1. If $f'(a) = \lim\limits_{h \to 0} \dfrac{(3+h)^{10} - 3^{10}}{h}$, what is a possible $f(x)$ and the value of a?

$= \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$f(x+h) = (3+h)^{10}$

$f(x) = 3^{10}$

$f(x) \stackrel{?}{=} \underline{x^{10}}$ and $a \stackrel{?}{=} \underline{3}$

guess: $f(x) = x^{10}$, $a = 3$

$f'(x) = \lim\limits_{h \to 0} \dfrac{(x+h)^{10} - x^{10}}{h}$

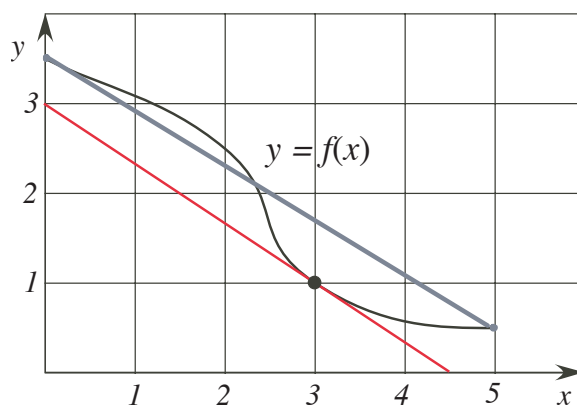$f'(3) = \lim\limits_{h \to 0} \dfrac{(3+h)^{10} - (3)^{10}}{h}$

my second guess would have been: $f(x) = 3^x$, $a = 10$

$f'(x) = \lim\limits_{h \to 0} \dfrac{3^{x+h} - 3^x}{h}$

$f'(10) = \lim\limits_{h \to 0} \dfrac{3^{10+h} - 3^{10}}{h}$

not the same

2.

$y = f(x)$

The figure above describes the graph of $y = f(x)$ and its tangent line at $x = 3$. Answer the problems below:

a. Estimate the average rate of change of $f(x)$ over the interval $[0, 5]$.

average rate of change: $\dfrac{f(b) - f(a)}{b - a}$

$f(5) = \frac{1}{2}$, $f(0) = \frac{7}{2}$: $\dfrac{f(5) - f(0)}{5 - 0} = \dfrac{\frac{1}{2} - \frac{7}{2}}{5 - 0} = \dfrac{-\frac{6}{2}}{5} = \dfrac{-3}{5}$ *slope of gray line*

b. $f(3) \stackrel{?}{=} \underline{1}$ and $f'(3) \stackrel{?}{=} \underline{-\frac{2}{3}}$ *slope of red line*

c. Find the equation of the tangent line at $x = 3$. Give your answer in slope-intercept form.

$y - 1 = -\frac{2}{3}(x - 3)$ => $y - 1 = -\frac{2}{3}x + 2$ => $y = -\frac{2}{3}x + 3$

3. The slope of the curve $y = ax^2 + bx$ at the point $(2, 4)$ is -8. Calculate the values of a and b.

4. Find the values of x for which both the graphs of the functions $f(x) = x^3 - 3x^2 + 7x + 8$ and $g(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 5x - 3$ have parallel tangent lines at x. Pick one such location on the graph of $f(x)$ and find the equation of the tangent line there.

3. The slope of the curve $y = ax^2 + bx$ at the point $(2, 4)$ is -8. Calculate the values of a and b.

derivative at $x=2$ is -8: $f'(2) = -8$
$$f'(x) = 2ax + b = -8$$
$$f'(2) = 2a(2) + b = -8$$
$$4a + b = -8$$

this is unsolvable w/o: $f(2) = 4$
$$a(2)^2 + b(2) = 4$$
$$4a + 2b = 4$$

solve the system of equations:
(i) $4a + b = -8$
(ii) $4a + 2b = 4$

method of cancellation:
$$4a + b = -8$$
$$-(4a + 2b = 4)$$
$$0 - b = -12$$
$$b = 12$$

substitute into eq (i):
$$4a + 12 = -8$$
$$4a = -20$$
$$a = -5$$

$a = -5, b = 12$

Check answers:
$f(x) = -5x^2 + 12x$
$f(2) = -5(2)^2 + 12(2)$
$\quad = -5(4) + 24$
$\quad = -20 + 24$
$\quad = 4$
$f'(x) = -10x + 12$ power rule
$f'(2) = -10(2) + 12$ $\frac{d}{dx}(ax^n) = a \cdot n x^{n-1}$
$\quad = -20 + 12$
$\quad = -8$

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4. Find the values of x for which both the graphs of the functions $f(x) = x^3 - 3x^2 + 7x + 8$ and $g(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 5x - 3$ have parallel tangent lines at x. Pick one such location on the graph of $f(x)$ and find the equation of the tangent line there.

parallel tangent lines means $f'(x) = g'(x)$
$3x^2 - 6x + 7 + 0 = \frac{1}{3} \cdot 3x^2 - \frac{1}{2} \cdot 2x + 5 - 0$ power rule
$3x^2 - 6x + 7 = x^2 - x + 5$ $\frac{d}{dx}(ax^n) = anx^{n-1}$
$2x^2 - 5x + 2 = 0$
$2x^2 - 4x - x + 2 = 0$
$2x(x-2) - 1(x-2) = 0$
$(2x-1)(x-2) = 0$

$2x-1 = 0 \qquad x-2 = 0$
$2x = 1 \qquad\quad x = 2$
$x = \frac{1}{2}$

Note: Since $f'(x) = g'(x)$, I can use either to find the slope of the tangent line. I choose $g(x)$ as it's simpler. But $(x, f(x)) \neq (x, g(x))$ so the tangent line of $f(x)$ at $x = \frac{1}{2}$ is parallel but not equal to the tangent line of $g(x)$ at $x = \frac{1}{2}$.

tangent line at $x = \frac{1}{2}$
$g'(x) = x^2 - x + 5$
$g'(\frac{1}{2}) = (\frac{1}{2})^2 - \frac{1}{2} + 5$
$\quad = \frac{1}{4} - \frac{2}{4} + \frac{20}{4}$
$\quad = \frac{21}{4}$

$f(x) = x^3 - 3x^2 + 7x + 8$
$f(\frac{1}{2}) = (\frac{1}{2})^3 - 3(\frac{1}{2})^2 + 7(\frac{1}{2}) + 8$
$\quad = \frac{1}{8} - 3(\frac{1}{4}) + \frac{7}{2} + 8$
$\quad = \frac{1}{8} - \frac{3}{4} + \frac{7}{2} + 8$
$\quad = \frac{1}{8} - \frac{6}{8} + \frac{28}{8} + \frac{64}{8}$
$\quad = \frac{87}{8}$

$y - (\frac{87}{8}) = \frac{21}{4}(x - 2)$
$y - \frac{87}{8} = \frac{21}{4}x - \frac{21}{2}$
$y - \frac{87}{8} = \frac{21}{4}x - \frac{84}{8}$
$y = \frac{21}{4}x + \frac{3}{8}$

tangent line at $x = 2$
$g'(x) = x^2 - x + 5$
$g'(2) = (2)^2 - (2) + 5$
$\quad = 4 - 2 + 5$
$\quad = 7$

$f(x) = x^3 - 3x^2 + 7x + 8$
$f(2) = (2)^3 - 3(2)^2 + 7(2) + 8$
$\quad = 8 - 12 + 14 + 8$
$\quad = 18$

$y - 18 = 7(x - 18)$
$y - 18 = 7x - 126$
$y = 7x - 108$