Exit Ticket Derivative and Integral Review

Fill in the derivatives and integrals:

1. $\frac{d}{dx}[k] =$

2. $\int k dx =$

3. $\frac{d}{dx}[kx^n] =$

4. $\int x^n dx =$

5. $\frac{d}{dx}[\ln(x)] =$

6. $\int \frac{1}{x} dx =$

7. $\frac{d}{dx}[\log_a(x)] =$

8. $\int \frac{1}{x \cdot \ln(a)} dx =$

9. $\frac{d}{dx}[e^x] =$

10. $\int e^x dx =$

11. $\frac{d}{dx}[a^x] =$

12. $\int a^x dx =$

13. $\frac{d}{dx}[\sin(x)] =$

14. $\int \cos(x) dx =$

15. $\frac{d}{dx}[\cos(x)] =$

16. $\int \sin(x) dx =$

17. $\frac{d}{dx}[\tan(x)] =$

18. $\int \sec^2(x) dx =$

19. $\frac{d}{dx}[\sec(x)] =$

20. $\int \sec(x)\tan(x) dx =$

Use the rules above to find the integrals below and check your answer:

1. $\int \cot(x)\sin(x) dx$

2. $\int \frac{1+\cos^2(\theta)}{\cos^2(\theta)} d\theta$

3. $\int \frac{2u^2 - 5u + \sqrt[3]{u}}{u^2} du$

4. $\int 6x(x^2+1)^2 dx$

Exit Ticket Natural Log

Fill in the following rules:

1. $\ln(a) + \ln(b) =$

2. $\ln(a) - \ln(b) =$

3. $\ln(x^a) =$

4. $\ln\left(ax^b\right) =$

5. $\frac{d}{dx}\left[\ln(ax+b)\right] =$

6. $\int \frac{a}{ax+b}dx =$

Use the above rules to solve the following equations for x:

1. $\int \frac{1}{2x+5}dx$

2. $\int \frac{1}{x+12}dx$

3. $\frac{d}{dx}\left[\ln\left(\frac{1-x}{1+x}\right)\right]$

4. $\frac{d}{dx}\left[\ln\left(\frac{2x^2-3}{3x^3-6}\right)\right]$

5. $\int \frac{2x}{4x^2+12}dx$

6. $\int \frac{5x+7}{5x^2+14x+6}dx$

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Exit Ticket Integral Review

Solve the following integrals and identify the integral rule used:

1. $\int \cot(x)\sin(x)dx$

2. $\int \frac{1+\cos^2(\theta)}{\cos^2(\theta)}d\theta$

3. $\int \frac{\sin(x)}{1+\cos^2(x)}dx$

4. $\int 6x(x^2+1)^{\frac{1}{2}}dx$

5. $\int \sin(2x)dx$

6. $\int \frac{1}{1+\sin(\theta)}d\theta$

7. $\int \frac{3x}{(2x^2+1)^2}dx$

8. $\int \frac{3x}{(2x^2+1)}dx$

Exit Ticket Inverse Trigonometric Functions

Fill in the derivatives and integrals:

1. $\dfrac{d}{dx}\left[\arcsin(x)\right] =$

2. $\displaystyle\int \frac{1}{\sqrt{1-x^2}}dx =$

3. $\dfrac{d}{dx}\left[\arctan(x)\right] =$

4. $\displaystyle\int \frac{1}{1+x^2}dx =$

Use the rules above to find the integrals below:

1. $\displaystyle\int \frac{1}{1+9x^2}dx$

2. $\dfrac{d}{dx}\left[\arcsin\left(\frac{3}{4}x\right)\right]$

3. $\displaystyle\int \frac{3}{\sqrt{9-4x^2}}dx$

4. $\dfrac{d}{dx}\left[\arctan\left(x^2\right)\right]$

5. $\displaystyle\int \frac{5x+1}{4+9x^2}dx$

6. $\dfrac{d}{dx}\left[\arcsin(x+1)\right]$

Exit Ticket Area Between Curves

> **Area Between curves** Assuming that $f(x) \geq g(x)$ for $a \leq x \leq b$, the area between the curves is:
>
> $$\int_a^b [f(x) - g(x)]\, dx$$

Set up but do NOT solve the integral that finds the areas bounded by the functions below:

1. $y = x^2 + 2,\ y = \sin(x),\ x = -1,\ x = 2$

2. $x = y^2 + 1,\ x = 5,\ y = -3,\ y = 3$

3. $y = \frac{1}{x+2},\ y = (x+2)^2,\ x = -\frac{3}{2},\ x = 1$

4. $x = y^2 - y - 6,\ x = 2y + 4$

Exit Ticket Volume of Solids with Uniform Cross-sections

> **Volume of Solids with Uniform Cross-sections** Consider a solid whose base is the region bounded by given function(s) with uniform cross-sections perpendicular to the x-axis. The volume of the solid is given by:
>
> $$V = \int_a^b [A(x)]\, dx$$
>
> where $A(x)$ is the area of the cross-section

Set up but do NOT solve the integral that finds the volume of the solid whose base is bounded by $y = x^2 + 2,\ y = \sin(x),\ x = -1,\ x = 2$ and has uniform cross-sections perpendicular to the x-axis in the shape of:

1. squares

2. triangles of height x^2

3. semicircles

4. rectangles of height $\sqrt{x}$

Exit Ticket Solids of Revolution

Solids of Revolution Consider a solid formed by rotating a bounded region about a line $y = c$ with cross-sectional area functions $A(x)$, then the volume formula is

$$V = \int_a^b [A(x)]\, dx.$$

Disk method: $A(x) = \pi r^2$ where r is a function of x
Washer method: $A(x) = \pi [R^2 - r^2]$ where R, r are a functions of x
Shell method: $A(x) = 2\pi rh$ where r, h are a functions of x

Set up but do NOT solve the integral that finds the volume of the solid formed by rotating the region bounded by:

1. $y = \sqrt{x}$, $y = 3$, and the $y-$axis about the $y-$axis

2. $y = 10 - 6x + x^2$, $y = -10 + 6x - x^2$, $x = 1$, and $x = 5$ about the line $y = 8$

3. $x = y^2 - 4$, $x = 6 - 3y$ about the line $y = 8$

Exit Ticket Work and Energy

> **Work and Energy** Suppose that the force at any given x is given by $F(x)$, then the work done by the force in moving the object from $x = a$ to $x = b$ is given by
>
> $$W = \int_a^b F(x)dx.$$

Set up but do NOT solve the following integral:

1. A uniform chain 10 m long weighing 30 kg lying completely at the foot of a building 50 m tall.
   (a) What is the work done against gravity to move one end to the top of the building with the rest of the chain danging free?
   (b) What is the work done to move one end only 30 m off the ground?
   (c) What is the work done to move the top end of the chain 5 meters off the ground with the rest of the chain still on the ground?

Exit Ticket Integration by Parts

Integration by Parts Let $u(x)$ and $v(x)$ be two differentiable functions. Integration by parts says

$$\int udv = ux - \int vdu$$

Evaluate the following integrals:

1. $\int 8xe^{6x}dx$

2. $\int 4x\cos(2-3x)dx$

3. $\int (2-x)^2\ln(4x)dx$

4. $\int \ln(x)dx$

5. $\int e^{-x}\sin(4x)dx$

6. $\int \frac{x^7}{\sqrt{x^4+1}}dx$

Exit Ticket Partial Fraction

Partial Fraction Decomposition:

denominator	partial fraction
$ax+b$	$\frac{A}{ax+b}$
$(ax+b)^n$	$\frac{A_1}{ax+b}+\frac{A_2}{(ax+b)^2}+...+\frac{A_n}{(ax+b)^n}$
ax^2+bx	$\frac{Ax+B}{ax^2+bx}$
$(ax^2+b)^n$	$\frac{A_1x+B_1}{ax^2+b}+\frac{A_2x+B_2}{(ax^2+b)^2}+...+\frac{A_nx+B_n}{(ax^2+b)^n}$

Solve the following integrals using partial fractions:

1. $\displaystyle\int \frac{x^2+x+1}{(x+1)(x+4)^2}dx$

2. $\displaystyle\int \frac{2x^4+x^3+4x^2+1}{x^3+x}dx$

Exit Ticket Improper Integrals

Improper Integrals

1. If $\int_a^c f(x)dx$ exists for every $t > a$, then $\int_a^\infty f(x)dx = \lim_{c \to \infty} \int_a^c f(x)dx$ provided that the limit exists and is finite.

2. If $\int_c^a f(x)dx$ exists for every $c < b$, then $\int_{-\infty}^b f(x)dx = \lim_{c \to -\infty} \int_c^b f(x)dx$ provided that the limit exists and is finite.

3. If $f(x)$ is continuous on the interval $[a, b)$ and not at $x = b$, then $\int_a^b f(x)dx = \lim_{c \to b^-} \int_a^c f(x)dx$ provided that the limit exists and is finite.

4. If $f(x)$ is continuous on the interval $(a, b]$ and not at $x = a$, then $\int_a^b f(x)dx = \lim_{c \to a^+} \int_c^b f(x)dx$ provided that the limit exists and is finite.

5. If $f(x)$ is not continuous $x = t$ where $a < t < b$, then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ provided that the limit exists and is finite.

The integral is considered **convergent** if the limit exists and is finite, and **divergent** if the limit doesn't exist or is infinite.

Solve the following integrals using the concept above:

1. $\displaystyle\int_0^\infty \frac{1}{x}dx$

2. $\displaystyle\int_{-5}^1 \frac{1}{10+2x}dx$

3. $\displaystyle\int_1^4 \frac{1}{x^2+x-6}dx$

4. $\displaystyle\int_{-\infty}^0 \frac{e^{\frac{1}{x}}}{x^2}dx$

Exit Ticket Numerical Integration

Numerical Integration We can estimate the integral $\int_a^b f(x)dx$ using the following formulas,

1. **midpoint:** $\int_a^b f(x)dx \approx \Delta x\left[f(x_1^*) + f(x_2^*) + ... + f(x_n^*)\right]$
2. **trapezoid:** $\int_a^b f(x)dx \approx \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + ... + 2f(x_{n-1} + f(x_n)\right]$
3. **simpson's:** $\int_a^b f(x)dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) ... + 2f(x_{n-1}) + 4f(x_{n-1}) + f(x_n)\right]$

where n is the number is subintervals and $\Delta x = \frac{b-a}{n}$

Estimate the following integrals using each of the rules above:

1. $\int_1^7 \frac{1}{x^3+1}dx$

2. $\int_0^4 \cos\left(1+\sqrt{x}\right)dx$

Exit Ticket Double Integrals

Double Integrals The integral over a horizontally simple region $D = \{(x, y) \mid h_1(y) \leq x \leq h_2(y),\ c \leq y \leq d\}$ is

$$\int\int_D f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dxdy$$

Find the integral $\int\int_D 2x\,dA$ for the regions bounded by the following functions:

1. $y = x^2 + 2,\ y = \sin(x),\ x = -1,\ x = 2$

2. $x = y^2 + 1,\ x = 5,\ y = \pm 2$

3. $y = \frac{1}{x+2},\ y = (x+2)^2,\ x = -1,\ x = 1$

4. $x = y^2 - y - 6,\ x = 2y + 4$

Exit Ticket Polar Integrals

Polar Integrals The integral over region $D = \{(r, \theta) \mid a \leq r \leq b,\ c \leq \theta \leq d\}$ is

$$\int\int_D f(r,\theta)\, dA = \int_c^d \int_a^b f(r,\theta) \cdot r \cdot dr d\theta$$

Find the integral $\int\int_D 2x^2 + y^2\, dA$ for the regions bounded by the following:

1. the circles of radius 1 and 3 centered at the origin

2. the circles of radius 1 and 3 centered at the origin contained in the third quadrant

3. $0 \leq y \leq 1;\ -\sqrt{1-y^2} \leq x \leq \sqrt{1-y^2}$

4. $-1 \leq y \leq 1;\ 0 \leq x \leq \sqrt{1-y^2}$

Exit Ticket Separable Differential Equations

> **Separable Differential Equations** The solution to the separable differential equations $\frac{dy}{dx} = p(x)q(y)$ is
>
> $$\begin{aligned} \frac{dy}{dx} &= p(x)q(y) \\ \frac{1}{q(y)}dy &= p(x)dx \\ \int \frac{1}{q(y)}dy &= \int p(x)dx \end{aligned}$$

Find the general solution to the following differential equations:

1. $\dfrac{dy}{dx} = 6y^2x$

2. $y' = \dfrac{3x^2 + 4x - 4}{2y - 4}$

3. $y' = 2xe^{-y} - 4e^{-y}$

4. $\dfrac{dy}{dt} = \dfrac{\cos^2(y)}{y}$

Produced by Audriana Houtz, Mathematics Ph.D. student at the University of Notre Dame.

Exit Ticket Euler's Method

Euler's Method Consider the initial value problem

$$y' = f(x, y);\ y(x_0) = y_0.$$

Using the step size h

$$x_n = x_{n-1} + h$$
$$y_n = y_{n-1} + h \cdot f(x_{n-1}, y_{n-1})$$

Find $(x_2, y_2$ for each of the following differential equations using the step size $h = \frac{1}{2}$.

1. $\dfrac{dy}{dx} = 1 + 2xy;\ y(0) = 3$

2. $y' = x^2 + y^2;\ y(1) = 2$

3. $y' = 3x + 3y^2;\ y(0) = 1$

4. $\dfrac{dy}{dt} = \dfrac{\sqrt{y}}{t+1};\ y(1) = 1$

Exit Ticket Partial Derivatives

> **Partial Derivatives** A derivative asks how much does y "moves" when we vary x. Partial derivatives are the multi-variable version of this process.
>
> - $\dfrac{\partial}{\partial x}[f(x,y)] = f_x =$ take the derivative with respect to x while keeping y constant
> - $\dfrac{\partial}{\partial y}[f(x,y)] = f_y =$ take the derivative with respect to y while keeping x constant

Find all four partial second derivatives f_{xx}, f_{yy}, f_{xy}, f_{yx}:

1. $8xe^{6x-y^2}$

2. $\ln\left(5x^2-y\right)$

3. $\cos(3x)y^2$

4. $\dfrac{x-1-2y}{x^2}$

Exit Ticket Estimating Partial Derivatives

Estimating Partial Derivatives There are three formulas for estimating partial derivatives with respect to x:

- **forward difference:** $\frac{\partial}{\partial x}[f(x,y)] \approx \frac{f(x+h,y)-f(x,y)}{h}$
- **backwards difference:** $\frac{\partial}{\partial x}[f(x,y)] \approx \frac{f(x,y)-f(x-h,y)}{h}$
- **central difference:** $\frac{\partial}{\partial x}[f(x,y)] \approx \frac{f(x+h,y)-f(x-h,y)}{2h}$

Below is a chart that describes the "Feels like" temperature ($F(T,W)$) given the wind speed (W) and air temperature (T), give all estimates of $\frac{\partial F}{\partial W}$ at the given points:

$W \backslash T$	40	35	30	25
5	36	31	25	19
10	34	27	21	15
15	31	25	19	13
20	30	24	17	11

1. $(35, 15)$

2. $(25, 20)$

3. $(40, 5)$

4. $(30, 10)$

Exit Ticket Chain Rule

Chain Rule Let $z = f(x, y)$, $x = g(s, t)$, and $y = h(s, t)$ be functions of two variables. The partial derivatives $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ can be found by the chain rules:

1. $\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}$
2. $\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}$

Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ for the functions below:

1. $z = x^2 + 2xy,\ y = s + t,\ x = s^2 + 4t$

2. $z = x\cos(x) + y^2,\ x = 3t + 1,\ y = s^2 + t^2$

3. $z = \dfrac{x^2 - x}{y^4},\ x = t^3,\ y = cos(2s)$

4. $z = \sqrt{x^2 + y^2} + \dfrac{y}{x},\ x = \sin(t),\ y = s^2 + t^2$