Fibre Orientation Estimation from kq-Space Sampling considering the motion-induced phase effects in Diffusion MRI

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***Abstract*—High spatio-angular resolution diffusion MRI (dMRI) has been shown to provide accurate identification of complex fiber configurations, albeit at the cost of long acquisition times. We propose a method to recover intra-voxel fiber configurations at high spatio-angular resolution relying on a kq-space under-sampling scheme to enable accelerated acquisitions. The inverse problem for reconstruction of the fiber orientation distribution (FOD) is regularized by a structured sparsity prior promoting simultaneously voxelwise sparsity and spatial smoothness of fiber orientation. Prior knowledge of the spatial distribution of white matter, gray matter and cerebrospinal fluid is also assumed. A minimization problem is formulated and solved via a forward-backward convex optimization algorithmic structure. Simulations and real data analysis suggest that accurate FOD mapping can be achieved from severe kq-space under-sampling regimes, potentially enabling high spatio-angular dMRI in the clinical setting.**

***Index Terms*—dMRI, FOD, kq-space, phase, under-sampling**

I. Introduction

In theory, diffusion images are real-valued. However, in practice they are often contaminated by phase factors during the diffusion encoding process. This phase contamination is mostly due to magnetic field inhomogeneities and biological motion (e.g. physiological and involuntary patient motion).

Usually, methods dealing with signals directly in q-space overcome this difficulty by simply taking the modulus of the complex diffusion signal, in order to obtain real diffusion-weighted images. However, in k-space, and consequently kq-space, the phase contamination breaks the Hermitian symmetry of the images, and can not be neglected. In particular, in the linear operator expressed in equation (3) of the main manuscript, the term $H^{(q,c)}$ takes into account this phase factor. More precisely, for each gradient q and coil receiver c, $H^{(q,c)}$ is a diagonal matrix whose diagonal elements consist of the phase factors arising during the acquisition.

In the proposed work, phase-distortions are estimated from the central portion of the k-space data [**?**], [**?**], which is always fully sampled. More specifically, by performing the inverse Fourier transform of a zero-padded version of the fully sampled central part of the k-space, we obtain a complex image whose phase provides the diagonal elements of $H^{(q,c)}$.

In our experiments on synthetic data, a linear phase term randomly generated has been considered in the acquisition of each different phantom slice and in the acquisition of each different diffusion gradient. The k-space shift produced by the linear phase has been limited within the range of ± 2 pixels along each dimensions.

Simulations on real data are performed considering a simplified scenario where no biological motion is assumed, i.e. $H^{(q,c)}$ is equal to identity.

II. Method

A. kq-Space measurement model

While the framework described in Section **??** gives a representation of the signal in q-space, we propose an under-sampled kq-space measurement model. In this context, the measurements $\hat{Y} \in \mathbb{C}^{MC\times K}$, with C and M being the numbers of coil receivers and diffusion gradients respectively, are expressed as follows:

$$\begin{cases} \hat{Y} = \left[\hat{Y}_{q,c}\right]_{\substack{1\leqslant q\leqslant M\\ 1\leqslant c\leqslant C}}, \\ \hat{Y}_{q,c} = \mathcal{A}_{q,c}(X) + \eta_{q,c}, \end{cases} \tag{1}$$

where $\hat{Y}_{q,c} \in \mathbb{C}^{1\times K}$ and $\eta_{q,c} \in \mathbb{C}^{1\times K}$ is a realization of an i.i.d. Gaussian noise. Indeed, while the noise contamination on the magnitude images is characterized by a Rician distribution, the original noise in k-space is Gaussian. The matrix $X \in \mathbb{R}^{(n+2)\times N}$ represents the unknown FOD field of interest and the linear operator $\mathcal{A}_{q,c}$ is given by:

$$\mathcal{A}_{q,c}(X) = \Phi_q X S_0 U^{(c)} H^{(q,c)} F M^{(q)}. \tag{2}$$

The various terms defining the operator $\mathcal{A}_{q,c}$ are described as follows. $\Phi_q \in \mathbb{R}^{1\times(n+2)}$ is the q^{th} row of the dictionary Φ that spans the response of a single fiber oriented along n different directions, to which 2 isotropic compartments are added, to represent the gray matter and the CSF. It is important to note

This work was supported by the UK Engineering and Physical Sciences Research Council (EPSRC, grants EP/M011089/1).

that, in the kq-space context, the complete acquisition of the $\mathbf{s}_0$ image needs to be available for calibration and normalization purposes. In order to implicitly force the FOD coefficients of each voxel to sum up to one, the Fourier coefficients of the s_0 image are intentionally introduced as first row of $\hat{Y}$. However, in the kq-space setting, measurements cannot be normalized, as the diffusion volumes themselves are not accessible but only an incomplete k-space counterpart. Thus, a matrix $S_0 \in \mathbb{R}^{N\times N}$, whose elements along the diagonal correspond to the $\mathbf{s}_0$ image intensities, is then explicitly introduced in the full measurement operator. The acquisition of the diffusion signal from multiple channels is taken into account through the diagonal matrix $U^{(c)} \in \mathbb{C}^{N\times N}$ which contains the sensitivity map of the corresponding channel c. Moreover, motion and magnetic field inhomogeneities generate phase distortions that are accounted in the diagonal matrix $H^{(q,c)} \in \mathbb{C}^{N\times N}$. The multi-channel sensitivities are assumed to be estimated from the $\mathbf{s}_0$ image, while the phase distortion is recovered from the low resolution diffusion images, available if the k-space is fully sampled in the central region. Finally, $F \in \mathbb{C}^{N\times N}$ represents the 2D Fourier matrix and $M^{(q)} \in \mathbb{R}^{N\times K_q}$ is a different realization of a binary mask that under-samples each slice of the acquired volume in a different way. It is important to notice that different realizations of $M^{(q)}$ are considered for each applied diffusion gradient. In addition, $M^{(1)}$ is chosen to be the identity matrix of $\mathbb{R}^N$ since $\hat{Y}_{1,c}$, which stores the Fourier coefficients of the $\mathbf{s}_0$ image, needs to be always fully acquired for normalization and calibration purposes.

As a result, $\hat{Y}_{q,c}$ corresponds to the under-sampled k-space of the diffusion image acquired with gradient q and by the receiver coil c.

B. Minimization problem

C. Algorithm

III. Simulations and Results

IV. Discussion and Conclusion

Acknowledgment

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