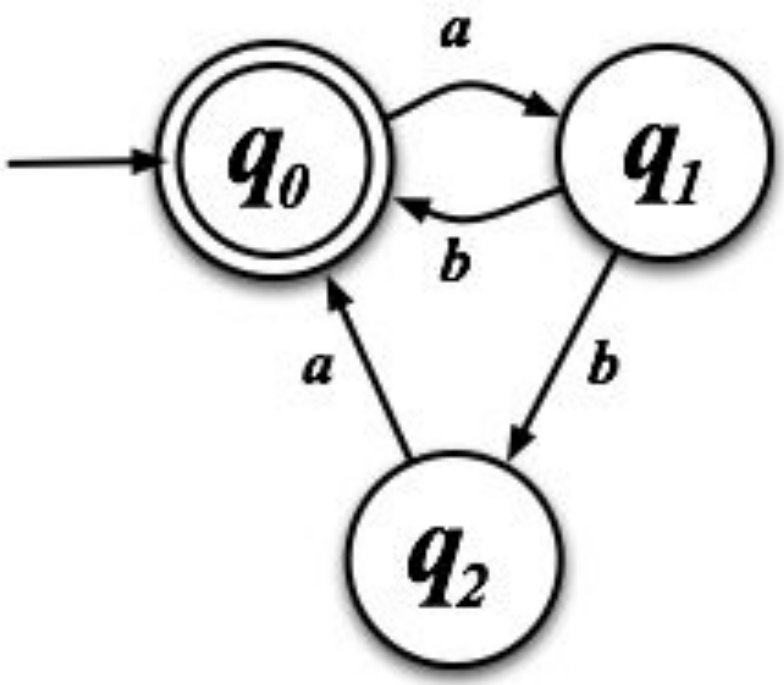
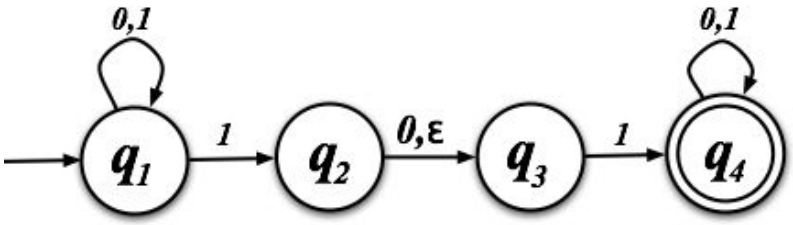
Regular expressions

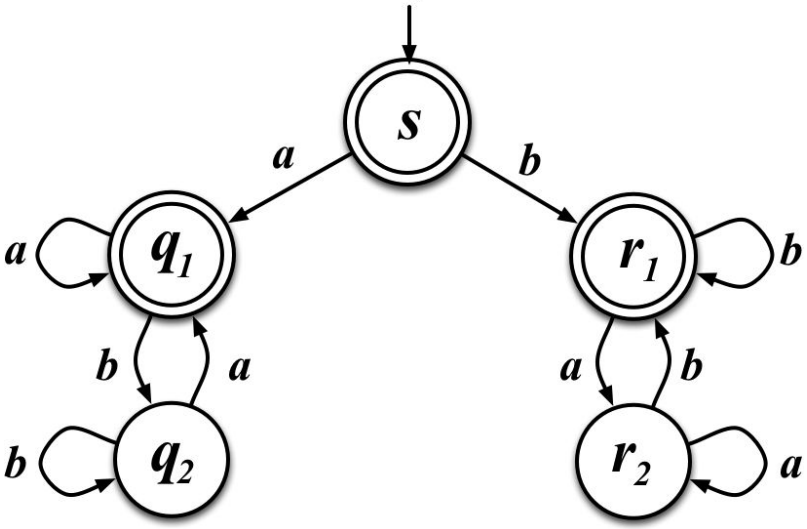
Sipser 1.3 (pages 63-76)

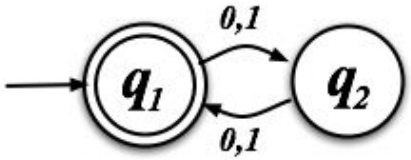
Looks familiar...



Your turn now!







Formally

- Definition 1.52:

  Say that R is a *regular expression* if R is

  1. a for some a in the alphabet Σ
  2. ε
  3. ∅
  4. $(R_1 \cup R_2)$, where R_1 and R_2 are regular expressions
  5. $(R_1 \circ R_2)$, where R_1 and R_2 are regular expressions
  6. (R_1^*), where R_1 is a regular expression

Examples

- $0^*10^* = \{w \mid \quad\}$
- $= \{w \mid w \text{ is a string of odd length}\}$
- $(0 \cup \varepsilon)(1 \cup \varepsilon) =$
- $(01)^*\varnothing =$
- $(+ \cup - \cup \varepsilon)(DD^* \cup DD^*.D^* \cup D^*.DD^*) =$

  $\text{where } D = \{0,1,2,3,4,5,6,7,8,9\}$

Identities

- Let R be a regular expression
  - $R \circ \emptyset$ =
  - $R \circ \varepsilon$ =
  - $R \cup \emptyset$ =
  - $R \cup \varepsilon$ =

Regular expressions describe...

regular languages!

Regular expressions and NFAs

- Theorem 1.54: A language is regular if and only if some regular expression describes it.
- Proof ($\Leftarrow$)
  1. **If *a* $\in \Sigma$, *then a is regular.***
  2. ***ε is regular.***
  3. $\emptyset$ ***is regular.***
  4. ***If*** R_1 ***and*** R_2 ***are regular, then*** $(R_1 \cup R_2)$ ***is regular.***
  5. ***If*** R_1 ***and*** R_2 ***are regular, then*** $(R_1 \circ R_2)$ ***is regular.***
  6. ***If*** R_1 ***is a regular, then*** (R_1^*) ***is regular***

$(0 \cup 1)^* 10 \cup \varepsilon$

$\bar{R}_6\ \bar{R}_7$

R_5 $\quad$ $\bar{R}_8\ \bar{R}_9$

R_3 $\quad$ R_4

R_1 $\quad$ $\bar{R}_2$

$R_6 : 0$

$R_7 : 1$

$R_5 : R_6 \cup R_7$

$R_3 : (R_5)^*$

$R_8 : 1$

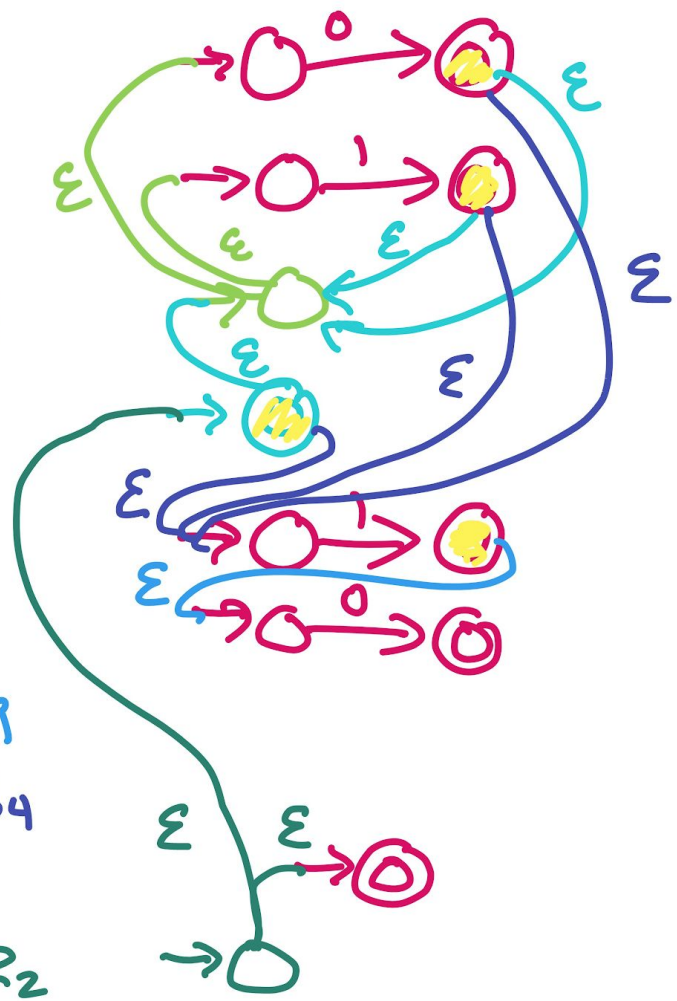
$R_9 : 0$

$R_4 : R_8 \circ R_9$

$R_1 : R_2 \circ R_4$

$R_2 : \varepsilon$

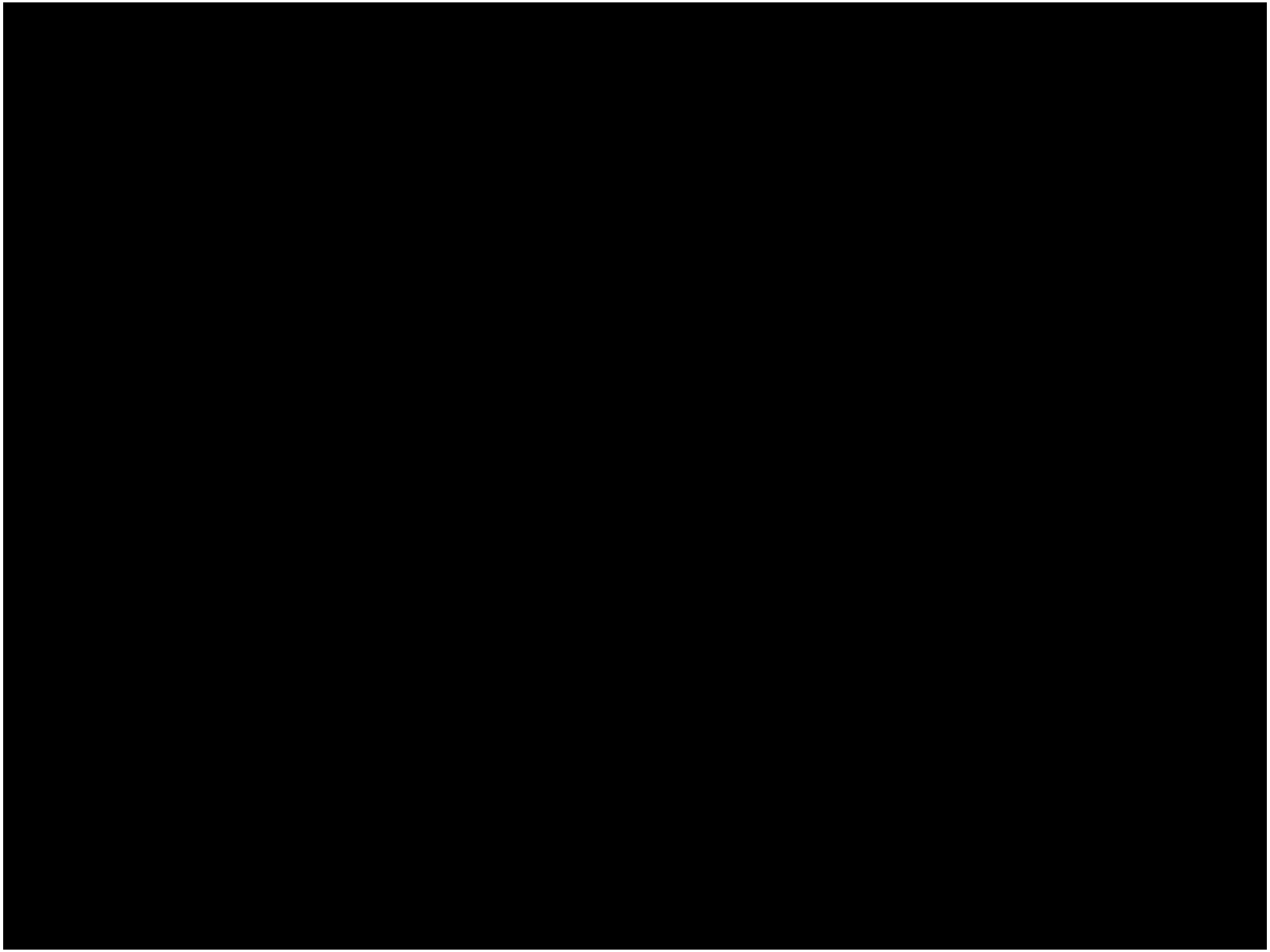
$R : R_1 \cup R_2$



Proof in action

- Build an NFA to that recognizes the regular expression

$$a(a \cup b)^*a$$



Video credit - Kexin Wang (F19)