Algorithms

Sipser 3.3 (pages 154-159)

Computability

- **Hilbert's Tenth Problem:**

  Find "a process according to which it can be determined by a finite number of operations" whether a given a *polynomial*

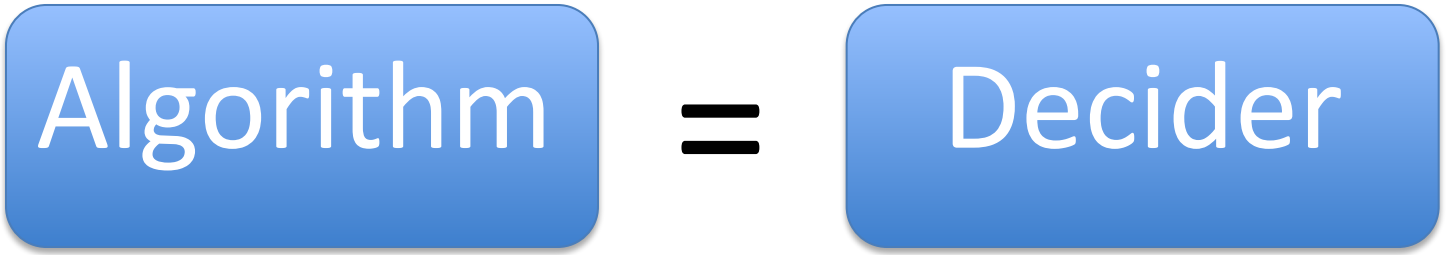
$$p(x_1, x_2, \ldots, x_n)$$

  has an integral root.

Algorithms

- Intuitively:
  - An *algorithm* is a finite sequence of operations, each chosen from a finite set of well-defined operations, that halts in a finite time.
  - Sometimes also called *procedures* or *recipes*

Church-Turing Thesis



Languages and Problems

- Let

$$D = \{p \mid p \text{ is a polynomial with an integral root}\}$$

- Hilbert's Tenth Problem:
  Determine if D is Turing-decidable

$D = \{p \mid p \text{ is a polynomial with an integral root}\}$

- Turing-recognizable
- M = "On input p,
  where p is a polynomial $p(x_1,x_2,\ldots,x_n)$.
  1. Lexicographically generate integer values for $(x_1,x_2,\ldots,x_n)$.
  2. Evaluate p as each set of values is generated.
  3. If, at any point, the polynomial evaluates to 0, *accept*."

Hierarchy of languages