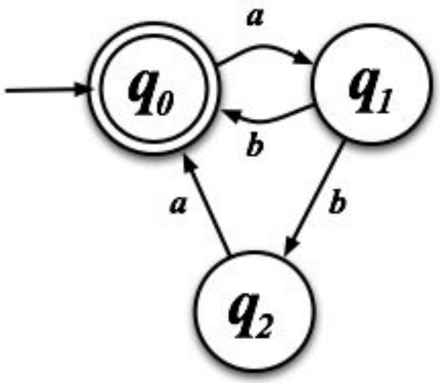
NFAs and DFAs

Sipser 1.2 (pages 47-63)

Last time…

NFA

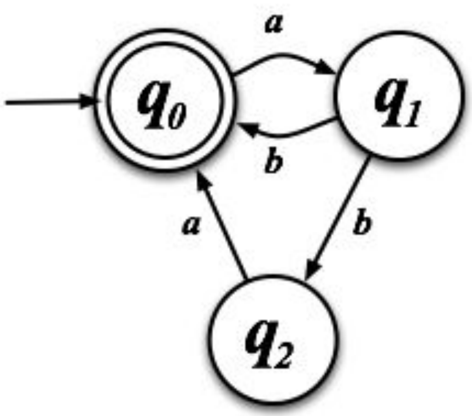
- A ***nondeterministic finite automaton (NFA)*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where
  - Q is a finite set called the **states**
  - Σ is a finite set called the **alphabet**
  - $\delta: Q \times \Sigma_\varepsilon \rightarrow P(Q)$ is the **transition function**
  - $q_0 \in Q$ is the **start state**
  - $F \subseteq Q$ is a set of **accept states**
- In-class exercise:



NFA computation

- Let $N=(Q, \Sigma, \delta, q_0, F)$ be a NFA and let w be a string over the alphabet Σ
- Then N **accepts** w if
  - w can be written as $w_1w_2w_3\ldots w_m$ with each $w_i \in \Sigma\varepsilon$ and
  - There exists a sequence of states $s_0, s_1, s_2, \ldots, s_m$ exists in Q with the following conditions:

1. $s_0 = q_0$
2. $s_{i+1} \in \delta(s_i, w_{i+1})$ for $i = 0, \ldots, m-1$
3. $s_m \in F$



One last operation

Kleene star operation

- Let A be a language.
- The Kleene star operation is
  $A^* = \{x_1x_2...x_k \mid k \geq 0$ and each $x_i \in A\}$

Exercise

- $A = \{w \mid w$ is a string of 0s and 1s containing an odd number of 1s$\}$
- $B = \{w \mid w$ is a string of 0s and 1s containing an even number of 1s$\}$
- $C = \{0,1\}$

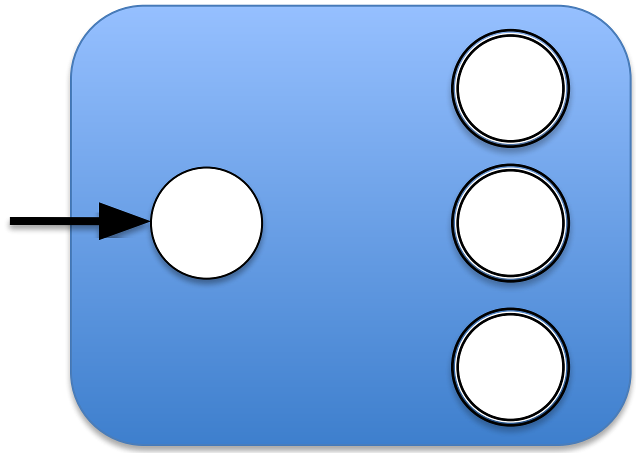
- What are A^*, B^*, and C^*?

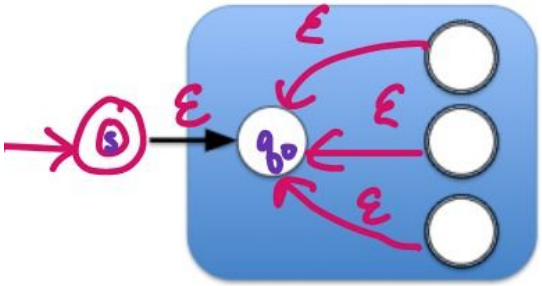
klay'nee

Kleene pronounced his last name klay'nee. His son, Ken Kleene, wrote: "As far as I am aware this pronunciation is incorrect in all known languages. I believe that this novel pronunciation was invented by my father."
- From http://visualbasic.about.com/od/usevb6/a/RegExVB6.htm

Kleene star

- Theorem: The class of languages recognized by NFAs is closed under the Kleene star operation.





<u>Proof</u> Let A be a language recognized by an NFA $N = (Q, \Sigma, \delta, q_0, F)$. We construct the NFA $N' = (Q \cup \{s\}, \Sigma, \delta', s, F \cup \{s\})$, where

$$\delta'(q,\sigma) = \begin{cases} \{q_0\} & \text{if } q = s \text{ \& } \sigma = \varepsilon \\ \emptyset & \text{if } q = s \text{ \& } \sigma \neq \varepsilon \\ \{q_0\} \cup \delta(q,\sigma) & \text{if } q \in F \text{ \& } \sigma = \varepsilon \\ \delta(q,\sigma) & \text{otherwise} \end{cases}$$

$q \neq s$ & ($q \in F, \sigma \neq \varepsilon$ or $q \notin F$

Since $L(N') = A^*$, the class of lang. recog. by NFAs is closed under the star operation. $\square$

Q.E.D.

If only…

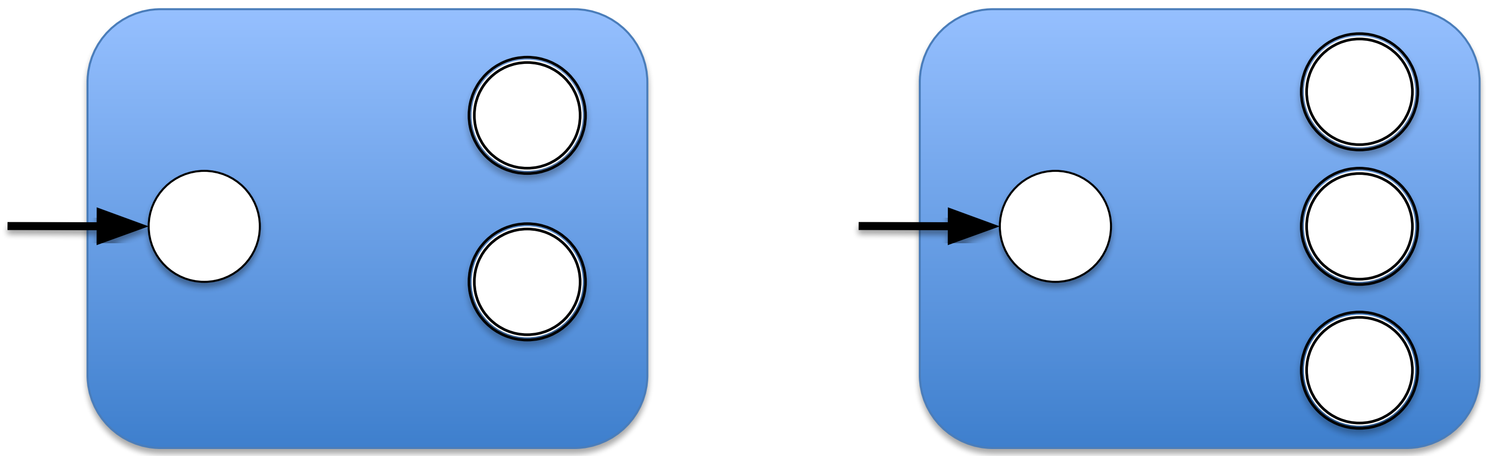
… somebody would prove that the
class of languages recognized by NFAs
and
the class of languages recognized by
DFAs
were equal…

Then…

- We'd have:
  - The class of regular languages is closed under:
    - Concatenation
    - Kleene star

And...

a cute proof for closure under union!



We can be that somebody!

- Theorem: A language is regular if and only if there exists an NFA that recognizes it.

- Proof:

  ($\Rightarrow$)

  Let A be a regular language…

Then there exists a DFA M

- M is a 5-tuple $(Q, \Sigma, \delta_M, q_0, F)$, where
  - Q is a finite set called the **states**
  - Σ is a finite set called the **alphabet**
  - $\delta: Q \times \Sigma \rightarrow Q$ is the **transition function**
  - $q_0 \in Q$ is the **start state**
  - $F \subseteq Q$ is a set of **accept states**

We need to show there exists an NFA N

- N is a 5-tuple $(Q, \Sigma, \delta_N, q_0, F)$, where
  - Q is a finite set called the **states**
  - Σ is a finite set called the **alphabet**
  - $\delta: Q \times \Sigma\varepsilon \rightarrow P(Q)$ is the **transition function**
  - $q_0 \in Q$ is the **start state**
  - $F \subseteq Q$ is a set of **accept states**

- **Can we define N from M?**

Now the other way!

- Theorem: A language is regular if and only if there exists an NFA that recognizes it.

- Proof:

  $(\Leftarrow)$

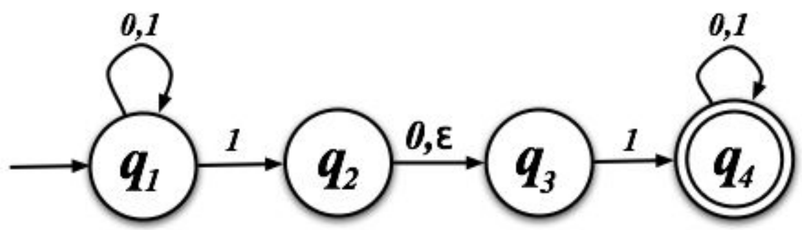
  Let A be a language accepted by NFA $N = (Q, \Sigma, \delta, q_0, F)$

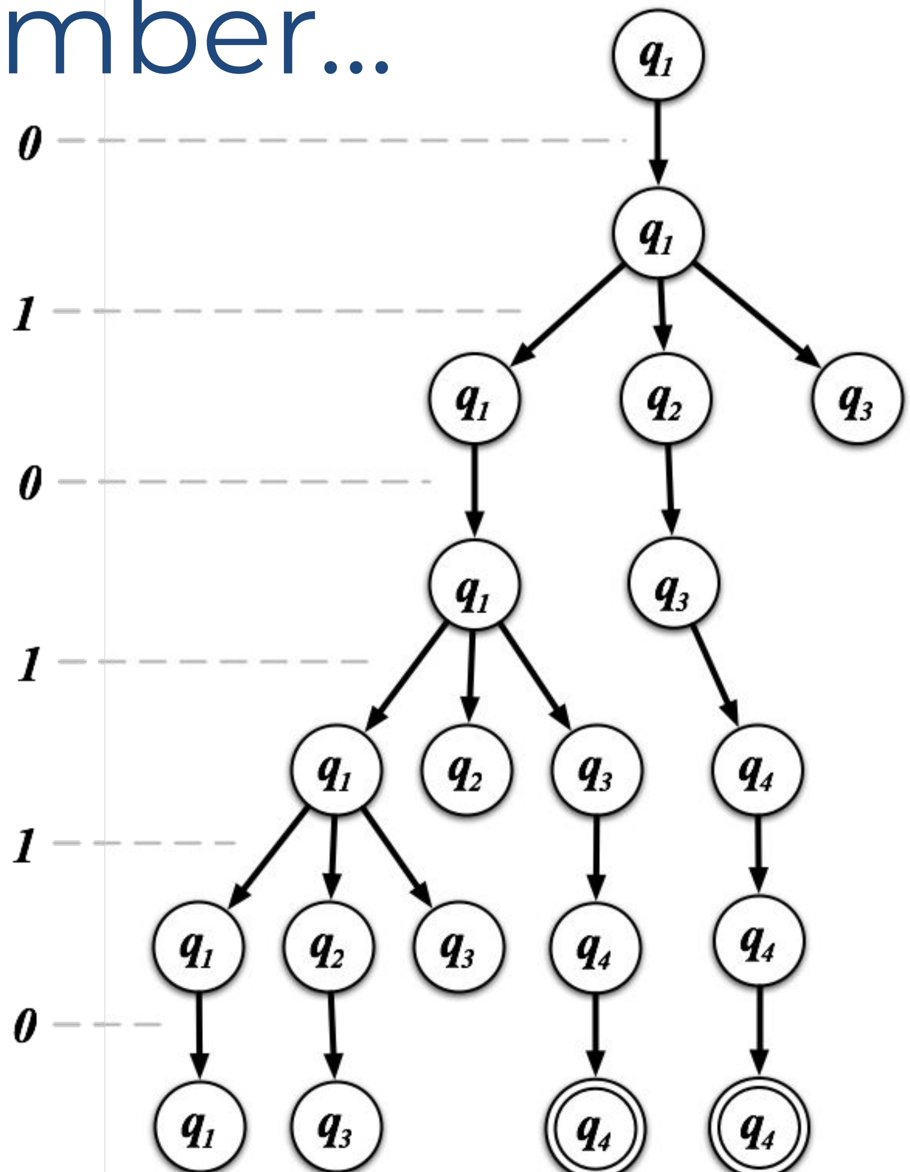
Equivalent machines

- Definition:
  Two machines are equivalent if they recognize the same language

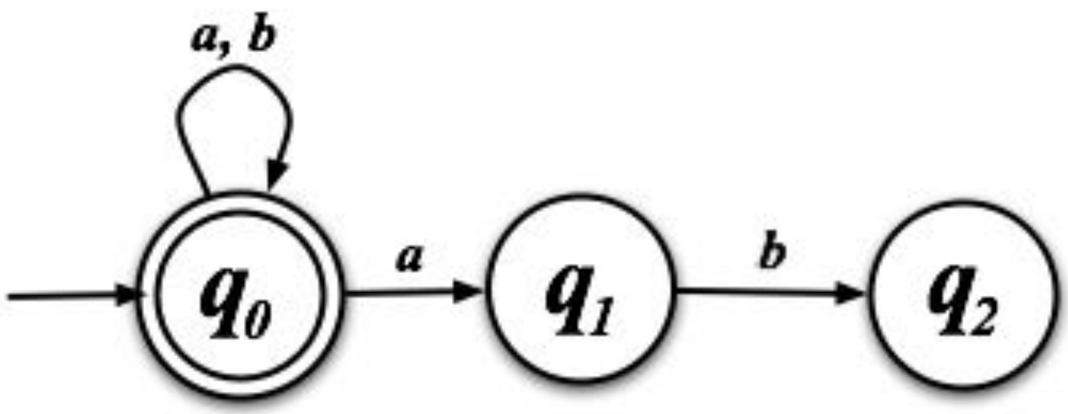
- Let's prove…
  Theorem:
  Every NFA has an equivalent DFA.

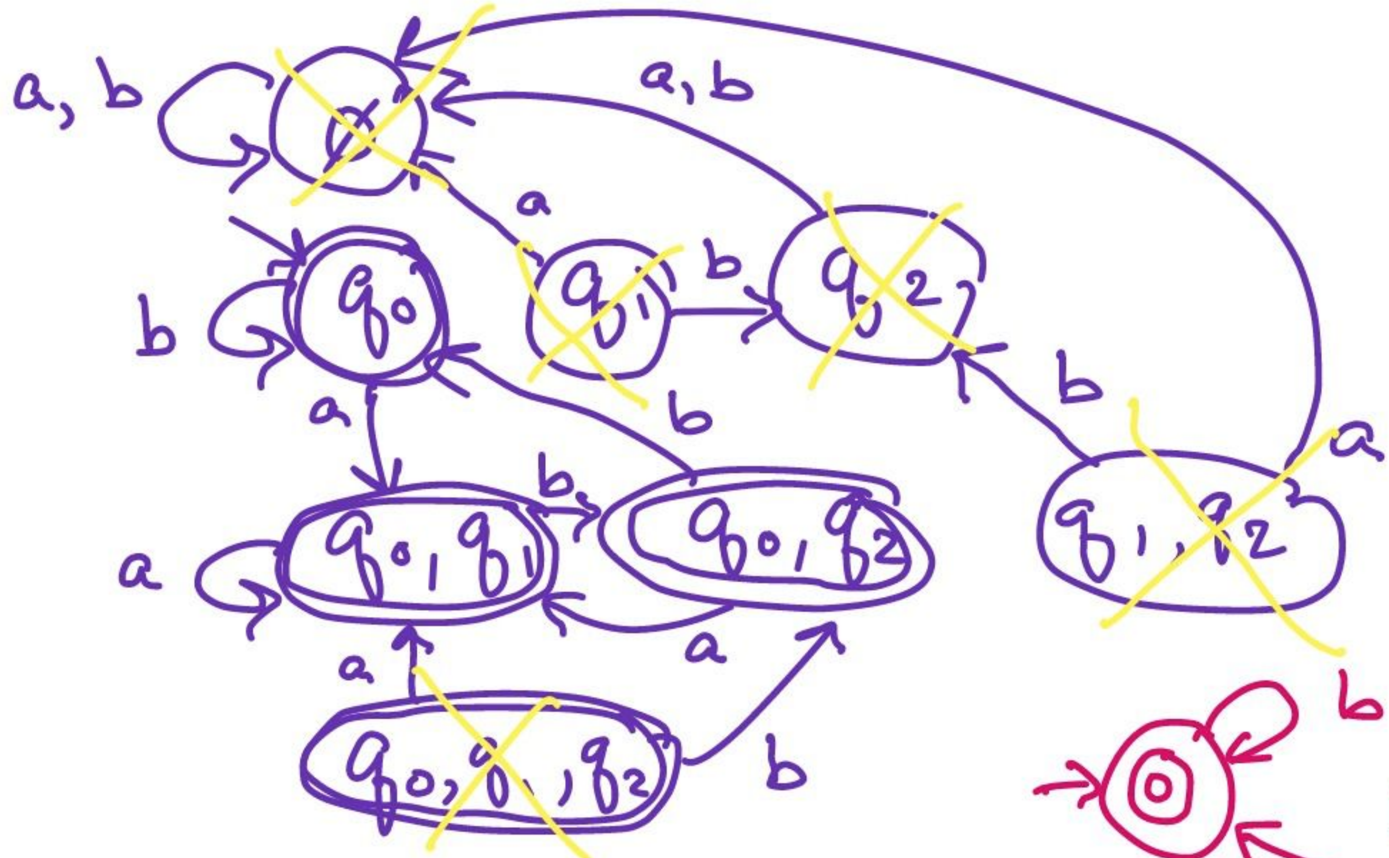
Remember…

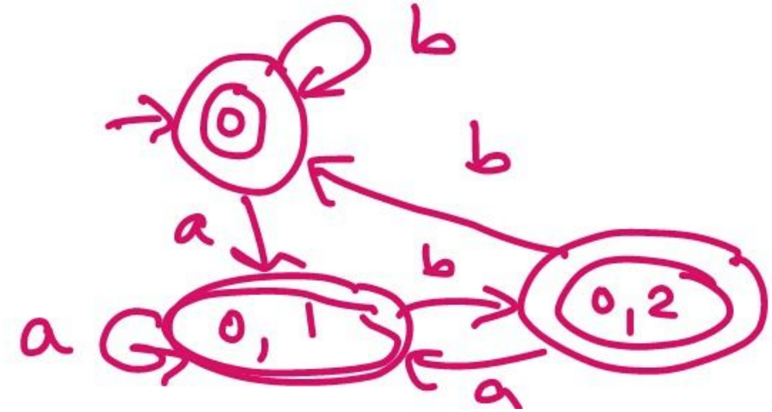




A simpler example







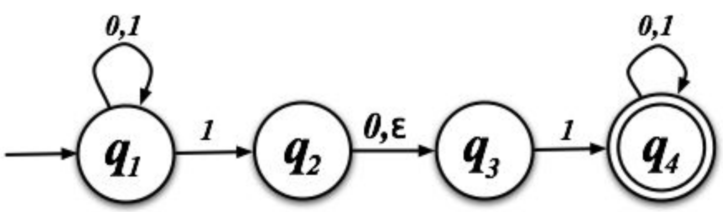
$$|P(Q)| = 2^{|Q|}$$

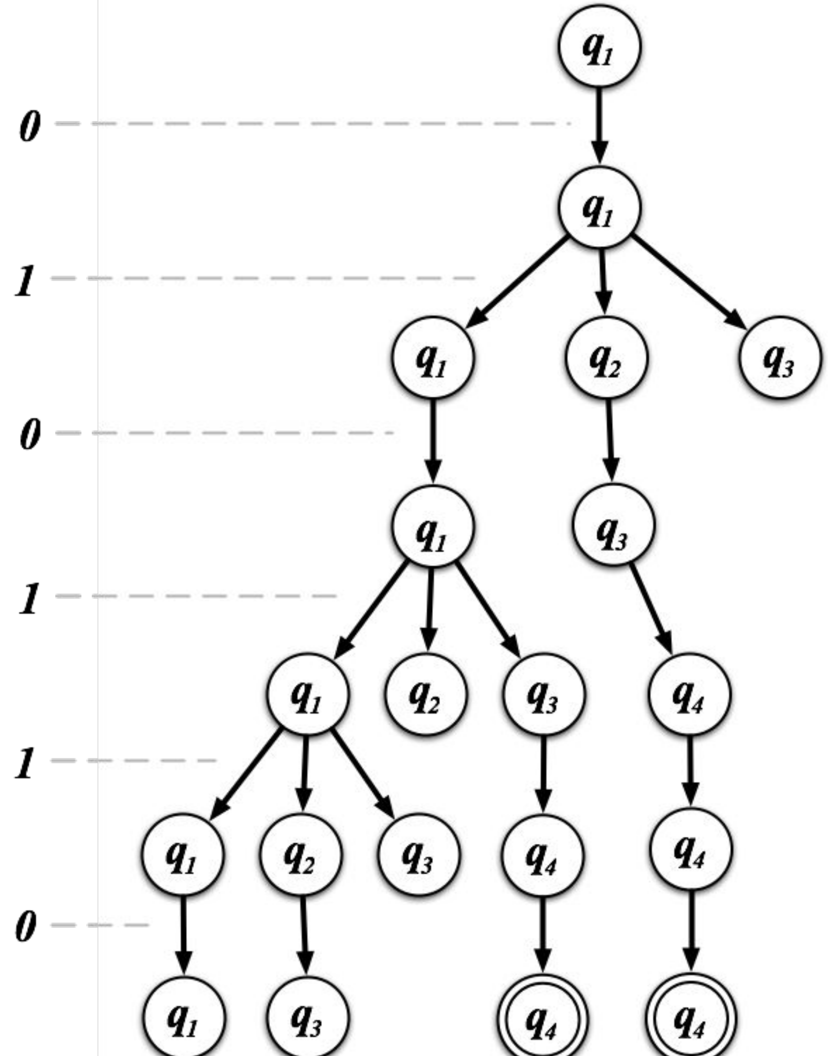
Removing choice

Proof:

- Let A be a language accepted by NFA $N = (Q, \Sigma, \delta, q_0, F)$
- We construct a DFA $M=(Q', \Sigma, \delta', q_0', F')$ recognizing A
  - $Q' = P(Q)$
  - For $R \in Q'$ and $a \in \Sigma$, define $\delta'(R,a) = \bigcup_{r \in R} \delta(r,a)$
  - $q_0' = \{q_0\}$
  - $F' = \{ R \in Q' \mid R \text{ contains an accept state from } F\}$

Okay, but what about ε arrows?





Modifying our construction

Proof:

- Let A be a language accepted by NFA
  $N = (Q, \Sigma, \delta, q_0, F)$
- We construct a DFA $M=(Q', \Sigma, \delta', q_0', F')$ recognizing A
  - $Q' = P(Q)$
  - For $R \in Q'$ and $a \in \Sigma$,
    define $\delta'(R,a) = \bigcup_{r \in R} E(\delta(r,a))$

    where $E(R) = \{q \mid q$ can be reached from R along 0 or more ε arrows$\}$
  - $q_0' = E(\{q_0\})$
  - $F' = \{ R \in Q' \mid R \text{ contains an accept state from } F\}$

Modifying our construction

Proof:

- Let A be a language accepted by NFA $N = (Q, \Sigma, \delta, q_0, F)$
- We construct a DFA $M=(Q', \Sigma, \delta', q_0', F')$ recognizing A
  - $Q' = P(Q)$
  - For $R \in Q'$ and $a \in \Sigma$,
    define $\delta'(R,a) = \bigcup_{r \in R} E(\delta(r,a))$
    where $E(R) = \{q \mid q$ can be reached from R along 0 or more ε arrows$\}$
  - $q_0' = E(\{q_0\})$
  - $F' = \{ R \in Q' \mid R \textit{ contains an accept state from } F\}$