Exit Ticket Log and Exponential Rules Practice

Fill in the following rules:

1. $\ln(a) + \ln(b) =$
2. $\ln(a) - \ln(b) =$
3. $\ln(x^a) =$
4. $\ln\left(ax^b\right) =$
5. $e^{\ln(x)} =$
6. $\ln(e^x) =$
7. $e^a \cdot e^b =$
8. $e^{\frac{a}{b}} =$

Use the above rules to solve the following equations for x:

1. $4 = \ln(x^2)$
2. $8 = \ln(x)^3$
3. $2 = \ln((xe)^2)$
4. $2 = \ln(xe^2)$
5. $6 = \ln(ex^2)$
6. $6 = \ln(2e^x)$
7. $e^{3x} - e^{5x+1} = 0$
8. $3e^{3x} - 5e^{5x} = 0$

Produced by Audriana Houtz, Mathematics Ph.D. student at the University of Notre Dame.

Exit Ticket Log and Exponential Rules Practice

Fill in the following rules:

1. $\ln(a) + \ln(b) = \ln(a \cdot b)$
2. $\ln(a) - \ln(b) = \ln(\frac{a}{b})$
3. $\ln(x^a) = a \cdot \ln(x)$
4. $\ln(ax^b) = \ln(a) + b \cdot \ln(x)$
5. $e^{\ln(x)} = x$
6. $\ln(e^x) = x \cdot \ln(e) = x$
7. $e^a \cdot e^b = e^{a+b}$
8. $e^{\frac{a}{b}} = \sqrt[b]{e^a}$

Use the above rules to solve the following equations for x:

1. $4 = \ln(x^2)$

$4 = 2 \cdot \ln(x)$ $\quad \ln(x^a) = a \cdot \ln(x)$

$2 = \ln(x)$

$e^2 = e^{\ln(x)}$ $\quad e^{\ln(x)} = x$

$e^2 = x$

2. $8 = \ln(x)^3$

$\sqrt{8} = \sqrt[3]{\ln(x)^3}$

$2 = \ln(x)$

$e^2 = e^{\ln(x)}$ $\quad e^{\ln(x)} = x$

$e^2 = x$

3. $2 = \ln((xe)^2)$

$2 = 2\ln(xe)$ $\quad \ln(x^a) = a \cdot \ln(x)$

$1 = \ln(xe)$

$1 = \ln(x) + \ln(e)$ $\quad \ln(a \cdot b) = \ln(a) + \ln(b)$

$1 = \ln(x) + 1$ $\quad \ln(e) = 1$

$0 = \ln(x) \longrightarrow e^0 = e^{\ln(x)}$ $\quad e^{\ln(x)} = x$

$1 = x$

4. $2 = \ln(xe^2)$

$2 = \ln(x) + \ln(e^2)$ $\quad \ln(a \cdot b) = \ln(a) + \ln(b)$

$2 = \ln(x) + 2\ln(e)$ $\quad \ln(x^a) = a \cdot \ln(x)$

$2 = \ln(x) + 2$ $\quad \ln(e) = 1$

$0 = \ln(x)$

$e^0 = e^{\ln(x)} \longrightarrow 1 = x$ $\quad e^{\ln(x)} = x$

5. $6 = \ln(ex^2)$

$6 = \ln(e) + \ln(x^2)$ $\quad \ln(a \cdot b) = \ln(a) + \ln(b)$

$6 = 1 + \ln(x^2)$ $\quad \ln(e) = 1$

$6 = 1 + 2\ln(x)$ $\quad \ln(x^a) = a \cdot \ln(x)$

$5 = 2\ln(x)$

$\frac{5}{2} = \ln(x) \longrightarrow e^{\frac{5}{2}} = e^{\ln(x)}$ $\quad e^{\ln(x)} = 1$

$e^{\frac{5}{2}} = x$

6. $6 = \ln(2e^x)$

$6 = \ln(2) + \ln(e^x)$ $\quad \ln(a \cdot b) = \ln(a) + \ln(b)$

$6 = \ln(2) + x\ln(e)$ $\quad \ln(x^a) = a \cdot \ln(x)$

$6 = \ln(2) + x$ $\quad \ln(e) = 1$

$6 - \ln(2) = x$

7. $e^{3x} - e^{5x+1} = 0$

$e^{3x} = e^{5x+1}$

$\ln(e^{3x}) = \ln(e^{5x+1})$

$3x\ln(e) = (5x+1)\ln(e)$ $\quad \ln(x^a) = a \cdot \ln(x)$

$3x = 5x + 1$ $\quad \ln(e) = 1$

$-2x = 1$

$x = -\frac{1}{2}$

8. $3e^{3x} - 5e^{5x} = 0$

$3e^{3x} = 5e^{5x}$

$\ln(3e^{3x}) = \ln(5e^{5x})$

$\ln(3) + \ln(e^{3x}) = \ln(5) + \ln(e^{5x})$ $\quad \ln(a \cdot b) = \ln(a) + \ln(b)$

$\ln(3) + 3x\ln(e) = \ln(5) + 5x\ln(e)$ $\quad \ln(x^a) = a \cdot \ln(x)$

$\ln(3) + 3x = \ln(5) + 5x$ $\quad \ln(e) = 1$

$\ln(3) - \ln(5) = 2x$ $\quad \ln(a) - \ln(b) = \ln(\frac{a}{b})$

$\ln(\frac{3}{5}) = 2x$

$\frac{1}{2}\ln(\frac{3}{5}) = x$

Exit Ticket Log and Exponential Rules Practice

Fill in the following rules:

1. $\ln(a) + \ln(b) =$

2. $\ln(a) - \ln(b) =$

3. $\ln(x^a) =$

4. $\ln\left(ax^b\right) =$

5. $e^a \cdot e^b =$

6. $e^{\frac{a}{b}} =$

7. $e^{\ln(x)} =$

8. $\ln(e^x) =$

Use the above rules to solve the following equations for x:

1. $\ln(x^2 + 2x + 1) = 8$

2. $\ln(x^2 + 2x + 1) = \ln(x^2) + 1$

3. $3e^{3x} - 5e^{-5x} = 0$

4. $3e^{3x} - 5e^{5x} = 0$

5. $2\ln(x) = \ln(2) + \ln(3x - 4)$

6. $\ln(x) + \ln(x - 1) = \ln(4x)$

7. $log_9(x - 5) + log_9(x + 3) = 1$

8. $log_2(x - 2) + log_2(x + 1) = 2$

Exit Ticket Log and Exponential Rules Practice

Fill in the following rules:

1. $\ln(a) + \ln(b) =$ $\ln(a \cdot b)$
2. $\ln(a) - \ln(b) =$ $\ln(\frac{a}{b})$
3. $\ln(x^a) =$ $a \cdot \ln(x)$
4. $\ln(ax^b) =$ $\ln(a) + b \cdot \ln(x)$
5. $e^a \cdot e^b =$ e^{a+b}
6. $e^{\frac{a}{b}} =$ $\sqrt[b]{e^a}$
7. $e^{\ln(x)} =$ x
8. $\ln(e^x) =$ $x \cdot \ln(e) = x$

Use the above rules to solve the following equations for x:

1. $\ln(x^2 + 2x + 1) = 8$

$\ln((x+1)^2) = 8$
$2\ln(x+1) = 8$
$\ln(x+1) = 4$
$e^{\ln(x+1)} = e^4$
$x+1 = e^4$
$x = e^4 - 1$

2. $\ln(x^2 + 2x + 1) = \ln(x^2) + 1$

$\ln((x+1)^2) = \ln(x^2) + 1$
$\ln((x+1)^2) - \ln(x^2) = 1$
$\ln(\frac{(x+1)^2}{x^2}) = 1$
$\ln((\frac{x+1}{x})^2) = 1$
$2\ln(\frac{x+1}{x}) = 1$
$\ln(\frac{x+1}{x}) = \frac{1}{2}$
$e^{\ln(\frac{x+1}{x})} = e^{\frac{1}{2}}$
$\frac{x+1}{x} = e^{\frac{1}{2}}$
$x+1 = xe^{\frac{1}{2}}$

quadratic formula

3. $3e^{3x} - 5e^{-5x} = 0$

$3e^{3x} = 5e^{-5x}$
$\ln(3e^{3x}) = \ln(5e^{-5x})$
$\ln(3) + 3x = \ln(5) - 5x$
$8x = \ln(\frac{5}{3})$
$x = \frac{1}{8}\ln(\frac{5}{3})$

4. $3e^{3x} - 5e^{5x} = 0$

$3e^{3x} = 5e^{5x}$
$\ln(3e^{3x}) = \ln(5e^{5x})$
$\ln(3) + 3x = \ln(5) + 5x$
$\ln(3) - \ln(5) = 2x$
$\ln(\frac{3}{5}) = 2x \longrightarrow \frac{1}{2}\ln(\frac{3}{5}) = x$

5. $2\ln(x) = \ln(2) + \ln(3x - 4)$

$\ln(x^2) = \ln(2(3x-4))$
$\ln(x^2) = \ln(6x-8)$
$e^{\ln(x^2)} = e^{\ln(6x-8)} = \frac{1}{3}x^{-3}$
$x^2 = 6x - 8$
$x^2 - 6x + 8 = 0 \longrightarrow (x-4)(x-2) = 0$
$x = 2, 4$

6. $\ln(x) + \ln(x - 1) = \ln(4x)$

$\ln(x(x-1)) = \ln(4x)$
$\ln(x^2 - x) = \ln(4x)$
$e^{\ln(x^2-x)} = e^{\ln(4x)}$
$x^2 - x = 4x$
$x^2 - 5x = 0 \longrightarrow x(x^2-5) = 0$
$= 0, \pm\sqrt{5}$

not allowed due to ln(x-1)

$-\sqrt{5}$ also not allowed due to ln(x)

7. $log_9(x - 5) + log_9(x + 3) = 1$

$\log_9((x-5)(x+3)) = 1$
$\log_9(x^2 - 2x - 15) = 1$
$9^{\log(x^2-2x-15)} = 9^1$
$x^2 - 2x - 15 = 9$
$x^2 - 2x - 24$
$(x-6)(x+4) = 0$
$x = -4, 6$

not allowed $\log_9(x-5)$

8. $log_2(x - 2) + log_2(x + 1) = 2$

$\log_2((x-2)(x+1)) = 2$
$2^{\log_2((x-2)(x+1))} = 2^2$
$(x-2)(x+1) = 4$
$x^2 - x - 2 = 4$
$x^2 - x - 6 = 0$
$(x-3)(x+2) = 0$
$x = -2, 3$

not allowed $\log_2(x+1)$

Exit Ticket Power Rule

> **Power Rule**
>
> $$\frac{d}{dx}[x^n] = nx^{n-1}$$

Use the product rule above to find the derivative of the following functions:

1. $y = x^3$

2. $y = 4x^2 + 5x - 6$

3. $y = 5x^{\frac{1}{4}} - 4x^{\frac{1}{2}} + 7$

4. $g(x) = \frac{1}{3}x^{-3}$

5. $g(x) = \frac{1}{x^5}$

6. $y(x) = \frac{1}{3\sqrt[3]{x}}$

7. $R = \frac{15x^7+18x^5-21x^4}{3x}$

8. $L = \dfrac{\frac{3}{4}x^{\frac{11}{3}} + \frac{2}{5}x^{\frac{5}{3}} + \frac{5}{11}x^{\frac{2}{3}}}{x^{\frac{-8}{3}}}$

Exit Ticket Power Rule

> **Power Rule**
> $$\frac{d}{dx}[x^n] = nx^{n-1}$$

Use the product rule above to find the derivative of the following functions:

1. $y = x^3$

$y' = 3x^{3-1}$

$= 3x^2$

2. $y = 4x^2 + 5x - 6$

$y' = 4 \cdot 2x^{2-1} + 5x^{1-1} - 0$

$= 8x + 5$

3. $y = 5x^{\frac{1}{4}} - 4x^{\frac{1}{2}} + 7$

$y' = 5 \cdot \frac{1}{4} x^{\frac{1}{4}-1} - 4 \cdot \frac{1}{2} x^{\frac{1}{2}-1} + 0$

$= \frac{5}{4} x^{-\frac{3}{4}} - 2x^{-\frac{1}{2}}$

4. $g(x) = \frac{1}{3}x^{-3}$

$g'(x) = \frac{1}{3} \cdot (-3) x^{-3-1}$

$= -x^{-4}$

5. $g(x) = \frac{1}{x^5} = x^{-5}$

$g'(x) = -5x^{-5-1}$

$= -5x^{-6}$

6. $y(x) = \frac{1}{3\sqrt[3]{x}} = \frac{1}{3} x^{-\frac{1}{3}}$

$y' = \frac{1}{3} \cdot (-\frac{1}{3}) x^{-\frac{1}{3}-1}$

$= -\frac{1}{9} x^{-\frac{4}{3}}$

7. $R = \frac{15x^7 + 18x^5 - 21x^4}{3x} = \frac{15x^7}{3x} + \frac{18x^5}{3x} - \frac{21x^4}{3x}$

$R = 5x^6 + 6x^4 - 7x^3$

$R' = 5 \cdot 6x^{6-1} + 6 \cdot 4x^{4-1} - 7\ 3x^{3-1}$

$= 30x^5 + 24x^3 - 21x^2$

8. $L = \frac{\frac{3}{4}x^{\frac{11}{3}} + \frac{2}{5}x^{\frac{5}{3}} + \frac{5}{11}x^{\frac{2}{3}}}{x^{\frac{-8}{3}}} = x^{\frac{8}{3}}\left(\frac{3}{4}x^{\frac{11}{3}} + \frac{2}{5}x^{\frac{5}{3}} + \frac{5}{11}x^{\frac{2}{3}}\right)$

$L = \frac{3}{4}x^{\frac{19}{3}} + \frac{2}{5}x^{\frac{13}{3}} + \frac{5}{11}x^{\frac{10}{3}}$

$L' = \frac{3}{4} \cdot \frac{19}{3} x^{\frac{19}{3}-1} + \frac{2}{5} \cdot \frac{13}{3} x^{\frac{13}{3}-1} + \frac{5}{11} \cdot \frac{2}{3} x^{\frac{10}{3}-1}$

$= \frac{57}{12} x^{\frac{16}{3}} + \frac{26}{15} x^{\frac{10}{3}} + \frac{10}{33} x^{\frac{7}{3}}$

Exit Ticket Quotient Rule

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\cdot g(x) - g'(x)\cdot f(x)}{\left[g(x)\right]^2}$$

Use the quotient rule above to find the (fully simplified) derivative of the following functions:

1. $y = \frac{x}{x+1}$

2. $y = \frac{x^2}{3x-1}$

3. $y = \frac{x^3}{\sqrt{x}+1}$

4. $y = \frac{x^2-1}{x^2+1}$

5. $g(x) = \frac{\ln(x)-1}{\ln(x)+1}$

6. $g(x) = \frac{e^x-1}{e^x+1}$

Exit Ticket Quotient Rule

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\cdot g(x) - g'(x)\cdot f(x)}{[g(x)]^2}$$

Use the quotient rule above to find the (fully simplified) derivative of the following functions:

1. $y = \frac{x}{x+1}$ $f(x) = x$ $g(x) = x+1$

$$y' = \frac{(1)(x+1)-(1)(x)}{(x+1)^2}$$
$$= \frac{x+1-x}{(x+1)^2}$$
$$= \frac{1}{(x+1)^2}$$

2. $y = \frac{x^2}{3x-1}$

$$y' = \frac{(2x)(3x-1)-(3)(x^2)}{(3x-1)^2}$$
$$= \frac{6x^2-2x-3x^2}{(3x-1)^2}$$
$$= \frac{3x^2-2x}{(3x-1)^2}$$
$$= \frac{x^2-1}{x^2+1}$$

3. $y = \frac{x^3}{\sqrt{x}+1}$

$$y' = \frac{(3x^2)(x^{\frac{1}{2}}+1)-(\frac{1}{2}x^{-\frac{1}{2}})(x^3)}{(x^{\frac{1}{2}}+1)^2}$$
$$= \frac{3x^{\frac{5}{2}}+3x^{\frac{1}{2}}-\frac{1}{2}x^{\frac{5}{2}}}{(x^{\frac{1}{2}}+1)^2}$$
$$= \frac{\frac{5}{2}x^{\frac{5}{2}}+3x^{\frac{1}{2}}}{(x^{\frac{1}{2}}+1)^2}$$

4. $y = \frac{x^2-1}{x^2+1}$

$$y' = \frac{(2x)(x^2+1)-(2x)(x^2-1)}{(x^2+1)^2}$$
$$= \frac{2x^3+2x-2x^3+2x}{(x^2+1)^2}$$
$$= \frac{4x}{(x^2+1)^2}$$

5. $g(x) = \frac{\ln(x)-1}{\ln(x)+1}$

$$g'(x) = \frac{(\frac{1}{x})(\ln(x)+1)-\frac{1}{x}(\ln(x)-1)}{(\ln(x)+1)^2}$$
$$= \frac{\frac{\ln(x)}{x}+\frac{1}{x}-\frac{\ln(x)}{x}+\frac{2}{x}}{(\ln(x)+1)^2}$$
$$= \frac{\frac{2}{x}}{(\ln(x)+1)^2}$$
$$= \frac{2}{x(\ln(x)+1)^2}$$

6. $g(x) = \frac{e^x-1}{e^x+1}$

$$g'(x) = \frac{e^x(e^x+1)-e^x(e^x-1)}{(e^x+1)^2}$$
$$= \frac{e^{2x}+e^x-e^{2x}+e^x}{(e^x+1)^2}$$
$$= \frac{2e^x}{(e^x+1)^2}$$

Exit Ticket Chain Rule

> **Chain Rule**
> $$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$$

Use the chain rule above to find the (fully simplified) derivative of the following functions:

1. $f(x) = (3x^2 - 1)^3(4x^2 + 3)^5$

2. $f(x) = (2x^2 - 4)^7(2x^2 + 4)^8$

3. $y = \frac{(x^2-1)^3}{x^2+1}$

4. $g(x) = \frac{\ln(x)-1}{\ln(x)+1}$

5. $g(x) = \ln\left(\frac{e^x-1}{e^x+1}\right)$

6. $y(x) = \frac{x^9-1}{\sqrt{x^2-1}}$

Exit Ticket Chain Rule

> **Chain Rule**
> $$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$$

Use the chain rule above to find the (fully simplified) derivative of the following functions:

1. $f(x) = (3x^2-1)^3(4x^2+3)^5$

$$f'(x) = 3(3x^2-1)^2(6x)(4x^2+3)^5 + 5(4x^2+3)^4(8x)(3x^2-1)^3$$
$$= (4x^2+3)^4(3x^2-1)^2(18x(4x^2+3)+40x(3x^2-1))$$
$$= (4x^2+3)^4(3x^2-1)^2(72x^3+54x+120x^3-40x)$$
$$= (4x^2+3)^4(3x^2-1)^2(192x^3+14x)$$

2. $f(x) = (2x^2-4)^7(2x^2+4)^8$

$$f'(x) = 7(2x^2-4)^6(4x)(2x^2+4)^8 + 8(2x^2+4)^7(4x)(2x^2-4)^7$$
$$= (2x^2-4)^6(2x^2+4)^7(28x(2x^2+4)+32x(2x^2-4))$$
$$= (2x^2-4)^6(2x^2+4)^7(56x^3+112x+64x^3-128x)$$
$$= (2x^2-4)^6(2x^2+4)^7(120x^3-16x)$$

3. $y = \frac{(x^2-1)^3}{x^2+1}$

$$y' = \frac{3(x^2-1)^2(2x)(x^2+1)-2x(x^2-1)^3}{(x^2+1)^2}$$
$$= \frac{6x(x^2-1)^2(x^2+1)-2x(x^2-1)^3}{(x^2+1)^2}$$
$$= \frac{(x^2-1)^2(6x(x^2+1)-2x(x^2-1))}{(x^2+1)^2}$$
$$= \frac{(x^2-1)^2(4x^3+8x)}{(x^2+1)^2}$$

4. $g(x) = \frac{\ln(x)-1}{\ln(x)+1}$

$$g'(x) = \frac{\frac{1}{x}(\ln(x)+1)-\frac{1}{x}(\ln(x)-1)}{(\ln(x)+1)^2}$$
$$= \frac{\frac{1}{x}\ln(x)+\frac{1}{x}-\frac{1}{x}\ln(x)+\frac{1}{x}}{(\ln(x)+1)^2}$$
$$= \frac{\frac{2}{x}}{(\ln(x)+1)^2}$$
$$= \frac{2}{x(\ln(x)+1)^2}$$

5. $g(x) = \ln\left(\frac{e^x-1}{e^x+1}\right)$

$$g(x) = \ln(e^x-1) - \ln(e^x+1)$$
$$g'(x) = \frac{e^x}{e^x-1} - \frac{e^x}{e^x+1}$$
$$= \frac{e^x(e^x+1)-e^x(e^x-1)}{(e^x-1)(e^x+1)}$$
$$= \frac{e^{2x}+e^x-e^{2x}+e^x}{(e^x-1)(e^x+1)}$$
$$= \frac{2e^x}{(e^x-1)(e^x+1)}$$

6. $y(x) = \frac{x^9-1}{\sqrt{x^2-1}}$

$$y'(x) = \frac{9x^8(x^2-1)^{1/2}-\frac{1}{2}(x^2-1)^{-1/2}(2x)(x^9-1)}{((x^2-1)^{1/2})^2}$$
$$= \frac{9x^8(x^2-1)^{1/2}-(x^2-1)^{-1/2}(x^{10}-x)}{(x^2-1)} \cdot \frac{(x^2-1)^{1/2}}{(x^2-1)^{1/2}}$$
$$= \frac{9x^8(x^2-1)-(x^{10}-x)}{(x^2-1)^{3/2}}$$
$$= \frac{9x^{10}-9x^8-x^{10}+x}{(x^2-1)^{3/2}}$$
$$= \frac{8x^{10}-9x^8+x}{(x^2-1)^{3/2}}$$

Exit Ticket Second Derivative Test

Second Derivative Test
Suppose $f(x)$ has a critical point ($f'(x) = 0$ or DNE) at $x = c$. We classify the critical points as follows:

- if $f''(c)$ is positive (concave up), then $f(c)$ is a **local minimum**
- if $f''(c)$ is negative (concave down), then $f(c)$ is a **local maximum**
- if $f''(x) = 0$, then we must use the first derivative test.

Use the second derivative test to solve the following problems:

1. What is the maximum product of two positive numbers whose sum is 100?

2. We have \$500 to build a fence around a rectangular field where the sides are made of different material. The vertical fencing cost \$10/ft, the bottom fencing is \$2/ft, and the top fencing is \$5/ft. What is the maximum area we can enclose?

3. A company's sale price is a linear function of their monthly demand. When they charge \$60 a piece they sale 100 pieces and when they charge \$50 a piece they sale 200 pieces. Their monthly cost is \$5000 fixed cost and \$10 per piece produced. What is their maximum monthly profit?

Exit Ticket Second Derivative Test

> **Second Derivative Test**
> Suppose $f(x)$ has a critical point ($f'(x) = 0$ or DNE) at $x = c$. We classify the critical points as follows:
>
> - if $f''(c)$ is positive (concave up), then $f(c)$ is a **local minimum**
> - if $f''(c)$ is negative (concave down), then $f(c)$ is a **local maximum**
> - if $f''(x) = 0$, then we must use the first derivative test.

Use the second derivative test to solve the following problems:

1. What is the maximum product of two positive numbers whose sum is 100?

Given: $x+y=100 \longrightarrow y = 100-x$

$P = x \cdot y$

$P = x(100-x)$

$= 100x - x^2$

$P' = 100 - 2x = 0$

$100 = 2x$

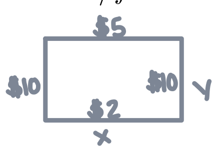
$50 = x$

$P(50) = 100(50) - (50)^2$

$= 5000 - 2500$

$= 2500$

2. We have \$500 to build a fence around a rectangular field where the sides are made of different material. The vertical fencing cost $\$10/ft$, the bottom fencing is $\$2/ft$, and the top fencing is $\$5/ft$. What is the maximum area we can enclose?



Given: $10y + 5x + 10y + 2x = 500$

$20y + 7x = 500$

$20y = 500 - 7x$

$y = \frac{500-7x}{20}$

$A = x\left(\frac{500-7x}{20}\right)$

$= 25x - \frac{7}{20}x^2$

$A' = 25 - \frac{7}{10}x = 0$

$25 = \frac{7}{10}x$

$\frac{250}{7} = x$

$A\left(\frac{250}{7}\right) = 25\left(\frac{250}{7}\right) - \frac{7}{20}\left(\frac{250}{7}\right)^2$

$= 25\left(\frac{250}{7}\right) - \frac{\cancel{7}}{\cancel{20}} \cdot \frac{\cancel{250}}{\cancel{7}} \cdot \frac{250}{7}$

$= 25\left(\frac{250}{7}\right) - \frac{25}{2} \cdot \left(\frac{250}{7}\right)$

$= \frac{25}{2}\left(\frac{250}{7}\right) = 25\left(\frac{125}{7}\right)$

3. A company's sale price is a linear function of their monthly demand. When they charge \$60 a piece they sale 100 pieces and when they charge \$50 a piece they sale 200 pieces. Their monthly cost is \$5000 fixed cost and \$10 per piece produced. What is their maximum monthly profit?

point-slope form:

$m = \frac{50-60}{200-100} = \frac{-10}{100} = -\frac{1}{10}$

$y - 50 = -\frac{1}{10}(x - 200)$

$s(x) = -\frac{1}{10}x + 70$

$R(x) = x \cdot s(x)$

$= x\left(-\frac{1}{10}x + 70\right)$

$= -\frac{1}{10}x^2 + 70x$

$C(x) = 10x + 5000$

$P(x) = R(x) - C(x)$

$= -\frac{1}{10}x^2 + 70x - (10x + 500)$

$= -\frac{1}{10}x^2 + 60x - 500$

$P'(x) = -\frac{1}{5}x + 60 = 0$

$60 = \frac{1}{5}x$

$300 = x$

$P(300) = -\frac{1}{10}(300)^2 + 60(300) - 500$

$= -9000 + 18{,}000 - 500$

$= 8{,}500$

Produced by Audriana Houtz, Mathematics Ph.D. student at the University of Notre Dame.

Exit Ticket L'Hopital

> **L'Hopital** If both $f(x)$ and $g(x)$ are differentiable functions such that:
>
> - $\lim_{x\to c} f(x) = 0 = \lim_{x\to c} g(x)$ such that $\lim_{x\to c} \frac{f'(x)}{g'(x)}$ exists then $\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$
> - $\lim_{x\to c} f(x) = \infty = \lim_{x\to c} g(x)$ such that $\lim_{x\to c} \frac{f'(x)}{g'(x)}$ exists then $\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$

Identify when you can use L'Hopital. If you can, evaluate the limit:

1. $\lim_{x\to\infty} \frac{3x^3 + 4x^2 - 3x + 5}{5x^4 + 3x^2 - 1}$

2. $\lim_{x\to 0^+} \frac{ln(x+1)}{\sqrt{x}}$

3. $\lim_{x\to\infty} \frac{-6e^{2x} + 7}{3x^{2x} + 5}$

4. $\lim_{x\to\infty} \frac{e^{2x} + 2e^x + 1}{e^x + 1}$

5. $\lim_{x\to 0} \frac{\sin(x) - \sin(2x)}{\sin(x) + \sin(3x)}$

6. $\lim_{x\to\infty} (1+x)^{\frac{1}{x}}$

Exit Ticket L'Hopital

> **L'Hopital** If both $f(x)$ and $g(x)$ are differentiable functions such that:
>
> - $\lim_{x\to c} f(x) = 0 = \lim_{x\to c} g(x)$ such that $\lim_{x\to c} \frac{f'(x)}{g'(x)}$ exists then $\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$
> - $\lim_{x\to c} f(x) = \infty = \lim_{x\to c} g(x)$ such that $\lim_{x\to c} \frac{f'(x)}{g'(x)}$ exists then $\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$

Identify when you can use L'Hopital. If you can, evaluate the limit:

1. $\lim_{x\to\infty} \frac{3x^3+4x^2-3x+5}{5x^4+3x^2-1}$ $\frac{\infty}{\infty}$

L'H x3

$= \lim_{x\to\infty} \frac{18}{120x} = 0$

2. $\lim_{x\to 0^+} \frac{ln(x+1)}{\sqrt{x}}$ $\frac{0}{0}$

$= \lim_{x\to 0^+} \frac{\frac{1}{x+1}}{\frac{1}{2}x^{-1/2}} = \lim_{x\to 0^+} \frac{1}{x+1} \cdot \frac{2x^{1/2}}{1}$

$= \lim_{x\to 0^+} \frac{x^{-1/2}}{1} = \lim_{x\to 0^+} \frac{1}{\sqrt{x}} = \infty$

3. $\lim_{x\to\infty} \frac{-6e^{2x}+7}{3x^{2x}+5}$ $\frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{-12e^{2x}}{6e^{2x}}$

$= \lim_{x\to\infty} -2$

$= -2$

4. $\lim_{x\to\infty} \frac{e^{2x}+2e^{x}+1}{e^{x}+1}$

$u = e^x$

hole

$= \lim_{u\to\infty} \frac{u^2+2u+1}{u+1} = \lim_{u\to\infty} \frac{(u+1)\cancel{(u+1)}}{\cancel{(u+1)}}$

$= \lim_{x\to\infty} e^x+1$

$= \infty$

5. $\lim_{x\to 0} \frac{\sin(x)-\sin(2x)}{\sin(x)+\sin(3x)}$ $\frac{0-0}{0-0} = \frac{0}{0}$

$= \lim_{x\to 0} \frac{\cos(x)-2\cos(2x)}{\cos(x)-3\cos(3x)}$

$= \frac{1-2(1)}{1-3(1)}$

$= \frac{-1}{-2}$

$= \frac{1}{2}$

6. $\lim_{x\to\infty} (1+x)^{\frac{1}{x}}$ ∞^0

$= \lim_{x\to\infty} e^{\ln((1+x)^{1/x})}$

$= \lim_{x\to\infty} e^{\frac{1}{x}\ln(1+x)}$ $e^{\frac{\infty}{\infty}}$

$= \lim_{x\to\infty} e^{\frac{\frac{1}{1+x}}{1}}$

$= \lim_{x\to\infty} e^{\frac{1}{1+x}}$

$= e^0 = 1$

Exit Ticket Curve Sketching

> **Curve Sketching Steps**
>
> 1. Intercepts (if given f(x))
>   → x-intercept: set $y = 0$ and solve
>   → y-intercept: set $x = 0$ and solve
> 2. First Derivative Sign Line (monotonicity)
>   → critical points: $f'(x) = 0$ or DNE
>   → draw line, label critical points, mark positive/negative ranges
> 3. Second Derivative Sign Line (concavity)
>   → inflection points: $f''(x) = 0$ or DNE
>   → draw line, label inflection points, mark positive/negative ranges
> 4. Asymptotes & End Behavior
>   → domain issues: $f(x)$ DNE
>   → vertical asymptotes: denominators=0
>   → end behavior: $\lim_{x\to\pm\infty} f(x)$

Find everything you need to graph the function: $f(x) = e^{3x} - e^{5x}$

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Find everything you need to graph the function: $f(x) = e^{3x} - e^{5x}$

Step 1: Intercepts

$0=e^{3x}-e^{5x}$
$e^{5x}=e^{3x}$
$\ln(e^{5x})=\ln(e^{3x})$
$5x=3x$
$2x=0$
$x=0$

$y=e^{3(0)}-e^{5(0)}$
$=e^{0}-e^{0}$
$=1-1$
$=0$

Step 2: y'-sign line

$f'(x)=3e^{3x}-5e^{5x}=0$
$3e^{3x}=5e^{5x}$
$\ln(3e^{3x})=\ln(5e^{5x})$
$\ln(3)+(3x)\ln(e)=\ln(5)+(5x)\ln(e)$
$\ln(3)+3x=\ln(5)+5x$
$\ln(3)-\ln(5)=2x$
$\ln(\frac{3}{5})=2x$
$\frac{1}{2}\ln(\frac{3}{5})=x$

+ | −
$\frac{1}{2}\ln(\frac{3}{5})\approx -0.25$

$\ln(1)=0$
$x<1 \Rightarrow \ln(x)<0$

Step3: y''-sign line

$f''(x)=9e^{3x}-25e^{5x}=0$
$9e^{3x}=25e^{5x}$
$\ln(9e^{3x})=\ln(25e^{5x})$
$\ln(9)+3x=\ln(25)+5x$
$\frac{1}{2}\ln(\frac{9}{25})=x$

+ | −
$\frac{1}{2}\ln(\frac{9}{25})$

Step4: Asymptotes

vertical: none

horizontal: negative positive

$\lim_{x\to\infty} e^{3x}-e^{5x} = -e^{3x}(-1+e^{2x}) = -\infty$

$\lim_{x\to-\infty} e^{3x}-e^{5x} = 0$

hint: try $x=-100$
$e^{-300}-e^{-500} = \frac{1}{e^{300}}-\frac{1}{e^{500}}$

Exit Ticket Extrema

First Derivative Test Suppose $f(x)$ has a critical point at $x = c$. We classify the critical points as follows:

- if $f'(x)$ changes its sign from positive to negative at $x = c$, then there is a **local maximum** at $x = c$.
- if $f'(x)$ changes its sign from negative to positive at $x = c$, then there is a **local minimum** at $x = c$.
- if $f'(x)$ does not change its sign at $x = c$, then there is neither a local minimum or maximum at $x = c$.

Second Derivative Test Let $f(x)$ be a function such that $f'(c) = 0$ and the function has a second derivative in an interval containing c. We can classify the critical point as follows:

- if $f''(c) > 0$ then f has a local minimum at the point $(c, f(c))$.
- if $f''(c) < 0$ then f has a local maximum at the point $(c, f(c))$.
- if $f''(c) = 0$ then the test is inconclusive

What can you tell me about the following functions (increasing, decreasing, maximums, minimums):

1. $f(x) = \frac{1}{3}x^3 - 3x^2 + 5x$

2. $f(t) = t - 3(t-1)^{\frac{1}{3}}$

3. $h'(x) = \frac{e^{3x}-1}{e^{5x}+1}$

4. $f'(x) = e^{4x} - e^{2x} - 2$

5. $f'(0) = 0$; $f''(x) = 6x + 1$

6. $f'(1) = 0$; $g''(t) = -2e^{-t} + te^{-t}$

Exit Ticket Extrema

First Derivative Test Suppose $f(x)$ has a critical point at $x = c$. We classify the critical points as follows:

- if $f'(x)$ changes its sign from positive to negative at $x = c$, then there is a **local maximum** at $x = c$.
- if $f'(x)$ changes its sign from negative to positive at $x = c$, then there is a **local minimum** at $x = c$.
- if $f'(x)$ does not change its sign at $x = c$, then there is neither a local minimum or maximum at $x = c$.

Second Derivative Test Let $f(x)$ be a function such that $f'(c) = 0$ and the function has a second derivative in an interval containing c. We can classify the critical point as follows:

- if $f''(c) > 0$ then f has a local minimum at the point $(c, f(c))$.
- if $f''(c) < 0$ then f has a local maximum at the point $(c, f(c))$.
- if $f''(c) = 0$ then the test is inconclusive

What can you tell me about the following functions (increasing, decreasing, maximums, minimums):

1. $f(x) = \frac{1}{3}x^3 - 3x^2 + 5x$

$f'(x) = x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$+ \quad 1 \quad - \quad 5 \quad +$

max (at 1) min (at 5)

2. $f(t) = t - 3(t-1)^{\frac{1}{3}}$

$f'(t) = 1 - (t-1)^{-2/3}$

$= 1 - \frac{1}{(t-1)^{2/3}}$

$t = 0, 1, 2$

$f' = 0$ (at 0), f' DNE (at 1), $f' = 0$ (at 2)

$+ \quad 0 \quad - \quad 1 \quad - \quad 2 \quad +$

max (at 0) neither (at 1) min (at 2)

3. $h'(x) = \frac{e^{3x}-1}{e^{5x}+1}$

$\frac{e^{3x}-1}{e^{5x}+1} = 0$

$e^{3x} - 1 = 0$

$- \quad 0 \quad +$ min

4. $f'(x) = e^{4x} - e^{2x} - 2$

$u = e^{2x}$: $u^2 - u - 2 = 0$

$(u-2)(u+1) = 0$

$e^{2x} = 2 \qquad e^{2x} = -1$

$x = \frac{1}{2}\ln(2) \qquad$ never

$- \quad \frac{1}{2}\ln(2) \quad +$ min

5. $f'(0) = 0$; $f''(x) = 6x + 1$

crit pt. $x=0$

$f''(0) = 6(0) + 1$

$= 1 > 0$

concave up

min.

6. $f'(1) = 0$; $g''(t) = -2e^{-t} + te^{-t}$

crit. pt. $x = 1$

$g''(1) = -2e^{-1} + 1e^{-1}$

$= -2 \cdot \frac{1}{e} + \frac{1}{e}$

$= -\frac{1}{e} < 0$

max concave down

Exit Ticket Concavity

Concavity Let $f(x)$ be a twice differentiable function with $f''(c) = 0$ or DNE (i.e. c is a *possible* inflection point). We say that:

- $f(x)$ **concave up** on an interval $I = (a, b)$ if $f''(x) > 0$ for all x such that $a < x < b$
- $f(x)$ **concave down** on an interval $I = (a, b)$ if $f''(x) < 0$ for all x such that $a < x < b$
- c is an inflection point if the function is continuous at the point and the concavity changes at that point

Identify when the function is concave up and concave down:

1. $f(x) = 3x^5 - 5x^3 + 3$

2. $f(t) = 3(t-1)^{\frac{1}{3}}$

3. $h(x) = \frac{9}{3}x^{\frac{4}{3}} - \frac{1}{6}x^3 + 3$

4. $g(t) = te^{-t}$

5. $f''(x) = \ln(3x) - \ln(5)$

6. $f'(x) = e^{4x} - e^{2x} - 2$

Exit Ticket Concavity

> **Concavity** Let $f(x)$ be a twice differentiable function with $f''(c) = 0$ or DNE (i.e. c is a *possible* inflection point). We say that:
>
> - $f(x)$ **concave up** on an interval $I = (a, b)$ if $f''(x) > 0$ for all x such that $a < x < b$
> - $f(x)$ **concave down** on an interval $I = (a, b)$ if $f''(x) < 0$ for all x such that $a < x < b$
> - c is an inflection point if the function is continuous at the point and the concavity changes at that point

Identify when the function is concave up and concave down:

1. $f(x) = 3x^5 - 5x^3 + 3$

$f'(x) = 15x^4 - 15x^2$

$f''(x) = 60x^3 - 30x = 0$

$30x(x^2-1) = 0$

− + − +

−1 0 1

down up down up

2. $f(t) = 3(t-1)^{\frac{1}{3}}$

$f'(t) = (t-1)^{-2/3}$

$f''(t) = -\frac{2}{3}(t-1)^{-5/3} = 0$

$-\frac{2}{3}\frac{1}{(t-1)^{5/3}} = 0$

+ −

1

up down

3. $h(x) = \frac{9}{3}x^{\frac{4}{3}} - \frac{1}{6}x^3 + 3$

$h'(x) = 4x^{1/3} - \frac{1}{2}x^2$

$h''(x) = \frac{4}{3}x^{-2/3} - x = 0$

always positive $\rightarrow$ $\frac{4}{3x^{2/3}} = x$

$\frac{4}{3} = x^{5/3}$

$\left(\frac{4}{3}\right)^{3/5} = x$

+ + −

0 $\left(\frac{4}{3}\right)^{3/5}$

up up down

4. $g(t) = te^{-t}$

$g'(t) = e^{-t} - te^{-t}$

$g''(t) = -e^{-t} - e^{-t} + te^{-t}$

$= -2e^{-t} + te^{-t}$

$= e^{-t}(-2+t) = 0$

$t = 2$

− +

2

5. $f''(x) = \ln(3x) - \ln(5)$

$\ln(3x) - \ln(5) = 0$

$\ln(3x) = \ln(5)$

$3x = 5$

$x = \frac{5}{3}$

− +

0 $\frac{5}{3}$

6. $f'(x) = e^{4x} - e^{2x} - 2$

$f''(x) = 4e^{4x} - 2e^{2x} = 0$

$4e^{4x} = 2e^{2x}$

$\ln(4) + 4x = \ln(2) + 2x$

$2x = \ln(\frac{1}{2})$

$x = \frac{1}{2}\ln(\frac{1}{2})$

− +

$\frac{1}{2}\ln(\frac{1}{2})$

Produced by Audriana Houtz, Mathematics Ph.D. student at the University of Notre Dame.

Exit Ticket Newton's Method

> **Newton's Method**
> If x_n is an approximation of a solution of $f(x) = 0$ and if $f'(x) \neq 0$ the next approximation is given by,
> $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Find the function you can apply Newton's method to:

1. $x^2 = cos(x)$

2. $2 - x^2 = sin(x)$

Find an initial guess and write the equation for x_1 using Newton's method:

1. $f(x) = x^3 - 7x^2 + 8x - 1$

2. $f(x) = x^3 - x^2 - 15x + 1$

Use Newton's method and the initial guess given to find x_2:

1. $f(x) = -x^3 + 4\ ;\ x_0 = 1$

2. $f(x) = cos(x) - 2x\ ;\ x_0 = 0$

Exit Ticket Newton's Method

Newton's Method
If x_n is an approximation of a solution of $f(x) = 0$ and if $f'(x) \neq 0$ the next approximation is given by,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Find the function you can apply Newton's method to:

1. $x^2 = cos(x)$

$f(x) = x^2 - \cos(x)$

$f(x) = \cos(x) - x^2$

2. $2 - x^2 = sin(x)$

$f(x) = 2 - x^2 - \sin(x)$

$f(x) = \sin(x) - 2 + x^2$

Find an initial guess and write the equation for x_1 using Newton's method:

1. $f(x) = x^3 - 7x^2 + 8x - 1$

$f(0) = -1 \quad f(1) = 1 - 7 + 8 - 1 = 1$

$x_0 = \frac{1}{2}$

$x_1 = \frac{1}{2} - \frac{(1/2)^3 - 7(1/2)^2 + 8(1/2) - 1}{3(1/2)^2 - 14(1/2) + 8}$

2. $f(x) = x^3 - x^2 - 15x + 1$

$f(0) = 1 \quad f(1) = 1 - 1 - 15 + 1 = -14$

$x_0 = \frac{1}{2}$ or $\frac{1}{8}$

$x_1 = \frac{1}{2} - \frac{(1/2)^3 - (1/2)^2 - 15(1/2) + 1}{3(1/2)^2 - 2(1/2) - 15}$

Use Newton's method and the initial guess given to find x_2:

1. $f(x) = -x^3 + 4$; $x_0 = 1$

$f'(x) = -3x^2$

$x_1 = 1 - \frac{-(1)^3 + 4}{-3(1)^2}$

$= 1 - \frac{3}{-3} = 2$

$x_2 = 2 - \frac{-(2)^3 + 4}{-3(2)^2}$

$= 2 - \frac{-4}{-12}$

$= 2 + \frac{1}{3}$

$= 7/3$

2. $f(x) = cos(x) - 2x$; $x_0 = 0$

$f'(x) = -\sin(x) - 2$

$x_1 = 0 - \frac{\cos(0) - 2(0)}{-\sin(0) - 2}$

$= 0 - \frac{1 - 2}{-2}$

$= -\frac{-1}{-2}$

$= \frac{1}{2}$

$x_2 = \frac{1}{2} - \frac{\cos(1/2) - 2(1/2)}{-\sin(1/2) - 2}$

$= \frac{1}{2} + \frac{\cos(1/2) - 1}{\sin(1/2) - 2}$

Exit Ticket Anti-derivatives

Fill in the derivatives and anti-derivatives:

1. $\frac{d}{dx}[kx] =$

2. $\int k dx =$

3. $\frac{d}{dx}[kx^n] =$

4. $\int x^n dx =$

5. $\frac{d}{dx}[\ln(x)] =$

6. $\int \frac{1}{x} dx =$

7. $\frac{d}{dx}[\log_a(x)] =$

8. $\int \frac{1}{x \cdot \ln(a)} dx =$

9. $\frac{d}{dx}[e^x] =$

10. $\int e^x dx =$

11. $\frac{d}{dx}[a^x] =$

12. $\int a^x dx =$

13. $\frac{d}{dx}[\sin(x)] =$

14. $\int \cos(x) dx =$

15. $\frac{d}{dx}[\cos(x)] =$

16. $\int \sin(x) dx =$

17. $\frac{d}{dx}[\tan(x)] =$

18. $\int \sec^2(x) dx =$

19. $\frac{d}{dx}[\sec(x)] =$

20. $\int \sec(x)\cos(x) dx =$

Use the rules above to find the integrals below and check your answer:

1. $\int 3^x - 3x^4 - \cos(x) dx$

2. $\int \frac{x^3 - e^2}{x^2} dx$

3. $\int \frac{1}{\sqrt{1-x}} dx$

4. $\int 6x(x^2+1)^2 dx$

Exit Ticket Anti-derivatives

Fill in the derivatives and anti-derivatives:

1. $\frac{d}{dx}[kx] = k$	**2.** $\int k dx = kx + C$
3. $\frac{d}{dx}[kx^n] = k \cdot n \cdot x^{n-1}$	**4.** $\int x^n dx = \frac{1}{n+1} x^{n+1} + C$
5. $\frac{d}{dx}[\ln(x)] = \frac{1}{x}$	**6.** $\int \frac{1}{x} dx = \ln\lvert x \rvert + C$
7. $\frac{d}{dx}[\log_a(x)] = \frac{1}{x \cdot \ln(a)}$	**8.** $\int \frac{1}{x \cdot \ln(a)} dx = \log_a(x) + C$
9. $\frac{d}{dx}[e^x] = e^x$	**10.** $\int e^x dx = e^x + C$
11. $\frac{d}{dx}[a^x] = a^x \ln(a)$	**12.** $\int a^x dx = \frac{1}{\ln(a)} \cdot a^x + C$
13. $\frac{d}{dx}[\sin(x)] = \cos(x)$	**14.** $\int \cos(x) dx = \sin(x) + C$
15. $\frac{d}{dx}[\cos(x)] = -\sin(x)$	**16.** $\int \sin(x) dx = -\cos(x) + C$
17. $\frac{d}{dx}[\tan(x)] = \sec^2(x)$	**18.** $\int \sec^2(x) dx = \tan(x) + C$
19. $\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$	**20.** $\int \sec(x)\cos(x) dx = \sec(x) + C$ tan(x)

Use the rules above to find the integrals below and check your answer:

1. $\int 3^x - 3x^4 - \cos(x) dx$

$= \frac{1}{\ln(3)} \cdot 3^x - \frac{3}{5} x^5 - \sin(x) + C$

check: $3^x - 3x^4 - \cos(x)$

2. $\int \frac{x^3 - e^2}{x^2} dx = \int \frac{x^3}{x^2} - \frac{e^2}{x^2} dx$

$= \int x - e^2 x^{-2} dx$

$= \frac{1}{2} x^2 + e^2 x^{-1} + C$

check: $x^2 - e^2 x^{-2}$

3. $\int \frac{1}{\sqrt{1-x}} dx = \int (1-x)^{-1/2} dx$

1st guess: $(1-x)^{1/2} + C$

check: $\frac{1}{2}(1-x)^{-1/2}$

2nd guess: $2(1-x)^{1/2} + C$

check: $(1-x)^{-1/2}$

4. $\int 6x(x^2+1)^2 dx$

1st guess: $(x^2+1)^3 + C$

check: $3(x^2+1)^2 \cdot 2x$