Taylor Series method for uncertainty propagation

- Rationale: Calculate variance of estimate where estimate is a function of several other variables

- Eg Case study in Atomic Force Microscopy, we are interested in calculating Young's Modulus (E) the Sneddon formula for E is:

- (1) $$E = \frac{F * \pi * (1 - v^2)}{2 * \tan(\alpha) * \delta^2} = \underbrace{E(F, \delta)}$$

Young's Modulus is a function of F and δ we get from experimental data

* A proof of concept * to obtain the variance of mean E

space left deliberately awesome

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- From (1), we assume $\perp$ of variables:

$$F = a \qquad \delta = b$$

- Rename variables for ease

$$\sigma_F = \sigma_a \qquad \sigma_\delta = \sigma_b$$

- here are standard deviations

- $\sigma_a = \underbrace{B_a}_{\text{Systematic error}} + \sigma_a$, $\sigma_b = B_b + \underbrace{\sigma_b}_{\text{Random error}}$

- $$\underbrace{E_{true} - E_{estimate}}_{\sigma_{Total}} = \frac{\partial E}{\partial a}\sigma_a + \frac{\partial E}{\partial b}\sigma_b$$

- $E(X) - X$ from STATS 101

- $$\frac{\sigma^2_{Total}}{N} = \frac{1}{N}\left[\frac{\partial E}{\partial a}\sigma_a + \frac{\partial E}{\partial b}\sigma_b\right]^2$$

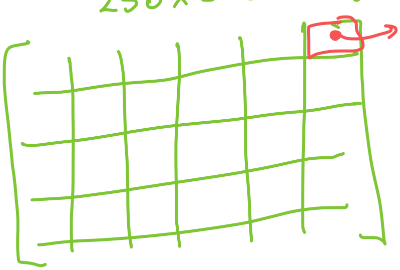
- (Sum of variances from STATS 101) is the Variance of the sum

- $$\frac{\sigma^2_{Total}}{N} = \frac{1}{N}\sum \sigma_a^2 + \frac{1}{N}\sum \sigma_b^2$$

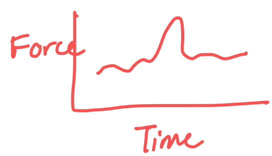
- $$\mathrm{var}(\sigma^2_{Total}) = \left[\frac{1}{N}\sum_{i}^{k} \sigma_k^2\right]$$ end of proof

- In Atomic Force Microscopy

256x256 image matrix



→ We obtain only 1 Force vs time curve



→ Therefore only 1 set of F and δ.

Simulation needed or multiple images needed to obtain μ_E and σ^2_E

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