Math 10350 – Example Set 15A
(Section 5.1 & 5.2)

(5.1) Right-endpoint Approximation. Estimate the area under the graph of $f(x) = e^{-x^2}$ over the interval $0 \le x \le 2$ using **4th right-endpoint approximation** (ie. with four sub-intervals). (Text notation: R_4).

How big should my step (Δx) be?

$\Delta x = \frac{b-a}{n}$ where $a \le x \le b$ and n = number of sub-intervals

$= \frac{2-0}{4} = \frac{1}{2}$

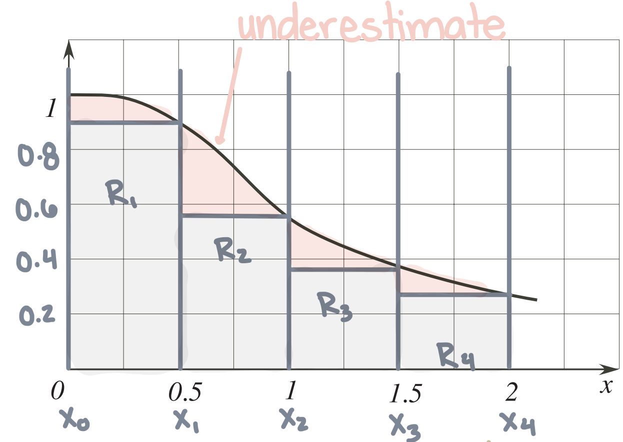
Right-endpoint means using right-hand value as the height of the rectangle:

$R_1 = f(x_1) \cdot \Delta x = (0.9)(\frac{1}{2})$

$R_2 = f(x_2) \cdot \Delta x = (0.55)(\frac{1}{2})$

$R_3 = f(x_3) \cdot \Delta x = (0.35)(\frac{1}{2})$

$R_4 = f(x_4) \cdot \Delta x = (0.25)(\frac{1}{2})$



General formula: (right-endpoint) $\sum_{i=0}^{n-1} f(x_{i+1}) \cdot \Delta x$ ← n counts; right-hand

(5.1) Left-endpoint Approximation. Estimate the area under the graph of $f(x) = e^{-x^2}$ over the interval $0 \le x \le 2$ using **4th left-endpoint approximation** (ie. with four sub-intervals). (Text notation: L_4).

How big should my step (Δx) be?

$\Delta x = \frac{b-a}{n}$ where $a \le x \le b$ and n = number of sub-intervals

$= \frac{2-0}{4} = \frac{1}{2}$

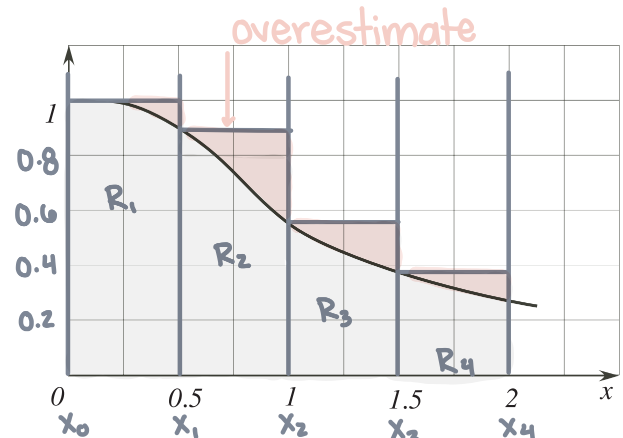
Left-endpoint means using left-hand value as the height of the rectangle:

$R_1 = f(x_0) \cdot \Delta x = (1)(\frac{1}{2})$

$R_2 = f(x_1) \cdot \Delta x = (0.9)(\frac{1}{2})$

$R_3 = f(x_2) \cdot \Delta x = (0.55)(\frac{1}{2})$

$R_4 = f(x_3) \cdot \Delta x = (0.39)(\frac{1}{2})$



General formula: (left-endpoint) $\sum_{i=0}^{n-1} f(x_i) \cdot \Delta x$ ← n counts; left-hand

(5.1) Midpoint Approximation. Estimate the area under the graph of $f(x) = e^{-x^2}$ over the interval $0 \le x \le 2$ using **Midpoint Rule** with four sub-intervals. (Text notation: M_4).

How big should my step (Δx) be?

$\Delta x = \frac{b-a}{n}$ where $a \le x \le b$ and n = number of sub-intervals

$= \frac{2-0}{4} = \frac{1}{2}$

Midpoint means using the midpoint value as the height of the rectangle:

$R_1 = f(x_{\frac{1}{2}}) \cdot \Delta x = (1)(\frac{1}{2})$

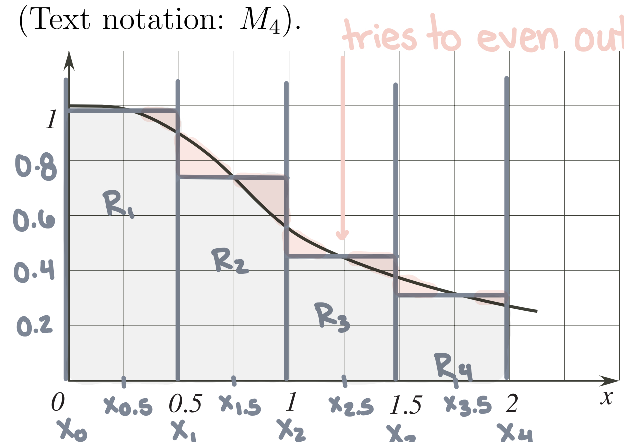
$R_2 = f(x_{1.5}) \cdot \Delta x = (0.65)(\frac{1}{2})$

$R_3 = f(x_{2.5}) \cdot \Delta x = (0.45)(\frac{1}{2})$

$R_4 = f(x_{3.5}) \cdot \Delta x = (0.3)(\frac{1}{2})$

$x_{n+\frac{1}{2}} = \frac{x_{n+1} - x_n}{2}$

so $x_{\frac{1}{2}} = \frac{x_1 - x_0}{2}$



General formula: $\sum_{i=0}^{n-1} f(x_{i+\frac{1}{2}}) \cdot \Delta x$ ← slight joke

1. Using the **Nth right-endpoint approximation**, express the area under the graph of $f(x) = e^{-x^2}$ over $0 \leq x \leq 2$ as a limit of right-endpoint approximations.

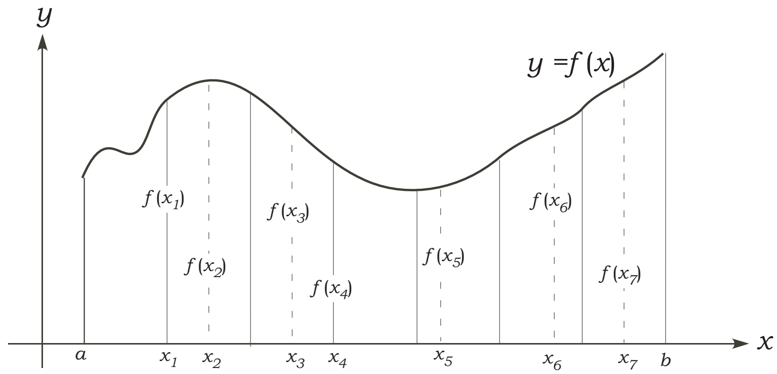
General formula: $\sum_{i=0}^{n-1} f(\underbrace{x_{i+1}}_{\text{right-hand}}) \cdot \Delta x$
(right-endpoint)

Ideally $n \to \infty$ that way we do not over/under-estimate:

$$\lim_{n\to\infty} \sum_{i=0}^{n-1} f(x_{i+1}) \cdot \Delta x = \lim_{n\to\infty} \sum_{i=0}^{n-1} f(x_{i+1}) \cdot \underbrace{\frac{b-a}{n}}_{\text{goes to } 0} = \lim_{n\to\infty} \sum_{i=0}^{n-1} e^{-(x_{i+1})^2} \cdot \left(\frac{b-a}{n}\right)$$

Remark. We denote the area under the graph of $f(x) = e^{-x^2}$ over $0 \leq x \leq 2$ with the definite integral notation: $\int_a^b f(x)\,dx = \int_0^2 e^{-x^2} dx$

Definite Integral of Positive Value functions. In general, we many select any point in a subinterval and do the same construction to obtain the area under the graph of $f(x)$.



These more general sums are called Riemann Sum. They give us a similar limiting formula for the value of the definite integral for a positive valued $f(x)$ over $[a, b]$. Write down the relation below:

Area $= \sum_{x=0}^{n-1} f(x)\,\Delta x$