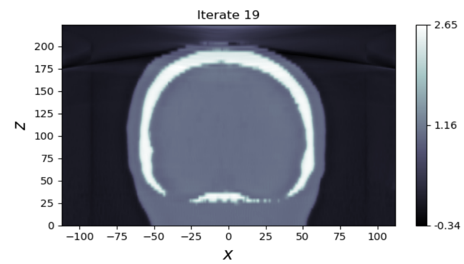

Figure 7: Iterative coronal

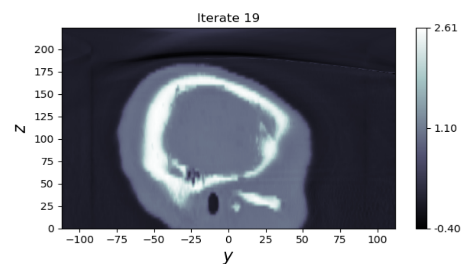

Figure 8: Iterative sagittal

λ=0.5

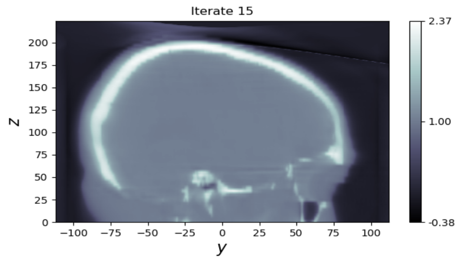

Figure 9: Iterative sagittal

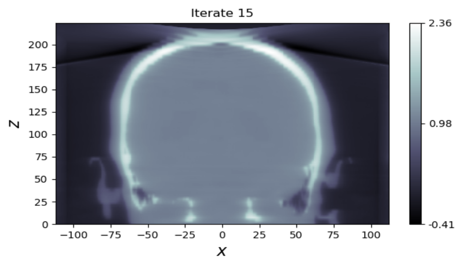

Figure 10: Iterative coronal

fbp

FBP method is used to compare with the iteractive reconstruction. We choose *Hann* as our filter type and ***frequency scaling*** was set to 0.6. The result is as follows:

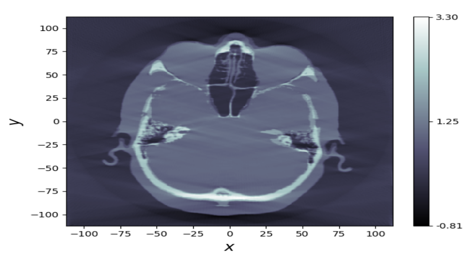

Figure 11: FBP transverse

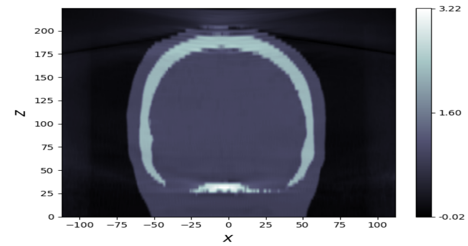

Figure 12: FBP coronal

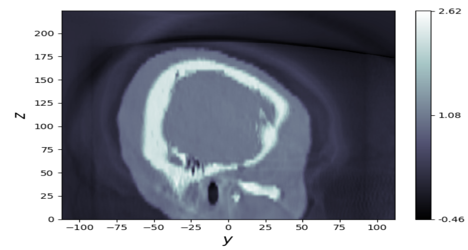

Figure 13: FBP sagittal

The figure convergents at last. And the image obviously has less detailed information compared to the previous iterative reconstruction. This may due to the filter removes some high frequency information. Meanwhile, the result has some artifacts as well.

Findings

Firstly, when it comes to the selection of iterative method, the conjugated gradient method has an edge over the normal steepest gradient as it can convergent faster. This is because the latter always takes the inverse direction of the gradient which is not efficient at all times.

Secondly, in terms of the selection of hyper parameter of the model,γ seems not to influence the final result a lot. Meanwhile, too much step length may lead to divergence.

Finally, the fbp method is computation-consuming and the result image has many artifacts and blur a bit, which is inferior to the result made by the iterative reconstruction.