Rec 08: Red-Black Trees

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s/o Anissa (sec 207) for animations!

What's wrong with BSTs?

- standard way of defining a BST:

```
type ‘a tree =
| Leaf
| Node of ‘a * ‘a tree * ‘a tree
```

- BST invariant w/ the following statements
  - all nodes must be larger than any node in its left subtree
  - all nodes must be smaller than any node in its right subtree

What's wrong with BSTs?

Both are valid trees:

```
        5              0
       / \              \
      /   \              1
     /     \              \
    2       7              2
   / \     / \              \
  1   4   6   9              3
 /   /       / \              \
0   3       8   10            ...
```

If a client puts nodes in increasing order, then we have a linked list!

- what we want to be `O(log(n))` becomes `O(n)` time!

Red Black Tree Invariant

- We say a node can either be red or black
  - no adjacent red nodes along any path
  - number of black nodes from root to any leaf is the same
    - number of black nodes is called black height BH
  - convention: root is black
- Example:

```
        B
       / \
      /   \
     R     R
    / \   / \
   B   B B   B
```

Why??

worst case:

```
           B
          / \
         /   \
        B     R
       / \   / \
      B ... ... B
     / \       / \
    B  ...   ...  R
   /               \
...                 ...
```

relative length of two paths:

- longest path alternates between R/B
- shortest path has B nodes
- both have same number of B nodes

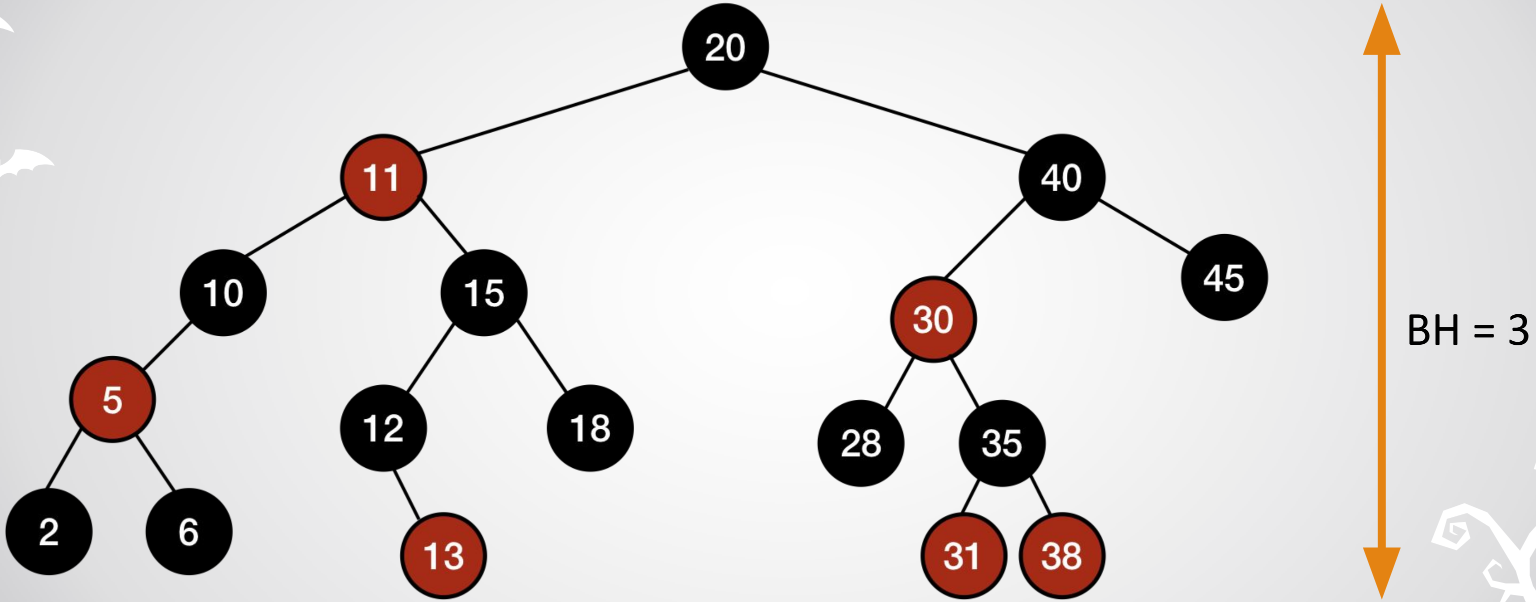
path length differs by factor of 2!

yay! `O(logn)` for operations

What are Red Black Trees?

- Balanced binary search tree
  - All nodes in the left subtree of a node are less than that node and all nodes in the right subtree are larger
  - If has n nodes, the height is O(logn)
- satisfies Red Black Tree Invariant

Red Black Tree Example



Note: a lot of times leaf nodes aren't actually drawn out

RB Trees in OCaml

Type Definition:

```
type color = Red | Black
type ‘a rb_tree =
| Leaf
| Node of color * ‘a * ‘a rb_tree * ‘a rb_tree
```

RB Trees in OCaml

Membership:

(same as BSTs!)

```
type rec mem x = function
   | Leaf -> false
   | Node (_, y, left, right) ->
       x = y || (x < y && mem x left) || (x > y && mem x right)
```

RB Trees Exercise

Exercise: given 3 nodes (not including leaves), what possible BST can you construct and how would you could you color them to satisfy the RB tree invariant?

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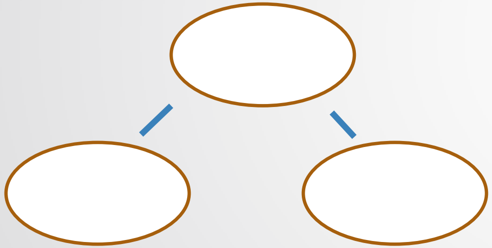

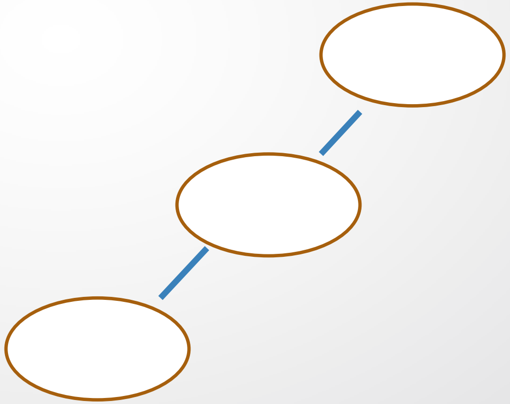

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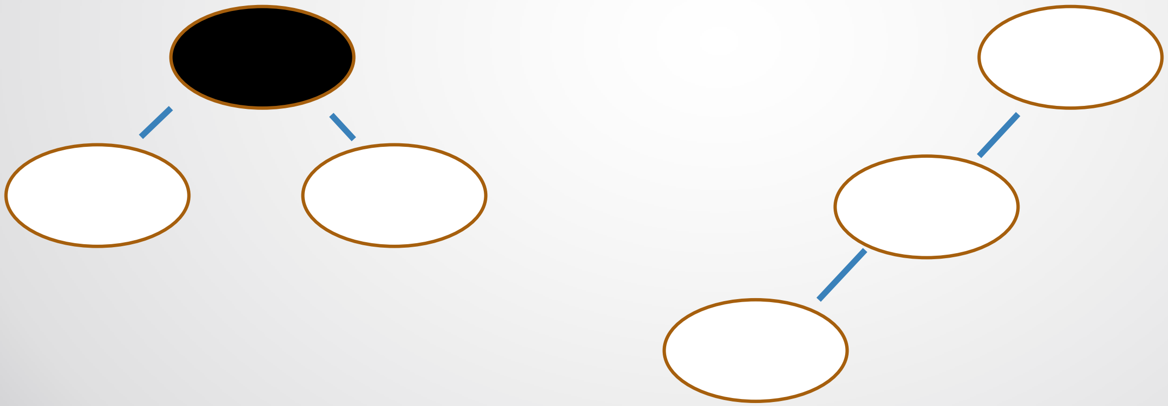

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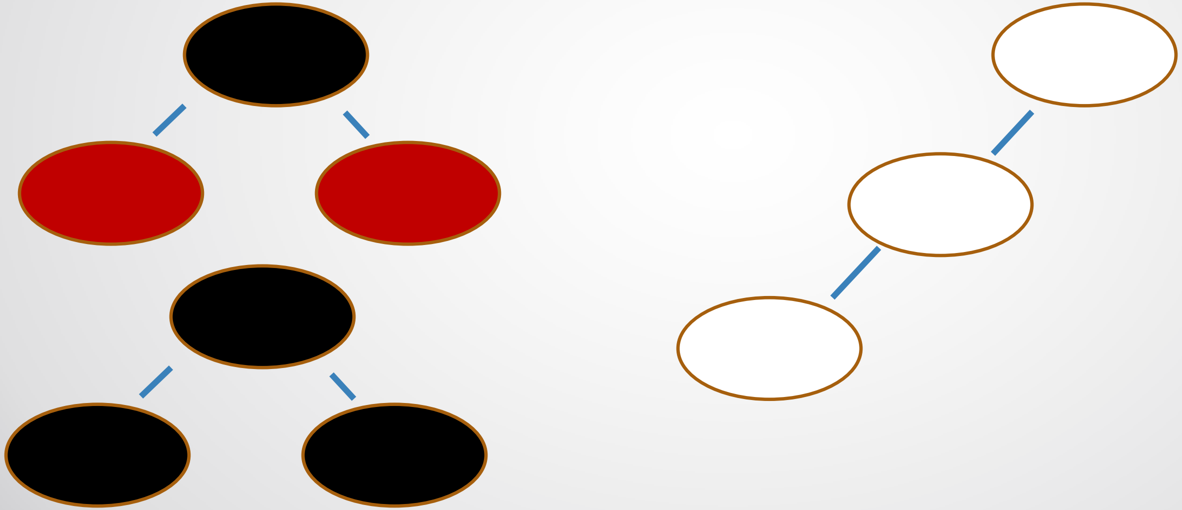

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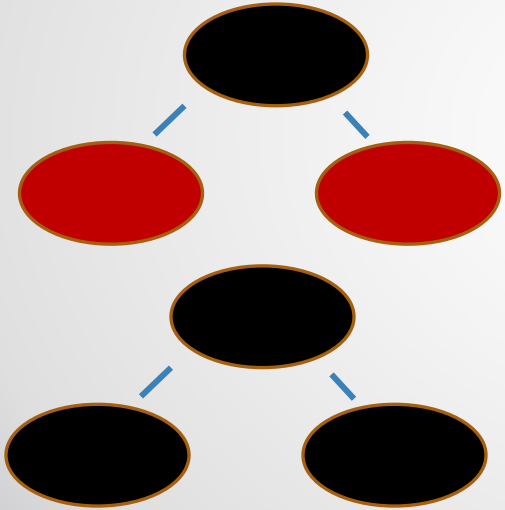

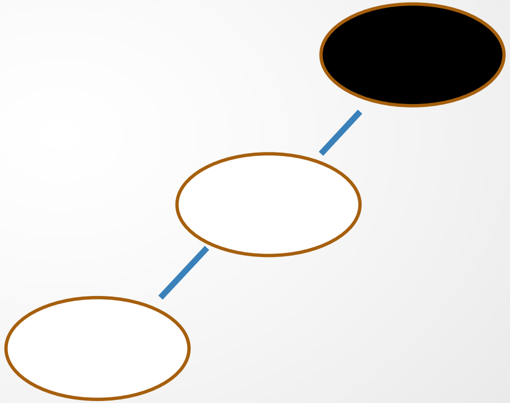

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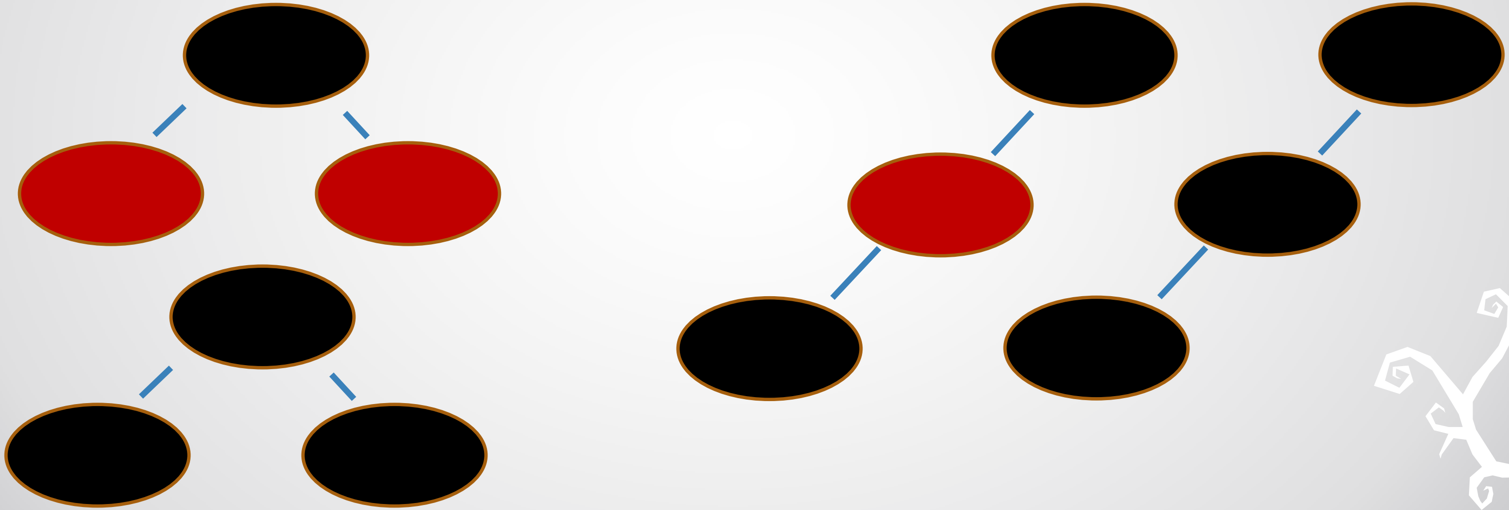

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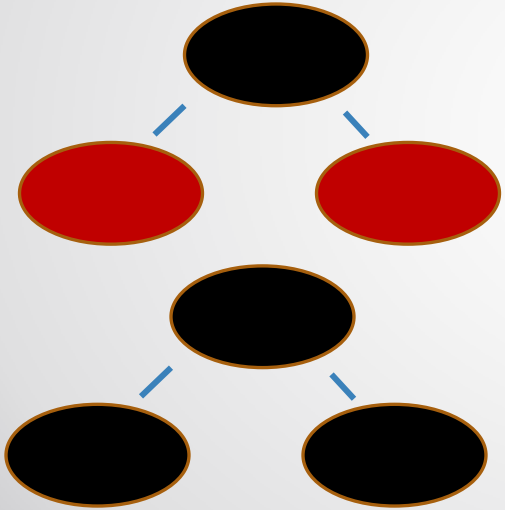

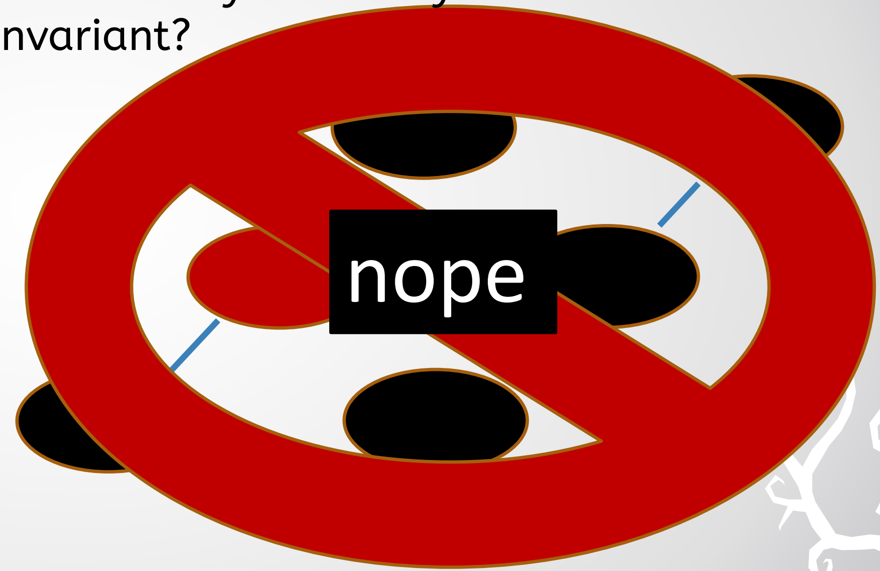

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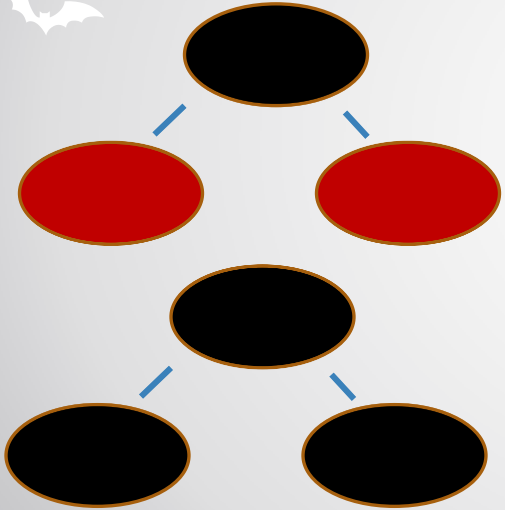
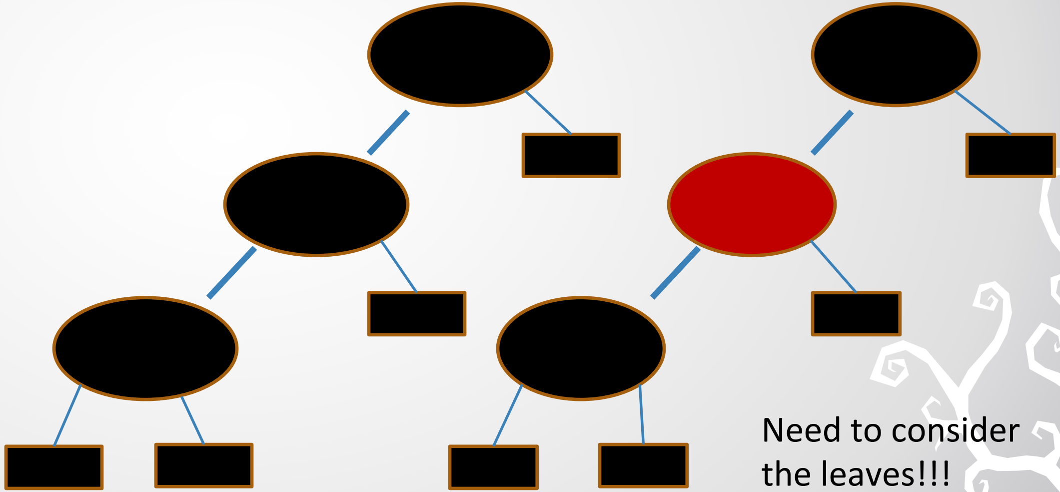
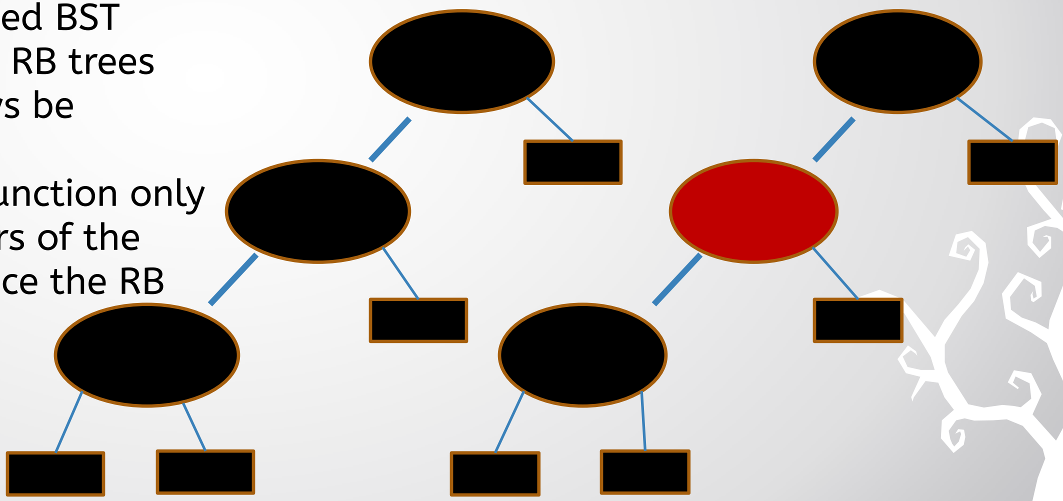




Need to consider the leaves!!!

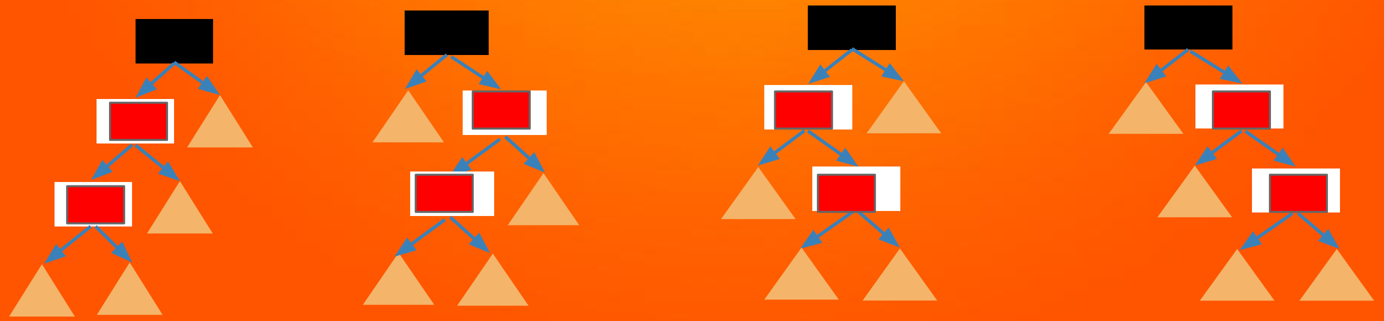
RB Trees Exercise

- To satisfy **global invariant** you could color both non-root nodes red
- However, this violates the **local invariant**
- Note: this BST is not a balanced BST
  - The coloring invariant for RB trees ensures that it will always be balanced!
  - This is why the balance function only needs to look at the colors of the nodes in order to rebalance the RB tree



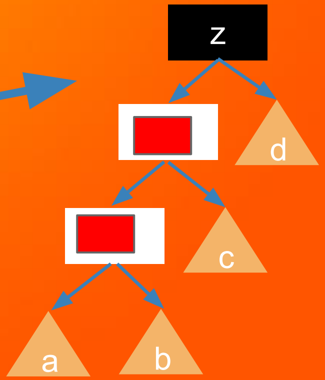
RB Trees Balance

- Okasaki's Algorithm: newly inserted node will always be red
- Creates 4 possible local invariant violations:



RB Trees Balance

```
let balance = function
  | Black, z, Node (Red, y, Node (Red, x, a, b), c), d
  | Black, z, Node (Red, x, a, Node (Red, y, b, c)), d
  | Black, x, a, Node (Red, z, Node (Red, y, b, c), d)
  | Black, x, a, Node (Red, y, b, Node (Red, z, c, d)) ->
    Node (Red, y, Node (Black, x, a, b), Node (Black, z, c, d))
  | a, b, c, d -> Node (a, b, c, d)
```



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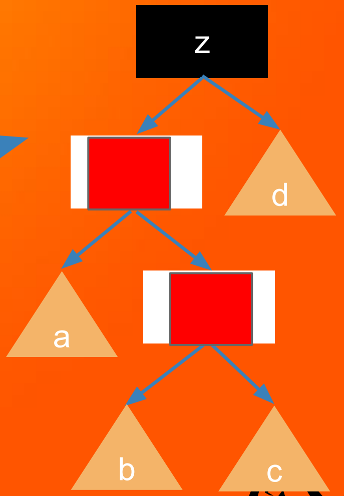

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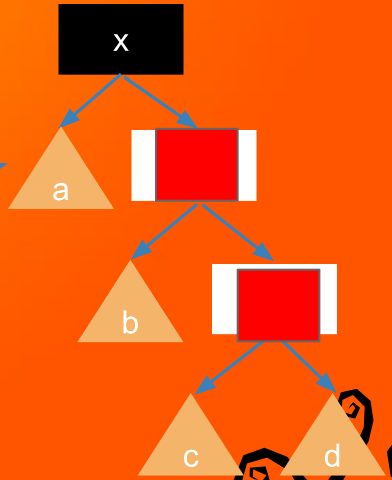

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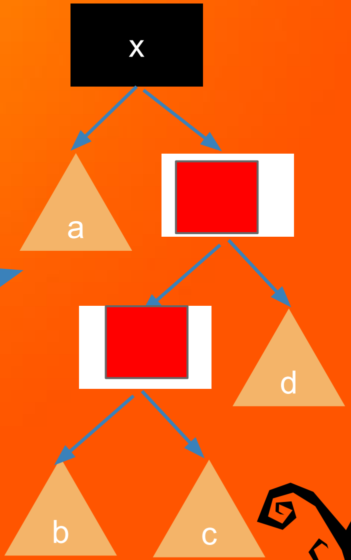

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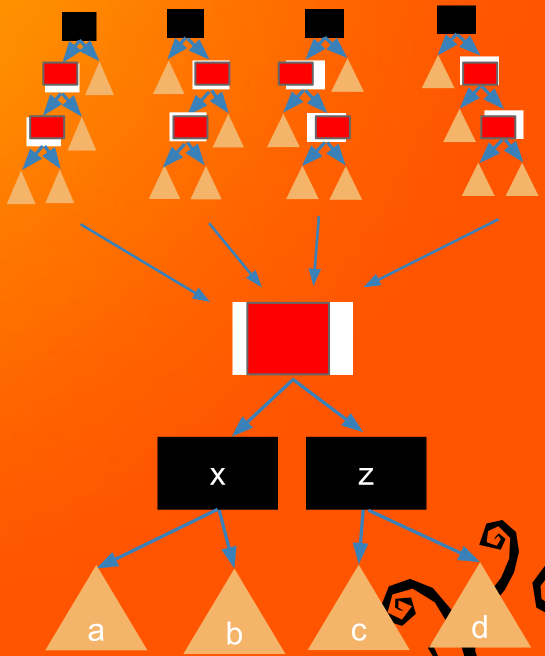

tree is already balanced!

RB Insert

- Insertion is a bit more tricky
- ex:

```
      B                   B
     / \     ==>         / \
    /   \    (B)        /   \
   R     R             R     R
                      /
                     B

      B                   B
     / \     ==>         / \
    /   \    (R)        /   \
   R     R             R     R
                      /
                     R
```

invariant broken!

Red-Black Trees - Insert

1. If the tree is empty, insert element at root as a red node
2. If the tree is non-empty:
   - If element is less than root node, recurse down the left subtree until reach a leaf and insert as red node
     - Rebalance tree
   - If element is greater than root, recurse down right subtree until you reach a leaf node and insert there as red node
     - Rebalance tree
3. Set root node to black

Red Black Trees Insert Example

- 🎃 Exercise: given an empty Red-Black Tree, insert (by hand) the numbers 1 through 7 into the tree following the insert algorithm

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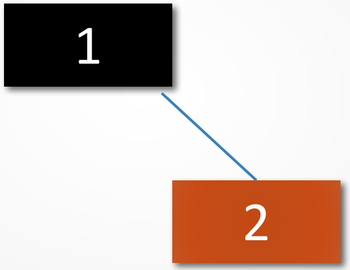

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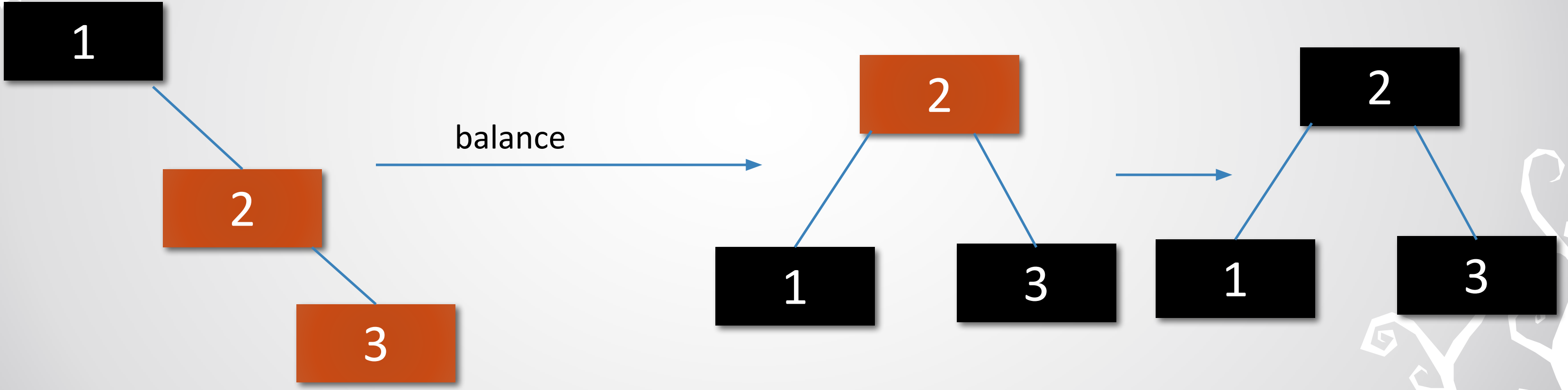

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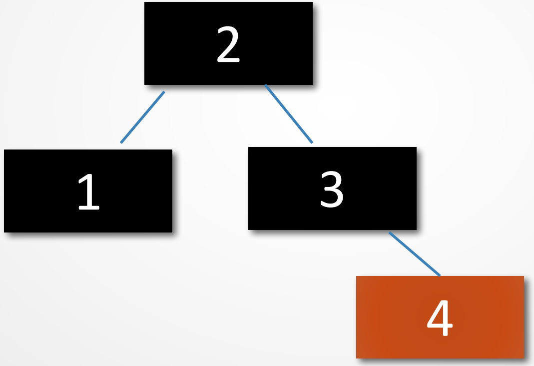

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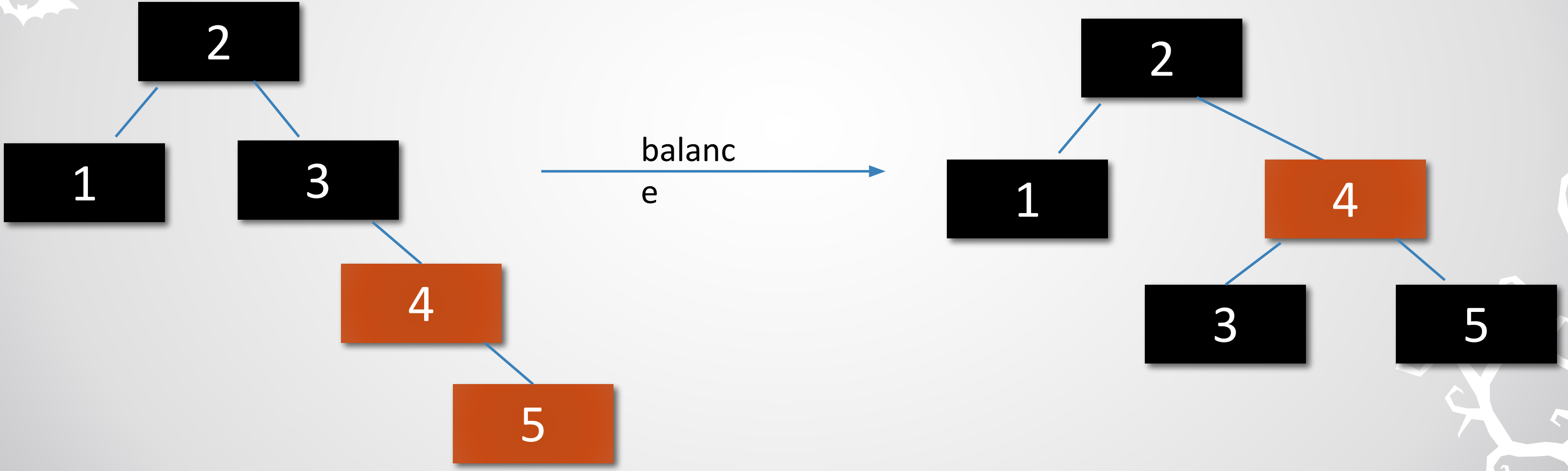

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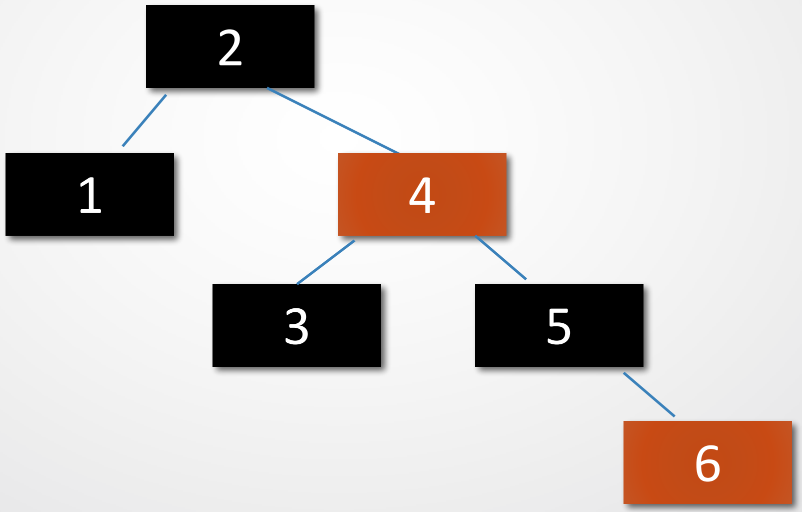

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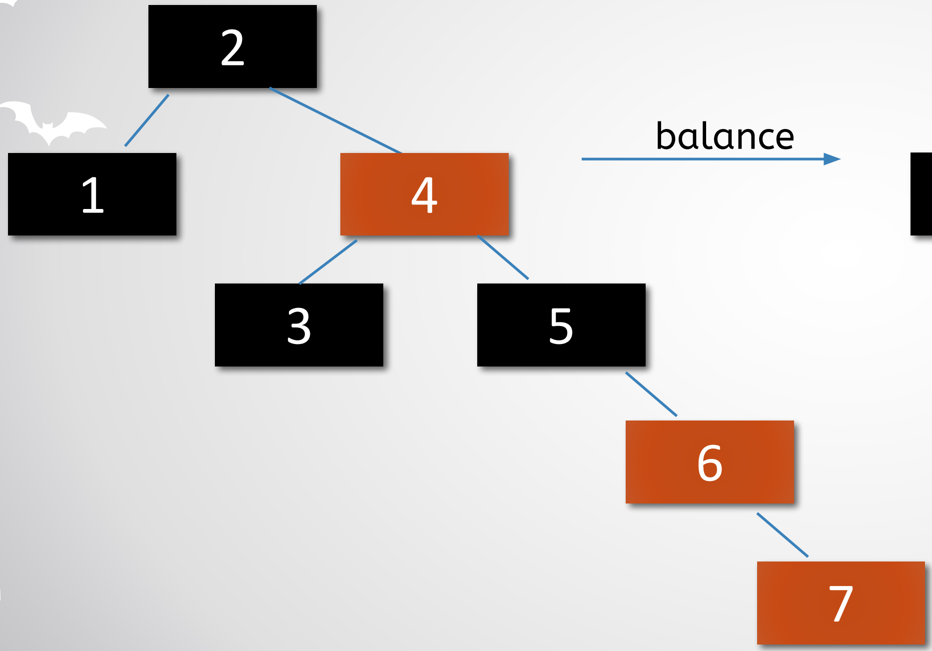
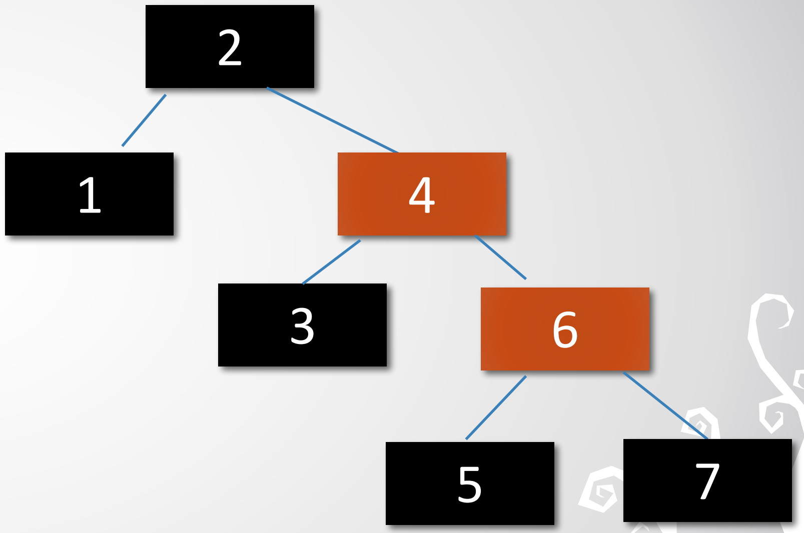
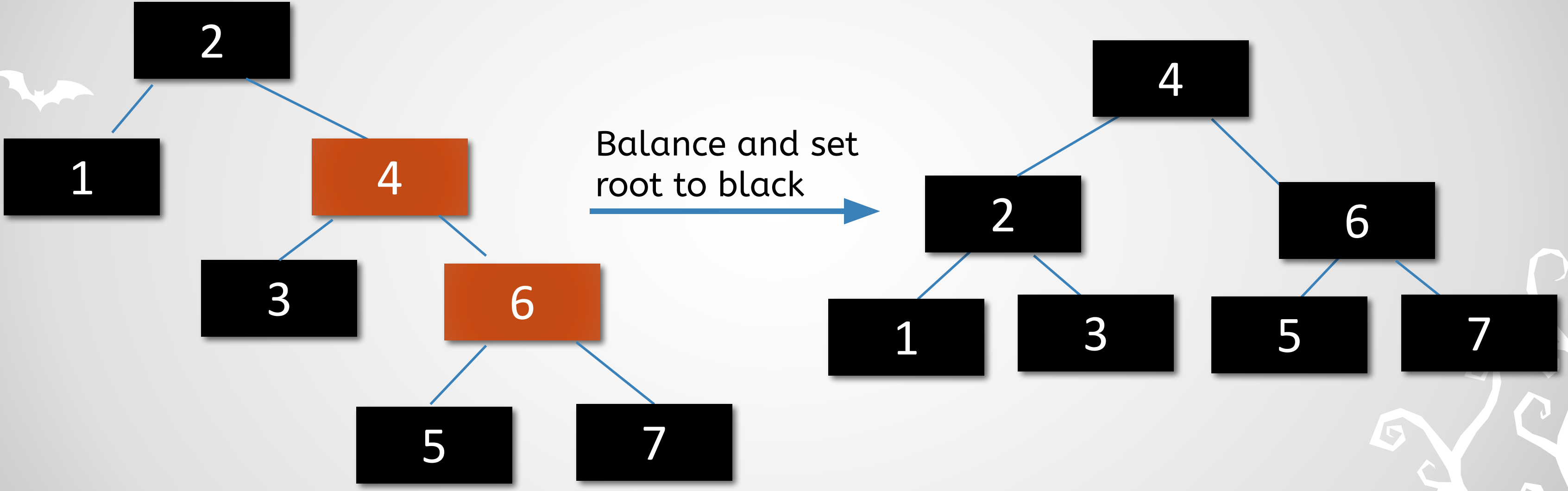


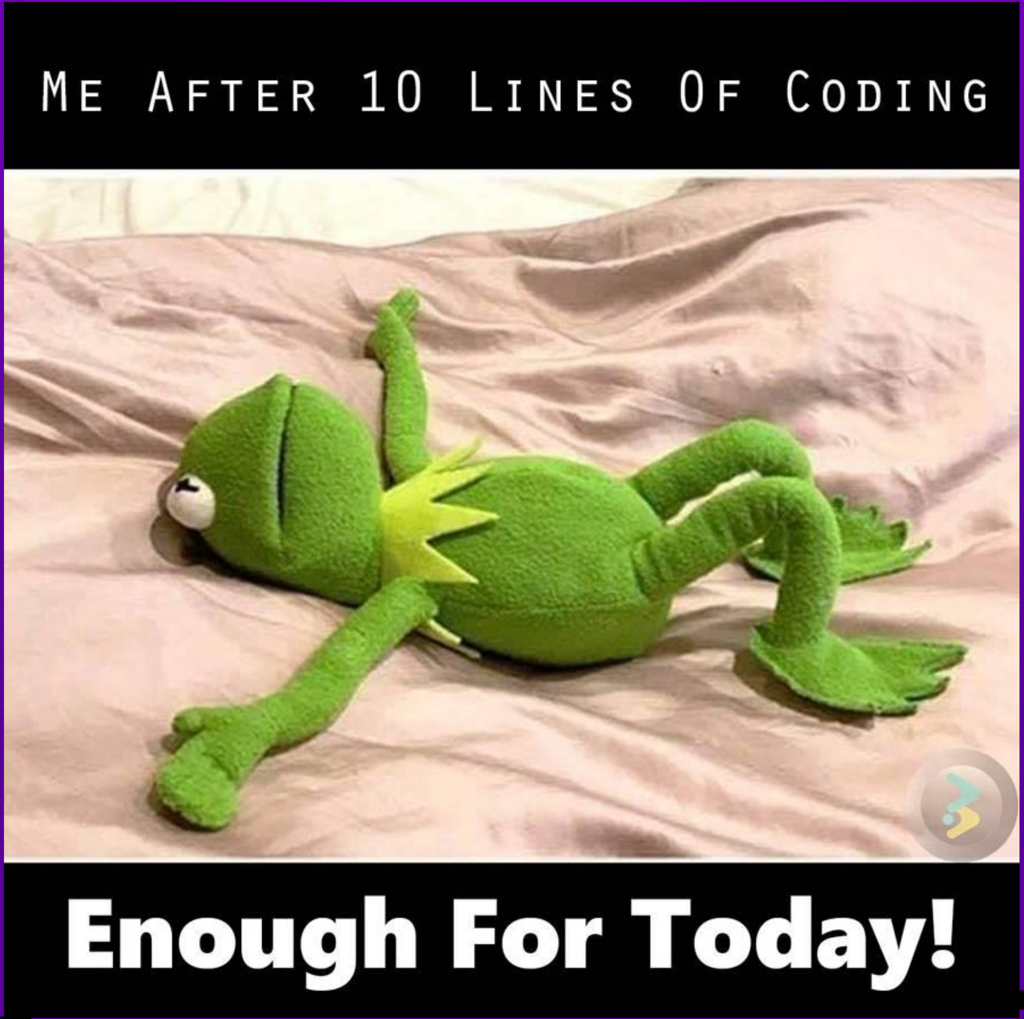


Not done yet!!

Red Black Trees Insert Example




Bernie
I am once again asking
you to let me know if you have any questions
imgflip.com


Me After 10 Lines Of Coding
Enough For Today!


Happy Halloween!
Stay safe!