Joint imaging and DDEs calibration for radio interferometry

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***Abstract*—In the context of radio interferometry, the objective is to find an estimate of an unknown image of the sky from degraded observations, acquired through an antenna array. In the theoretical case of a perfectly calibrated array, it has been shown that solving the corresponding imaging problem by iterative algorithms based on convex optimization and Compressive Sensing (CS) theory can be competitive with classical algorithms such as CLEAN. However, in practice, antenna-based gains are unknown and have to be modelled during the calibration process. The latter aims to estimate the DIEs and DDEs related to each antenna of the interferometer. In this work, we propose an alternated minimization algorithm to estimate jointly the DIEs/DDEs and the image while promoting its sparsity in a dictionary, leveraging the CS theory.**

In radio interferometry, the imaging problem of finding an estimation of an original unknown image $\overline{\mathbf{x}} = (\overline{\mathsf{x}}(n))_n \in \mathbb{R}_+^N$ from complex visibilities $\mathbf{y} \in \mathbb{C}^M$ can be formulated as an inverse problem. More precisely, for an interferometer with n_a antennas, we have $M = n_a(n_a-1)/2$ measurements acquired in the Fourier domain of the image of interest, from antenna pairs indexed by $(\alpha,\beta) \in \{1,\ldots,n_a\}^2$, with $\alpha < \beta$. Therefore, each degraded complex measurement $\mathsf{y}_{(\alpha,\beta)} \in \mathbb{C}$ acquired by the antenna pair (α,β) at the spatial frequency $\mathsf{u}_{\alpha,\beta} = \mathsf{u}_\alpha - \mathsf{u}_\beta$ can be modelled as

$$\mathsf{y}_{(\alpha,\beta)} = \sum_{n=-N/2}^{N/2-1} \mathsf{d}_\alpha(n)\mathsf{d}_\beta^*(n)\overline{\mathsf{x}}(n)\mathsf{e}^{-2\mathrm{i}\pi(\mathsf{u}_\alpha-\mathsf{u}_\beta)\frac{n}{N}} + \mathsf{b}_{(\alpha,\beta)}, \quad (1)$$

where $\mathbf{d}_\alpha = (\mathsf{d}_\alpha(n))_n \in \mathbb{C}^N$ represents a direction dependent effect (DDE) related to antenna α, and $\mathsf{b}_{(\alpha,\beta)}$ is a realization of a Gaussian additive noise. Note that direction independent effects (DIEs) can be seen as a special case of DDEs where $\mathbf{d}_\alpha = \delta_\alpha \mathbf{1}_N$, with $\delta_\alpha \in \mathbb{C}$ and $\mathbf{1}_N$ the unitary vector of dimension N.

When the antenna array is perfectly calibrated, i.e. when the DDEs are known, new methods based on both convex optimization and CS theory have been developed recently to find an estimation of $\overline{\mathbf{x}}$ from the observations (1) [1]. In particular, the estimated image can be defined as a solution to:

$$\underset{\mathbf{x}\in\mathbb{R}_+^N}{\text{minimize}} \ \frac{1}{2}\|\mathbf{G}\mathbf{F}\mathbf{x}-\mathbf{y}\|^2 + \eta\|\boldsymbol{\Psi}^\top\mathbf{x}\|_1, \quad (2)$$

where $\eta > 0$, $\boldsymbol{\Psi}^\top \in \mathbb{R}^{D\times N}$ is a given dictionary, $\mathbf{F} \in \mathbb{C}^{N\times N}$ denotes the Fourier matrix, and $\mathbf{G} \in \mathbb{C}^{M\times N}$ is a matrix containing on each line the antenna-based gain for the pair (α,β). Then, each line of $\mathbf{G}$ corresponds to the convolution of the Fourier transforms $\widehat{\mathbf{d}}_\alpha$ and $\widehat{\mathbf{d}}_\beta$ of $\mathbf{d}_\alpha$ and $\mathbf{d}_\beta$ respectively, centered at the frequency $\mathsf{u}_{\alpha,\beta}$.

However, in practice, antenna-based gains $(\mathbf{d}_\alpha)_\alpha$ have to be calibrated. The last years, several methods have been developed to estimate DIEs and/or DDEs, when the image is assumed to be known. In particular, in the StEFCal method [2] only DIEs are considered and the complex visibilities are rewritten as a data matrix $\mathbf{Y} \in \mathbb{C}^{n_a\times n_a}$ where, for every (α,β), $\mathsf{Y}_{\alpha,\beta} = \mathsf{y}_{(\alpha,\beta)}$. Note that due to the symmetry of measurements in (1) and reality of $\overline{\mathbf{x}}$, we have $\mathsf{Y}_{\beta,\alpha} = \mathsf{y}^*_{(\alpha,\beta)}$. The corresponding least squares minimization problem can be recasted as follows:

$$\underset{\mathbf{D}_1\in\mathbb{C}^{n_a\times N},\mathbf{D}_2\in\mathbb{C}^{n_a\times N}}{\text{minimize}} \ \frac{1}{2}\|\mathbf{D}_1\widehat{\mathbf{X}}\mathbf{D}_2^\top - \mathbf{Y}\|^2, \quad (3)$$

where $\mathbf{D}_1$ (resp. $\mathbf{D}_2$) is the matrix such that, each line α contains $\widehat{\mathbf{d}}_\alpha$ (resp. $\widehat{\mathbf{d}}_\alpha^*$) centered in u_α (resp. $-\mathsf{u}_\alpha$), and $\widehat{\mathbf{X}} \in \mathbb{C}^{N\times N}$ is the matrix containing on each line/column a shifted version of the Fourier transform of the image, to model the convolution operation. Note that for DIEs, each vector $\widehat{\mathbf{d}}_\alpha$ has one non-zero value. Thus, to solve (3), the StEFCal method needs to estimate only $2n_a$ values.

In this work, we propose a new method for the joint estimation of the original image, and the DDEs when both are unknown, based on a block-coordinate forward-backward algorithm [3]. It involves alternating between the estimation of the image and the DDEs, which are given as a solution to (2) and (3) respectively. In our approach, we assume that the DDEs have a bounded support in the Fourier domain, thus for the sparse matrices $\mathbf{D}_1$ and $\mathbf{D}_2$, we estimate only the coefficient values within this support (of size 1 when DDEs reduce to DIEs). Moreover, we consider constraints on the coefficient amplitudes of the DDEs, assuming that the amplitude of the central coefficient of $\widehat{\mathbf{d}}_\alpha$ is larger than the others. Finally, we make use of the prior information on the bright sources of the image to be estimated.

We analyze the performance of our method on simulated sky images of size 128×128, consisting of point sources, generated randomly on three intensity levels (Fig.1). While the first level is assumed to be known, the aim is to estimate the other two levels. We consider $n_a = 200$ randomly distributed antennas, and $\widehat{\mathbf{d}}_\alpha$ of support size 5×5. We performed simulations to reconstruct (i) the DIEs and the image (a) by combining StEFCal with an imaging algorithm, (b) using our method, and (ii) jointly the DDEs and the image with our method. The results show that the second level given in Fig. 1(center) is recovered only in case (ii). Finally, reconstructions have been compared in terms of SNR, on an average of 10 simulations, varying the random images, antennas distributions, DDE values and noise realizations. The SNR of the prior image (Fig.1 (left)) is equal to 38.8 dB, while for the reconstructed images, we have SNR= 37.8 dB for cases (i)(a-b) and SNR=53.2 dB for case (ii).

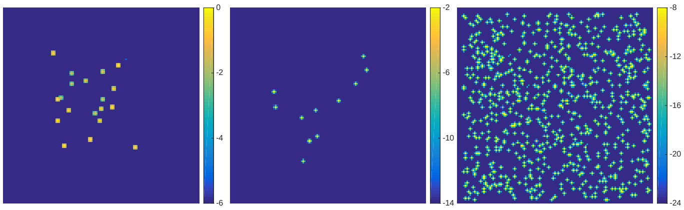

Figure 1. Images, in log scale, corresponding to the first level with the known bright sources $\overline{\mathbf{x}}_1$ (left), and to the fainter sources belonging to the second $\overline{\mathbf{x}}_2$ (center) and third $\overline{\mathbf{x}}_3$ (right) levels. We have $\overline{\mathbf{x}} = \overline{\mathbf{x}}_1 + \overline{\mathbf{x}}_2 + \overline{\mathbf{x}}_3$, with energy of $\overline{\mathbf{x}}_1$, $\overline{\mathbf{x}}_2$ and $\overline{\mathbf{x}}_3$ of the order of 1, 10^{-2} and 10^{-6}, respectively.

References

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