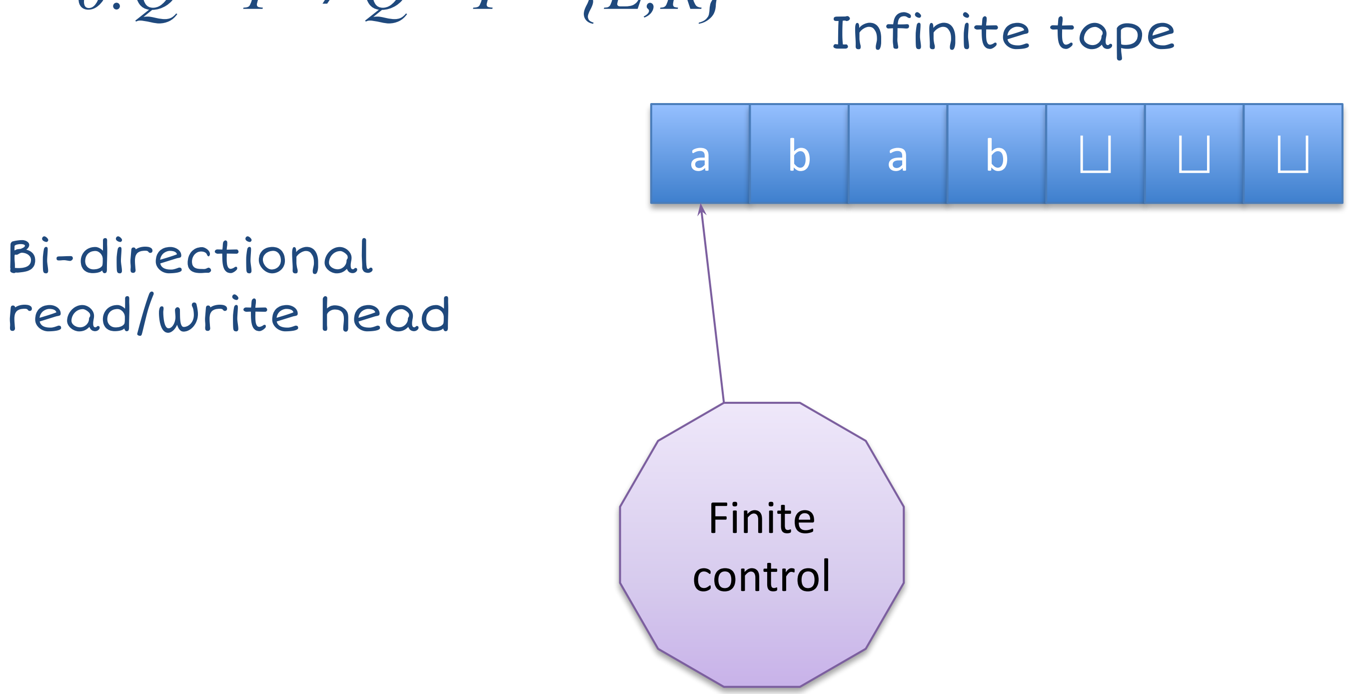
# More Turing Machines

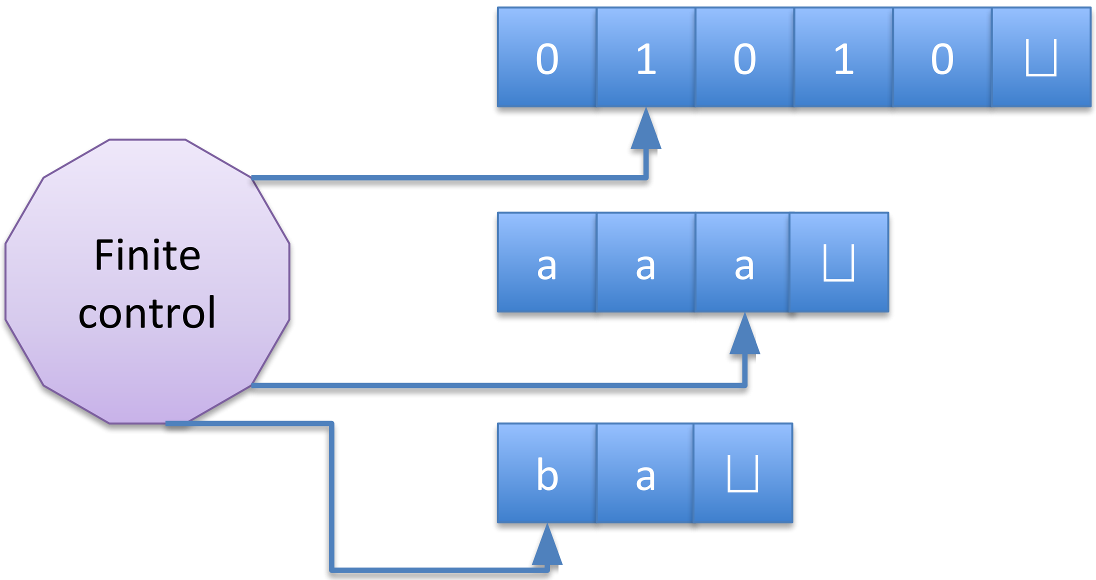
Sipser 3.2 (pages 148-154)

# Turing machines

- A Turing machine is a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$
  - $\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L,R\}$



# Multitape Turing Machines




- Formally, we need only change the transition function to

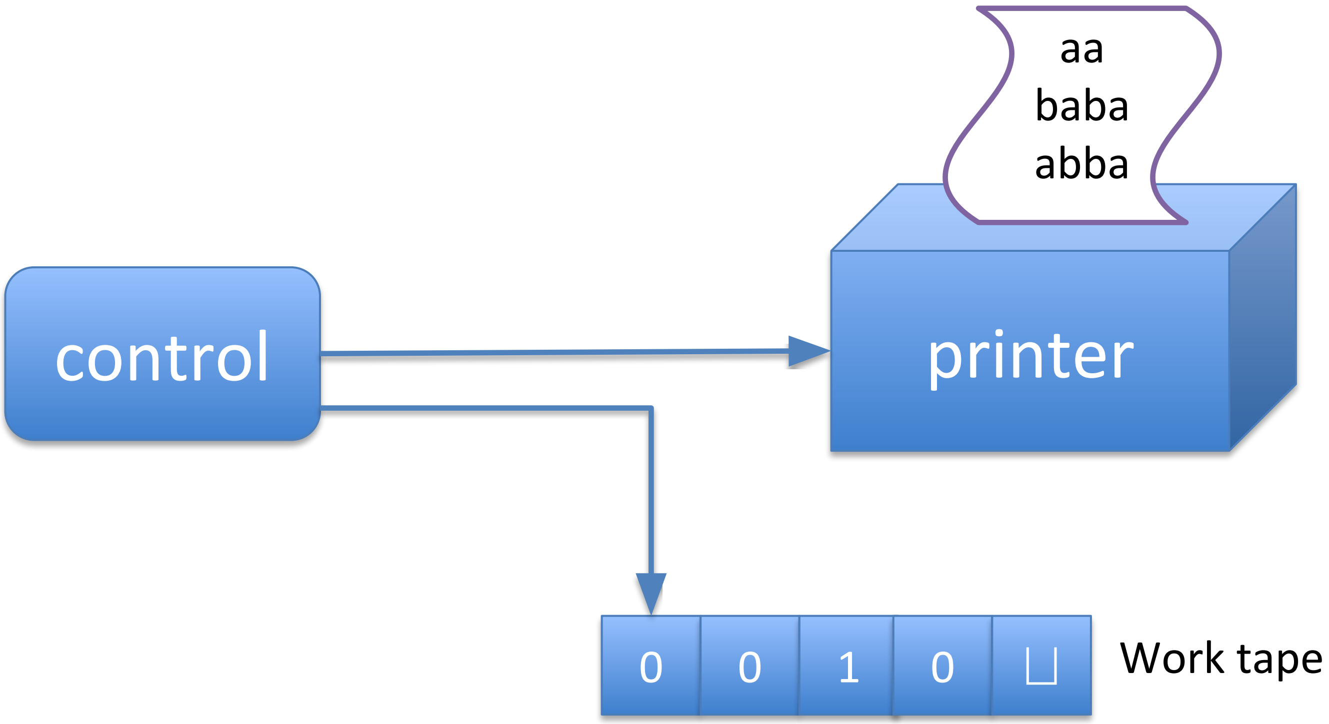
$$\delta: Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L,R\}^k$$

# Nondeterministic Turing machines

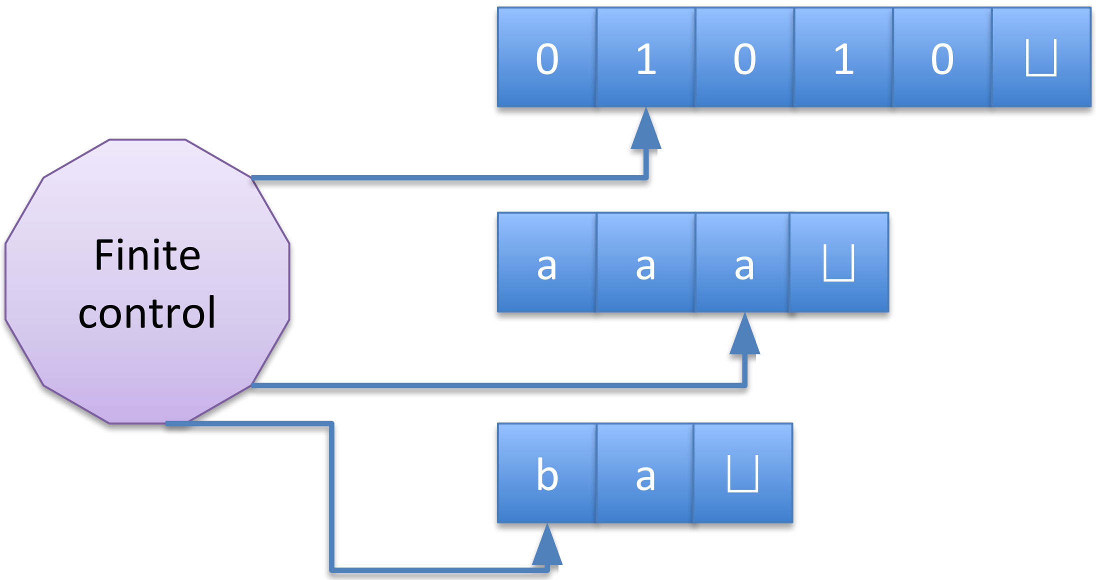
- Simply modify the transition function to satisfy:

$$\delta: Q \times \Gamma \rightarrow P(Q \times \Gamma \times \{L,R\})$$

# Enumerator



# Multitape Turing Machines

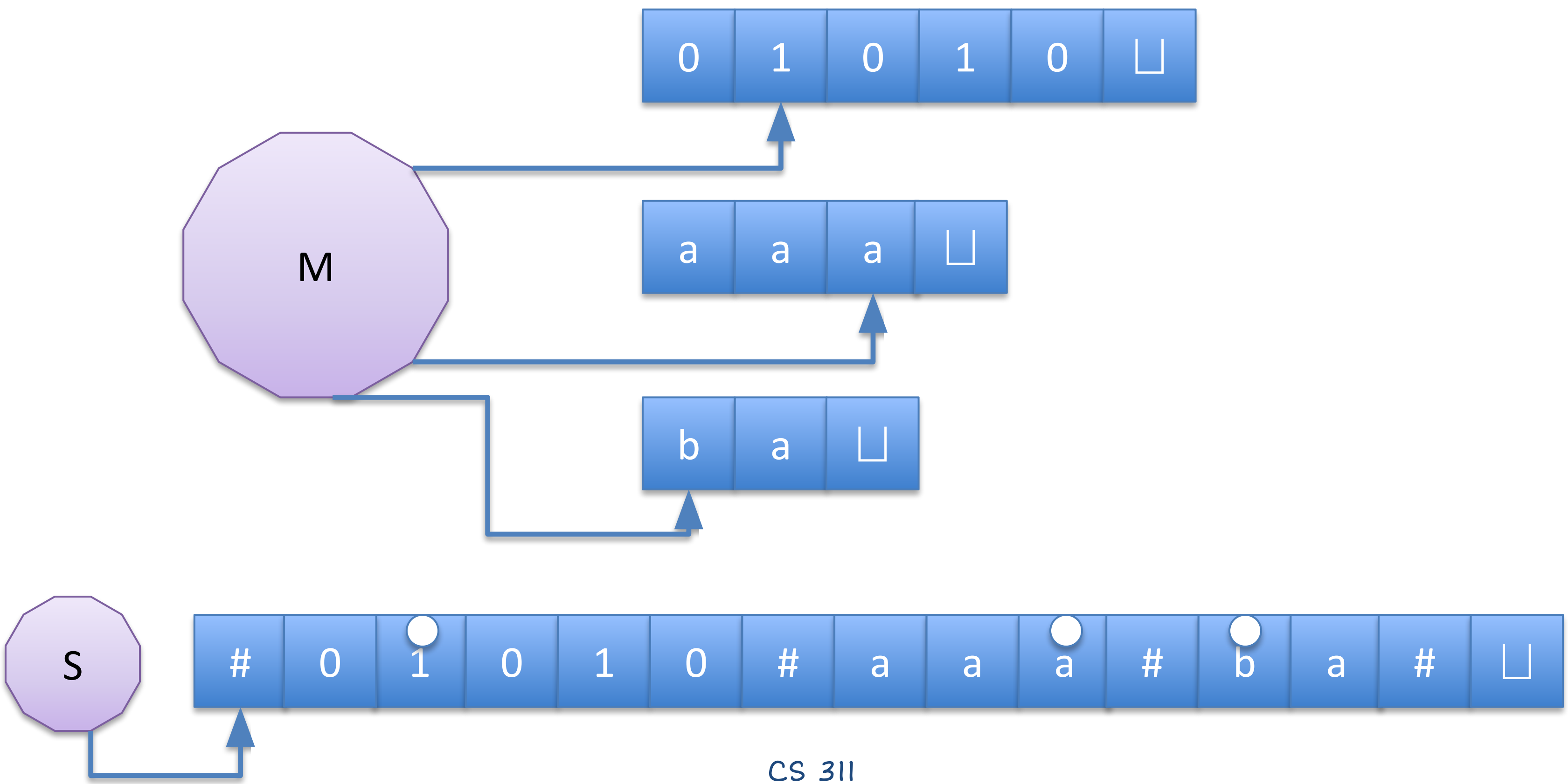



- Formally, we need only change the transition function to

$$\delta: Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L,R\}^k$$

# Seems more powerful, but…

- Theorem 3.13: Every multitape Turing machine has an equivalent single-tape Turing machine.



# Turing-recognizable languages

- Corollary 3.15: A language is Turing-recognizable if and only if some multitape Turing machine recognizes it.

# Recognizing *Composite* Numbers

- Let $L = \{1^n \mid n \text{ is a composite number}\}$
- Designing a Turing machine to accept $L$ would seem to involve factoring $n$
- However, if we could guess …

# Guessing

- Design a machine M that on input $1^n$ performs the following steps:
  1. Nondeterministically choose two numbers $p, q > 1$ and transform the input into $\#1^n\#1^p\#1^q\#$
  2. Multiply $p$ by $q$ to obtain $\#1^n\#1^{pq}\#$
  3. Checks the number of $1$'s before and after the middle $\#$ for equality
     - Accepts if equal, and rejects otherwise

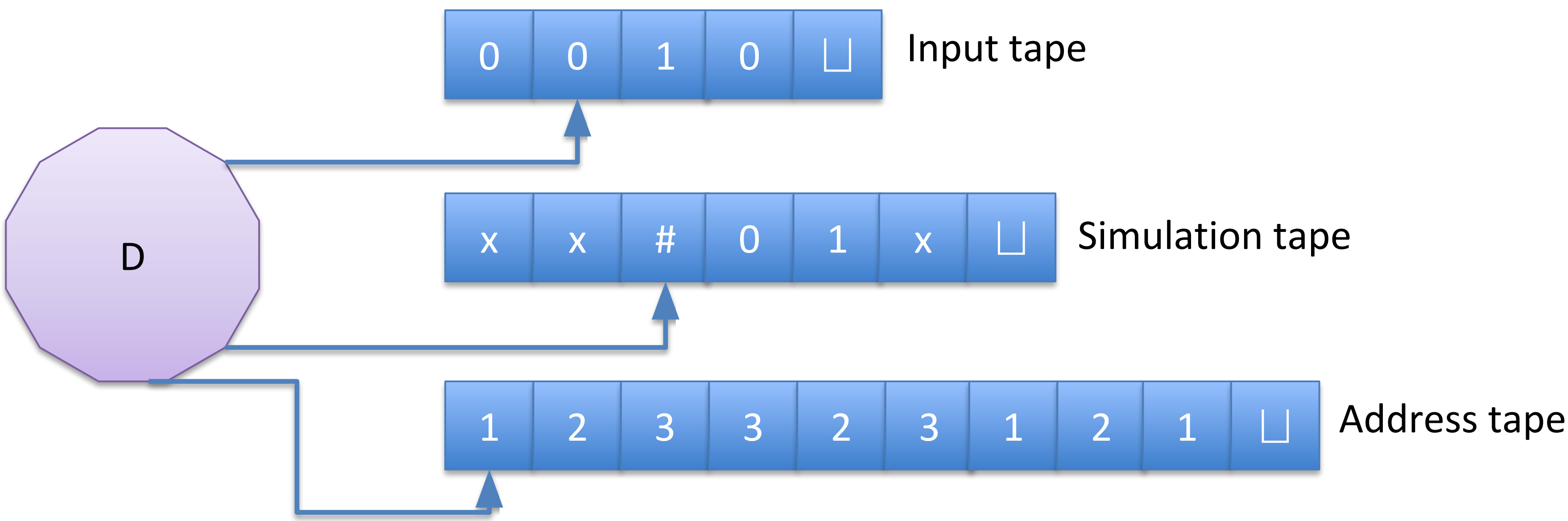
# Nondeterministic Turing machines

- Simply modify the transition function to satisfy:

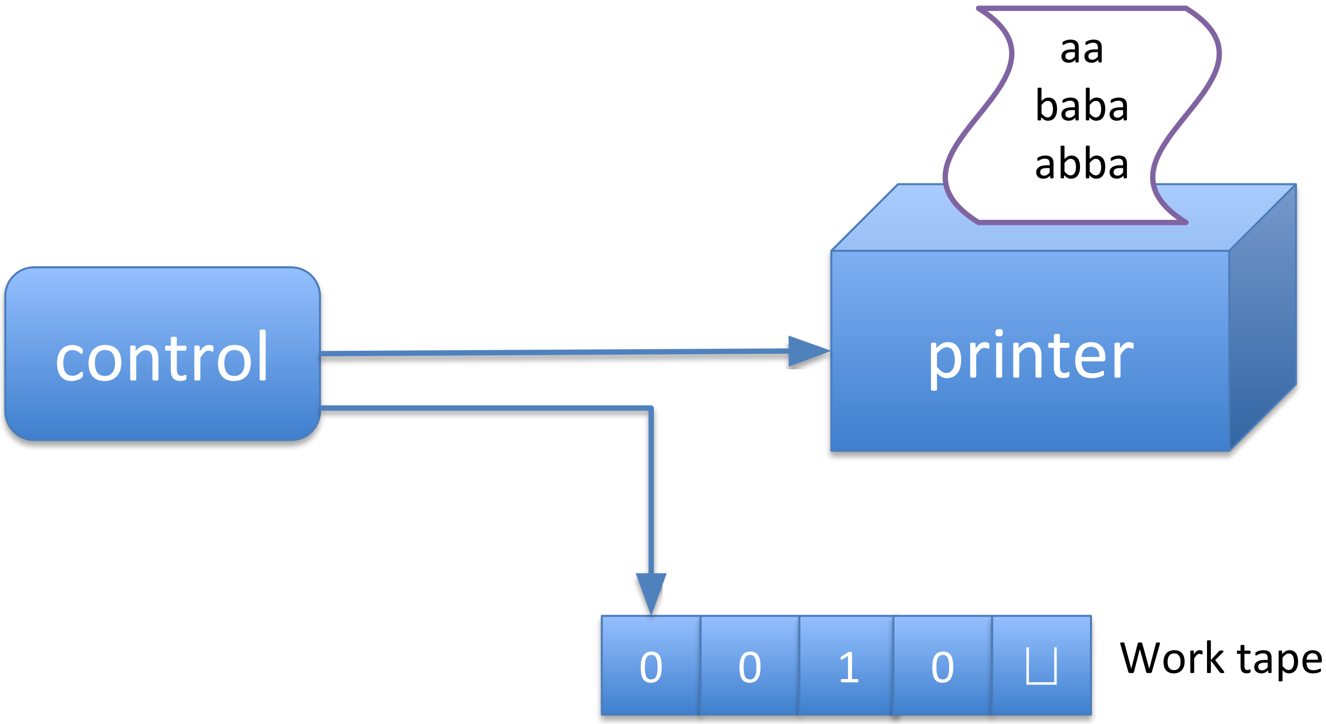
$$\delta: Q \times \Gamma \rightarrow P(Q \times \Gamma \times \{L, R\})$$

# Guessing doesn't buy us anything!

- Theorem 3.16: Every nondeterministic Turing machine has an equivalent deterministic Turing machine.



# Enumerator



# Equivalence with Turing machines

- Theorem 3.21: A language is Turing-recognizable if and only if some enumerator enumerates it.
- Proof

  (⇐) Suppose enumerator $E$ enumerates $L$. Define $M$ = "On input $w$:

  1. Run E. Every time that $E$ outputs a string, compare it with $w$.
  2. If $w$ ever appears in the output of E, *accept*."

# Equivalence with Turing machines

- Theorem 3.21: A language is Turing-recognizable if and only if some enumerator enumerates it.
- Proof

(⇒) Suppose TM $M$ recognizes $L$. Build a lexicographic enumerator to generate the list of all possible strings $s_1, s_2, \ldots$ over $\Sigma^*$.

Define $E$ = "Ignore input.

1. Repeat the following for $i = 1, 2, 3, \ldots$
2. Run $M$ for $i$ steps on each input $s_1, s_2, \ldots, s_i$.
3. If any computations accept, print out corresponding $s_i$."

# TMs take their own sweet time…

- Recognizers (like enumerators) may take a while to answer *yes* …
  and even longer if the answer is *no*

- A TM that *halts* on *all* inputs is called a ***decider***

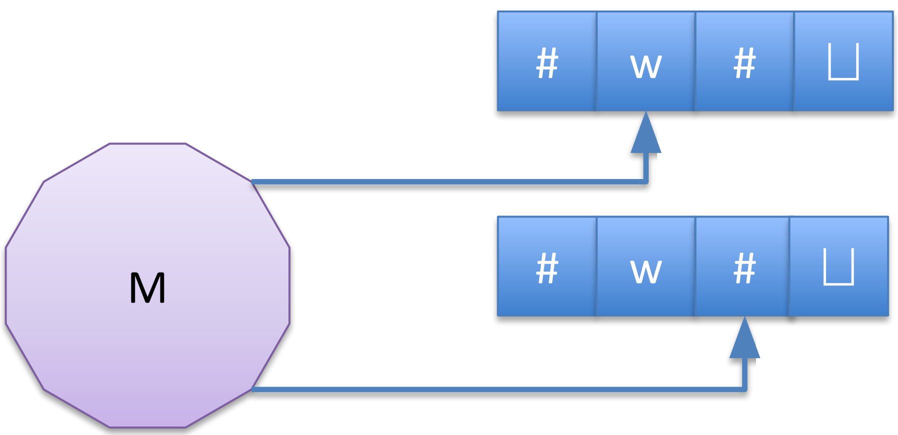
- A decider that recognizes a languages is said to ***decide*** that language

- Call a language ***Turing-decidable*** if some Turing machine decides it

# Recognizers and Deciders

- Theorem: A language is Turing-decidable if and only if both it and its complement are Turing-recognizable
- Proof:

  (⇒) By definition and swapping accept/reject.




  (⇐) Simulate, in parallel, $M_1$ on tape 1 and $M_2$ on tape 2.