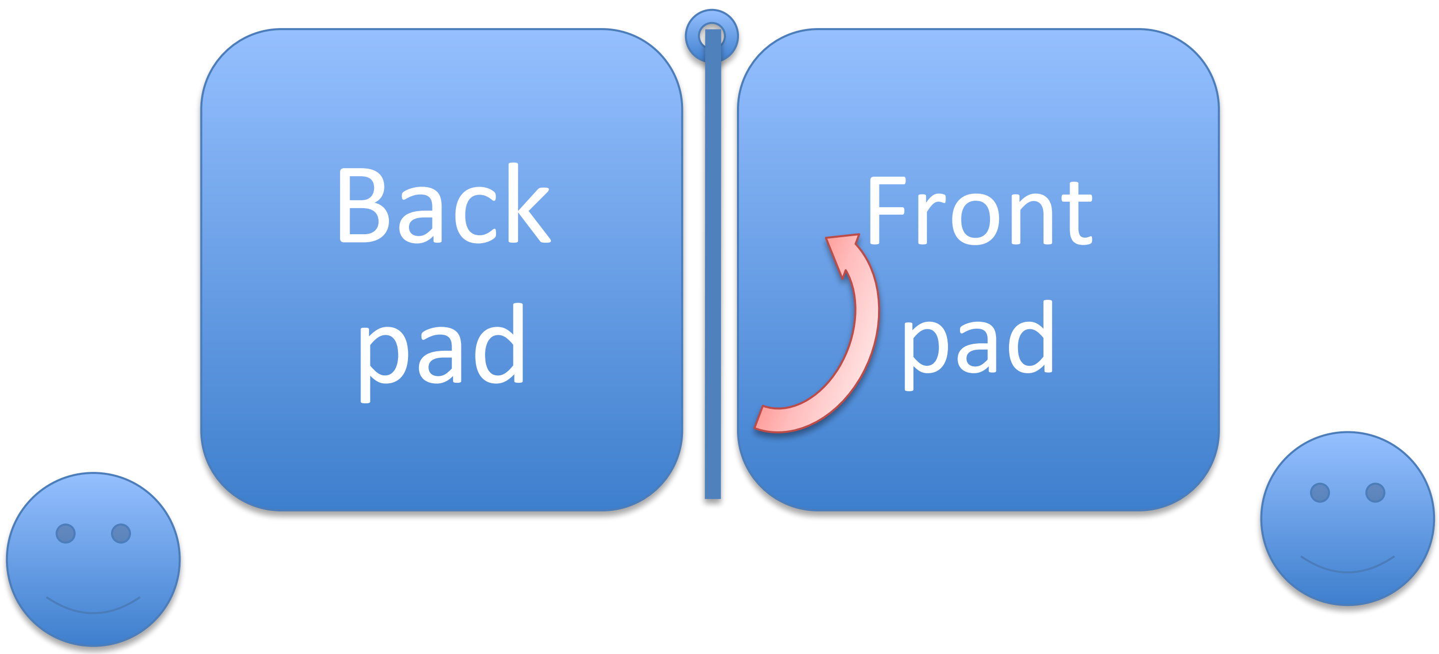
Intro to DFAs

Readings: Sipser 1.1 (pages 31-44)

With basic background from Sipser 0

Intuition: finite automata

- Don't hit anyone!



Intuition: finite automata

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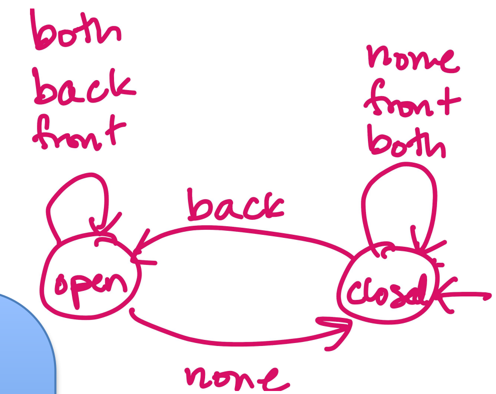
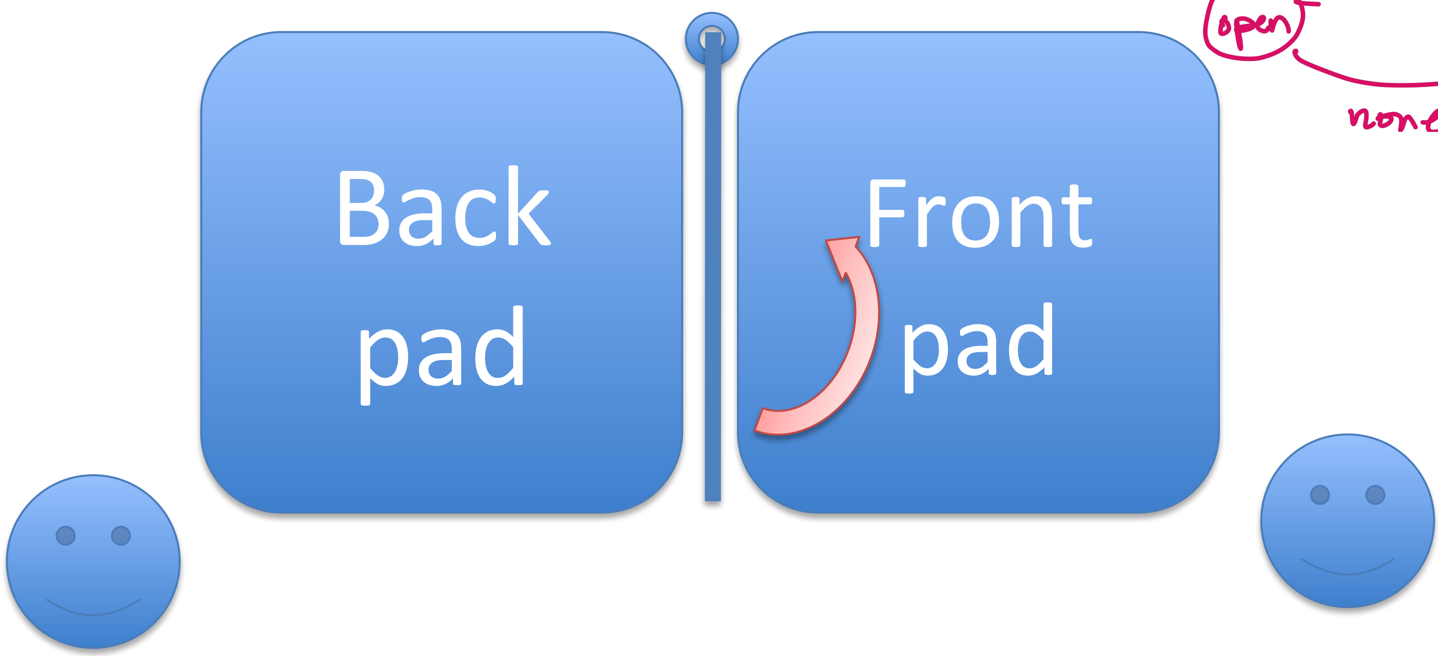
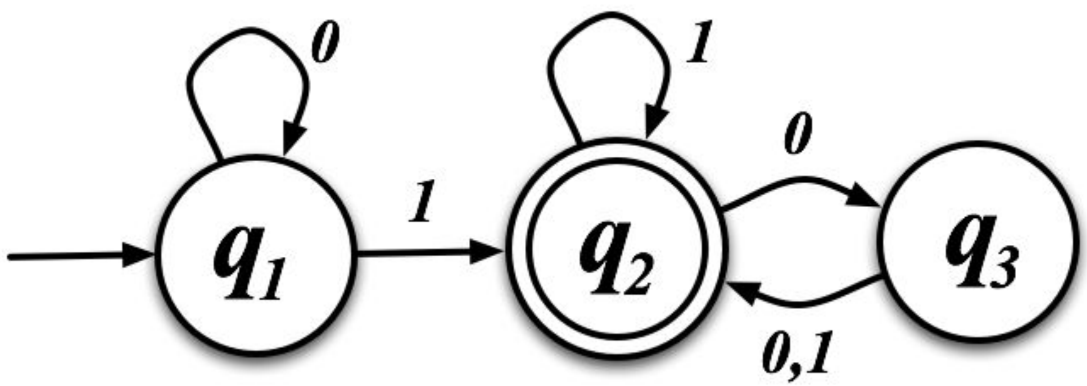




State diagram

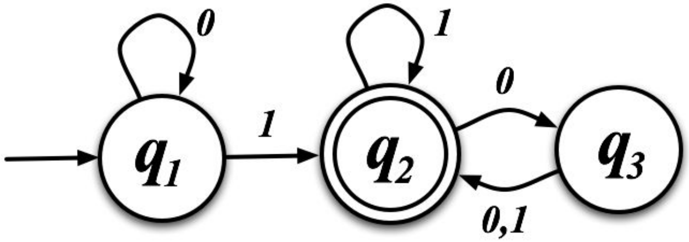
- What is accepted?
- 001? 000? 010?



- Can we come up with a description of the *language* accepted by this machine?

More formally…

- A ***(deterministic) finite automaton (DFA)*** is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where
  - Q is a finite set called the **states**
  - Σ is a finite set called the **alphabet**
  - $\delta: Q \times \Sigma \rightarrow Q$ is the **transition function**
  - $q_0 \in Q$ is the **start state**
  - $F \subseteq Q$ is a set of **accept states**
- In-class exercise:

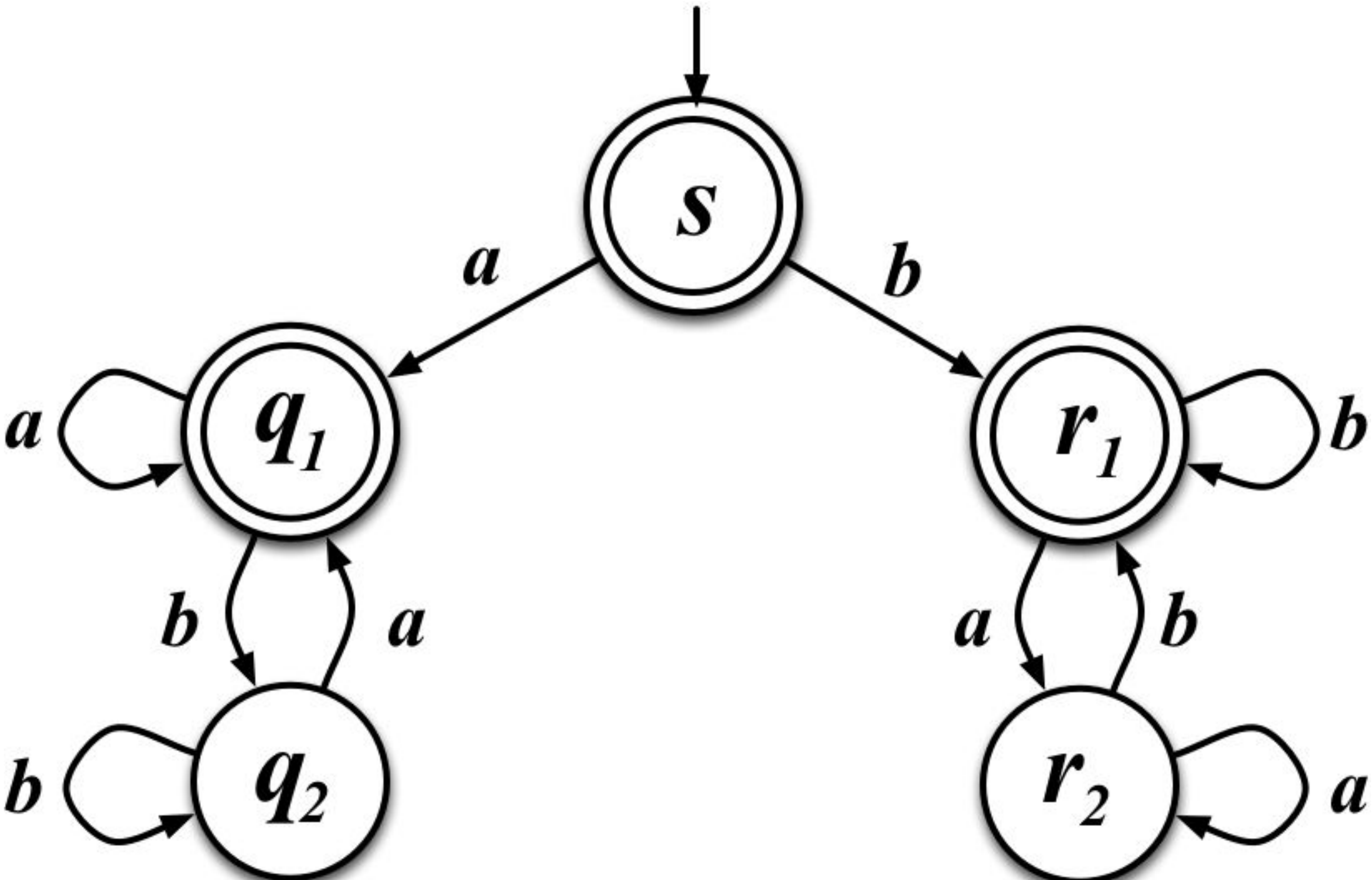


Languages

- The set of all strings accepted by a DFA M is called the **language of M** and is denoted $L(M)$
- We say that

  "M recognizes the language $L(M)$"

What language is accepted here?



Automata computation

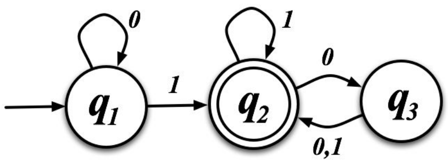
- More formally:
- Let $M=(Q, \Sigma, \delta, q_0, F)$ be a DFA and let

  $$w=w_1w_2w_3\ldots w_n$$

  be a string over the alphabet Σ
- Then M **accepts** w if a sequence of states $s_0, s_1, s_2, \ldots, s_n$ exists in Q with the following conditions:

1. $s_0=q_0$
2. $\delta(s_i, w_{i+1}) = s_{i+1}$ for $i = 0, \ldots, n-1$
3. $s_n \in F$

Automata computation



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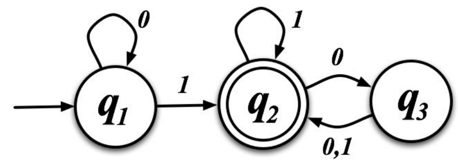

- More formally:
- Let $M=(Q, \Sigma, \delta, q_0, F)$ be a DFA and let

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$$w=w_1w_2w_3\ldots w_n$$

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Regular languages

- A language is a **regular language** if some DFA recognizes it
- Examples:
  - $L(M_1)$={w | w contains at least one 1 and an even number of 0s follow the last 1}
  - $L(M_2)$={w | w is a string over {a,b} that starts and ends with the same symbol}

Designing your own

- Is $\{w \mid w$ is a string of 0s and 1s containing an even number of 1s$\}$ a regular language?
- Is $\{w \mid w$ is a string of as and bs containing the substring $aba\}$ a regular language?

- How could we **prove** a “yes” answer?