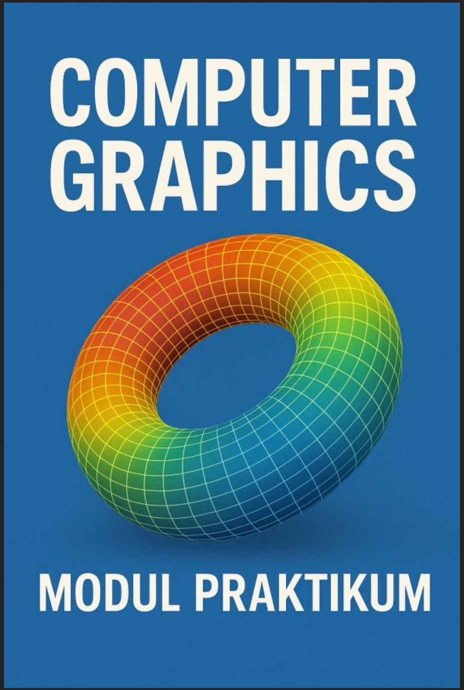
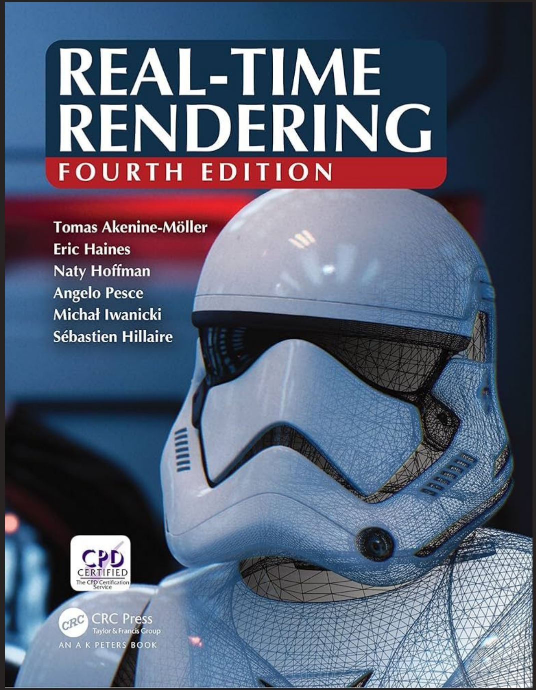
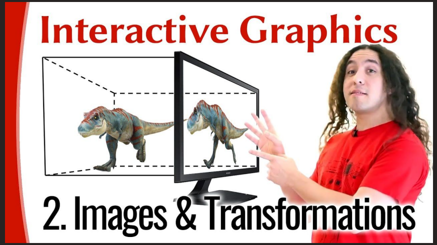
Praktikum Grafika Komputer

Transformasi

Referensi







Interactive Graphics 02 –
Images & Transformations
Cem Yuksel

Vektor & Matriks

Notasi

Notasi column-major form

$$a = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

$$A = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{bmatrix}$$

Notasi row-major form

$$a = \begin{bmatrix} a_x & a_y & a_z \end{bmatrix}$$

$$A = \begin{bmatrix} a_{00} & a_{10} & a_{20} \\ a_{01} & a_{11} & a_{21} \\ a_{02} & a_{12} & a_{22} \end{bmatrix}$$

[The (not-so) Major Confusion of Row-major and Column-major Matrices]

Perkalian dengan Notasi Column-Major

Perkalian matriks dilakukan dari kanan ke kiri

$$Ap = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$$

$$= p_x \begin{bmatrix} a_{00} \\ a_{10} \\ a_{20} \end{bmatrix} + p_y \begin{bmatrix} a_{01} \\ a_{11} \\ a_{21} \end{bmatrix} + p_z \begin{bmatrix} a_{02} \\ a_{12} \\ a_{22} \end{bmatrix}$$

Perkalian dengan Notasi Row-Major

Perkalian matriks dilakukan dari kiri ke kanan

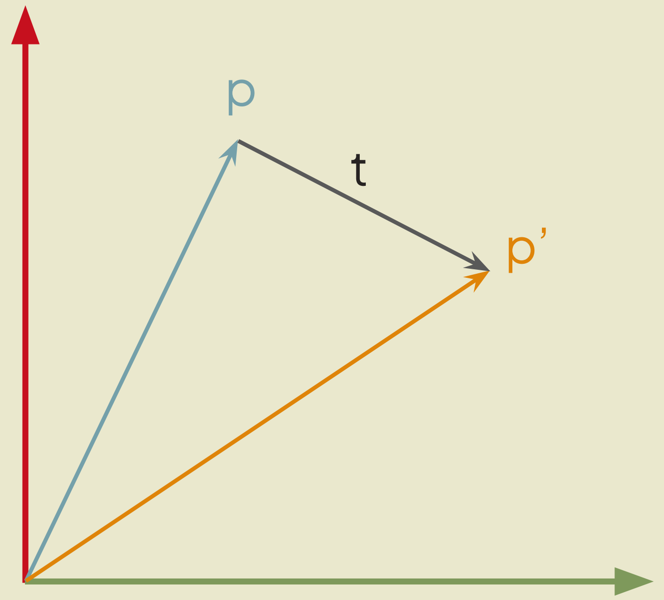
$$pA = \begin{bmatrix} p_x & p_y & p_z \end{bmatrix} \begin{bmatrix} a_{00} & a_{10} & a_{20} \\ a_{01} & a_{11} & a_{21} \\ a_{02} & a_{12} & a_{22} \end{bmatrix}$$

$$\begin{aligned} &= p_x \begin{bmatrix} a_{00} & a_{10} & a_{20} \end{bmatrix} + \\ &\quad p_y \begin{bmatrix} a_{01} & a_{11} & a_{21} \end{bmatrix} + \\ &\quad p_z \begin{bmatrix} a_{02} & a_{12} & a_{22} \end{bmatrix} \end{aligned}$$

Transformasi Affine

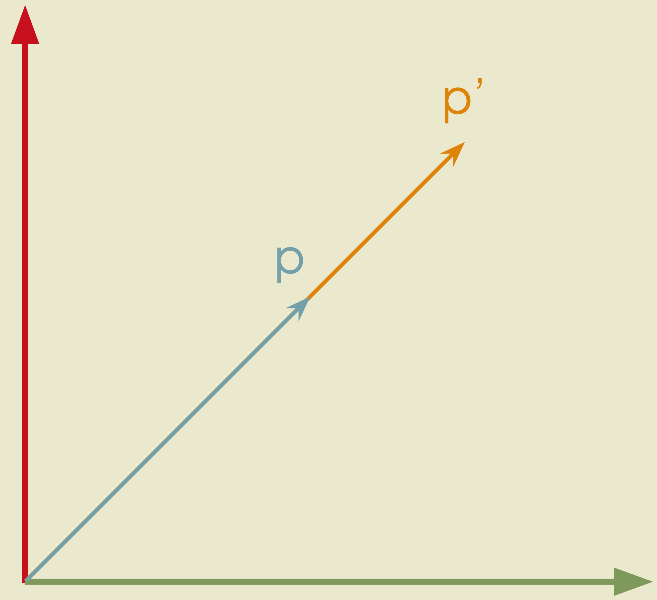
Translasi

$$\begin{bmatrix} p'_x \\ p'_y \end{bmatrix} = \begin{bmatrix} p_x + t_x \\ p_y + t_y \end{bmatrix}$$



Scaling (Seragam)

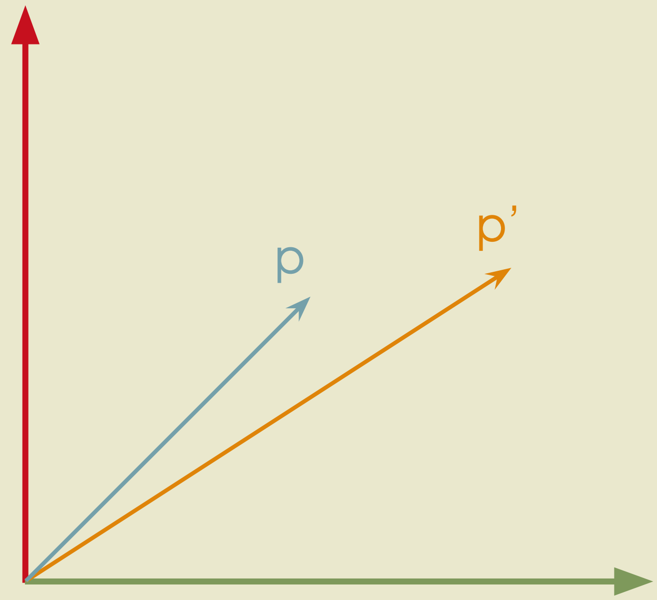
$$\begin{bmatrix} p'_x \\ p'_y \end{bmatrix} = \begin{bmatrix} sp_x \\ sp_y \end{bmatrix}$$



Scaling (Tidak Seragam)

$$\begin{bmatrix} p'_x \\ p'_y \end{bmatrix} = \begin{bmatrix} s_x p_x \\ s_y p_y \end{bmatrix}$$

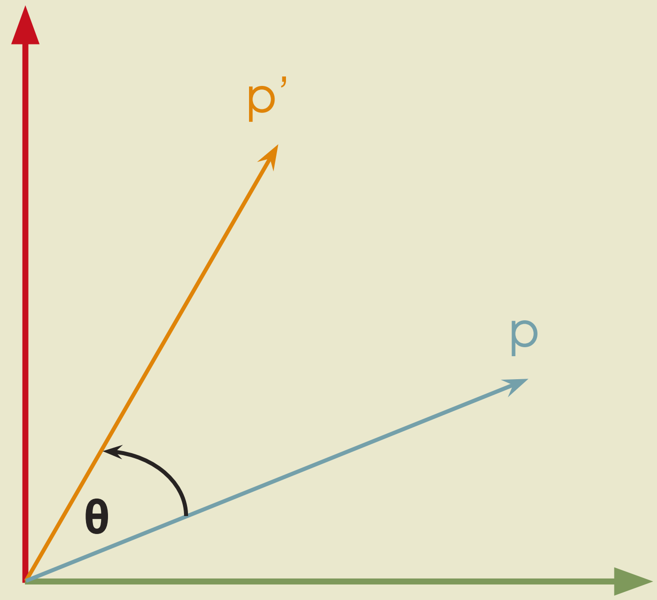
$$= \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$



Rotasi

$$\begin{bmatrix} p'_x \\ p'_y \end{bmatrix} = p_x \begin{bmatrix} cos\theta \\ sin\theta \end{bmatrix} + p_y \begin{bmatrix} -sin\theta \\ cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$



Matriks Komposisi dan Notasi Homogen

Matriks Komposisi

Sebuah matriks komposisi **M** bisa dibuat dari perkalian matriks-matriks transformasi

$$
\begin{aligned}
p' &= (RSRSRS)p \\
&= R(S(R(S(R(Sp))))) \\
&= Mp
\end{aligned}
$$

$$M \neq SRSRSR$$

[Singular Value Decomposition (SVD)]

Matriks Komposisi Dengan Translasi?

Contoh pembuatan matriks komposisi dengan translasi

$$p' = M_3(M_2(M_1(p + t)))$$

Ribet!

Solusi: operasi perkalian

$$\begin{bmatrix} p_x \\ p_y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix}$$

Translasi dengan Notasi Homogen

Translasi vektor 2D bisa direpresentasikan dengan matriks 3×3

$$\begin{bmatrix} p'_x \\ p'_y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix}$$

Lebih simple!

Matriks Komposisi Dengan Notasi Homogen

Contoh pembuatan matriks komposisi dengan translasi

$$p' = M_3(M_2(M_1(p + t)))$$

Sebelumnya

$$\begin{aligned} p' &= M_3(M_2(M_1(Tp))) \\ &= (M_3 M_2 M_1 T)p \\ &= Mp \end{aligned}$$

Dengan notasi homogen

Matriks Transformasi dengan Notasi Homogen

Translasi

2D

$$T = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

3D

$$T = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matriks Transformasi dengan Notasi Homogen

Scaling

2D

$$S = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{S'} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{s} \end{bmatrix}$$

$$S = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3D

$$S = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad S' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{s} \end{bmatrix}$$

$$S = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matriks Transformasi dengan Notasi Homogen

Rotasi

2D

$$R = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$$

3D

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\theta & -sin\theta & 0 \\ 0 & sin\theta & cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ sumbu-x

$$R_y = \begin{bmatrix} cos\theta & 0 & -sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ sin\theta & 0 & cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ sumbu-y

$$R_z = \begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ sin\theta & cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ sumbu-z

Transformasi Rigid-Body

Transformasi Rigid-Body

Transformasi yang mempertahankan jarak antara titik, sudut, dan *handedness*

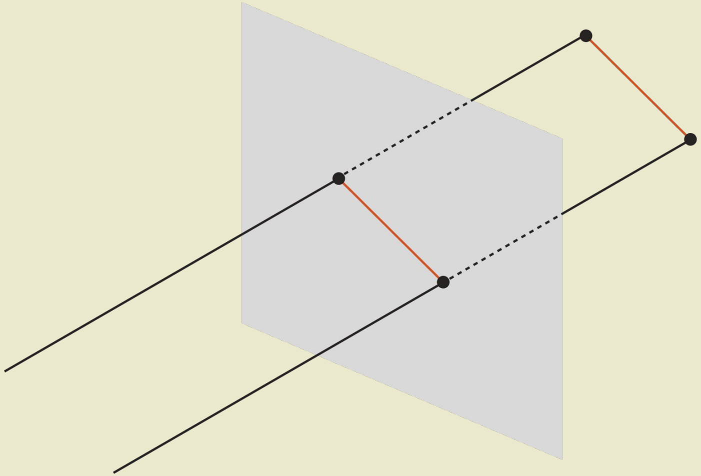
$$X = T(t)R = \begin{bmatrix} r_{00} & r_{01} & r_{02} & t_x \\ r_{10} & r_{11} & r_{12} & t_y \\ r_{20} & r_{21} & r_{22} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix}$$

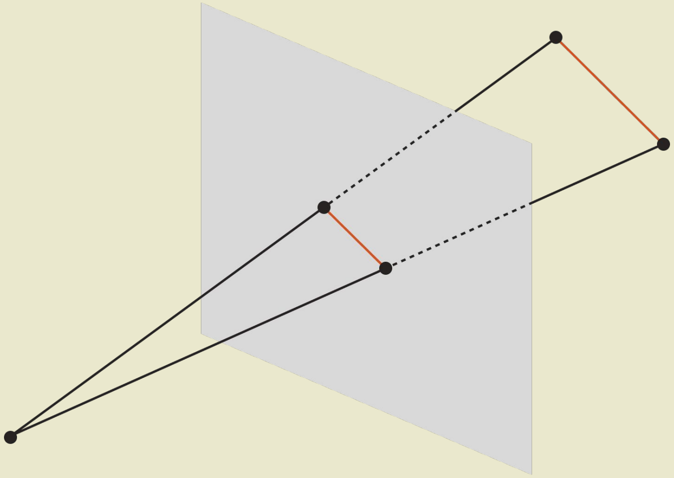
Proyeksi

Jenis-jenis proyeksi

Ortogonal



Perspektif

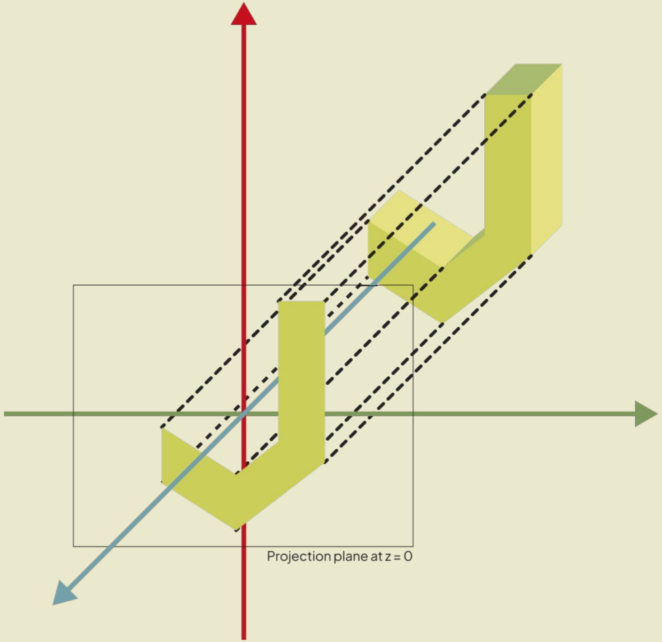


Konsep Proyeksi Ortogonal

Matriks proyeksi ortogonal pada bidang z = 0

$$P_o = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$|P_o| = 0$ → Tidak invertible

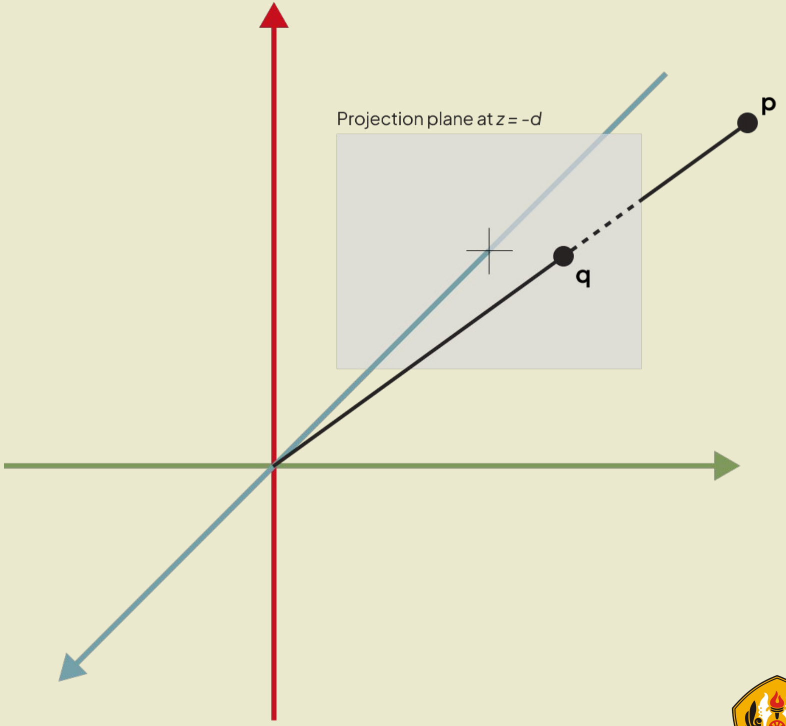


Konsep Proyeksi Perspektif

Matriks proyeksi perspektif pada bidang z = -d

$$P_p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{d} & 0 \end{bmatrix}$$

$|P_p| = 0$ → Tidak invertible

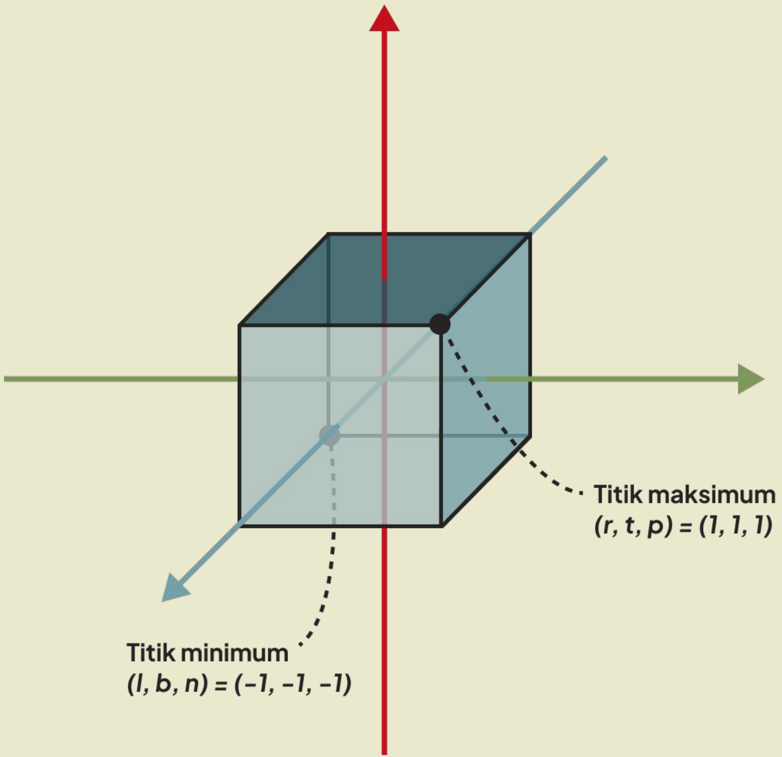


Ortogonal-Ruang

Matriks proyeksi ortogonal dari kubus satuan

$$P_o = S(s)T(t) = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & \frac{2}{f-n} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & -\frac{r+l}{2} \\ 0 & 0 & 0 & -\frac{t+b}{2} \\ 0 & 0 & 0 & -\frac{f+n}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

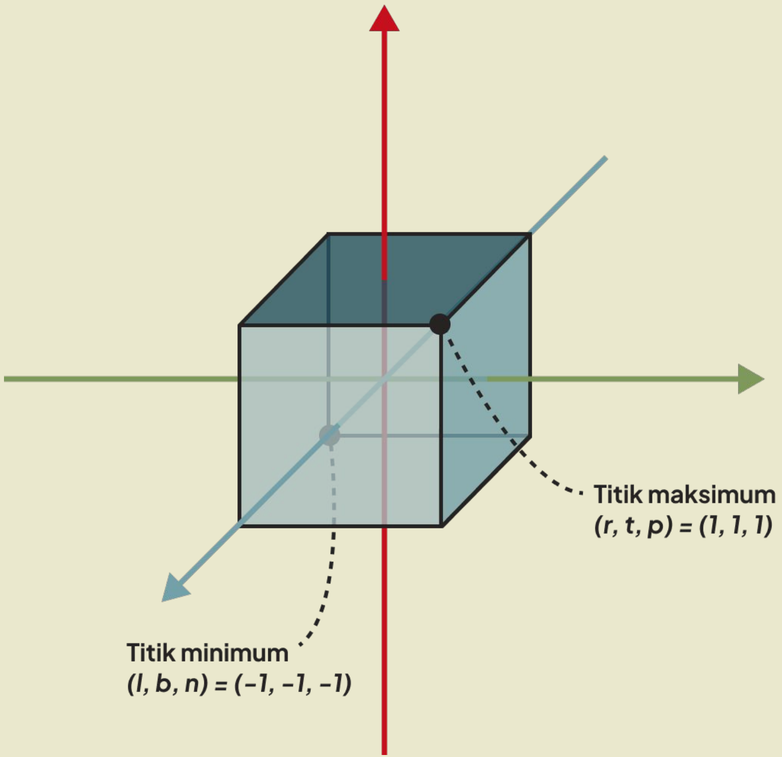


Perspektif-Ruang

Matriks transformasi perspektif yang memetakan titik ke dalam kubus satuan

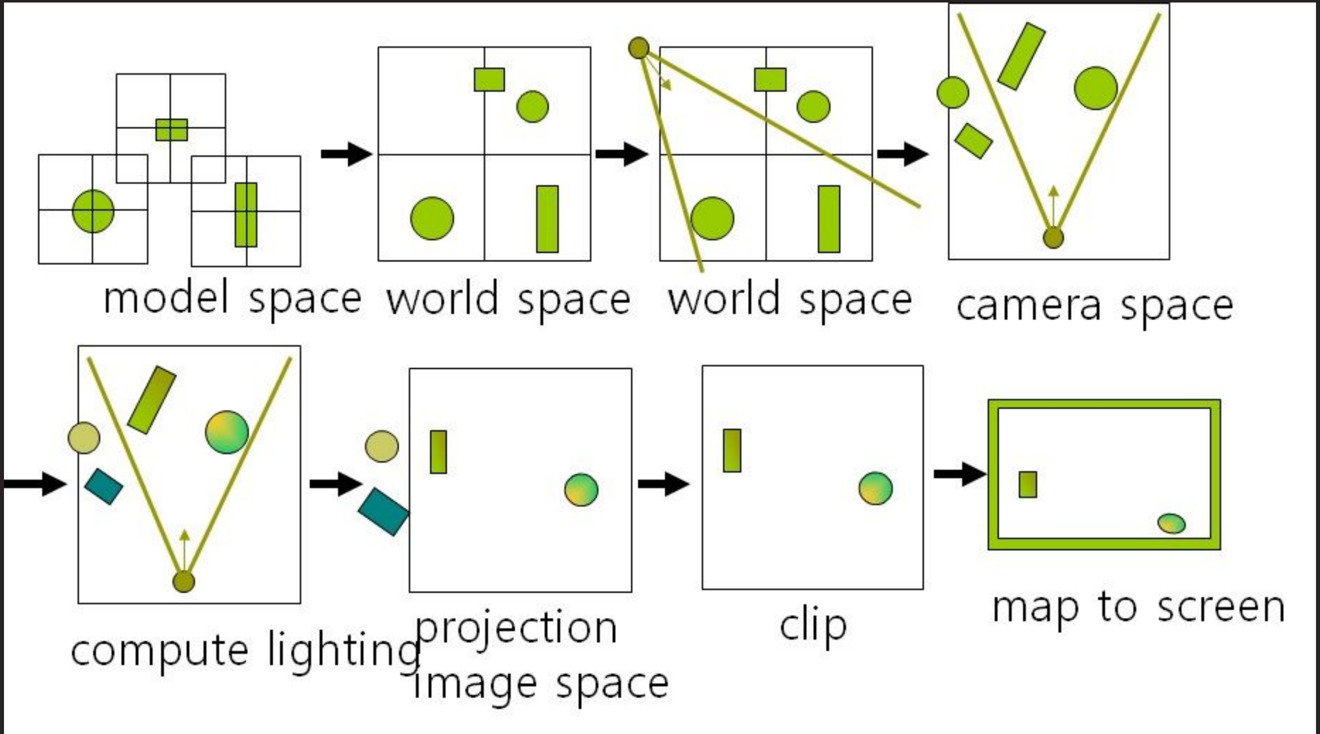
$$P_o = S(s)T(t) = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & \frac{2}{f-n} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & -\frac{r+l}{2} \\ 0 & 0 & 0 & -\frac{t+b}{2} \\ 0 & 0 & 0 & -\frac{f+n}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Sistem Koordinat

Objek → Layar

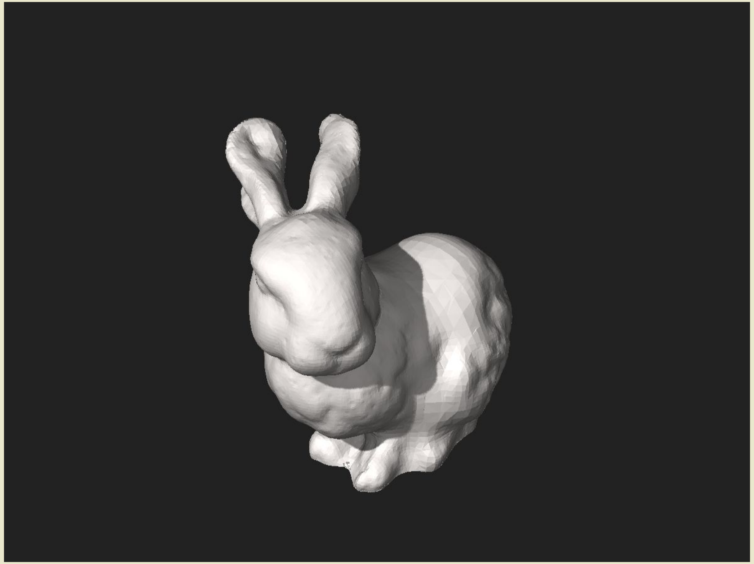

model space
world space
world space
camera space
compute lighting
projection
image space
clip
map to screen

Tugas </3

Tugas Pertemuan 3

Jelaskan proses rendering titik (x, y, z) suatu objek dalam world-space ke dalam viewport coordinates beserta matriks-matriks transformasinya

*penjelasannya harus sekreatif mungkin!



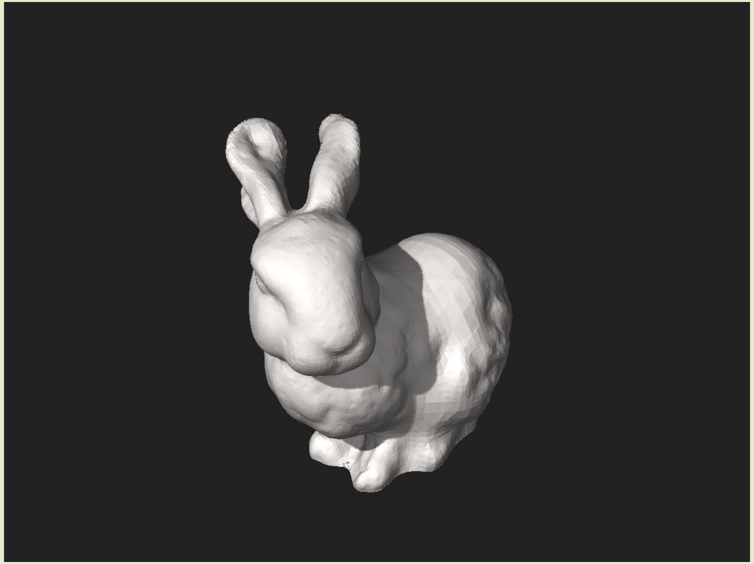
Stanford Bunny

Teknis Pengumpulan Tugas

Medium pengerjaan tugas dibebaskan asalkan penjelasan yang diberikan jelas

Tugas dikumpulkan dalam bentuk format file PDF dengan penamaan file: <NPM>_Tugas3.pdf

Kode Classroom: mwv472kg



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Terima Kasih

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