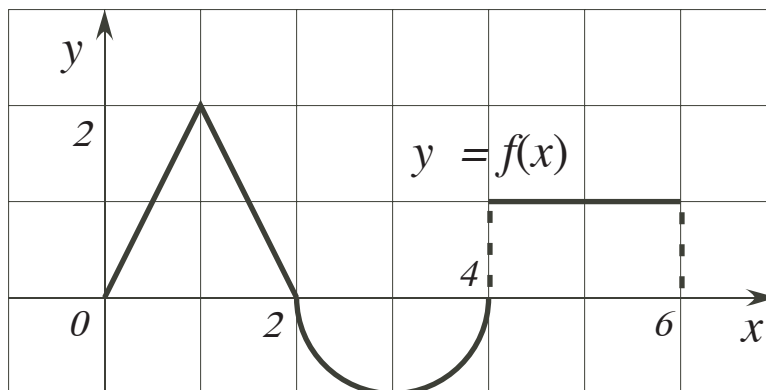


Math 10350 – Example Set 15B
(Section 5.2, 5.4, 5.5, 5.6 & 5.7)

1. Use geometry to compute the definite integral $\int_{-5}^0 \sqrt{25-x^2} dx$ ← left hemisphere $y = \sqrt{25-x^2}$
 $y^2 = 25 - x^2$



$x^2 + y^2 = 25$
 i.e. circle of radius 5
 positive semi-circle
 $\int_{-5}^0 \sqrt{25-x^2} dx$ is the
 area bounded by the
 left-upper quarter
 $= \frac{1}{4} \pi r^2 = \frac{1}{4} \pi (5)^2 = \frac{25}{4} \pi$

2. Consider the graph of $f(x)$ above. Using geometry, find the value of all the definite integrals below:

- a. $\int_0^2 f(x) dx \stackrel{?}{=} \triangle = \frac{1}{2}bh = \frac{1}{2}(2)(2) = 2$ the hemisphere has negative area under the graph
 c. $\int_2^4 f(x) dx \stackrel{?}{=} -\triangle = -\frac{1}{2} \pi r^2 = -\frac{1}{2} \pi (1)^2 = -\frac{1}{2} \pi$
 b. $\int_1^4 f(x) dx \stackrel{?}{=} \triangle - \triangle = 2 - \frac{1}{2} \pi$
 d. $\int_0^6 f(x) dx \stackrel{?}{=} \triangle - \triangle + \square = 2 - \frac{1}{2} \pi + (2)(1) = 4 + \frac{1}{2} \pi$

Properties of Definite Integral (5.2). Let $a < b < c$ and k be real numbers. Let $f(x)$ and $g(x)$ be continuous functions. Then we have the following:

- i. $\int_a^b [f(x) + g(x)] dx = \int_a^b f(x) dx + \int_a^b g(x) dx$ ii. $\int_a^b k \cdot f(x) dx = k \cdot \int_a^b f(x) dx$
 iii. $\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$

We also define:

- iv. $\int_a^a f(x) dx = 0$ v. $\int_b^a f(x) dx = -\int_a^b f(x) dx$

3. Given that $\int_0^2 f(x) dx = \int_2^3 f(x) dx = 5$, find

- a. $\int_0^3 f(x) dx \stackrel{?}{=} \int_0^2 f(x) dx + \int_2^3 f(x) dx = 5 + 5 = 10$ c. $\int_0^2 f(x) dx + \int_3^2 f(x) dx \stackrel{?}{=} \int_0^2 f(x) dx - \int_2^3 f(x) dx$
 b. $\int_0^2 [4f(x) + 2] dx \stackrel{?}{=} 4 \int_0^2 f(x) dx + \int_0^2 2 dx$ = 5 + 5 = 10
 $= 4(5) + \int_0^2 2 dx$ ← we don't know how to do this

Fundamental Theorem of Calculus (5.4). Let $F(x)$ be an anti-derivative of $f(x)$. Then

$$\int_a^b f(x) dx = \underline{F(b) - F(a)} = \underline{F(x) \Big|_a^b} \quad (\text{shorthand notation}).$$

In other words: Note that $F(x)$ includes a plus c , this disappears as $F(b)$ would have plus c and $-F(a)$ would have $-(\text{plus } c)$

$$\text{Total change in } F(x) \text{ over } [a, b] = \underline{\int_a^b f(x) dx} = \underline{\hspace{2cm}}$$

4. Evaluate the following definite integrals:

a. $\int_{-1}^0 (1 + 3x - e^{-x}) dx$

$$F(x) = x + \frac{3}{2}x^2 + e^{-x} + c$$

$$F'(x) = 1 + 3x - e^{-x} = f(x)$$

$$\int_{-1}^0 1 + 3x - e^{-x} dx = F(0) - F(-1)$$

$$= x + \frac{3}{2}x^2 + e^{-x} \Big|_{-1}^0$$

$$= 0 + \frac{3}{2}(0)^2 + e^{-0} - \left((-1) + \frac{3}{2}(-1)^2 + e^{-(-1)} \right)$$

$$= 0 + 0 + 1 + 1 - \frac{3}{2} + e$$

$$= \frac{1}{2} + e$$

b. $\int_{\pi/2}^{\pi} \cos \theta d\theta$

$$F(\theta) = \sin(\theta) + c$$

$$F'(\theta) = \cos(\theta)$$

$$\int_{\pi/2}^{\pi} \cos(\theta) d\theta = \sin(\theta) \Big|_{\pi/2}^{\pi}$$

$$= \sin(\pi) - \sin(\pi/2)$$

$$= 0 - \frac{\sqrt{2}}{2}$$

$$= -\frac{\sqrt{2}}{2}$$

c. $\int \sqrt{x-1} dx = \int (x-1)^{1/2} dx$

The format suggest power rule:

$$\int ax^n = \frac{a}{n+1} x^{n+1} + c$$

But we have $(x-1)^{1/2}$ and not $x^{1/2}$

$$\text{Try: } F(x) = \frac{1}{3/2} (x-1)^{3/2} + c = \frac{2}{3} (x-1)^{3/2} + c$$

$$F'(x) = \frac{2}{3} \cdot \frac{3}{2} (x-1)^{1/2} = (x-1)^{1/2}$$

$$\int \sqrt{x-1} dx = \frac{2}{3} (x-1)^{3/2} + c$$

d. $\int_1^5 \sqrt{x-1} dx$

$$F(x) = \frac{2}{3} (x-1)^{3/2} + c$$

$$F'(x) = (x-1)^{1/2}$$

$$\int_1^5 \sqrt{x-1} dx = \frac{2}{3} (x-1)^{3/2} \Big|_1^5$$

$$= \frac{2}{3} (5-1)^{3/2} - \frac{2}{3} (1-1)^{3/2}$$

$$= \frac{2}{3} [(4)^{1/2}]^3 - \frac{2}{3} [(1)^{1/2}]^3$$

$$= \frac{2}{3} (2)^3 - \frac{2}{3} (1)^3$$

$$= \frac{16}{3} - \frac{2}{3}$$

$$= \frac{14}{3}$$