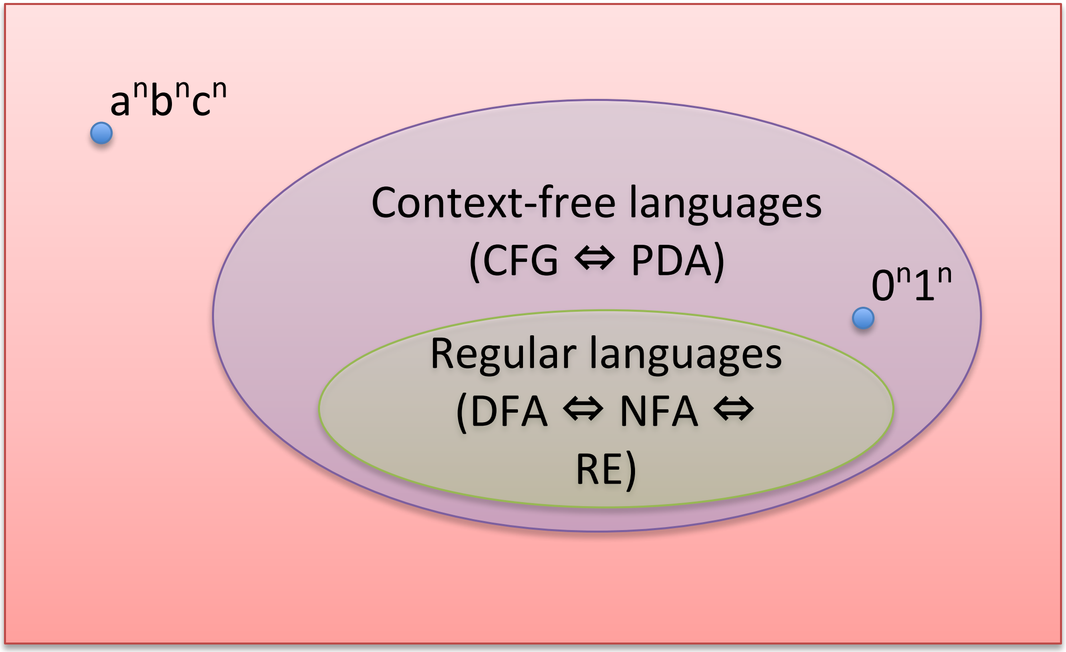
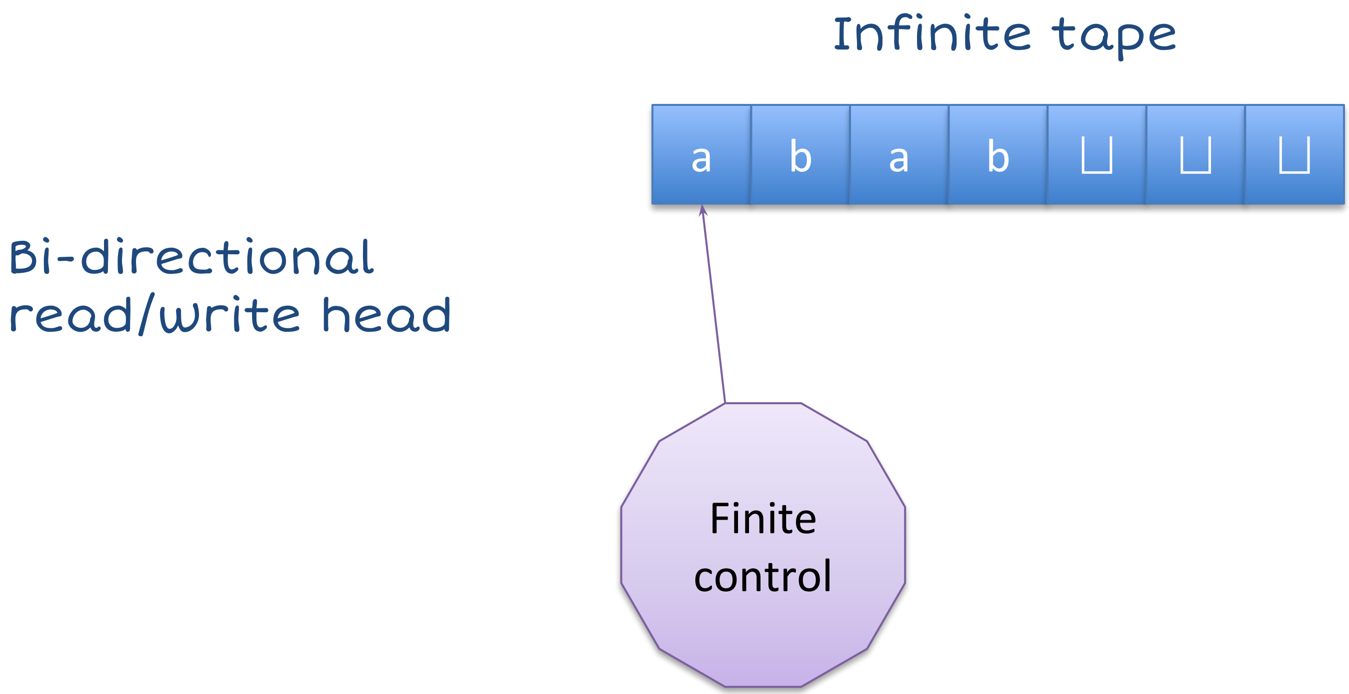
Turing machines

Sipser 3.1
(pages 137-144)

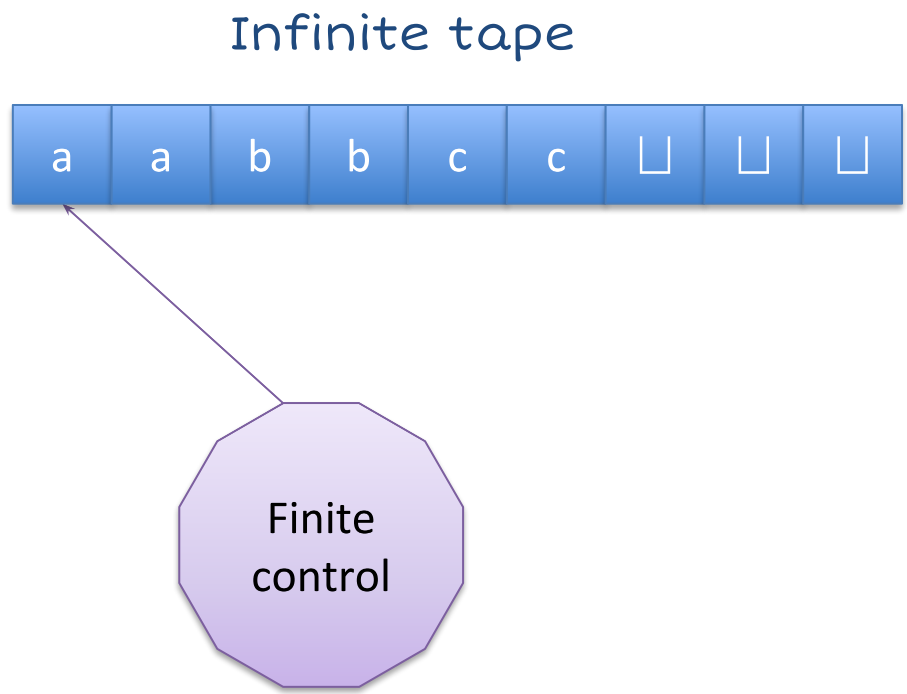
Chomsky hierarchy



Introducing… Turing machines



Recognizing $\{a^n b^n c^n \mid n \geq 0\}$



Formally…

- A Turing machine is a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$, where
  - Q is a finite set called the states
  - Σ is a finite set *not containing the blank symbol* $\sqcup$ called the input alphabet
  - Γ is a finite set called the tape alphabet with $\sqcup \in \Gamma$ and $\Sigma \subseteq \Gamma$
  - $\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L,R\}$
  - $q_0 \in Q$ is the start state
  - $q_{accept} \in Q$ is the accept state
  - $q_{reject} \in Q$ is the reject state

Configurations

- A *configuration* is
  - Current state
  - Current tape contents
  - Current head location
- $u\ q\ v$ means
  - Current state is q
  - Current tape contents is uv
  - Current head points at first symbol of v
- Example
  - $\hat{a}aq_1bbcc$
  - In state q_1
  - Tape contents are $\hat{a}abbcc$
  - Tape head is on first b

Yields

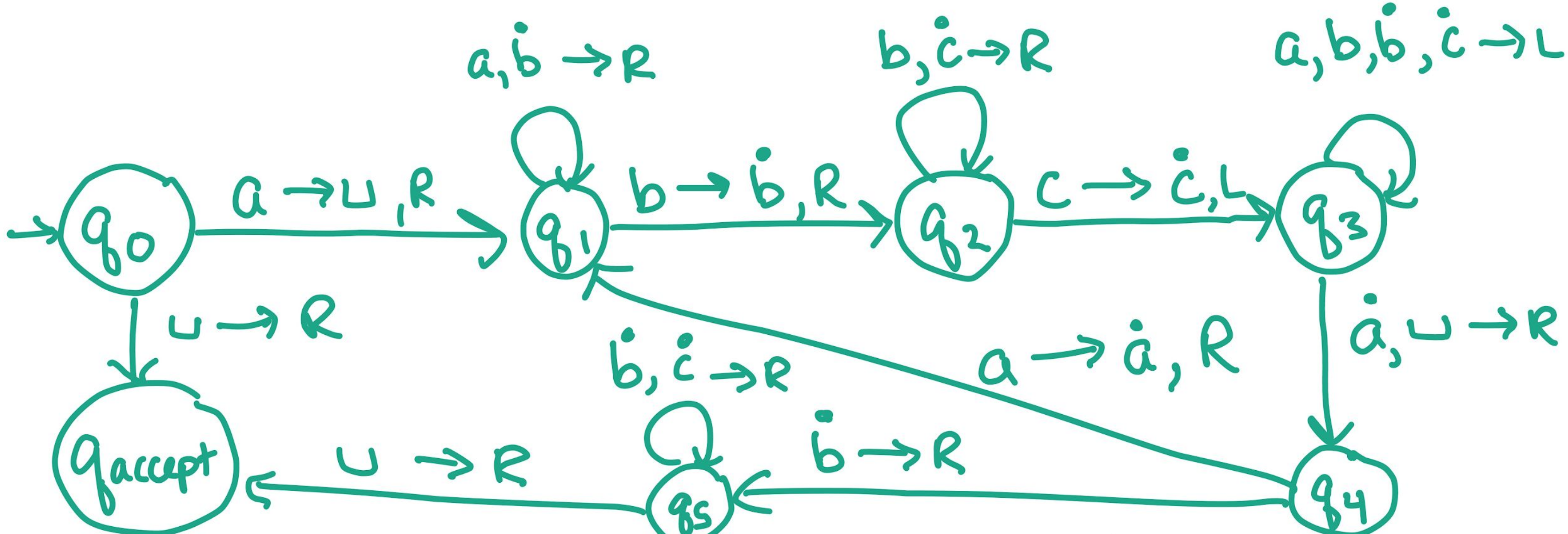
- A configuration C_1 yields configuration C_2 if the Turing machine can legally go from C_1 to C_2 in a single step
- $\hat{a}aq_1bbcc$ yields $\hat{a}a\hat{b}q_2bcc$
- Written $\hat{a}aq_1bbcc \vdash \hat{a}a\hat{b}q_2bcc$

Turing-recognizable languages

- A Turing machine accepts input w if a sequence of configurations $C_1, C_2, \ldots, C_k$ exists where
  1. C_1 is the start configuration of M on input w
  2. Each C_i yields C_{i+1}
  3. C_k is an accepting configuration

- Defn 3.5: A language is Turing-recognizable if it is accepted by some Turing machine.

$\{a^n b^n c^n \mid n \geq 0\}$ is Turing-recognizable



$u: \varepsilon, v: abc$

$u: \sqcup, v: bc$

$u: \sqcup\dot{b}, v: c$

$u: \sqcup, v: \dot{b}\dot{c}$

$u: \varepsilon, v: \sqcup\dot{b}\dot{c}$

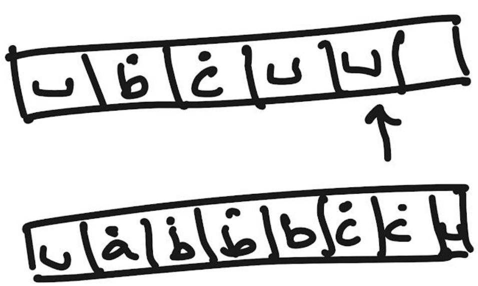
$q_0\, abc$

$\vdash \sqcup\, q_1\, bc$

$\vdash \sqcup\, \dot{b}\, q_2\, c$

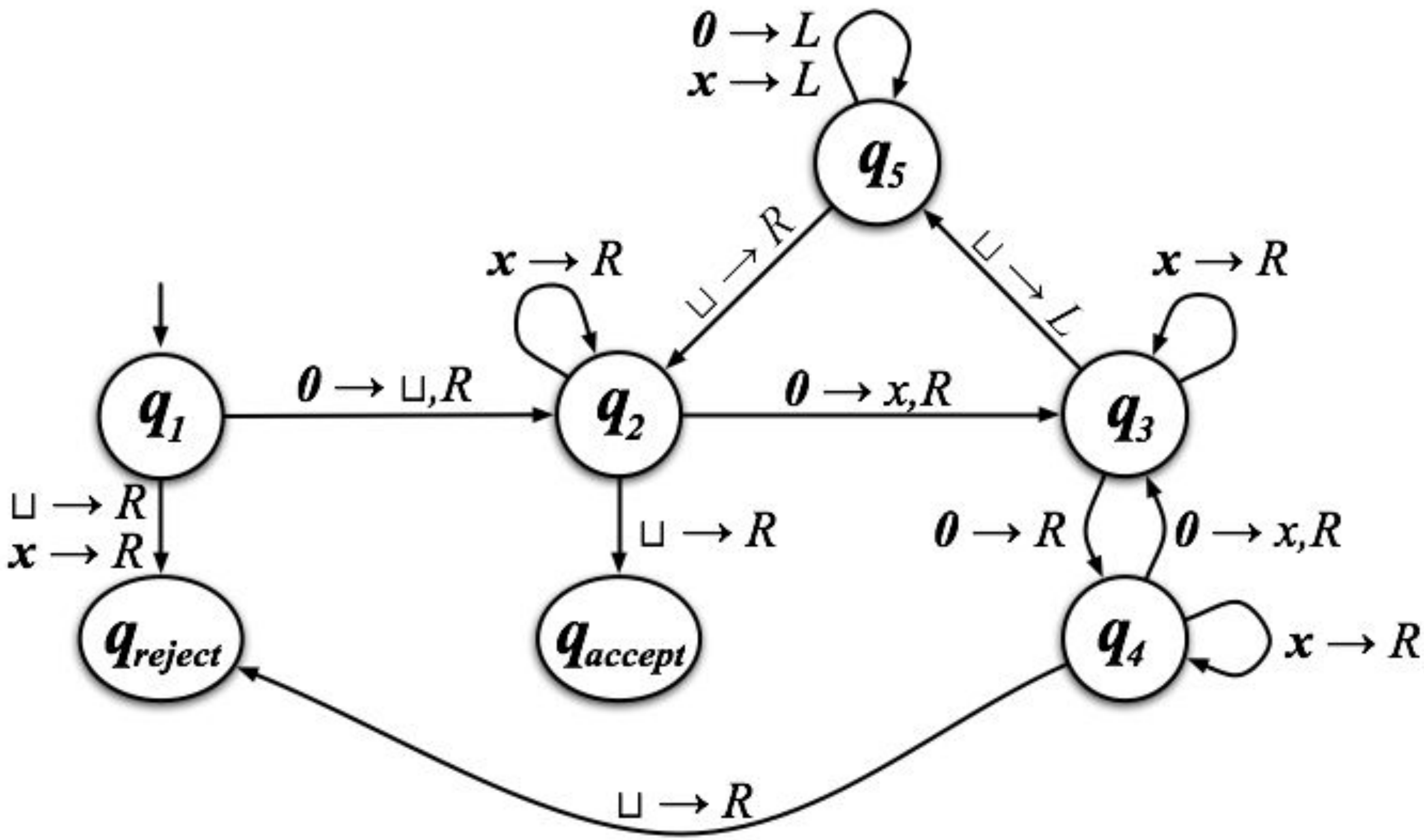
$\vdash \sqcup\, q_3\, \dot{b}\, \dot{c}$

$\vdash q_3\, \sqcup\, \dot{b}\, \dot{c}$



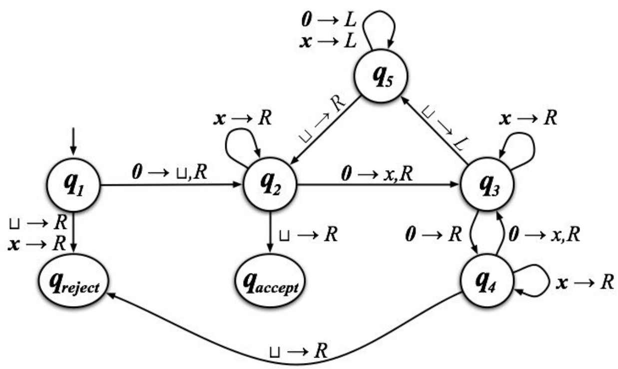
Recognizing $\{0^{2^n} | n \geq 0\}$.

Describing Turing machines



Describing Turing machines

- *Formal:*



- *Implementation:*
  - M = "On input string w.
    1. Sweep across tape, crossing off every other 0.
    2. If tape contained one 0, accept.
    3. Else, if number of 0's is odd, reject.
    4. Return head to left-hand end of tape.
    5. Go to step 1.“
- *High-level:*

  repeat until n=1

  exit if n mod 2 != 0

  set n = n / 2