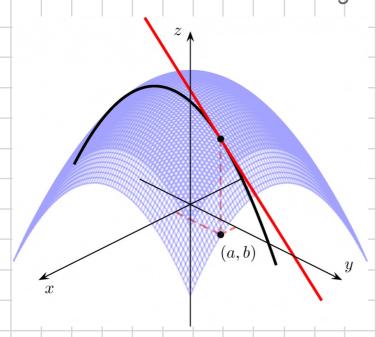
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## Applications

Derivatives and integrals have a lot of applications, in fact the next standard is dedicated to one application of the derivative, we will discuss three short applications in this section.

## equation of the tangent line

Recall from Calculus I that the derivative of a function is the slope of the tangent line. For vector-valued functions, the derivative gives a tangent vector that points in the direction of increasing t-values. This vector is used as a direction vector for the tangent.



Given the vector-valued function,  $\vec{\tau}(t)$ , we call  $\vec{\tau}'(t)$  the tangent vector provided it exists and is not  $\vec{0}$ . The tangent line to  $\vec{\tau}(t)$  at the point P is then the line that passes through the point P and is parallel to the tangent vector. If  $\vec{\tau}'(t) = \vec{0}$  we would have a vector with no magnitude and no direction.

Given that  $\vec{r}'(t) \neq \vec{0}$ , the unit tangent vector to the curve is given by  $\vec{T}(t) = \frac{\vec{r}(t)}{||\vec{r}'(t)||}$ .

デ(t) shown in black, tangent line shown in red i.e. v(t)=ア+tデ(t)

example. Find the general formula for the unit tangent vector and the vector equation of the tangent line to the curve given by  $\vec{r}(t) = \langle t\cos(t), t, t\sin(t) \rangle$  at  $t = \pi$ .

(i) r(t) = < toos(t), t, tsin(t)>

 $\vec{r}'(t) = \langle \cos(t) - \sin(t), 1, \sin(t) + \cos(t) \rangle$ 

 $||\vec{r}(t)|| = \int (\cos(t) - \sin(t))^2 + (1)^2 + (\sin(t) + \cos(t))^2$ 

=  $\int \cos^2(t) - 2\cos(t) \sin(t) + \sin^2(t) + 1 + \sin^2(t) + 2\cos(t) \sin(t) + \cos^2(t)$ 

 $=\sqrt{2\cos^2(t)+2\sin^2(t)+1}$ 

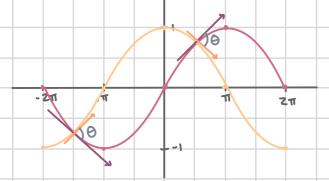
 $T(t) = \frac{r(t)}{||\vec{r}(t)||} < \frac{\cos(t) + \sin(t)}{||z\cos^2(t) + z\sin^2(t) + 1|}, \frac{1}{||z\cos^2(t) + z\sin^2(t) + 1|} > \frac{\sin(t) - \cos(t)}{||z\cos^2(t) + z\sin^2(t) + 1|} > \frac{1}{||z\cos^2(t) + z\sin^2(t) + 1|}$ 

 $P = \vec{r}(\pi) = \langle \pi \cos(\pi), \pi, \pi \sin(\pi) \rangle = \langle -\pi, \pi, 0 \rangle$ 

tangent line:  $\vec{v}(t) = \langle -\pi, \pi, 0 \rangle + t \langle -\frac{1}{13}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \rangle$ 

## angle of intersection

Suppose two curves  $\vec{r}$ , and  $\vec{r}_z$  intersect at a point P. Then the angle they intersect at can be determined by finding the angle of intersection of the tangent vector vectors at the point P.



Curves can intersect multiple times (sometimes with different angles) so we must be careful when inputting the intersection point. Curves can also run at different "speeds" i.e. the parameters could be different for each curve at the point of intersection.

example. Find the angle of intersection for the curves  $\vec{r}(t) = \langle \cos(t), -\sin(t), t \rangle$  and  $\vec{r}_2(s) = \langle -s, s^2 - 1, \ln(s) + \pi \rangle$ at the point  $P=(-1,0,\pi)$ First solve for the values t and  $v: \vec{r}_1(\pi) = \langle \cos(\pi), -\sin(\pi), \pi \rangle = \langle +1, 0, \pi \rangle + \vec{r}_2(1) = \langle -1, (1)^2 - 1, |n(1) + \pi \rangle = \langle -1, 0, \pi \rangle$ Next find the tangent vectors: F; (t) = <-sin(t), -cos(t), 1 > { +2(s) = <-1, 2s, 5 > Tangent vectors at our given  $t \in S$ :  $\vec{r}$ ,  $(\pi) = \langle 0, 1, 1 \rangle$   $\vec{r}$   $\vec{r}$   $(1) = \langle -1, 2, 1 \rangle$  $\langle 0, -1, 1 \rangle \circ \langle -1, 2, 1 \rangle = ||\langle 0, -1, 1 \rangle|| \cdot ||\langle -1, 2, 1 \rangle|| \cdot ||\langle 0, -1, 1 \rangle||$  $(0)(-1) + (-1)(z) + (1)(1) = \sqrt{(0)^2 + (-1)^2 + (1)^2} \sqrt{(-1)^2 + (2)^2 + (1)^2} \cos(\theta)$  $0-2+1=\sqrt{2}\sqrt{6}\cos\theta$  $\frac{1}{2\sqrt{3}} = \cos(\theta)$  $\theta = \arccos(-\frac{\sqrt{3}}{6})$ arclenath There are two types of distance that are commonly discussed: · displacement - the "direct" or shortest distance between the starting point and end point. · total distance traveled - distance that takes into account the path followed. Here is a 2D photo to show why both are important. | | r(b) - r(b) | = the two points rub ; ru) subtracted Salif'(+) 11dt = the distance traveled along the curve FLE) In Calculus I, we found that the arclength for a two-dimensional curve is given by L= So [fit] + [glt] dt The natural extension to three-dimensions is L= So [f'(t)] + [g'(t)] dt. We can simplify this equation to be L= Sa 1171(t)11 dt example. Find the arclength of 7(t)=<t,3cost,3sint> where -5465. 7'(t)= <1,-3sint, 3cost>  $|\vec{r}|_{\{t\}}|_{t} = |\vec{r}|_{t}^{2} + (-3\sin t)^{2} + (3\cos t)^{2} = |\vec{r}|_{t}^{4} = |\vec{r}|_{t}^{4}$ L= 10 (5-(-5)) = 1010 The last concept is the arclength function which tells us the distance traveled at time t. We define the arclength function as  $s(t) = \int_0^t ||\vec{r}'(t)|| dt$ . example. Determine the arclength function for \$\(\darkappa\) = < t, 3 cos(t), 3 sin(t)>.  $\vec{r}'(t) = \langle 1, -3 \sin(t), 3 \cos(t) \rangle$ 117 (4)11= 110 s(t) = 50 10 dt = t 10 1 = t 10 We can also ask, where are we on the curve if we have traveled a specified distance? To find this we solve the arclength function for t and plug the result into the parameterization. example. Reparameterize the function into \$\forall (t(s)).  $s = t \cdot 10^{1} = t = \frac{10^{1}}{10^{1}}$   $r^{2}(s(t)) = (\frac{5}{10}, 3\cos(\frac{3}{10}), 3\cos(\frac{3}{10}))$