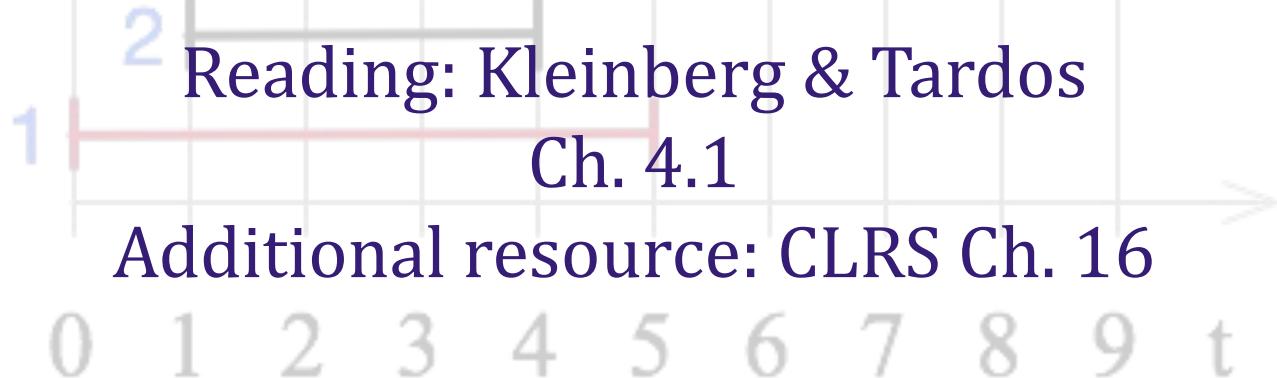


# Greedy algorithms (stay ahead) - scheduling



# So much to do, so little time...

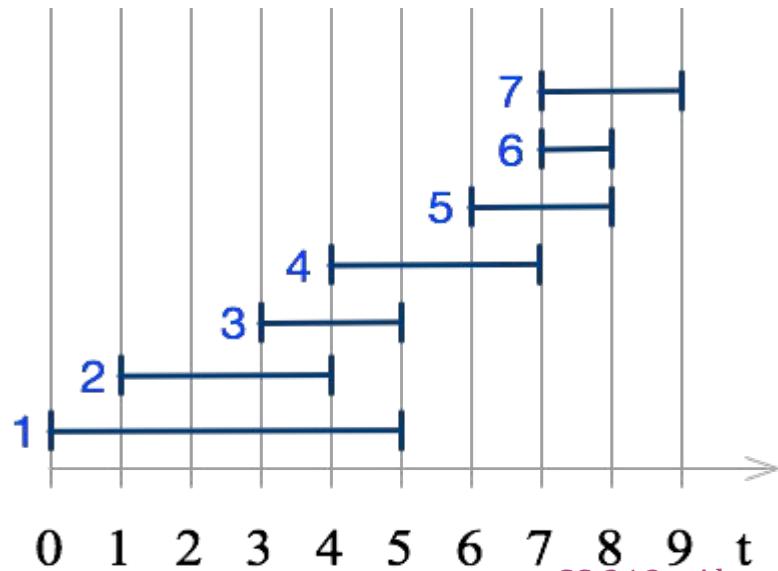
Senior symposium is only a couple months away! Your plan is to look at the schedule and mark down sessions and talks that you want to attend. Unfortunately, some will overlap, and you'll be forced to make a choice.



*How do you choose sessions and talks so that you maximize the total number that you attend?*

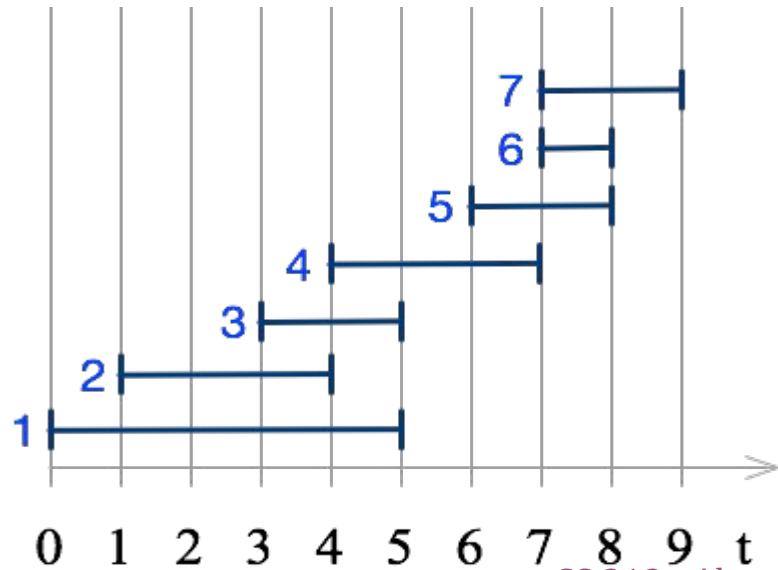
# Formalizing the problem

- Set of  $1, \dots, n$  sessions/talks



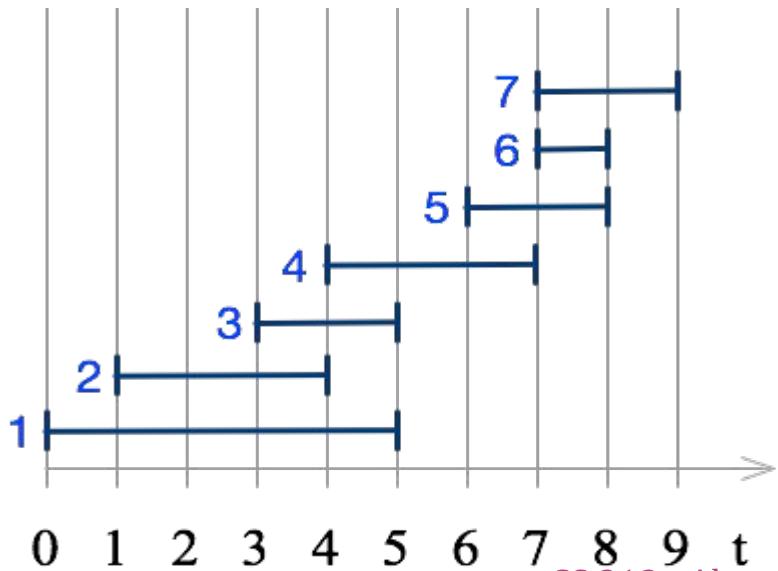
# Formalizing the problem

- Set of  $1, \dots, n$  requests
- $s(i), f(i)$  start and finish times



# Problem 1: interval scheduling

- Set of  $1, \dots, n$  requests
- $s(i), f(i)$  start and finish times
- i and j **compatible**: no overlap
- **compatible set**: no pair overlaps
- **optimal**: maximum-sized compatible set



$i$	$s(i)$	$f(i)$
1	0	5
2	1	4
3	3	5
4	4	7
5	6	8
6	7	8
7	7	9

# How to pick?

- Build up set by adding one request at a time
- Choose the locally “best” request
- Will it lead to an optimal solution?

$R$  = all requests sorted by some property

$S = \{\}$  // selected requests

while ( $R$  is not empty)

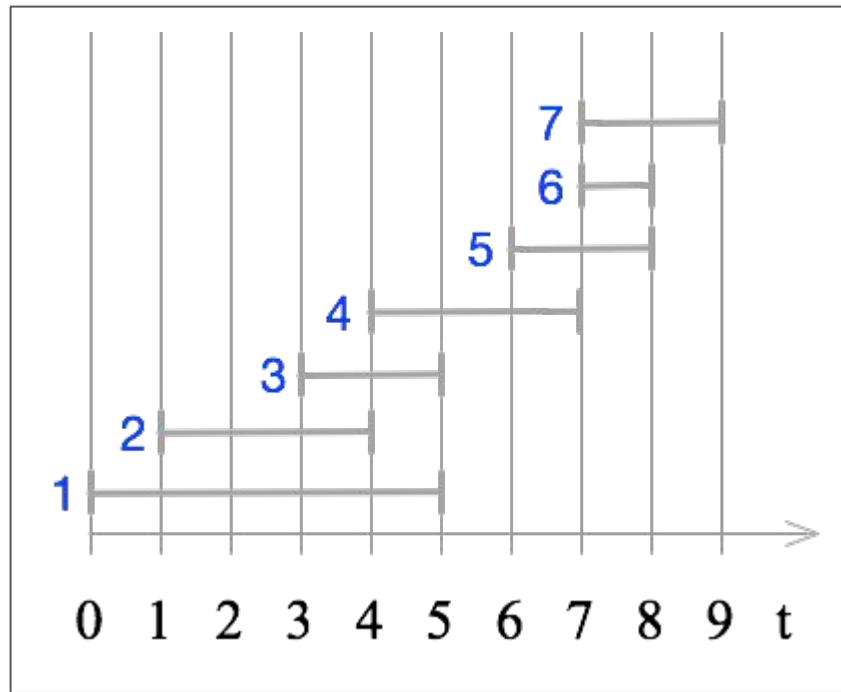
    remove next request  $i$  from  $R$

    add  $i$  to  $S$

    remove all overlapping requests from  $R$

# Next request please!

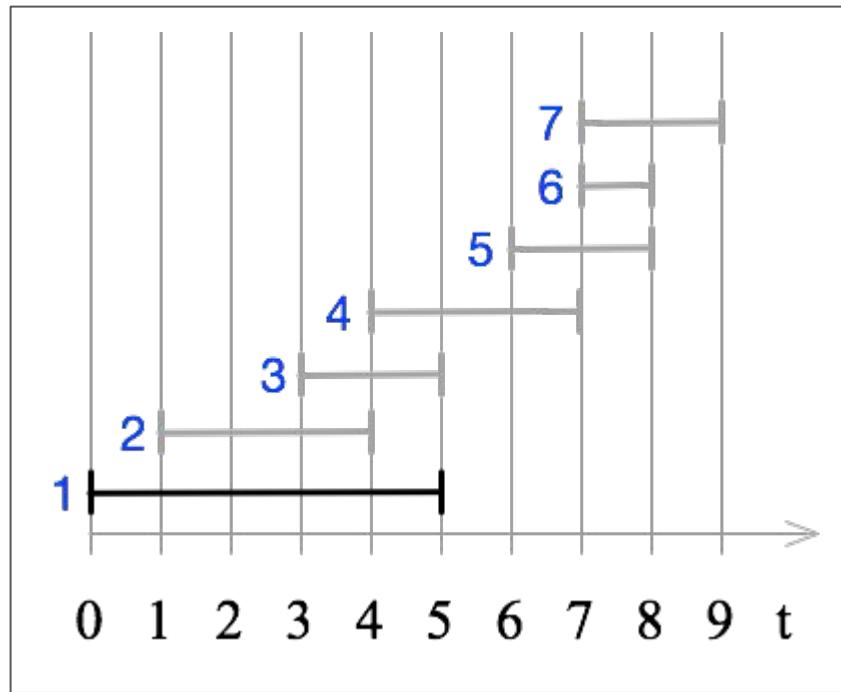
Let's try taking them in order of next starting time:



$i$	$s(i)$	$f(i)$
1	0	5
2	1	4
3	3	5
4	4	7
5	6	8
6	7	8
7	7	9

# Next request please!

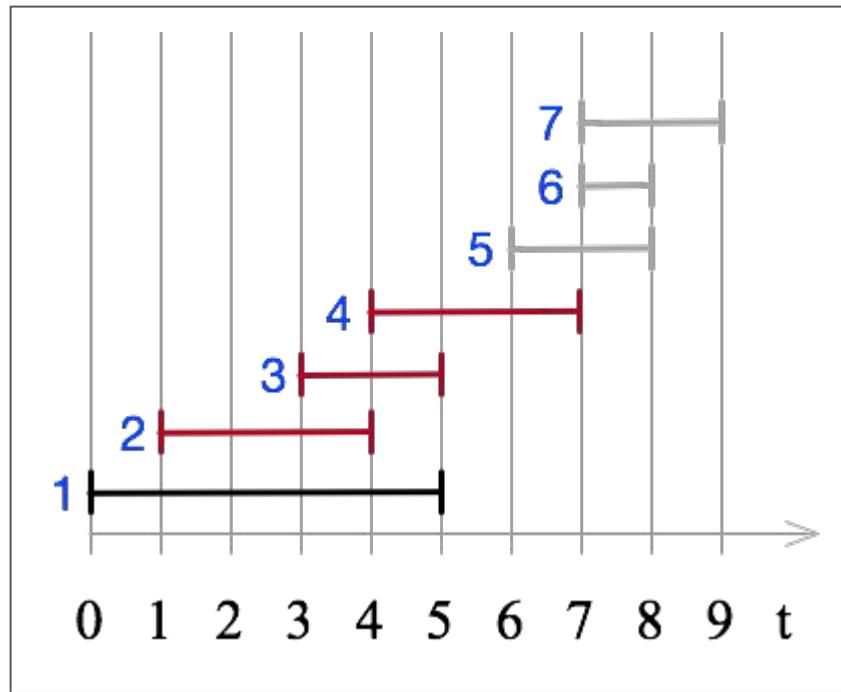
Let's try taking them in order of next starting time:



$i$	$s(i)$	$f(i)$
1	0	5
2	1	4
3	3	5
4	4	7
5	6	8
6	7	8
7	7	9

# Next request please!

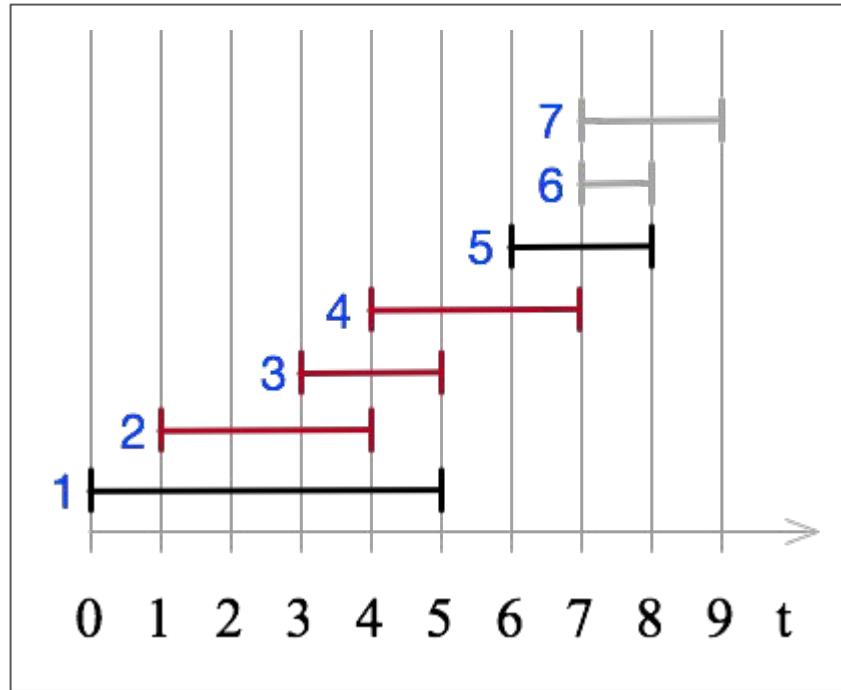
Let's try taking them in order of next starting time:



$i$	$s(i)$	$f(i)$
1	0	5
2	1	4
3	3	5
4	4	7
5	6	8
6	7	8
7	7	9

# Next request please!

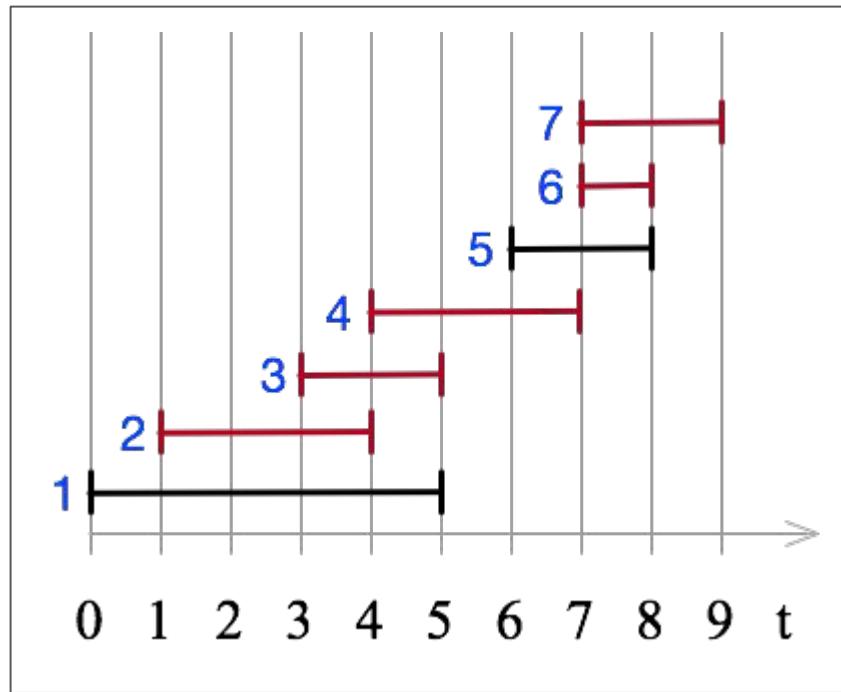
Let's try taking them in order of next starting time:



$i$	$s(i)$	$f(i)$
1	0	5
2	1	4
3	3	5
4	4	7
5	6	8
6	7	8
7	7	9

# Next request please!

Let's try taking them in order of next starting time:

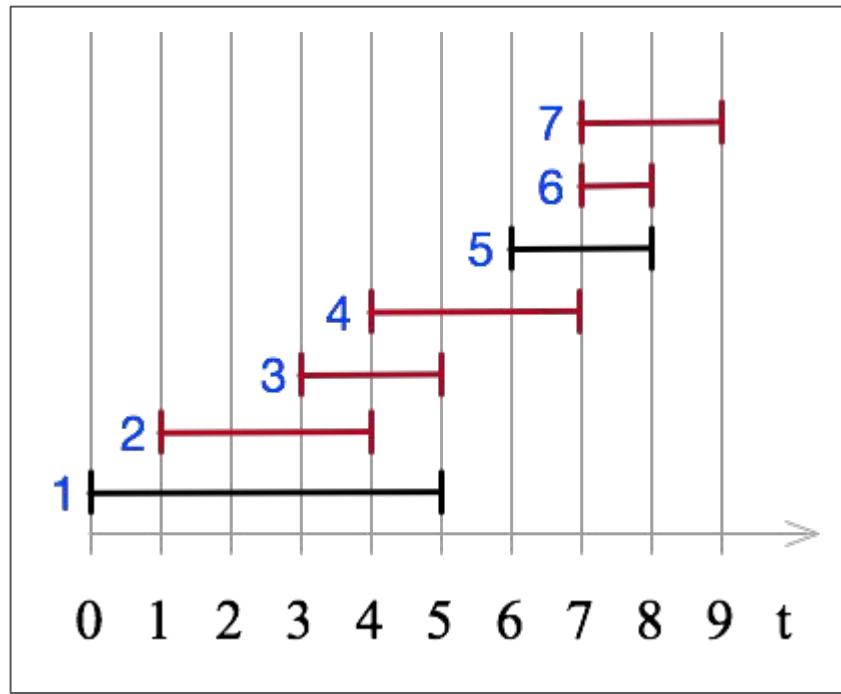


$i$	$s(i)$	$f(i)$
1	0	5
2	1	4
3	3	5
4	4	7
5	6	8
6	7	8
7	7	9

# Next request please!

Let's try taking them in order of next starting time:

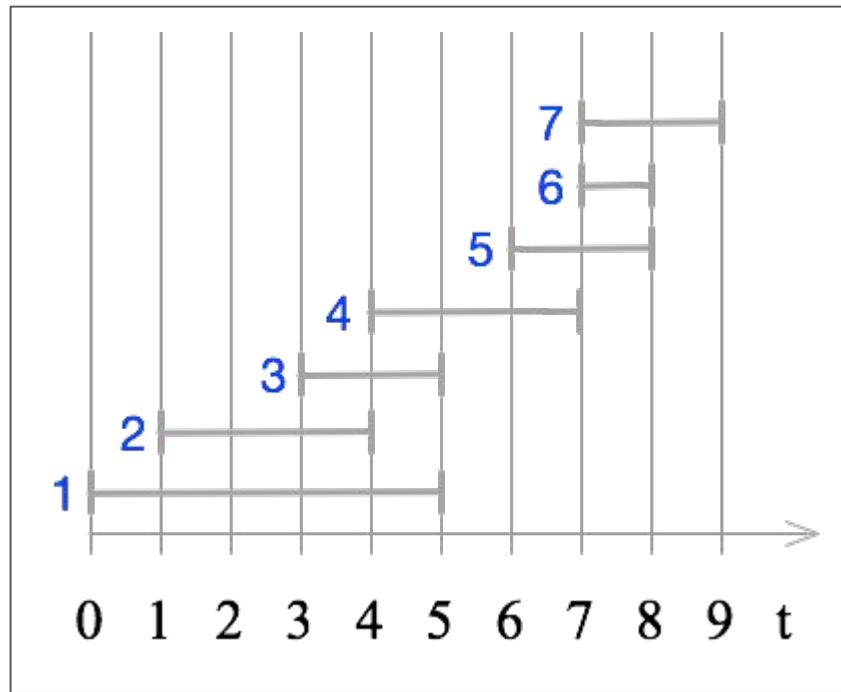
**2 requests can be fulfilled**



$i$	$s(i)$	$f(i)$
1	0	5
2	1	4
3	3	5
4	4	7
5	6	8
6	7	8
7	7	9

# Less is more?

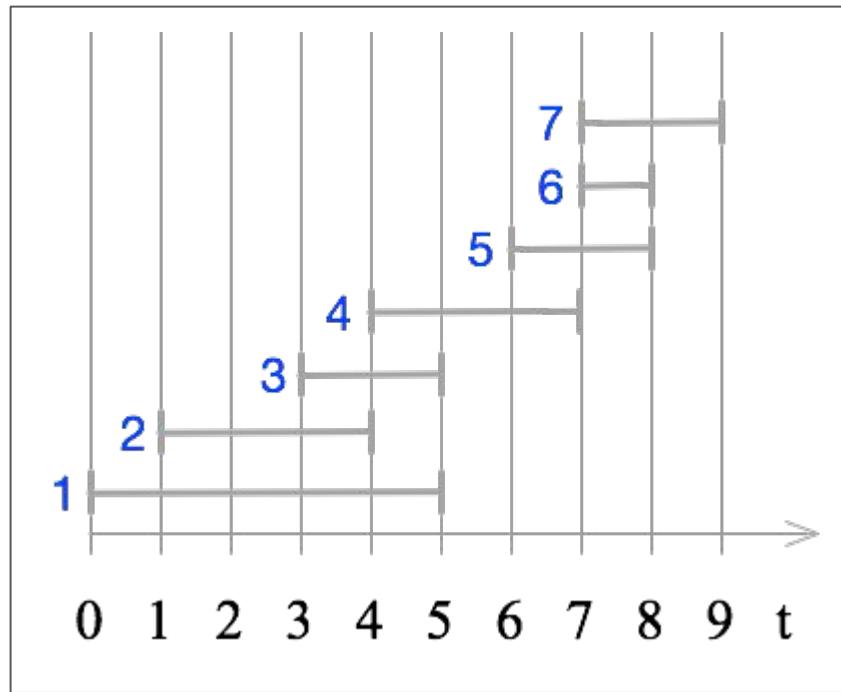
Let's try taking them in order of duration:



$i$	$s(i)$	$f(i)$
1	0	5
2	1	4
3	3	5
4	4	7
5	6	8
6	7	8
7	7	9

# Less is more?

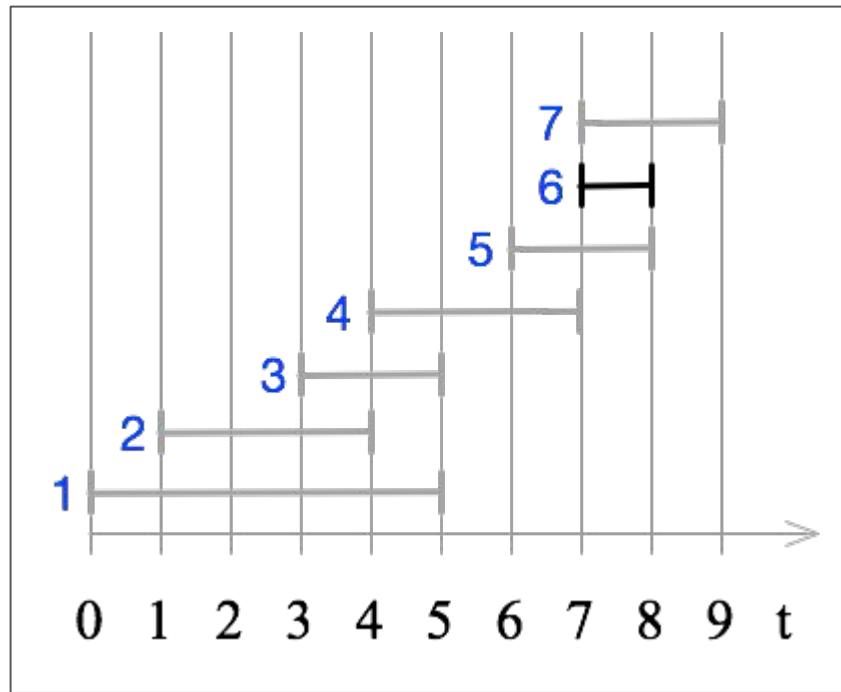
Let's try taking them in order of duration:



$i$	$s(i)$	$f(i)$	$dur$
6	7	8	1
3	3	5	2
5	6	8	2
7	7	9	2
2	1	4	3
4	4	7	3
1	0	5	5

# Less is more?

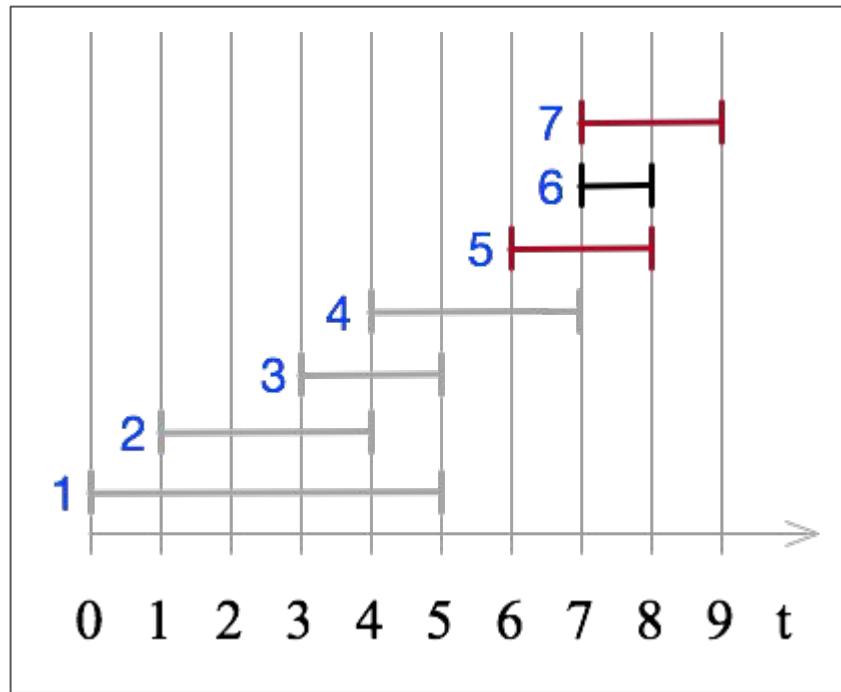
Let's try taking them in order of duration:



$i$	$s(i)$	$f(i)$	$dur$
6	7	8	1
3	3	5	2
5	6	8	2
7	7	9	2
2	1	4	3
4	4	7	3
1	0	5	5

# Less is more?

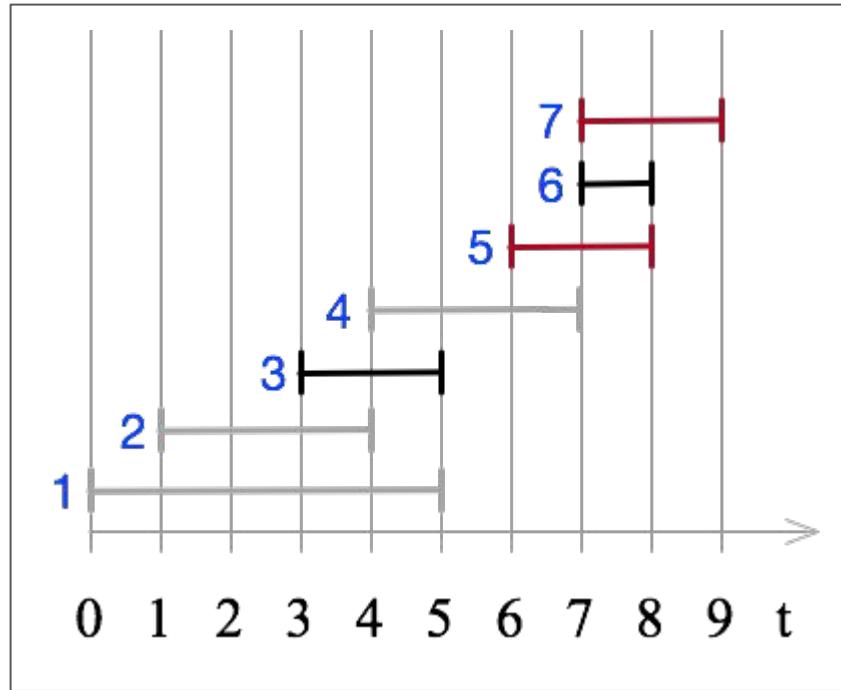
Let's try taking them in order of duration:



$i$	$s(i)$	$f(i)$	$dur$
6	7	8	1
3	3	5	2
5	6	8	2
7	7	9	2
2	1	4	3
4	4	7	3
1	0	5	5

# Less is more?

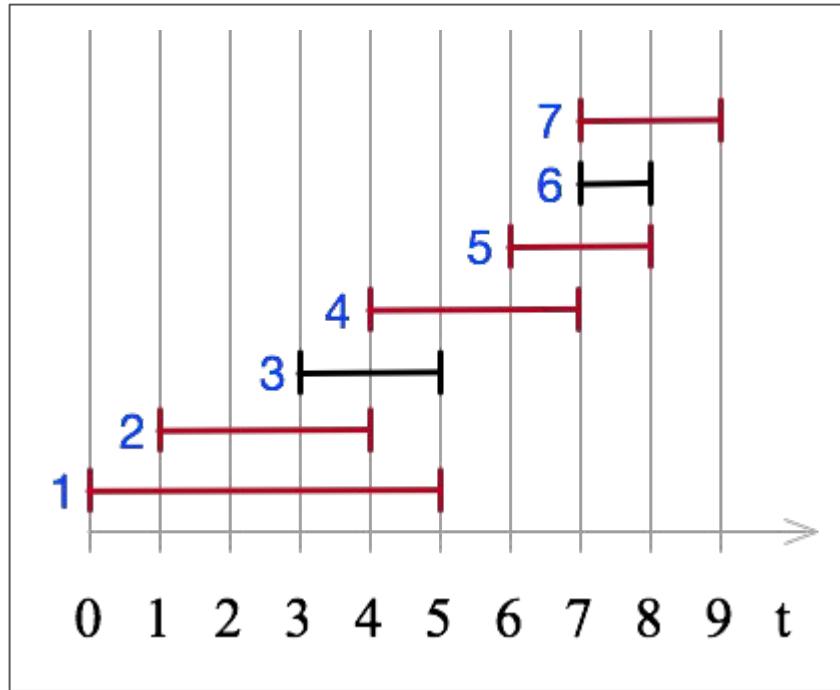
Let's try taking them in order of duration:



$i$	$s(i)$	$f(i)$	$dur$
6	7	8	1
3	3	5	2
5	6	8	2
7	7	9	2
2	1	4	3
4	4	7	3
1	0	5	5

# Less is more?

Let's try taking them in order of duration:

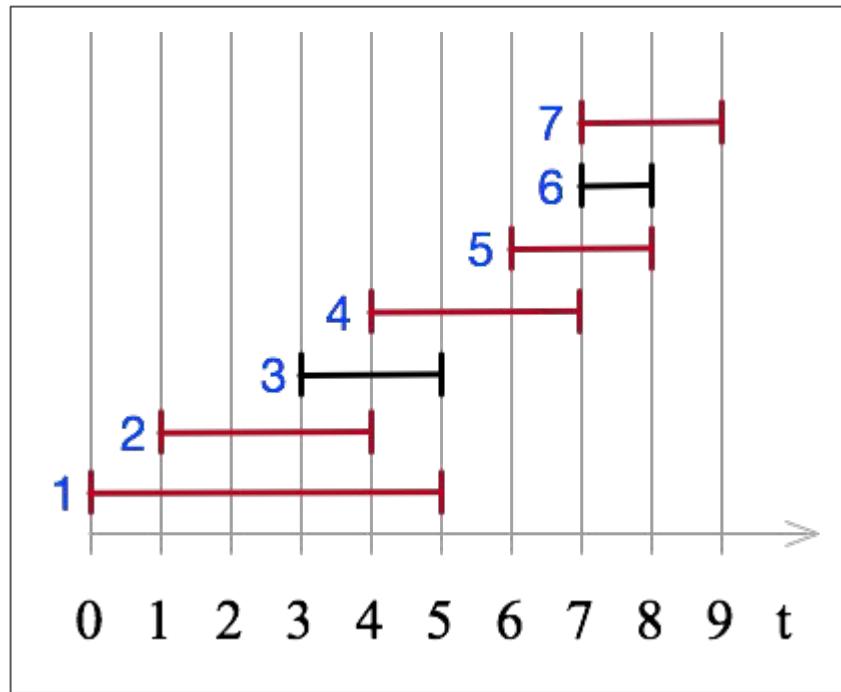


$i$	$s(i)$	$f(i)$	$dur$
6	7	8	1
3	3	5	2
5	6	8	2
7	7	9	2
2	1	4	3
4	4	7	3
1	0	5	5

# Less is more?

Let's try taking them in order of duration:

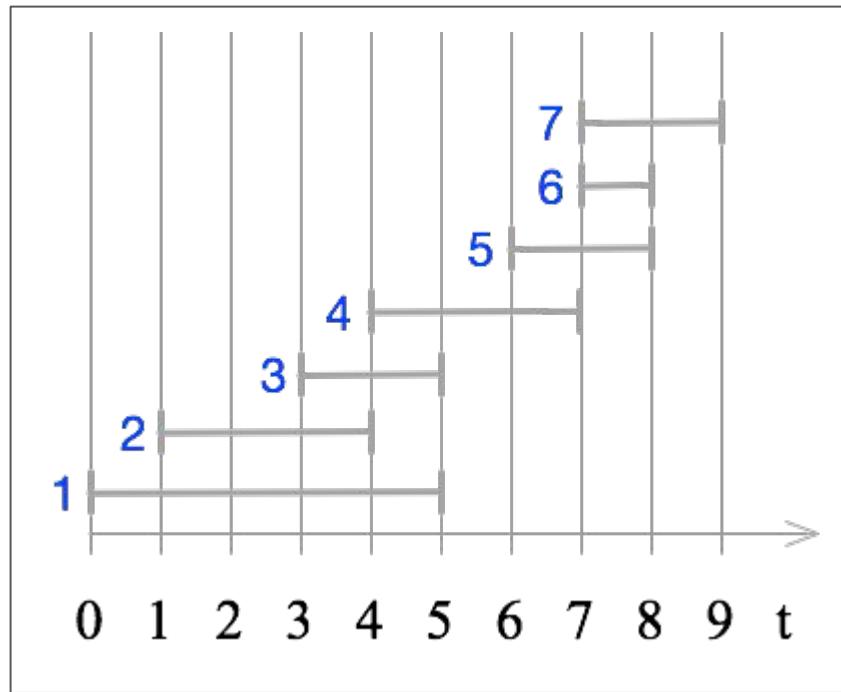
**2 requests can be fulfilled**



$i$	$s(i)$	$f(i)$	$dur$
6	7	8	1
3	3	5	2
5	6	8	2
7	7	9	2
2	1	4	3
4	4	7	3
1	0	5	5

# Not so pushy...

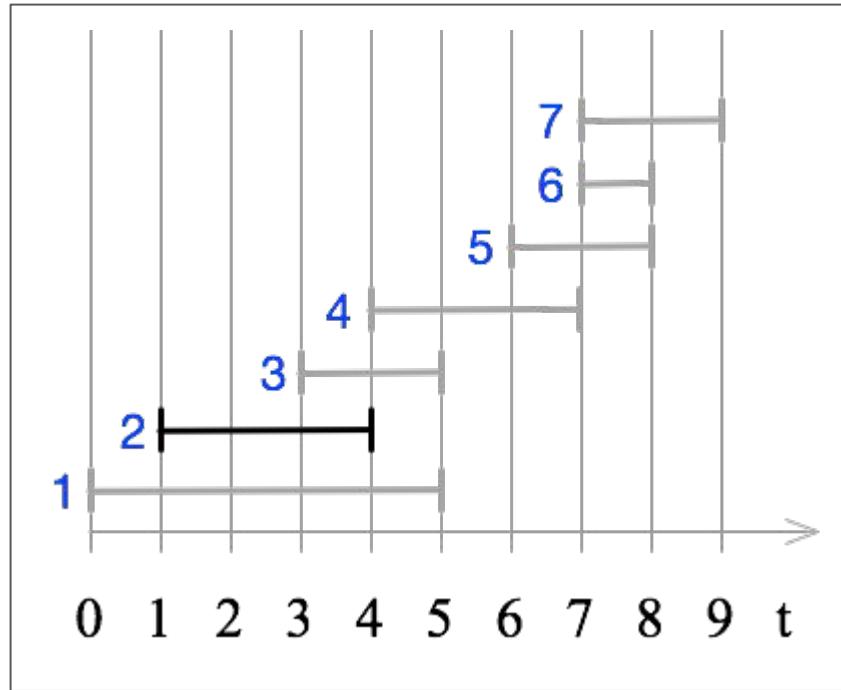
Let's try taking them in order of conflicts/overlaps:



<i>i</i>	<i>s(i)</i>	<i>f(i)</i>	<i>conflicts</i>
2	1	4	2
6	7	8	2
7	7	9	2
1	0	5	3
3	3	5	3
4	4	7	3
5	6	8	3

# Not so pushy...

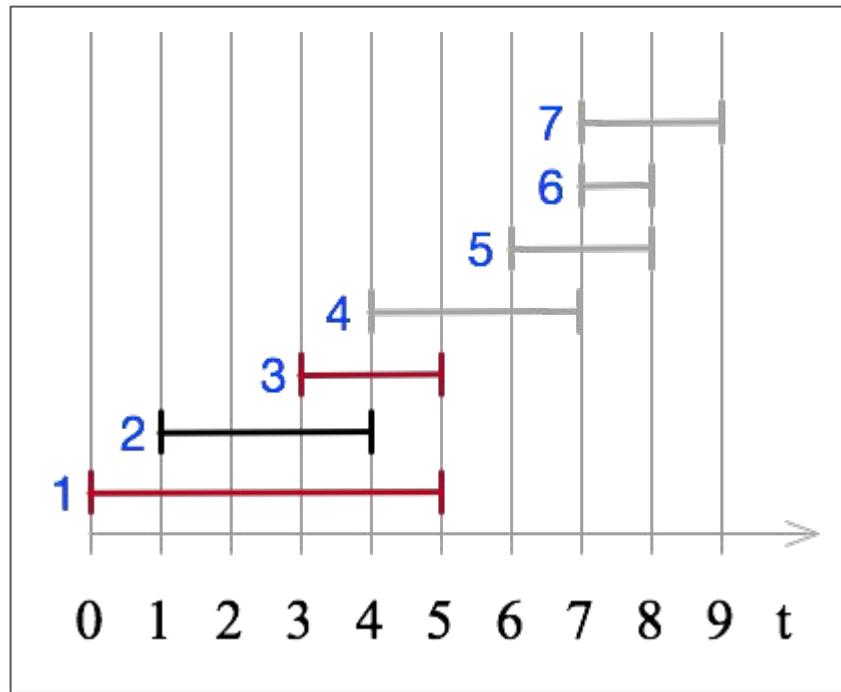
Let's try taking them in order of conflicts/overlaps:



<i>i</i>	<i>s(i)</i>	<i>f(i)</i>	<i>conflicts</i>
2	1	4	2
6	7	8	2
7	7	9	2
1	0	5	3
3	3	5	3
4	4	7	3
5	6	8	3

# Not so pushy...

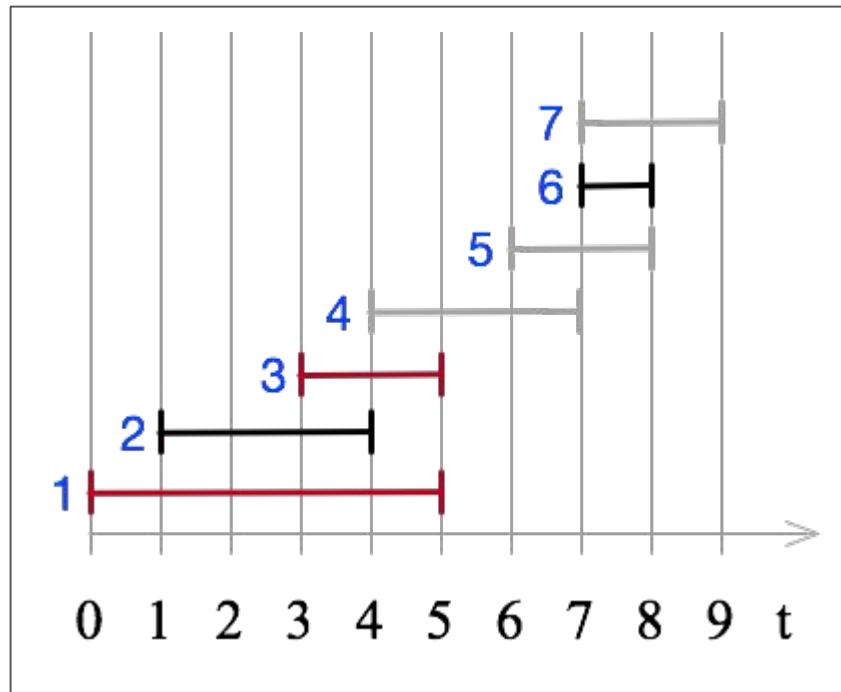
Let's try taking them in order of conflicts/overlaps:



<i>i</i>	<i>s(i)</i>	<i>f(i)</i>	<i>conflicts</i>
2	1	4	2
6	7	8	2
7	7	9	2
1	0	5	3
3	3	5	3
4	4	7	3
5	6	8	3

# Not so pushy...

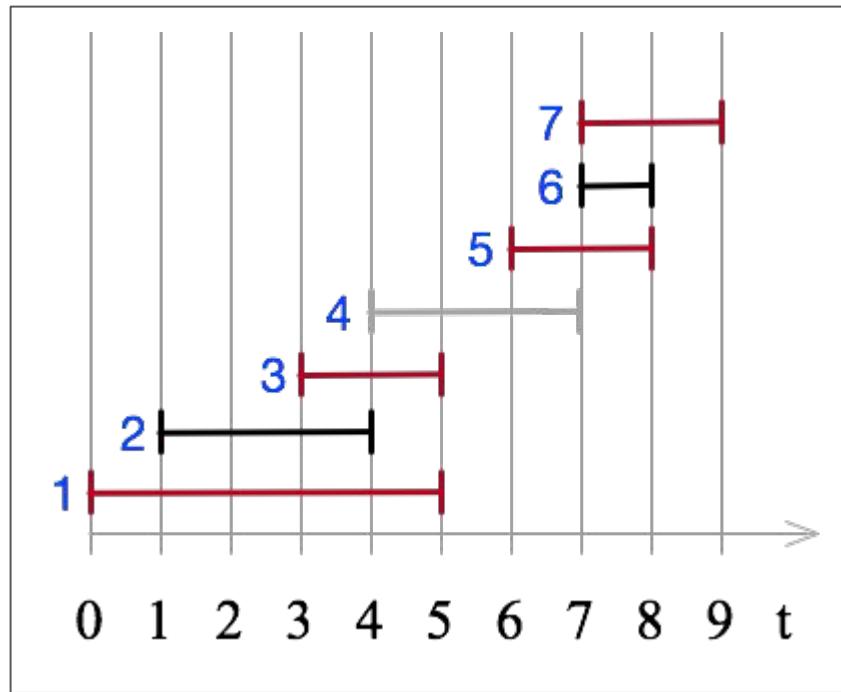
Let's try taking them in order of conflicts/overlaps:



$i$	$s(i)$	$f(i)$	<i>conflicts</i>
2	1	4	2
6	7	8	2
7	7	9	2
1	0	5	3
3	3	5	3
4	4	7	3
5	6	8	3

# Not so pushy...

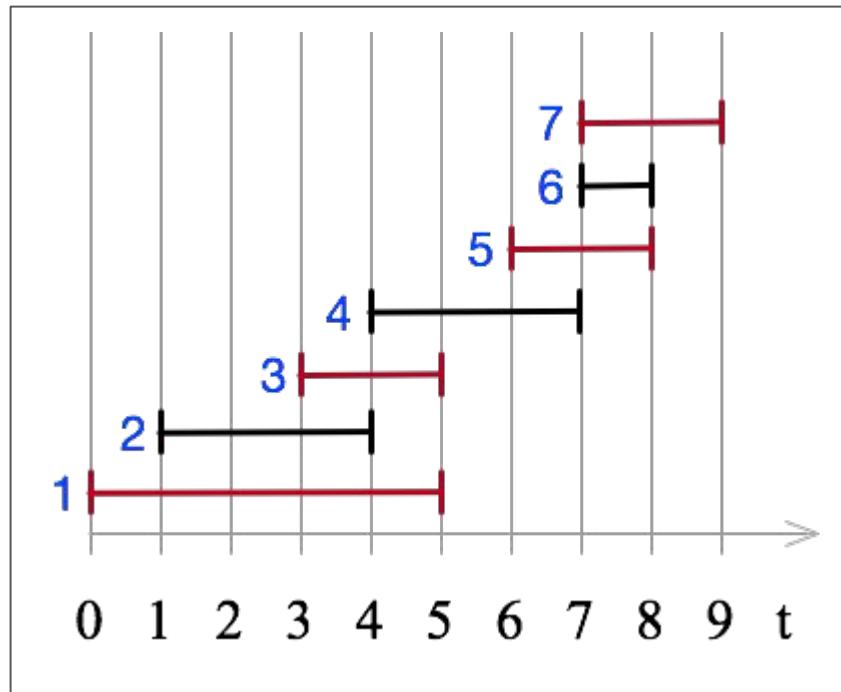
Let's try taking them in order of conflicts/overlaps:



$i$	$s(i)$	$f(i)$	<i>conflicts</i>
2	1	4	2
6	7	8	2
7	7	9	2
1	0	5	3
3	3	5	3
4	4	7	3
5	6	8	3

# Not so pushy...

Let's try taking them in order of conflicts/overlaps:

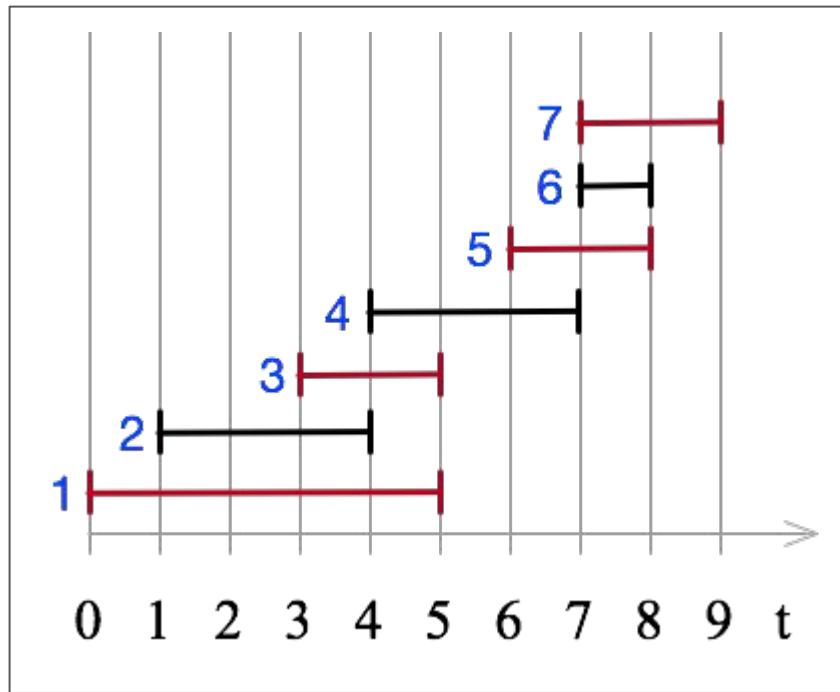


$i$	$s(i)$	$f(i)$	<i>conflicts</i>
2	1	4	2
6	7	8	2
7	7	9	2
1	0	5	3
3	3	5	3
4	4	7	3
5	6	8	3

# Not so pushy...

Let's try taking them in order of conflicts/overlaps:

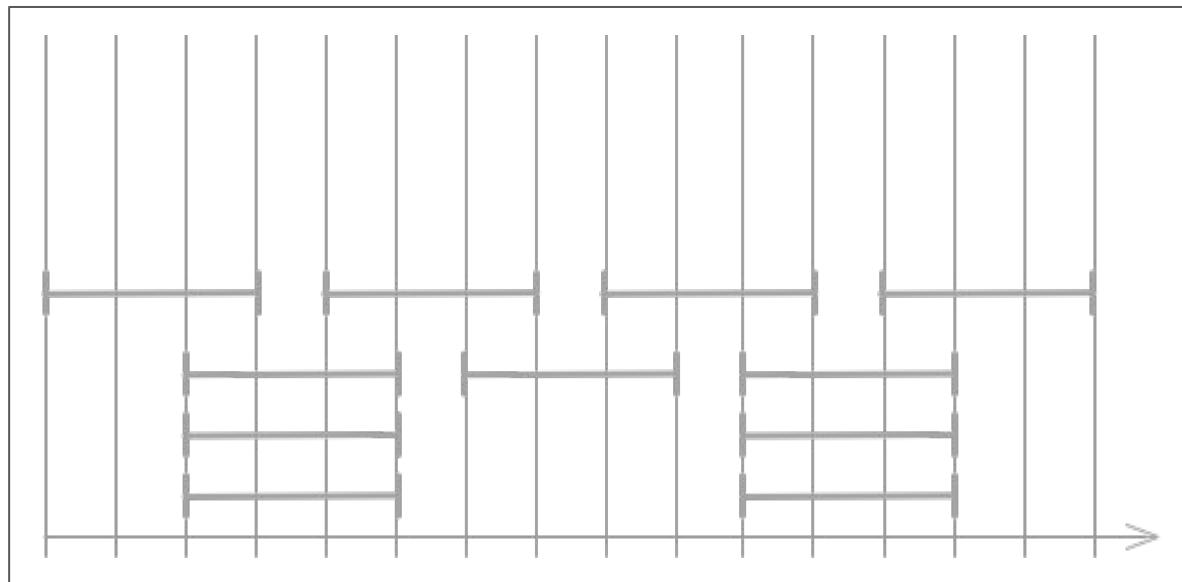
**3 requests fulfilled!**



$i$	$s(i)$	$f(i)$	<i>conflicts</i>
2	1	4	2
6	7	8	2
7	7	9	2
1	0	5	3
3	3	5	3
4	4	7	3
5	6	8	3

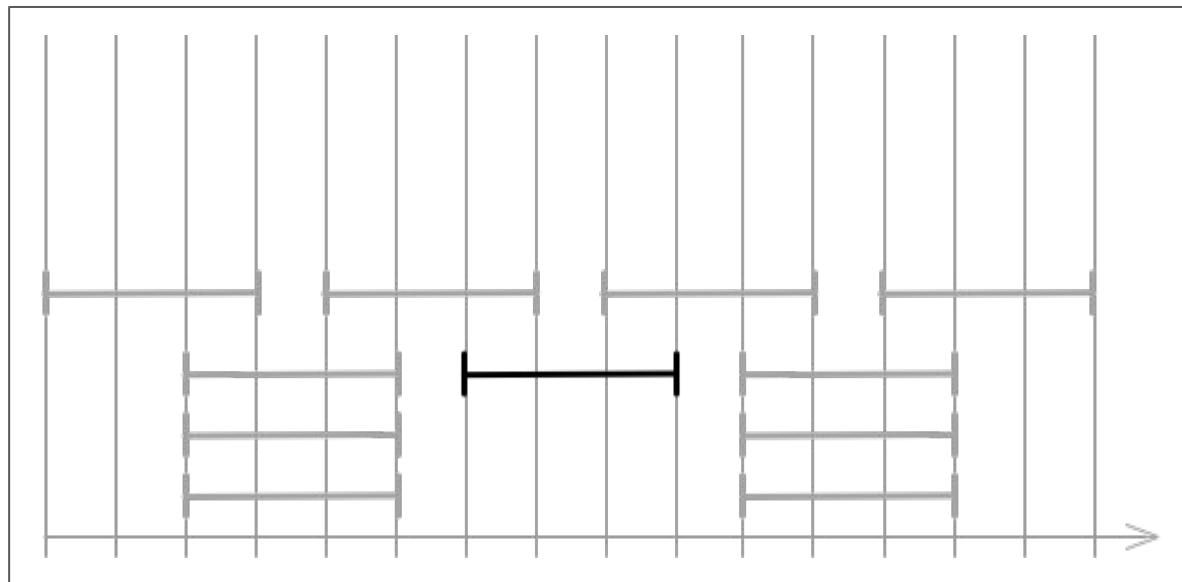
# Will this always work?

Let's try taking them in order of conflicts/overlaps:



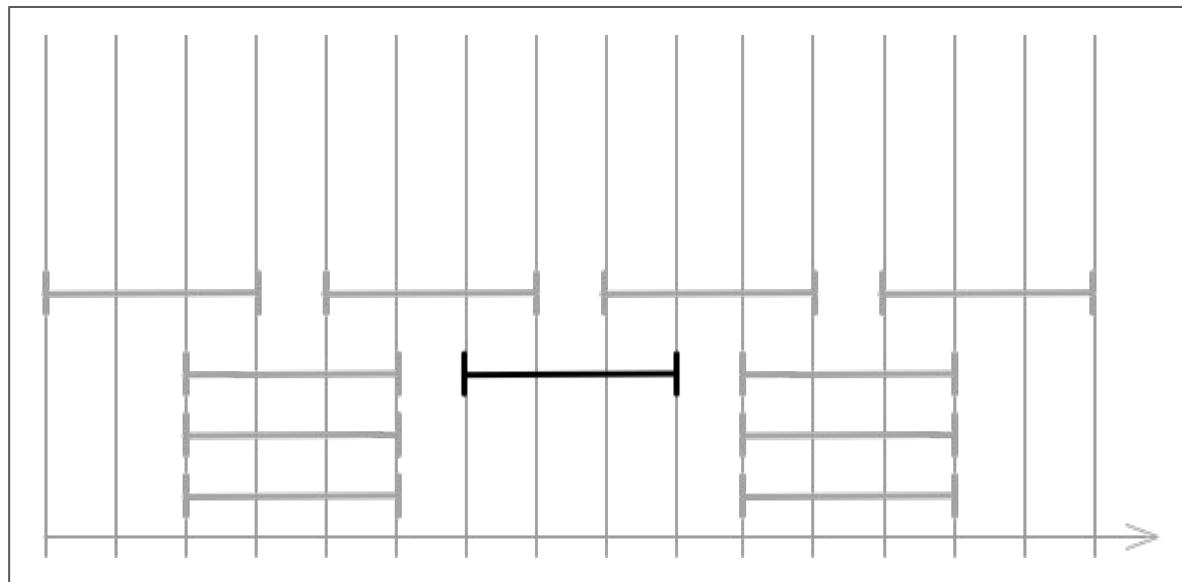
# Will this always work?

Let's try taking them in order of conflicts/overlaps:



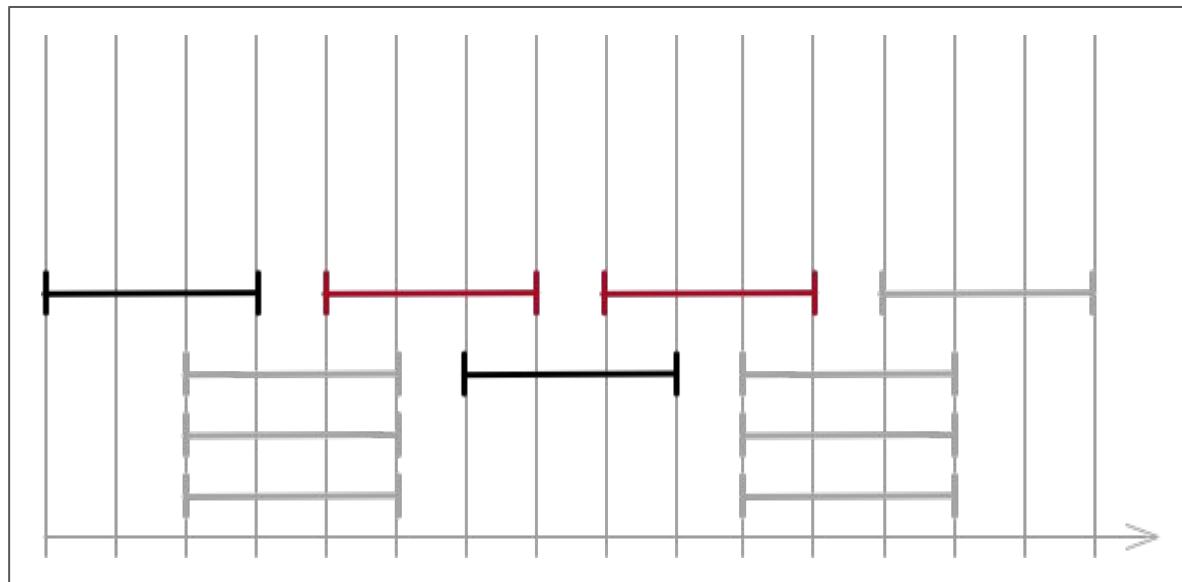
# Will this always work?

Let's try taking them in order of conflicts/overlaps:



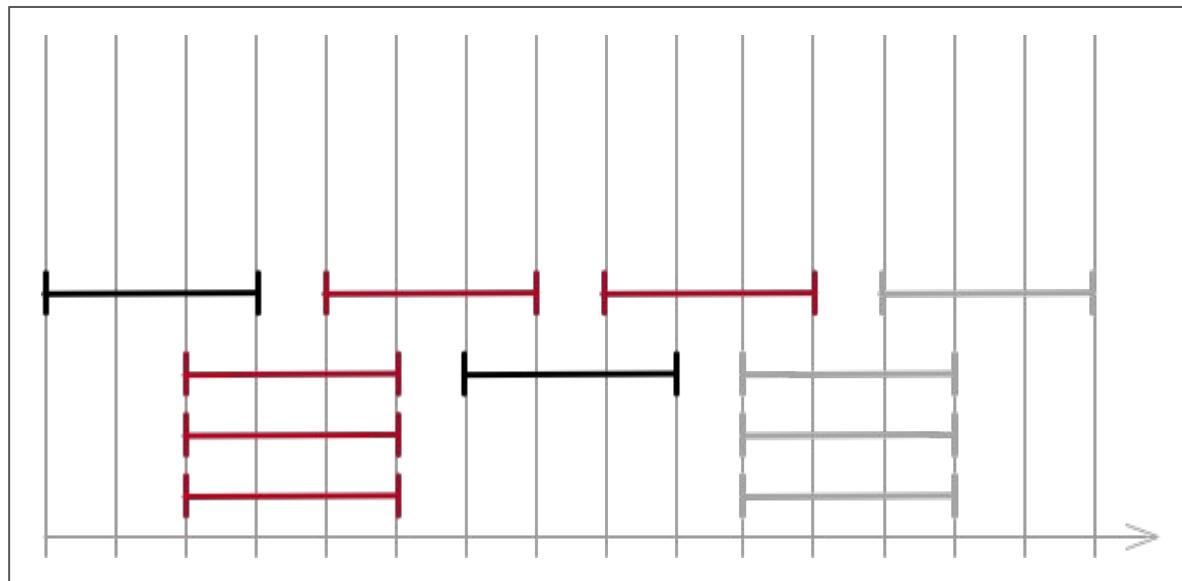
# Will this always work?

Let's try taking them in order of conflicts/overlaps:



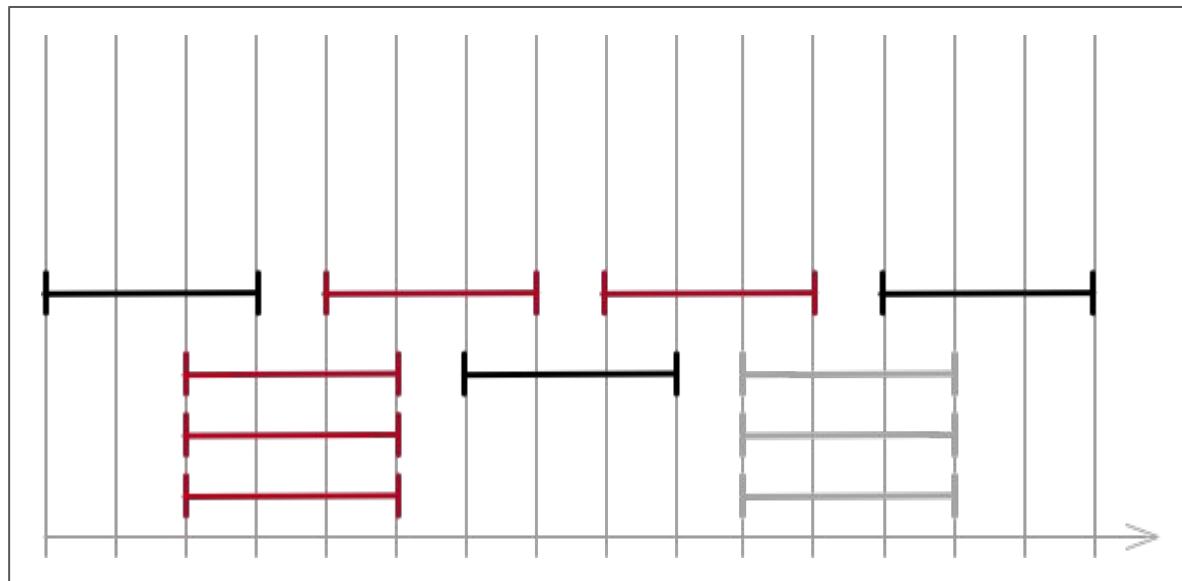
# Will this always work?

Let's try taking them in order of conflicts/overlaps:



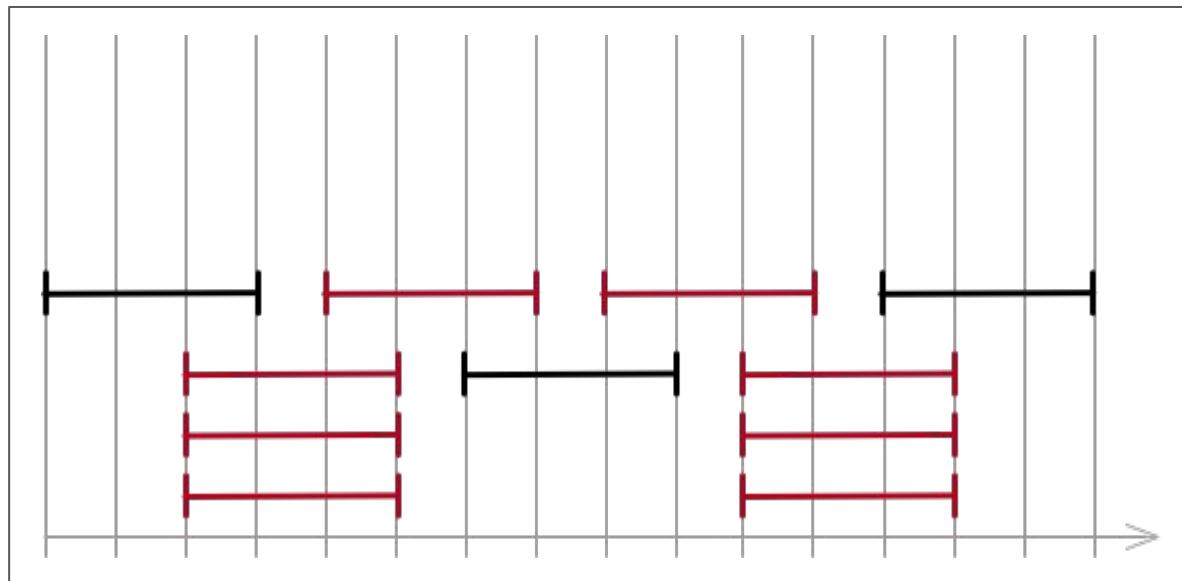
# Will this always work?

Let's try taking them in order of conflicts/overlaps:



# Will this always work?

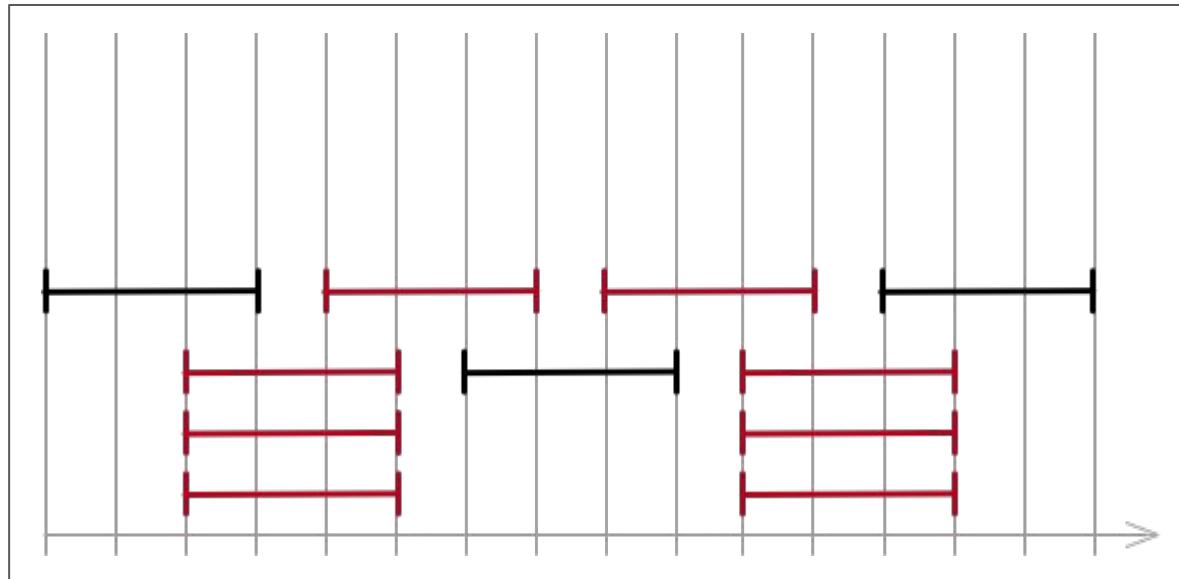
Let's try taking them in order of conflicts/overlaps:



# Will this always work?

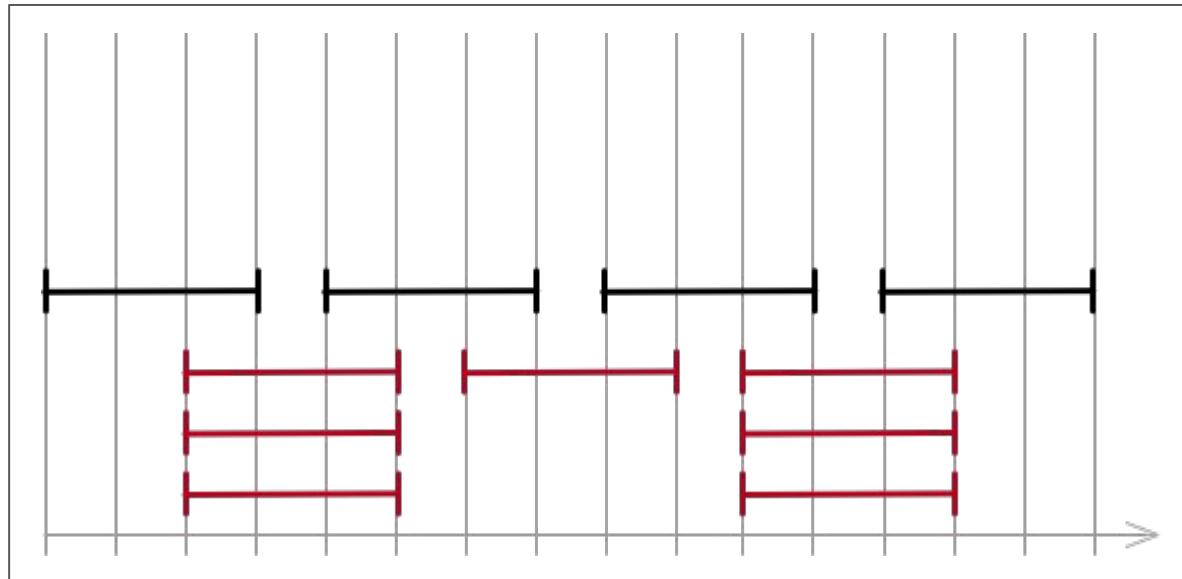
Let's try taking them in order of conflicts/overlaps:

**3 requests fulfilled**



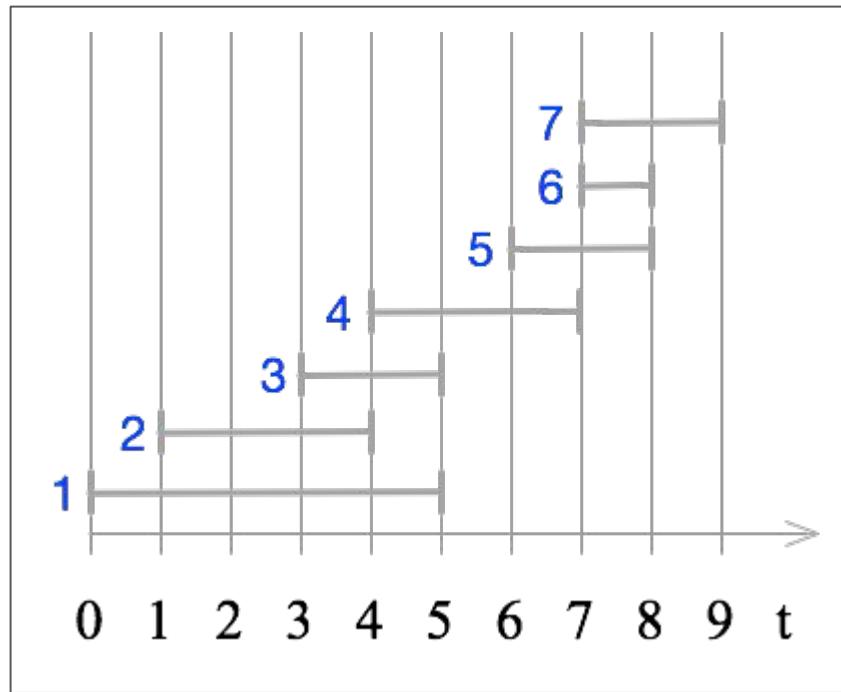
# Will this always work?

Let's try taking them in order of conflicts/overlaps:  
... but **4 requests fulfilled is optimal!**



# Okay, the sooner this ends, the better!

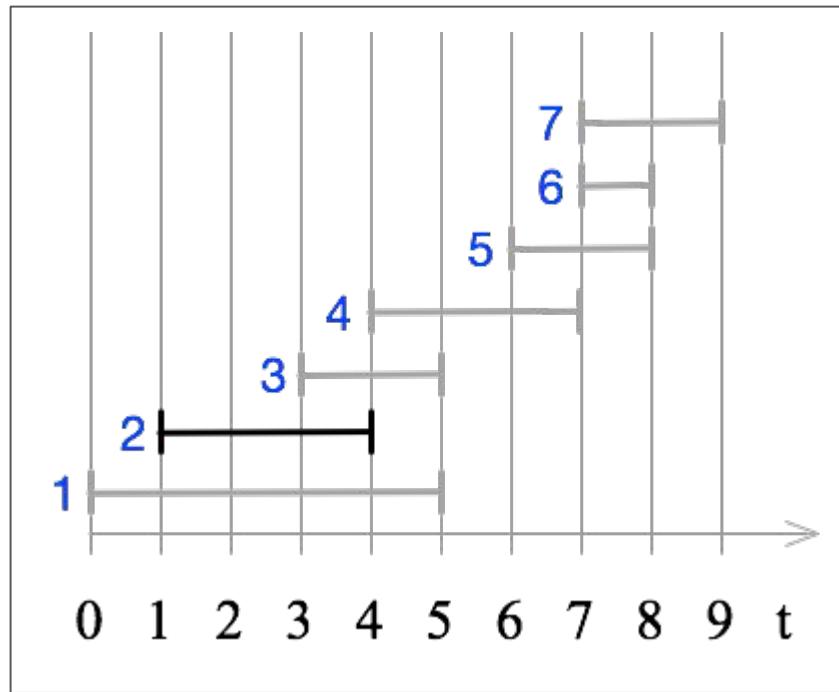
Let's try taking them in order of finish time:



$i$	$s(i)$	$f(i)$
2	1	4
1	0	5
3	3	5
4	4	7
5	6	8
6	7	8
7	7	9

# Okay, the sooner this ends, the better!

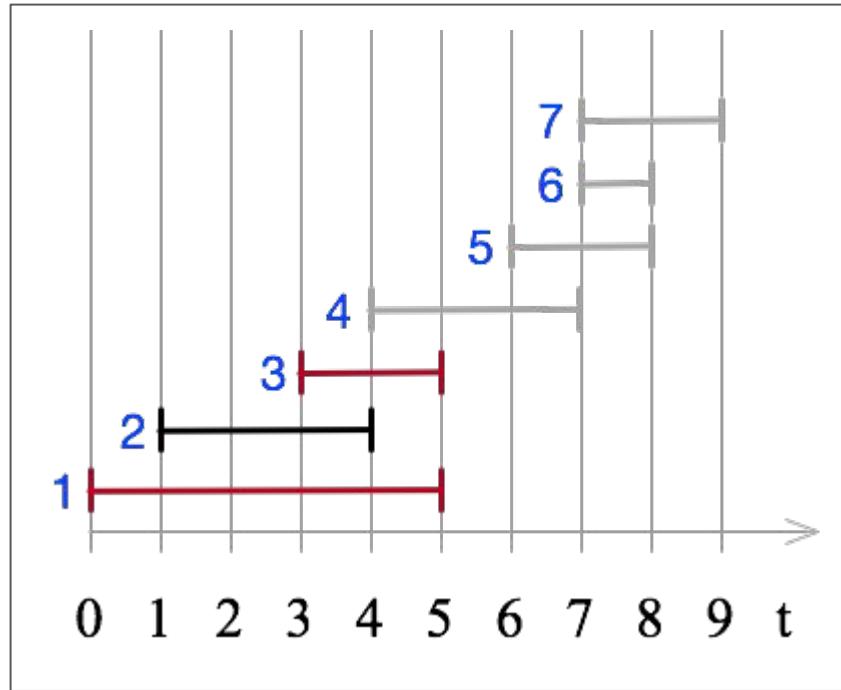
Let's try taking them in order of finish time:



$i$	$s(i)$	$f(i)$
2	1	4
1	0	5
3	3	5
4	4	7
5	6	8
6	7	8
7	7	9

# Okay, the sooner this ends, the better!

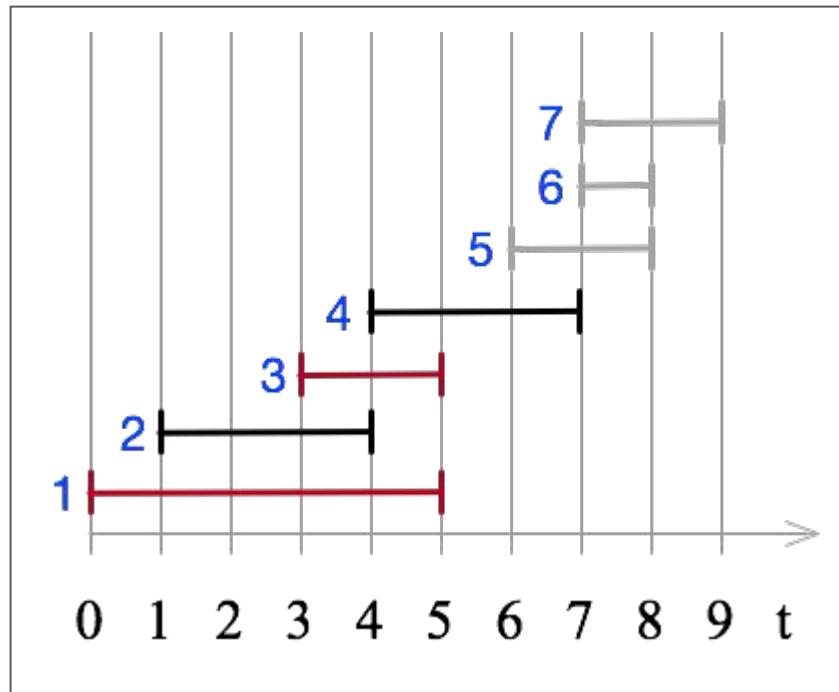
Let's try taking them in order of finish time:



$i$	$s(i)$	$f(i)$
2	1	4
1	0	5
3	3	5
4	4	7
5	6	8
6	7	8
7	7	9

# Okay, the sooner this ends, the better!

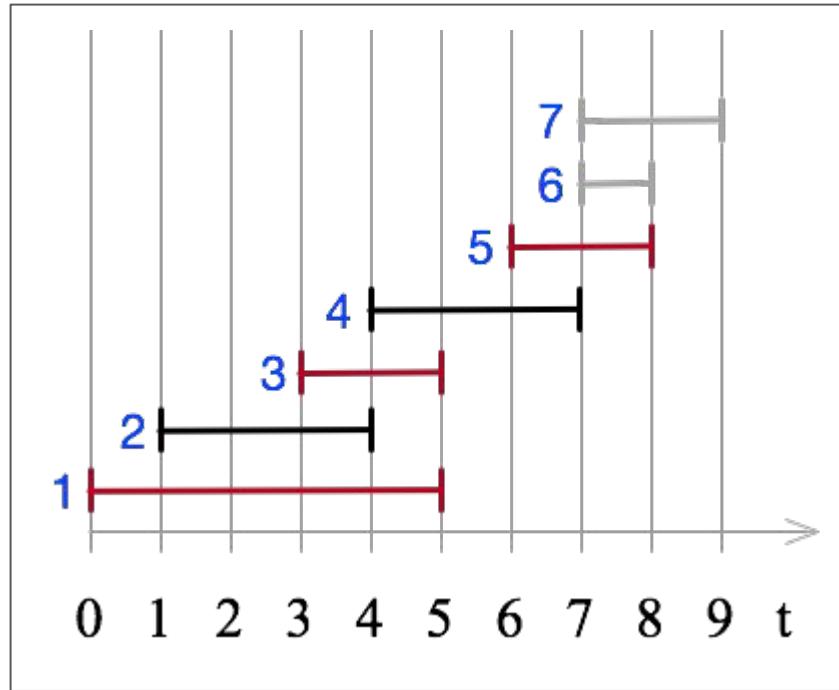
Let's try taking them in order of finish time:



$i$	$s(i)$	$f(i)$
2	1	4
1	0	5
3	3	5
4	4	7
5	6	8
6	7	8
7	7	9

# Okay, the sooner this ends, the better!

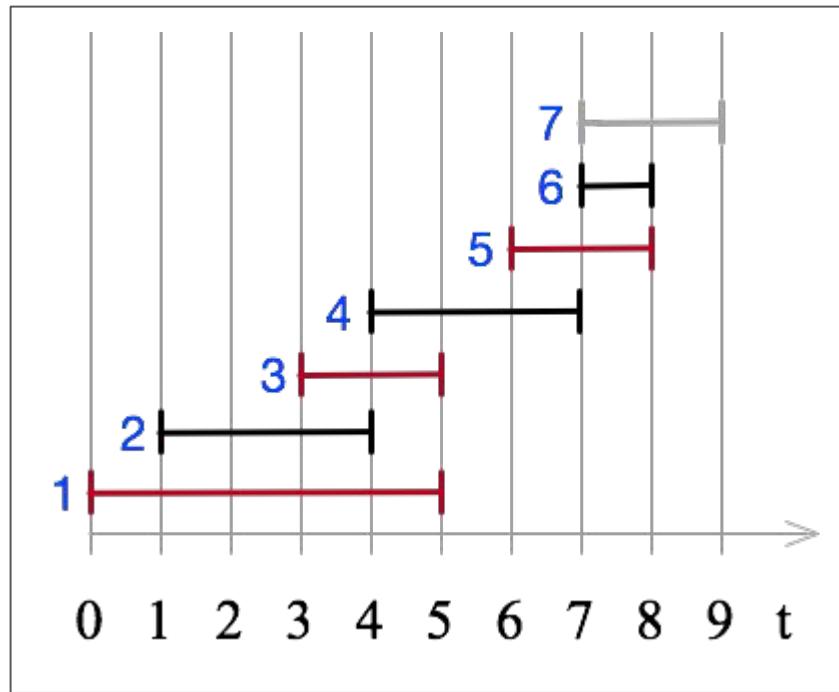
Let's try taking them in order of finish time:



$i$	$s(i)$	$f(i)$
2	1	4
1	0	5
3	3	5
4	4	7
5	6	8
6	7	8
7	7	9

# Okay, the sooner this ends, the better!

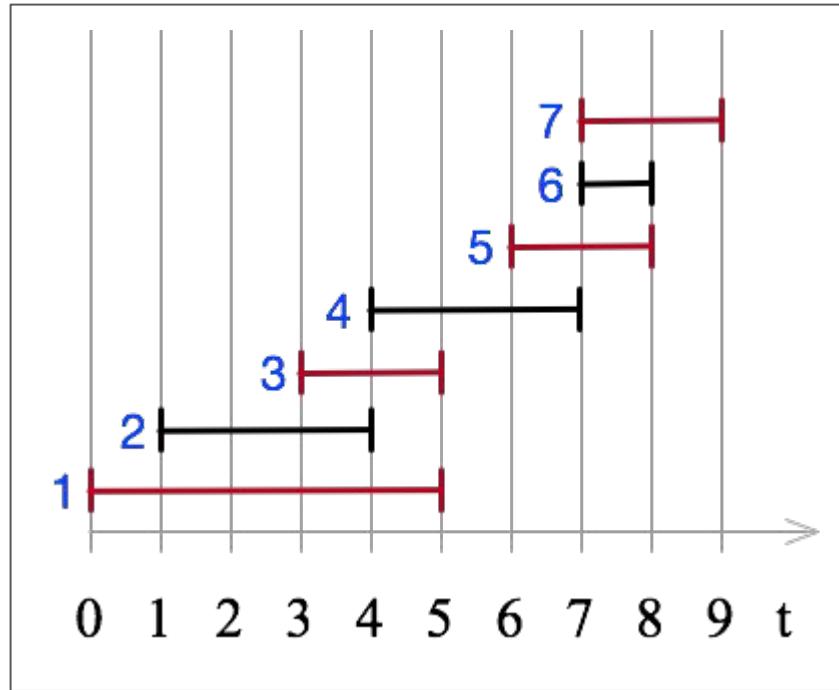
Let's try taking them in order of finish time:



$i$	$s(i)$	$f(i)$
2	1	4
1	0	5
3	3	5
4	4	7
5	6	8
6	7	8
7	7	9

# Okay, the sooner this ends, the better!

Let's try taking them in order of finish time:

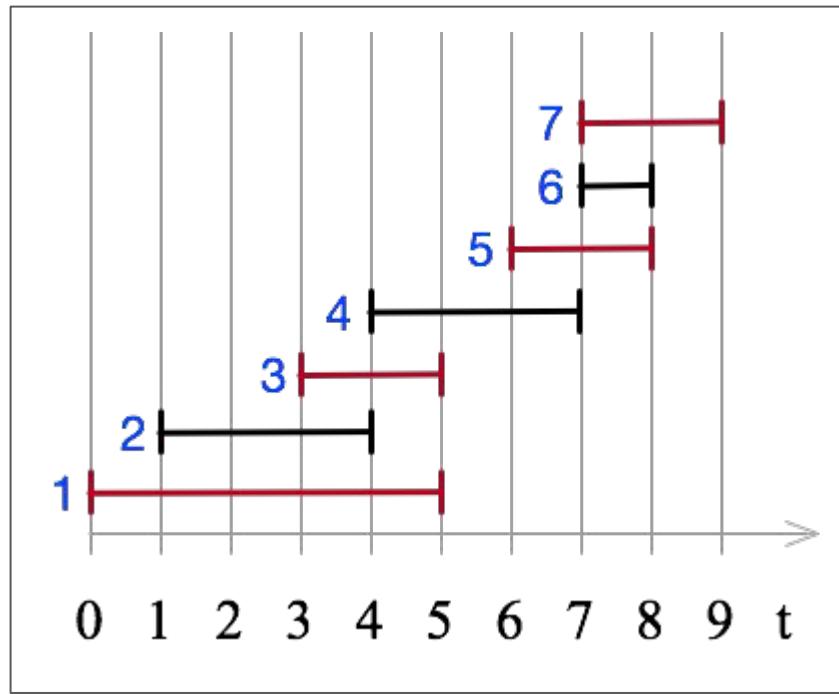


$i$	$s(i)$	$f(i)$
2	1	4
1	0	5
3	3	5
4	4	7
5	6	8
6	7	8
7	7	9

# Okay, the sooner this ends, the better!

Let's try taking them in order of finish time:

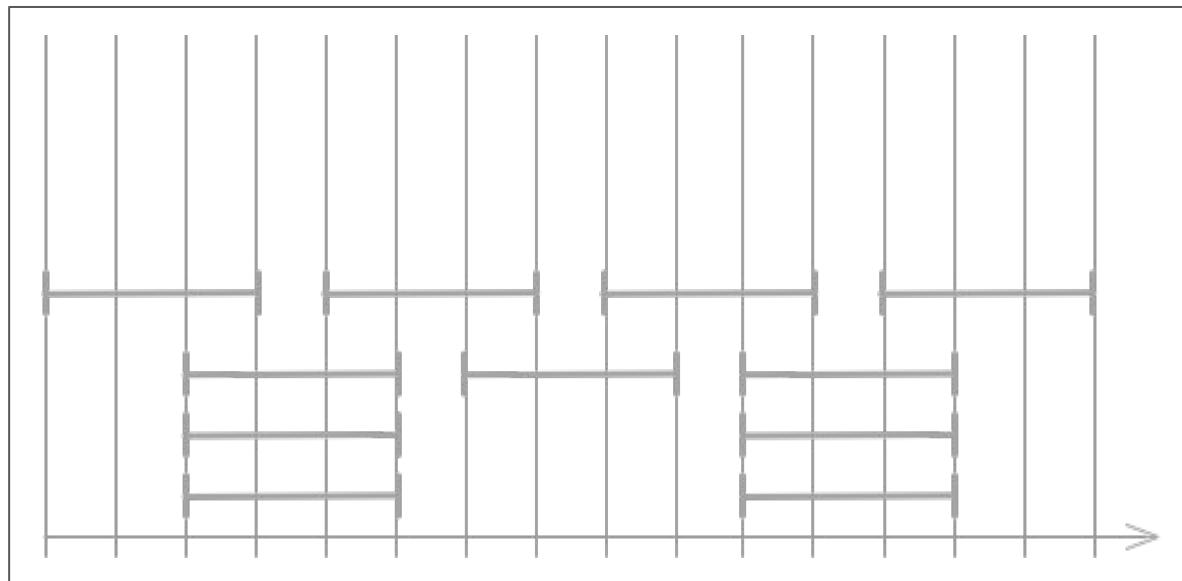
**3 requests fulfilled!**



$i$	$s(i)$	$f(i)$
2	1	4
1	0	5
3	3	5
4	4	7
5	6	8
6	7	8
7	7	9

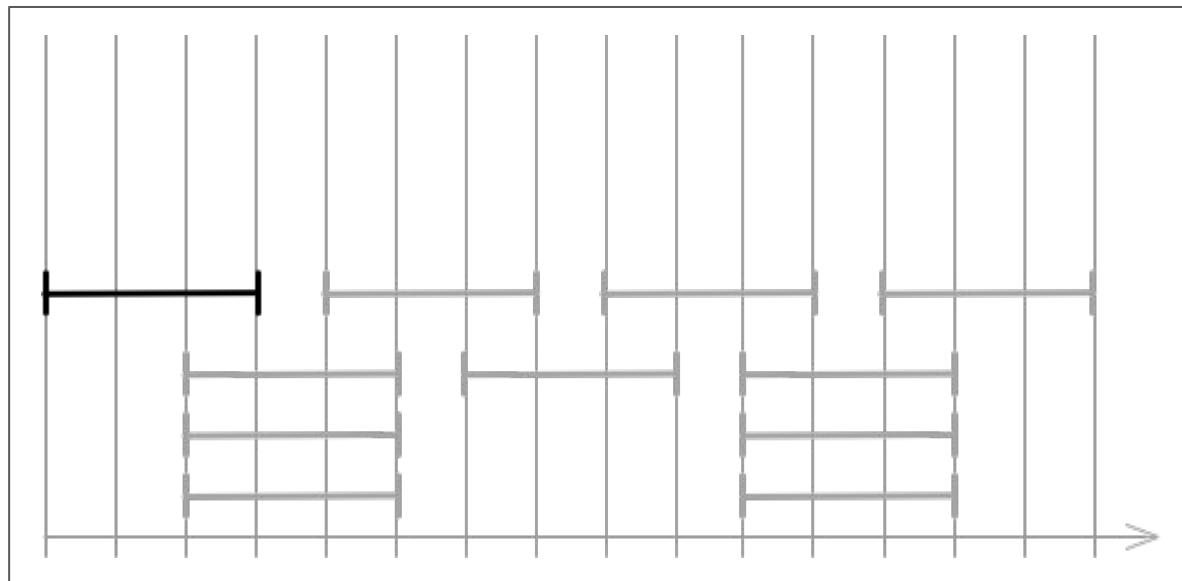
# Will this always work?

Let's try taking them in order of finish time:



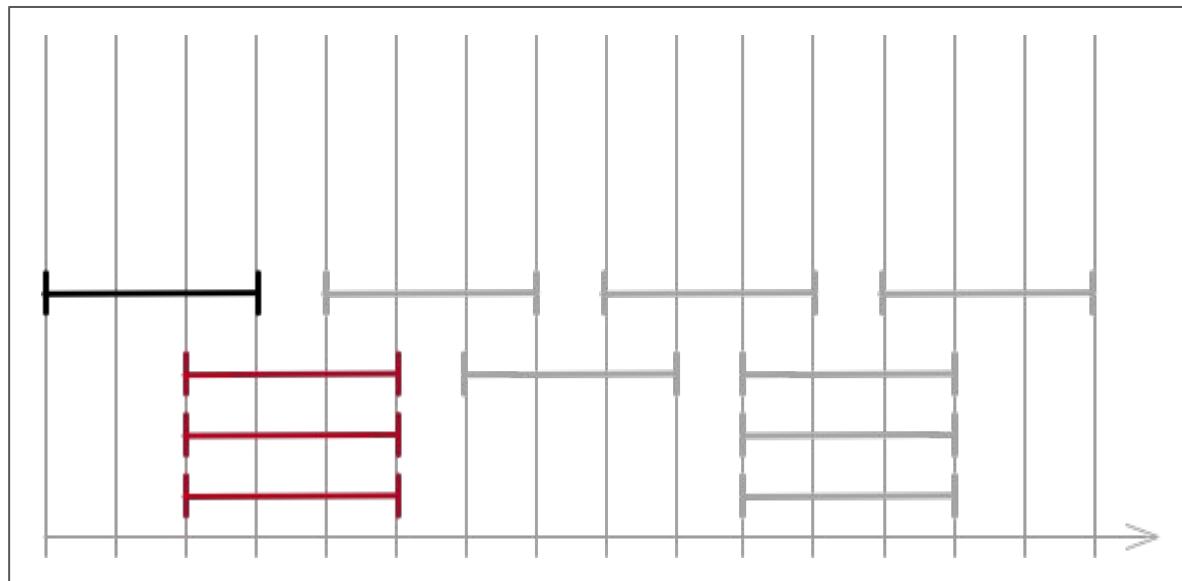
# Will this always work?

Let's try taking them in order of finish time:



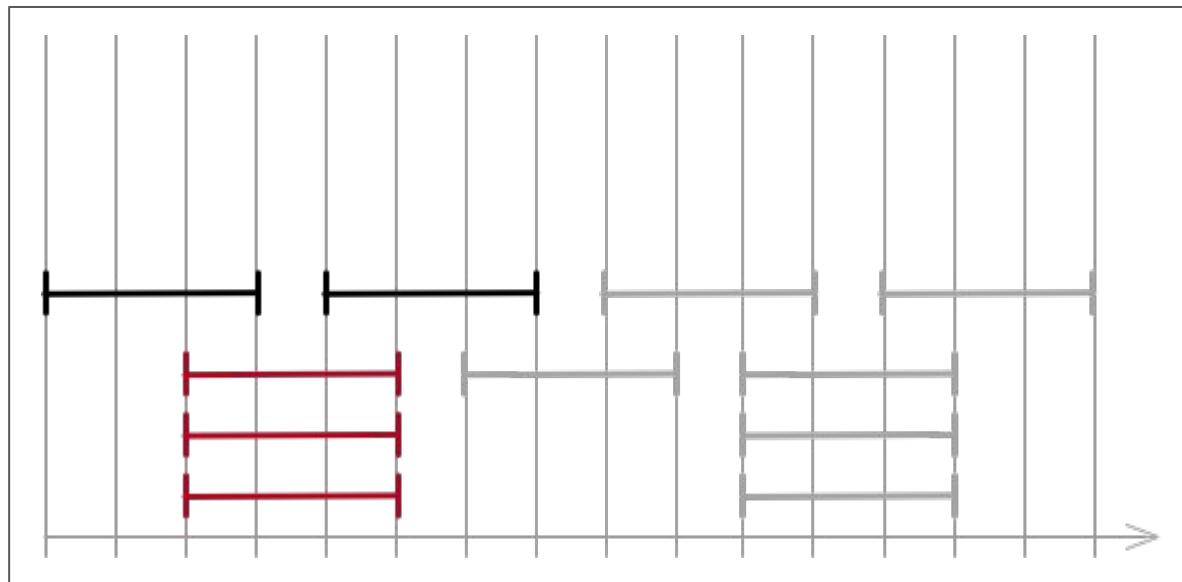
# Will this always work?

Let's try taking them in order of finish time:



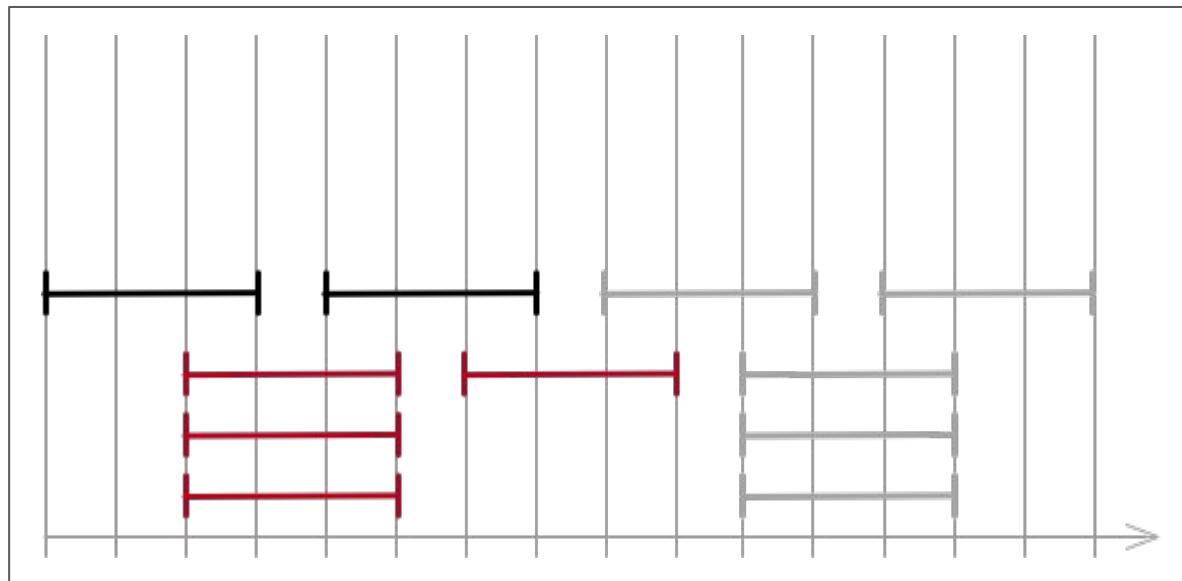
# Will this always work?

Let's try taking them in order of finish time:



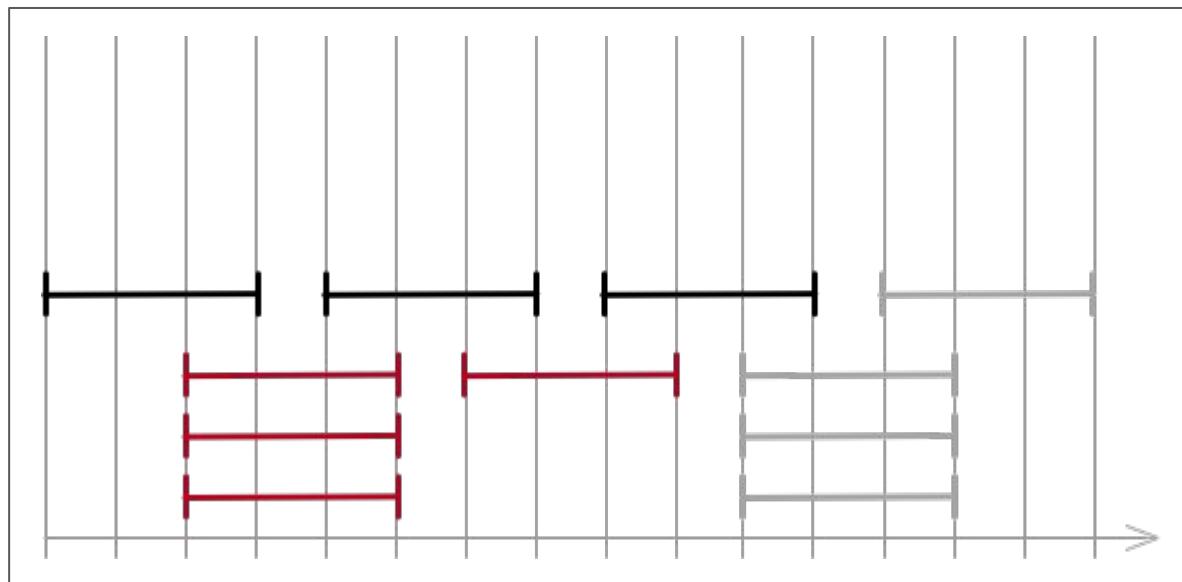
# Will this always work?

Let's try taking them in order of finish time:



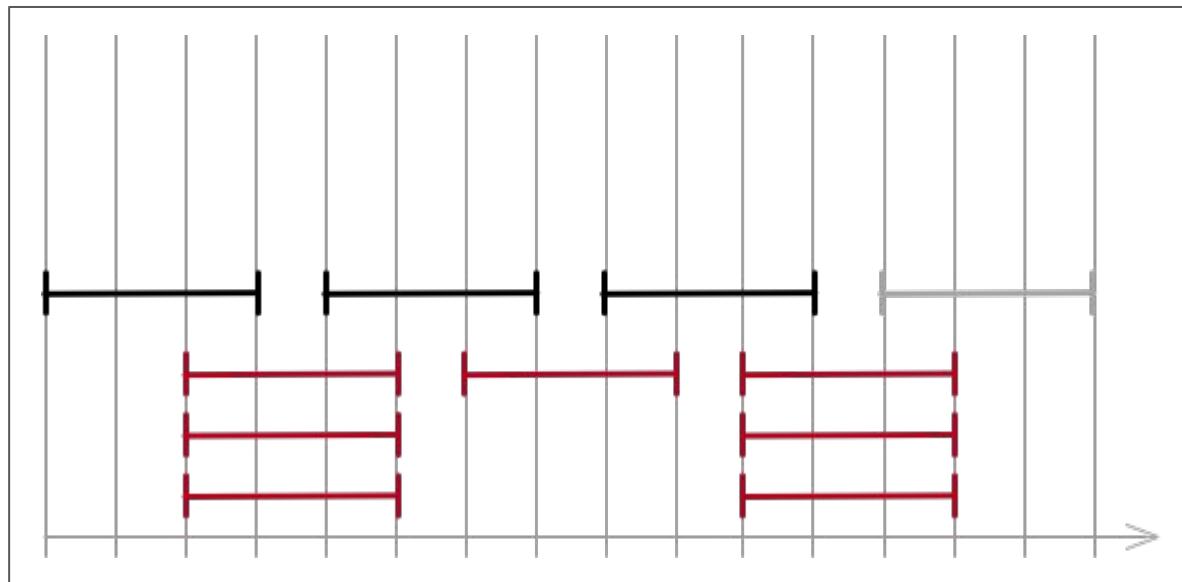
# Will this always work?

Let's try taking them in order of finish time:



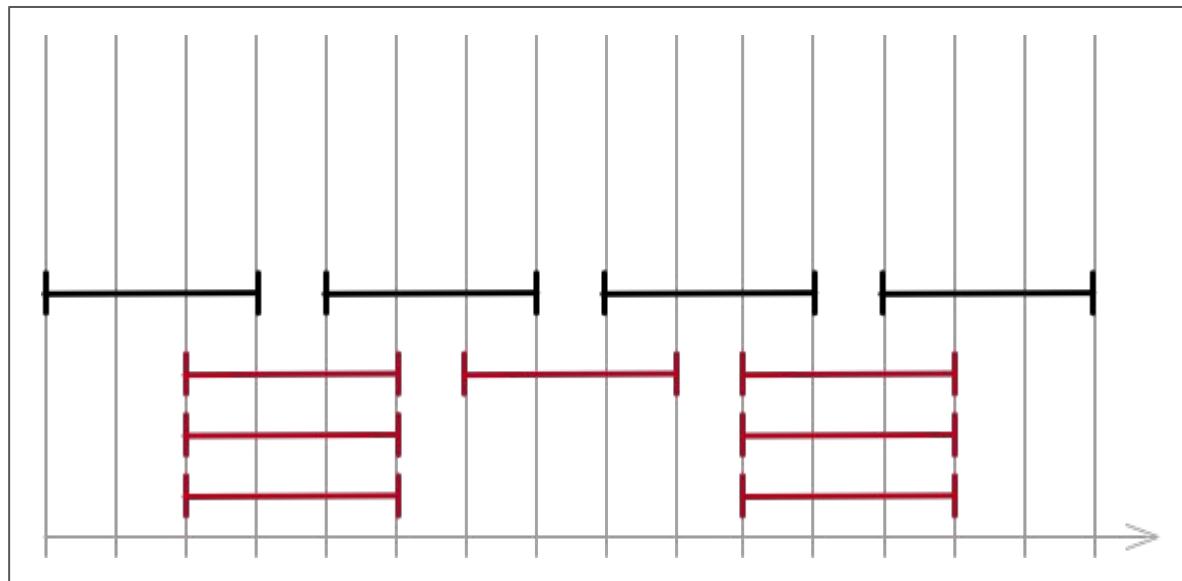
# Will this always work?

Let's try taking them in order of finish time:



# Will this always work?

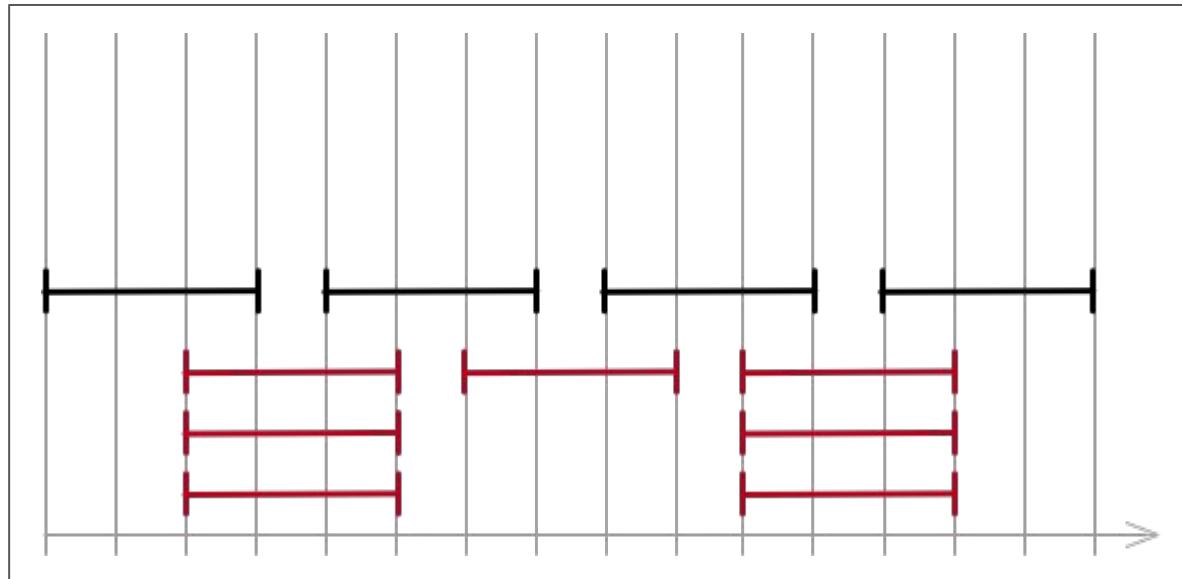
Let's try taking them in order of finish time:



# Will this always work?

Let's try taking them in order of finish time:

**4 requests fulfilled!**



# Finish time sorting is optimal

- Recall: **optimal** set
  - Compatible (no overlap)
  - Maximum-sized
- Proof
  - Let  $S$  be set produced by algorithm using finish time to sort
  - Observe  $S$  is **compatible** by construction

$R$  = all requests sorted by finish time

$S = \{\}$  // selected requests

while ( $R$  is not empty)

    remove next request  $i$  from  $R$

    add  $i$  to  $S$

    remove all overlapping requests from  $R$

# How do we prove maximum-sized?

- Let  $k = \#$  requests selected by algorithm;  $|S| = k$
- Let  $i_1, i_2, \dots, i_k$  be indices of selected requests,  
ordered by start/finish time

requests are compatible =>

$$s(i_r) < f(i_r) \leq s(i_{r+1})$$

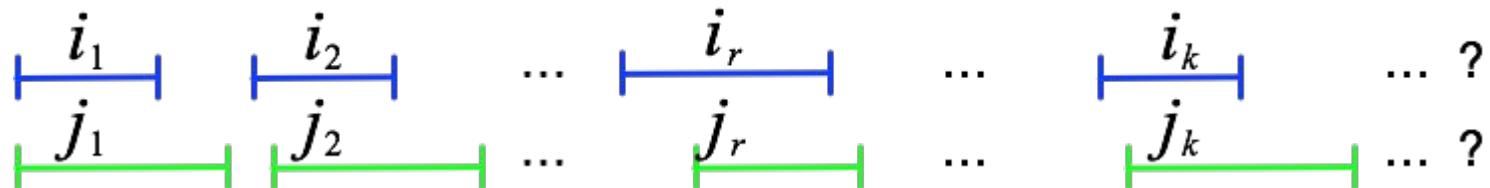
sorting by start = sorting by finish

# How do we prove maximum-sized?

- Let  $k = \#$  requests selected by algorithm;  $|S| = k$
- Let  $i_1, i_2, \dots, i_k$  be indices of selected requests, ordered by start/finish time
- Let  $O$  be other compatible set of  $m$  requests;  $|O| = m$
- Let  $j_1, j_2, \dots, j_m$  be indices of selected requests, ordered by start/finish time

Claim “greedy stays ahead”

$$f(i_r) \leq f(j_r) \text{ for all } r = 1, 2, \dots$$

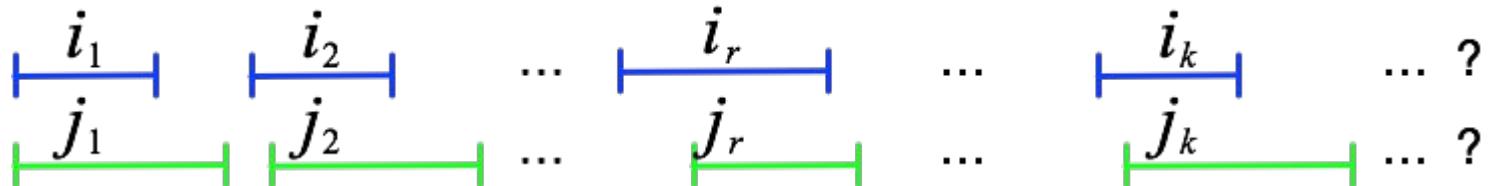


# How do we prove maximum-sized?

- Let  $k = \#$  requests selected by algorithm;  $|S| = k$
- Let  $i_1, i_2, \dots, i_k$  be indices of selected requests,  
 $r^{\text{th}}$  request in  $S$  finishes no later than  $r^{\text{th}}$  request in  $O$
- Let  $J_1, J_2, \dots, J_m$  be indices of selected requests,  
ordered by start finish time

Claim “greedy stays ahead”

$$f(i_r) \leq f(J_r) \text{ for all } r = 1, 2, \dots$$



Greedy stays ahead

The diagram illustrates a sequence of requests. It consists of two rows of horizontal bars. The top row, labeled with blue brackets, represents the sequence of requests  $i_1, i_2, \dots, i_r, \dots, i_k$ . The bottom row, labeled with green brackets, represents the sequence of requests  $j_1, j_2, \dots, j_r, \dots, j_k$ . The bars are positioned such that each bar in the top row overlaps with the corresponding bar in the bottom row. Ellipses between the bars indicate that there are more requests in the sequence.

Claim  $f(i_r) \leq f(j_r)$  for all  $r = 1, 2, \dots$

Proof by induction on index  $r$

Base case:  $r = 1$

- Algorithm chose  $i_1$  to be request with smallest finish =>  
 $f(i_1) \leq f(j_1)$

Inductive hypothesis: assume  $f(i_{r-1}) \leq f(j_{r-1})$  for index  $r-1 (>1)$

Inductive step: show for index  $r$

Greedy stays ahead

The diagram illustrates a sequence of requests. It consists of two rows of horizontal bars. The top row, labeled with blue bars, represents the sequence of requests  $i_1, i_2, \dots, i_r, \dots, i_k, \dots, ?$ . The bottom row, labeled with green bars, represents the sequence of requests  $j_1, j_2, \dots, j_r, \dots, j_k, \dots, ?$ . The bars are positioned such that each bar in the top row overlaps with the corresponding bar in the bottom row, indicating that the requests in the top row start at or after the end of the requests in the bottom row.

Claim  $f(i_r) \leq f(j_r)$  for all  $r = 1, 2, \dots$

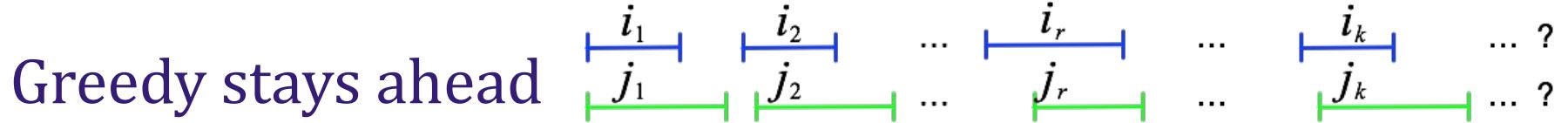
Proof by induction on index  $r$

Base case:  $r = 1$

- Algorithm chose  $i_1$  to be request with smallest finish =>  
 $f(i_1) \leq f(j_1)$

Inductive hypothesis: assume  $f(i_{r-1}) \leq f(j_{r-1})$  for index  $r-1 (>1)$

Inductive step: show for index  $r$



Claim  $f(i_r) \leq f(j_r)$  for all  $r = 1, 2, \dots$

Proof by induction on index  $r$

Inductive hypothesis (IH): assume  $f(i_{r-1}) \leq f(j_{r-1})$  for index  $r-1$  ( $>1$ )

Inductive step: show for index  $r$

- $O$  compatible, so  $f(j_{r-1}) \leq s(j_r)$
- By (IH),  $f(i_{r-1}) \leq f(j_{r-1})$ , so  $f(i_{r-1}) \leq s(j_r) \leq f(j_r)$
- Then request  $j_r$  compatible with  $S$ , so  $j_r$  in set  $R$  when choosing  $r^{\text{th}}$  request for  $S$
- Algorithm chooses next request with smallest finish (greedily), so  $f(i_r) \leq f(j_r)$

# Completing the proof: finish time sorting is optimal

- Claim algorithm outputs an **optimal** set
  - Compatible (no overlap)
  - Maximum-sized

$R$  = all requests sorted by finish time

$S = \{\}$  // selected requests

while ( $R$  is not empty)

    remove next request  $i$  from  $R$

    add  $i$  to  $S$

    remove all overlapping requests from  $R$

# Completing the proof: finish time sorting is optimal

- Claim algorithm outputs an **optimal** set
  - Compatible (no overlap)
  - Maximum-sized
- Proof
  - $S$  is **compatible** by construction
  - Suppose, for contradiction, there exists  $O$  with  $|O| > |S|$ ;  $m > k$ 
    - Since  $m > k$ ,  $O$  has request  $j_{k+1}$
    - $O$  compatible  $\Rightarrow f(j_k) \leq s(j_{k+1})$
    - Just proved Claim:  $f(i_r) \leq f(j_r)$  for all  $r = 1, 2, \dots, k$   
 $\Rightarrow f(i_k) \leq f(j_k) \leq s(j_{k+1})$
    - Then requests  $R$  included  $j_{k+1} \Rightarrow$  algorithm would add at least 1 more request, contradicting size  $k$

# Running time analysis

$R = \text{all requests sorted by finish time}$	$O(n \log n)$
$S = \{\} // \text{selected requests}$	$O(1)$
while ( $R$ is not empty)	$O(n)$
remove next request $i$ from $R$	$O(1)$
add $i$ to $S$	$O(1)$
remove all overlapping requests from $R$	$O(n)$
-----	
	$O(n^2)$

# Can we do better?

$R = \text{all requests sorted by finish time}$	$O(n \log n)$
$S = \{\} // \text{selected requests}$	$O(1)$
while ( $R$ is not empty)	$O(n)$
remove next request $i$ from $R$	$O(1)$
add $i$ to $S$	$O(1)$
<b>remove all overlapping requests from <math>R</math></b>	<b><math>O(n)</math></b>
<hr/>	
	$O(n^2)$

# Can we do better?

$R = \text{all requests sorted by finish time}$	$O(n \log n)$
$S = \{\} // \text{selected requests}$	$O(1)$
while ( $R$ is not empty)	$O(n)$
remove next request $i$ from $R$	$O(1)$
add $i$ to $S$	$O(1)$
<del>remove all overlapping requests from <math>R</math></del>	<b><math>\Theta(n)</math></b>
-----	
	$O(n^2)$

# Can we do better?

$R$ = all requests sorted by finish time	$O(n \log n)$
$S = \{\}$ // selected requests	$O(1)$
<b>current_finish_time = 0</b>	<b><math>O(1)</math></b>
while ( $R$ is not empty)	$O(n)$
remove next request $i$ from $R$	$O(1)$
<b>if <math>s(i) \geq \text{current\_finish\_time}</math></b>	<b><math>O(1)</math></b>
add $i$ to $S$	$O(1)$
<b>current_finish_time = <math>f(i)</math></b>	<b><math>O(1)</math></b>
-----	
	$O(n \log n)$

# Can we do even better?

$R$  = all requests sorted by finish time

$O(n \log n)$

$S = \{\}$  // selected requests

$O(1)$

$\text{current\_finish\_time} = 0$

$O(1)$

while ( $R$  is not empty)

$O(n)$

    remove next request  $i$  from  $R$

$O(1)$

    if  $s(i) \geq \text{current\_finish\_time}$

$O(1)$

        add  $i$  to  $S$

$O(1)$

$\text{current\_finish\_time} = f(i)$

$O(1)$

---

$O(n \log n)$

# Can we do even better?

**$R$  = all requests sorted by finish time**

$S = \{\}$  // selected requests

`current_finish_time = 0`

`while ( $R$  is not empty)`

`remove next request  $i$  from  $R$`

`if  $s(i) \geq \text{current\_finish\_time}$`

`add  $i$  to  $S$`

`$\text{current\_finish\_time} = f(i)$`

$\Theta(n \log n)$

$O(1)$

$O(1)$

$O(n)$

$O(1)$

$O(1)$

$O(1)$

$O(1)$

---

$\Theta(n \log n)$

# Greedy algorithm design

- Build up set one item at a time
- Choose next item “greedily”
- No going back

How do we know when it works?

- Proof techniques
  - Greedy stays ahead
  - Exchange argument

# Where to put the talks?

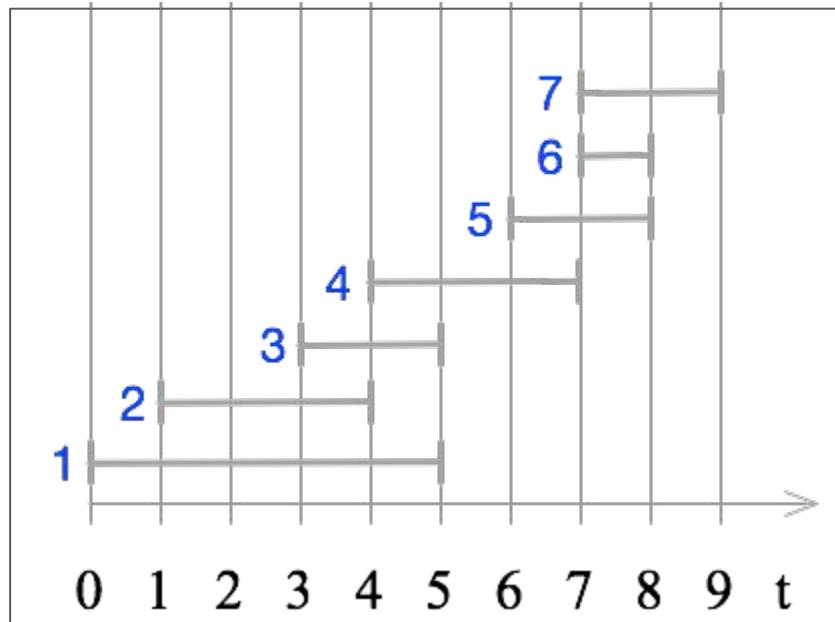
Senior symposium is coming up! As the organizer, you must assign rooms to the talks. Of course, you can't schedule two talks in the same room at one time!



*How do you assign talks to rooms so that you minimize the total number of rooms used for the event?*

## Problem 2: interval partitioning

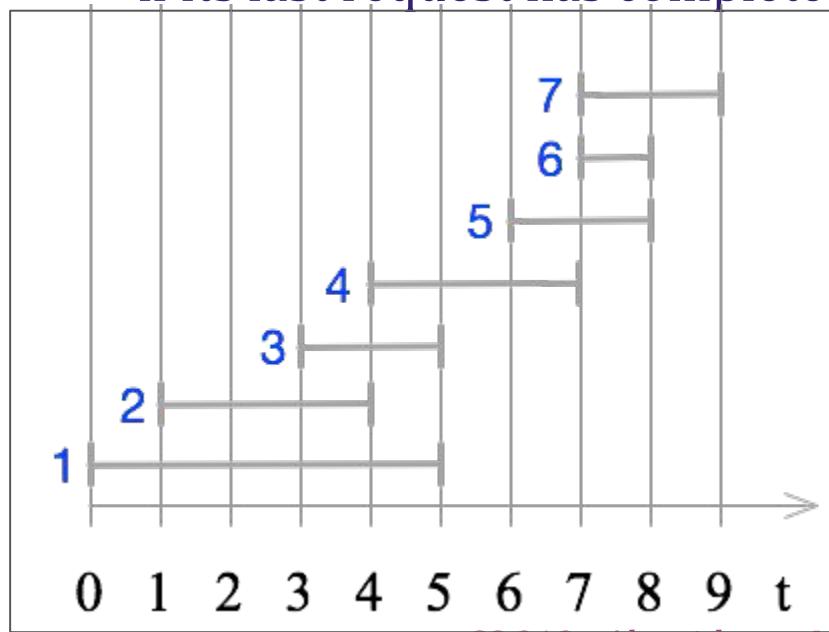
- Set of  $1, \dots, n$  requests
- $s(i), f(i)$  start and finish times



$i$	$s(i)$	$f(i)$
1	0	5
2	1	4
3	3	5
4	4	7
5	6	8
6	7	8
7	7	9

# Greedy algorithm for interval partitioning

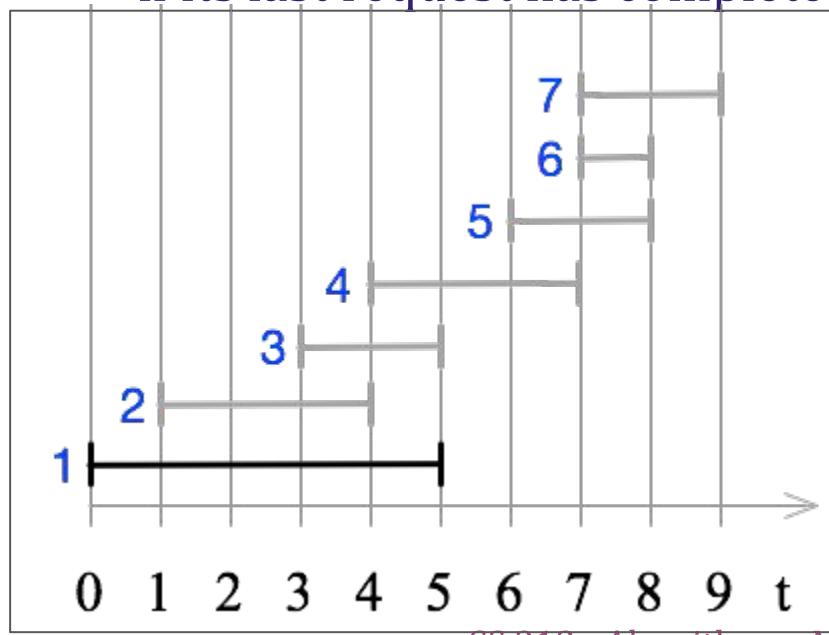
- Process requests ordered by start time
- For each request
  - allocate a new resource, or
  - reuse previously allocated resource if its last request has completed



$i$	$s(i)$	$f(i)$
1	0	5
2	1	4
3	3	5
4	4	7
5	6	8
6	7	8
7	7	9

# Greedy algorithm for interval partitioning

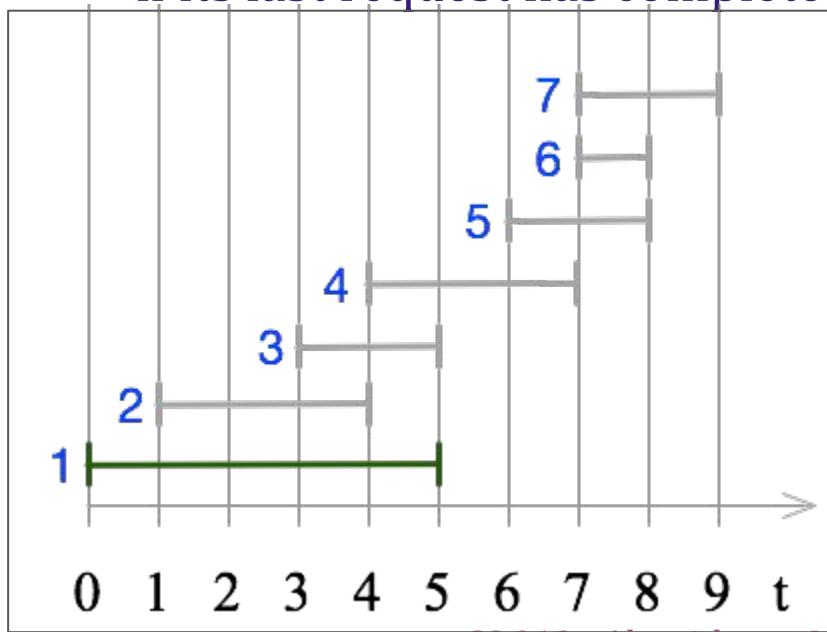
- Process requests ordered by start time
- For each request
  - allocate a new resource, or
  - reuse previously allocated resource if its last request has completed



$i$	$s(i)$	$f(i)$
1	0	5
2	1	4
3	3	5
4	4	7
5	6	8
6	7	8
7	7	9

# Greedy algorithm for interval partitioning

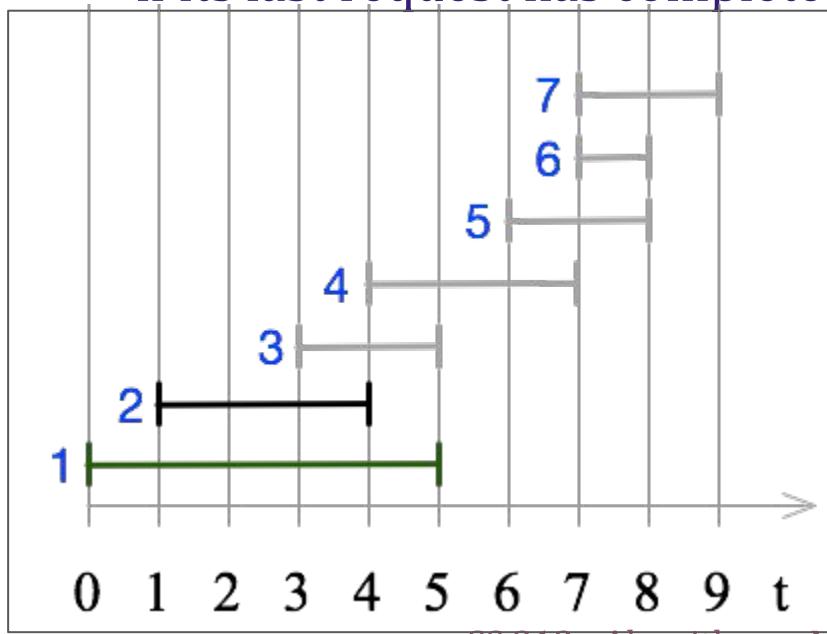
- Process requests ordered by start time
- For each request
  - **allocate a new resource**, or
  - reuse previously allocated resource if its last request has completed



$i$	$s(i)$	$f(i)$	
1	0	5	A
2	1	4	
3	3	5	
4	4	7	
5	6	8	
6	7	8	
7	7	9	

# Greedy algorithm for interval partitioning

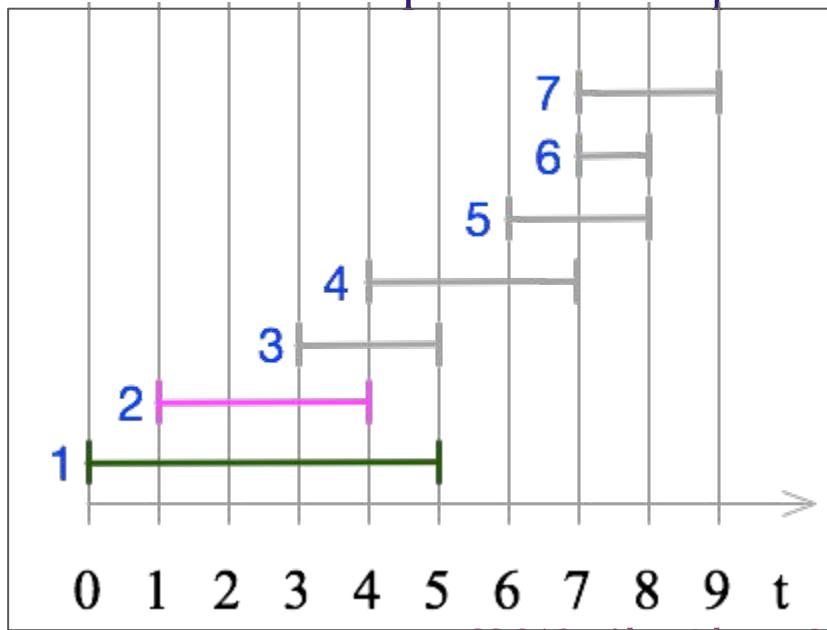
- Process requests ordered by start time
- For each request
  - allocate a new resource, or
  - reuse previously allocated resource if its last request has completed



$i$	$s(i)$	$f(i)$	
1	0	5	A
2	1	4	
3	3	5	
4	4	7	
5	6	8	
6	7	8	
7	7	9	

# Greedy algorithm for interval partitioning

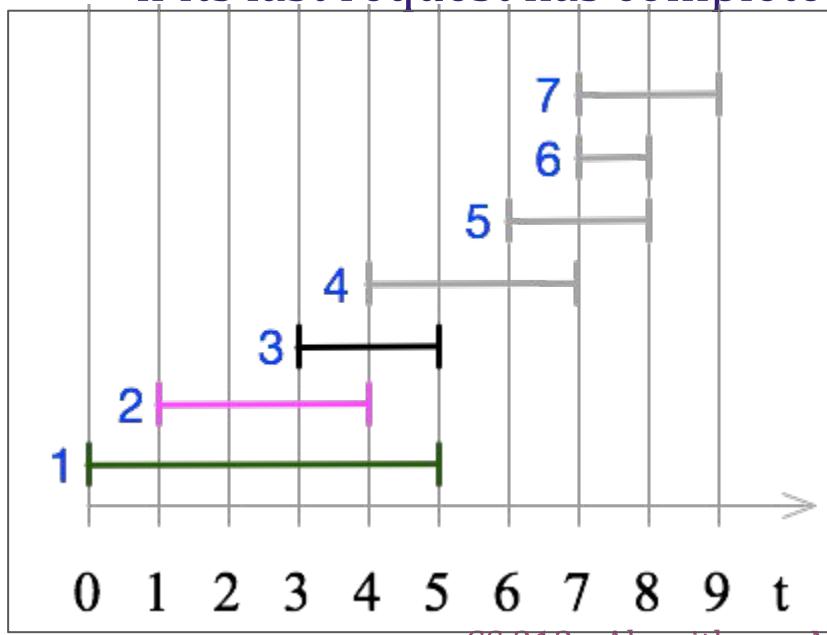
- Process requests ordered by start time
- For each request
  - **allocate a new resource**, or
  - reuse previously allocated resource if its last request has completed



$i$	$s(i)$	$f(i)$	
1	0	5	A
2	1	4	B
3	3	5	
4	4	7	
5	6	8	
6	7	8	
7	7	9	

# Greedy algorithm for interval partitioning

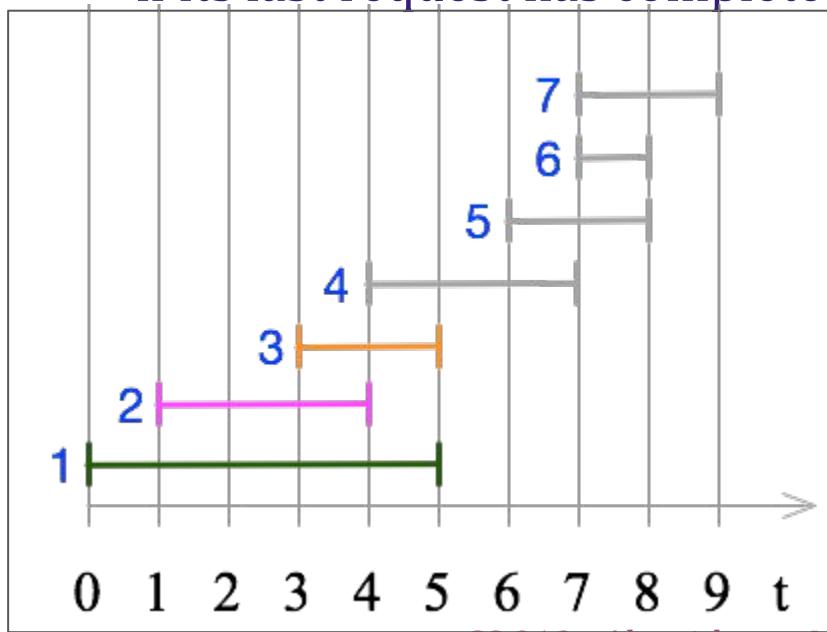
- Process requests ordered by start time
- For each request
  - allocate a new resource, or
  - reuse previously allocated resource if its last request has completed



$i$	$s(i)$	$f(i)$	
1	0	5	A
2	1	4	B
3	3	5	
4	4	7	
5	6	8	
6	7	8	
7	7	9	

# Greedy algorithm for interval partitioning

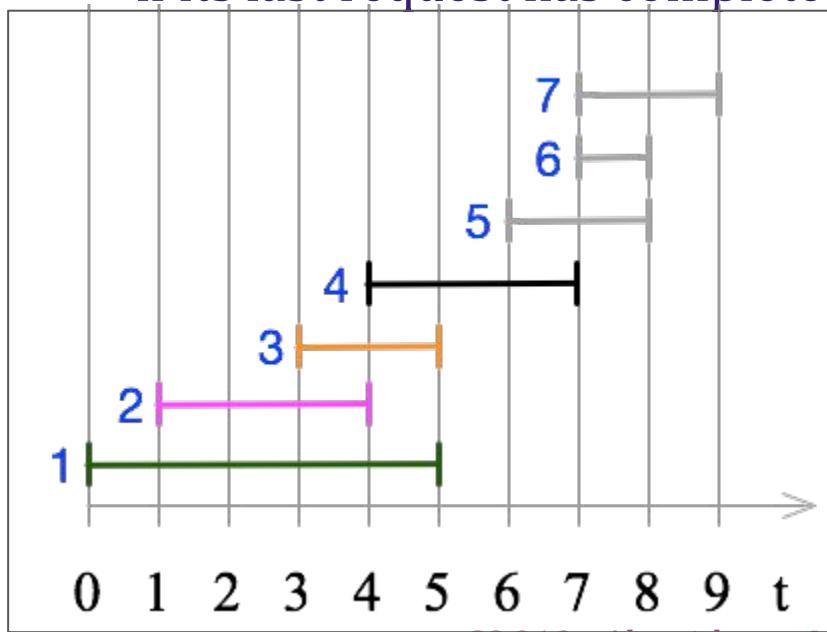
- Process requests ordered by start time
- For each request
  - **allocate a new resource**, or
  - reuse previously allocated resource if its last request has completed



$i$	$s(i)$	$f(i)$	
1	0	5	A
2	1	4	B
3	3	5	C
4	4	7	
5	6	8	
6	7	8	
7	7	9	

# Greedy algorithm for interval partitioning

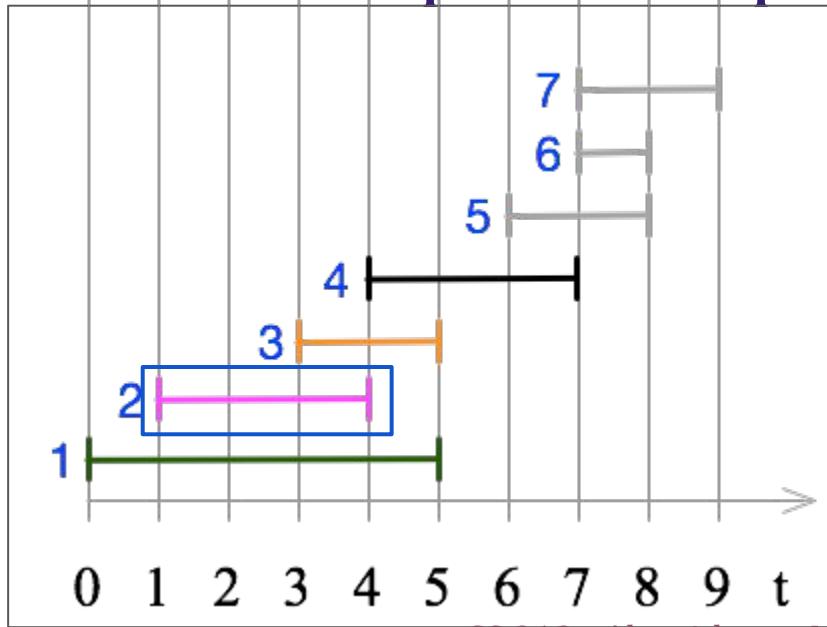
- Process requests ordered by start time
- For each request
  - allocate a new resource, or
  - reuse previously allocated resource if its last request has completed



$i$	$s(i)$	$f(i)$	
1	0	5	A
2	1	4	B
3	3	5	C
4	4	7	
5	6	8	
6	7	8	
7	7	9	

# Greedy algorithm for interval partitioning

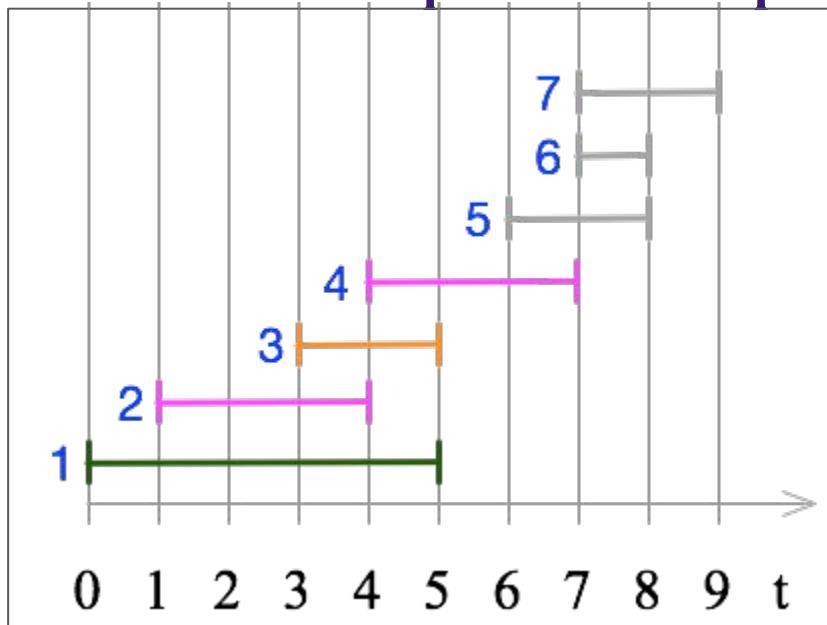
- Process requests ordered by start time
- For each request
  - allocate a new resource, or
  - reuse previously allocated resource  
**if its last request has completed**



$i$	$s(i)$	$f(i)$	
1	0	5	A
2	1	4	B
3	3	5	C
4	4	7	
5	6	8	
6	7	8	
7	7	9	

# Greedy algorithm for interval partitioning

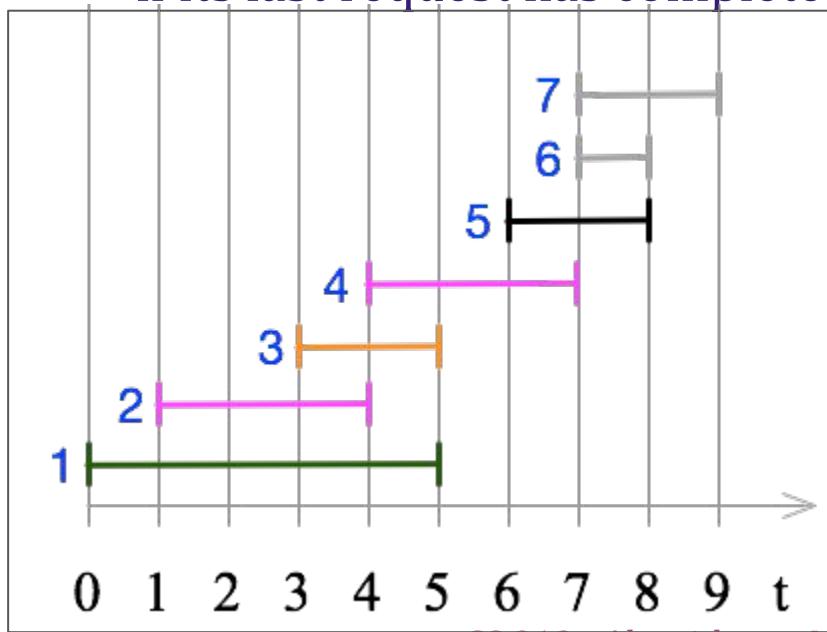
- Process requests ordered by start time
- For each request
  - allocate a new resource, or
  - reuse previously allocated resource  
**if its last request has completed**



$i$	$s(i)$	$f(i)$	
1	0	5	A
2	1	4	B
3	3	5	C
4	4	7	B
5	6	8	
6	7	8	
7	7	9	

# Greedy algorithm for interval partitioning

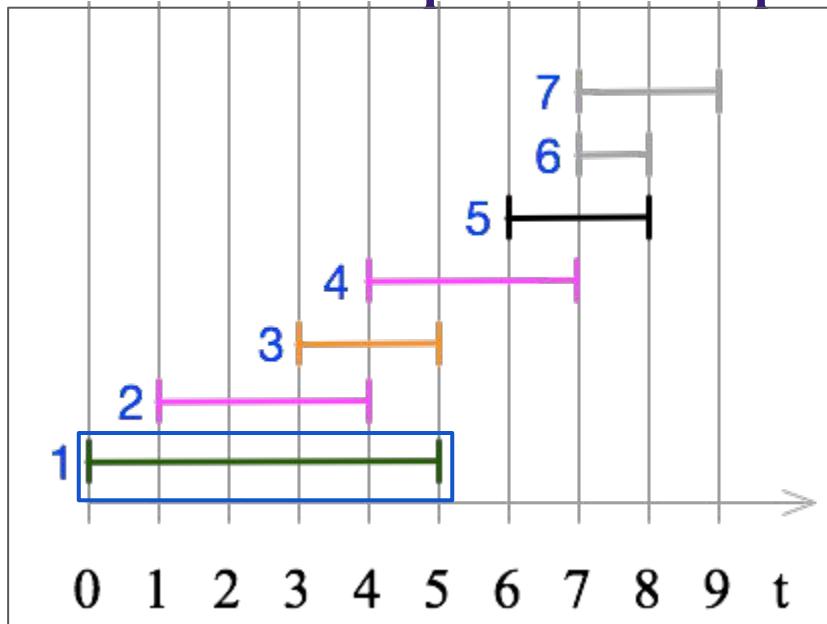
- Process requests ordered by start time
- For each request
  - allocate a new resource, or
  - reuse previously allocated resource if its last request has completed



$i$	$s(i)$	$f(i)$	
1	0	5	A
2	1	4	B
3	3	5	C
4	4	7	B
5	6	8	
6	7	8	
7	7	9	

# Greedy algorithm for interval partitioning

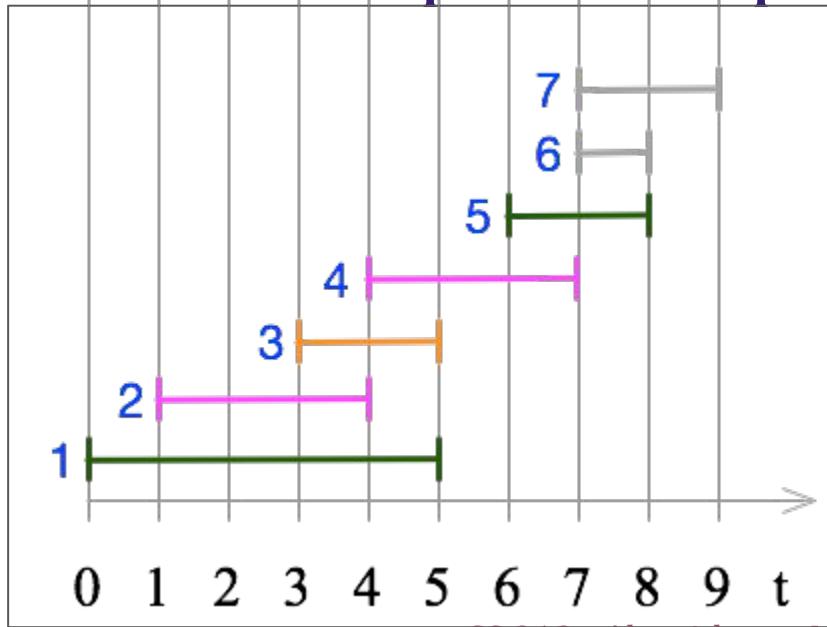
- Process requests ordered by start time
- For each request
  - allocate a new resource, or
  - reuse previously allocated resource  
**if its last request has completed**



$i$	$s(i)$	$f(i)$	
1	0	5	A
2	1	4	B
3	3	5	C
4	4	7	B
5	6	8	
6	7	8	
7	7	9	

# Greedy algorithm for interval partitioning

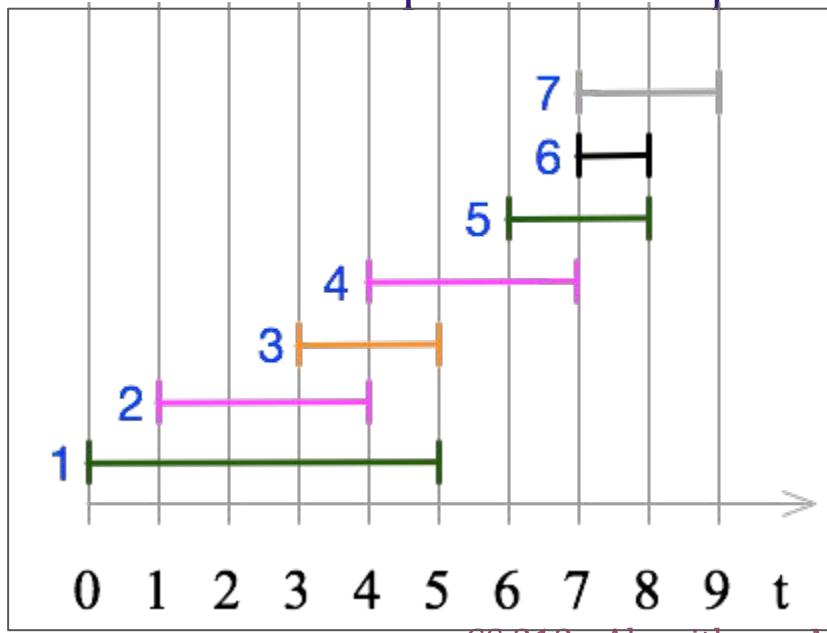
- Process requests ordered by start time
- For each request
  - allocate a new resource, or
  - reuse previously allocated resource  
**if its last request has completed**



$i$	$s(i)$	$f(i)$	
1	0	5	A
2	1	4	B
3	3	5	C
4	4	7	B
5	6	8	A
6	7	8	
7	7	9	

# Greedy algorithm for interval partitioning

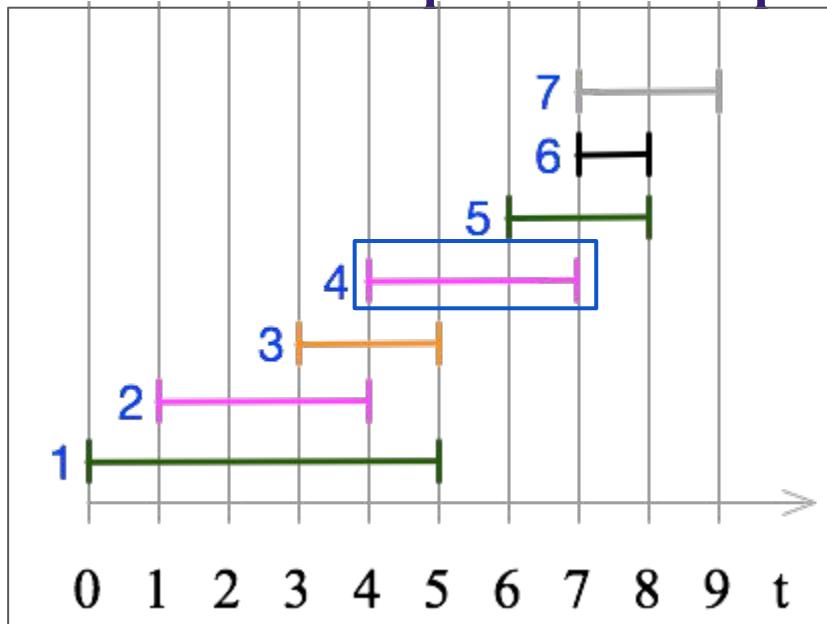
- Process requests ordered by start time
- For each request
  - allocate a new resource, or
  - reuse previously allocated resource if its last request has completed



$i$	$s(i)$	$f(i)$	
1	0	5	A
2	1	4	B
3	3	5	C
4	4	7	B
5	6	8	A
6	7	8	
7	7	9	

# Greedy algorithm for interval partitioning

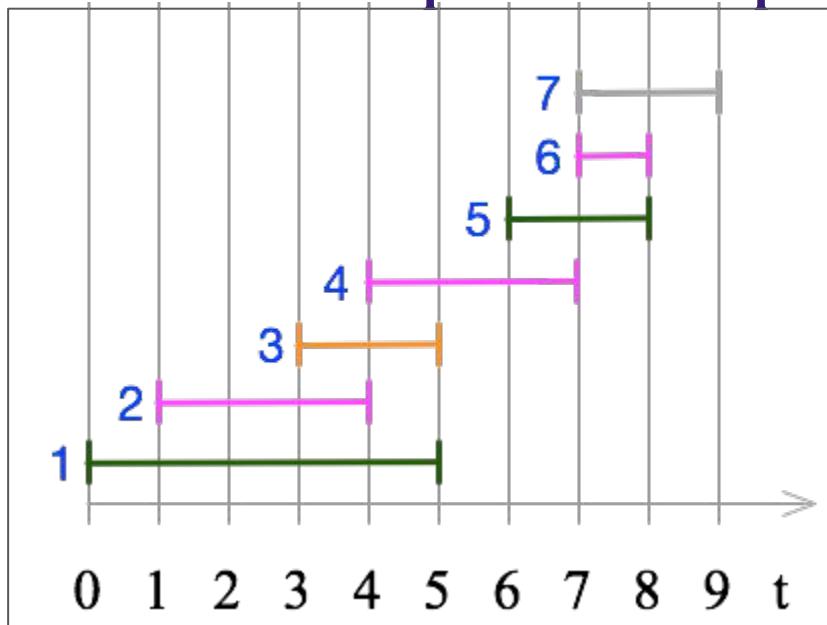
- Process requests ordered by start time
- For each request
  - allocate a new resource, or
  - reuse previously allocated resource  
**if its last request has completed**



$i$	$s(i)$	$f(i)$	
1	0	5	A
2	1	4	B
3	3	5	C
4	4	7	B
5	6	8	A
6	7	8	
7	7	9	

# Greedy algorithm for interval partitioning

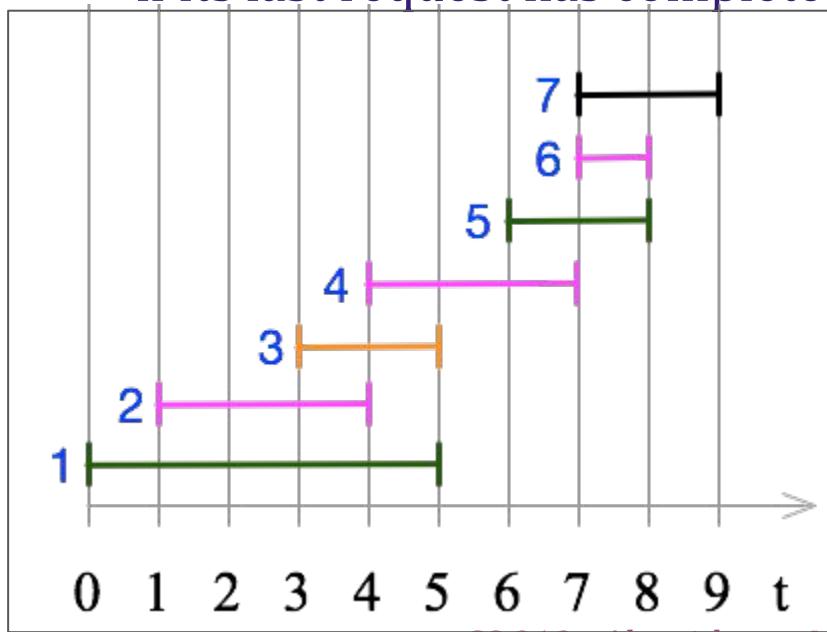
- Process requests ordered by start time
- For each request
  - allocate a new resource, or
  - reuse previously allocated resource  
**if its last request has completed**



$i$	$s(i)$	$f(i)$	
1	0	5	A
2	1	4	B
3	3	5	C
4	4	7	B
5	6	8	A
6	7	8	B
7	7	9	

# Greedy algorithm for interval partitioning

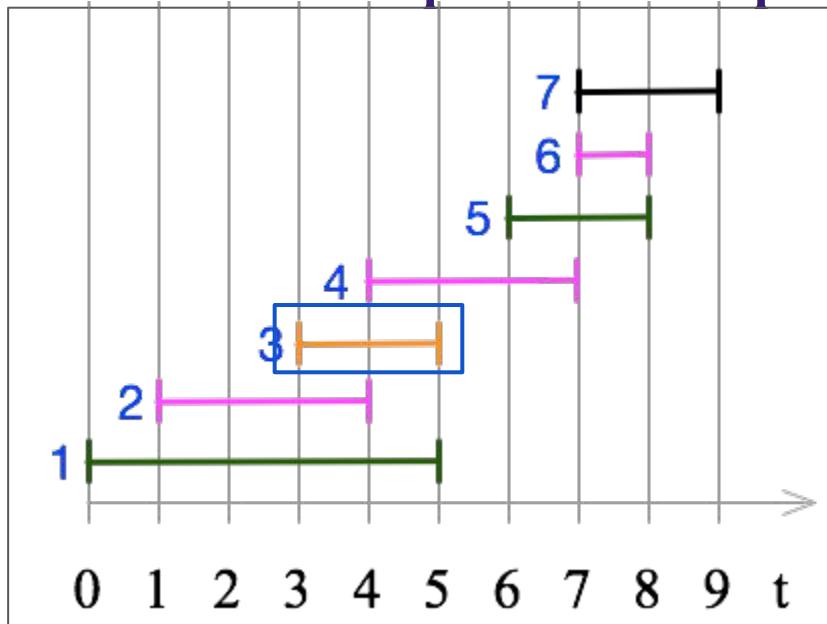
- Process requests ordered by start time
- For each request
  - allocate a new resource, or
  - reuse previously allocated resource if its last request has completed



$i$	$s(i)$	$f(i)$	
1	0	5	A
2	1	4	B
3	3	5	C
4	4	7	B
5	6	8	A
6	7	8	B
7	7	9	

# Greedy algorithm for interval partitioning

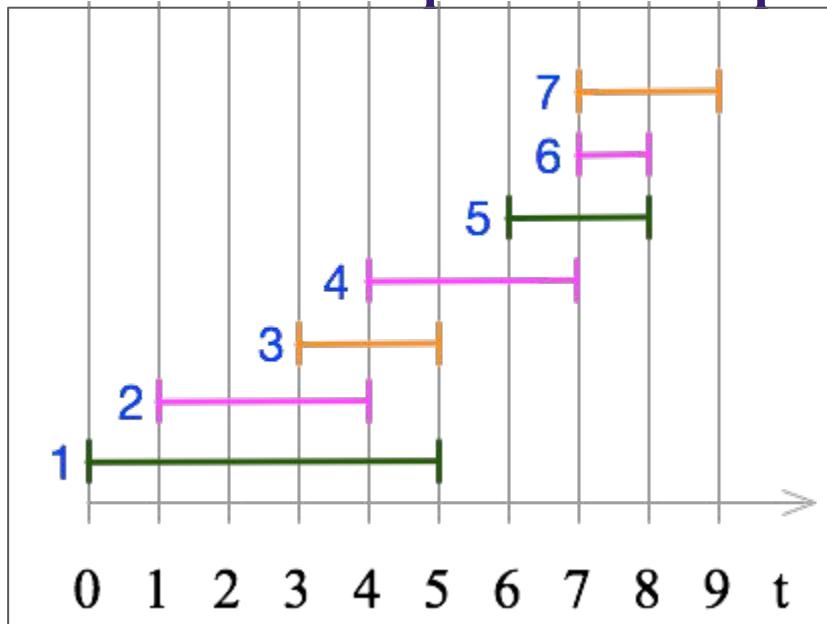
- Process requests ordered by start time
- For each request
  - allocate a new resource, or
  - reuse previously allocated resource  
**if its last request has completed**



$i$	$s(i)$	$f(i)$	
1	0	5	A
2	1	4	B
3	3	5	C
4	4	7	B
5	6	8	A
6	7	8	B
7	7	9	

# Greedy algorithm for interval partitioning

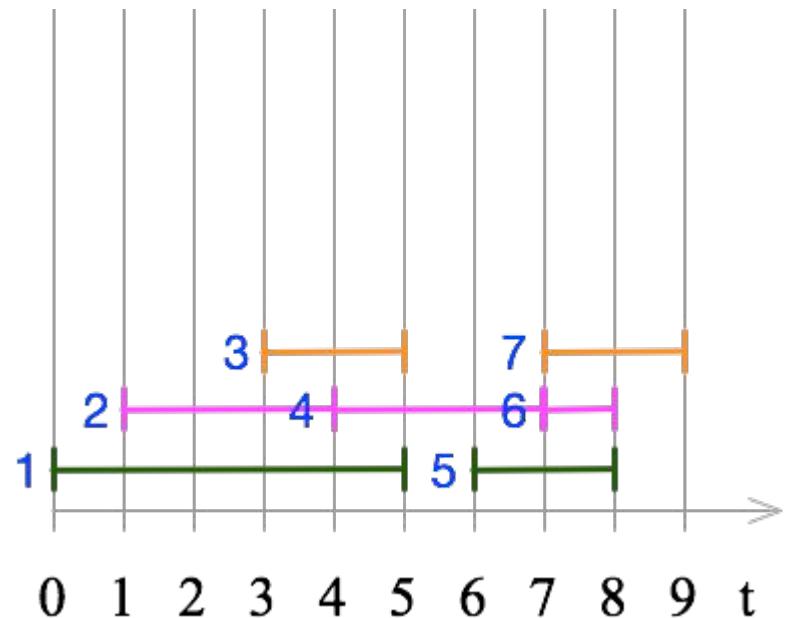
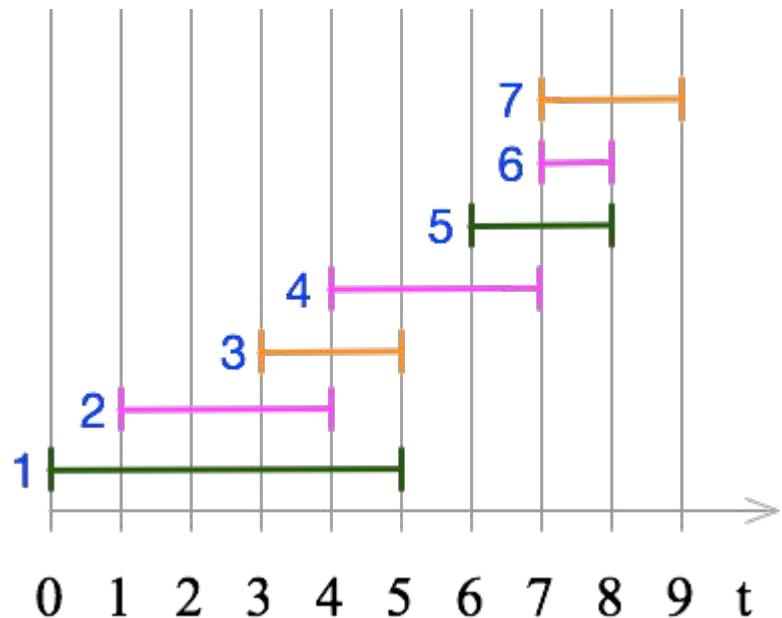
- Process requests ordered by start time
- For each request
  - allocate a new resource, or
  - reuse previously allocated resource  
**if its last request has completed**



$i$	$s(i)$	$f(i)$	
1	0	5	A
2	1	4	B
3	3	5	C
4	4	7	B
5	6	8	A
6	7	8	B
7	7	9	C

# Final schedule

**3 rooms used**



# Analysis: do we get an optimal schedule?

- Let  $d$  be **depth**: maximum number of requests that overlap
- Any solution requires at least this many resources
- Claim greedy algorithm uses exactly  $d$  resources and is therefore optimal.
- Proof: By contradiction.
  - Suppose greedy uses more than  $d$  resources
  - There must have been  $d$  requests running when  $(d+1)^{\text{st}}$  resource is allocated
  - This set of  $d+1$  requests overlap => depth >  $d$
  - Contradiction!

# Running time analysis

- Exercise