
Exit Ticket Derivative and Integral Review

Fill in the derivatives and integrals:

1. $\frac{d}{dx} [k] =$

2. $\int k dx =$

3. $\frac{d}{dx} [kx^n] =$

4. $\int x^n dx =$

5. $\frac{d}{dx} [\ln(x)] =$

6. $\int \frac{1}{x} dx =$

7. $\frac{d}{dx} [\log_a(x)] =$

8. $\int \frac{1}{x \cdot \ln(a)} dx =$

9. $\frac{d}{dx} [e^x] =$

10. $\int e^x dx =$

11. $\frac{d}{dx} [a^x] =$

12. $\int a^x dx =$

13. $\frac{d}{dx} [\sin(x)] =$

14. $\int \cos(x) dx =$

15. $\frac{d}{dx} [\cos(x)] =$

16. $\int \sin(x) dx =$

17. $\frac{d}{dx} [\tan(x)] =$

18. $\int \sec^2(x) dx =$

19. $\frac{d}{dx} [\sec(x)] =$

20. $\int \sec(x) \tan(x) dx =$

Use the rules above to find the integrals below and check your answer:

1. $\int \cot(x) \sin(x) dx$

2. $\int \frac{1 + \cos^2(\theta)}{\cos^2(\theta)} d\theta$

3. $\int \frac{2u^2 - 5u + \sqrt[3]{u}}{u^2} du$

4. $\int 6x(x^2 + 1)^2 dx$

Exit Ticket Natural Log

Fill in the following rules:

1. $\ln(a) + \ln(b) =$

2. $\ln(a) - \ln(b) =$

3. $\ln(x^a) =$

4. $\ln(ax^b) =$

5. $\frac{d}{dx} [\ln(ax + b)] =$

6. $\int \frac{a}{ax + b} dx =$

Use the above rules to solve the following equations for x:

1. $\int \frac{1}{2x + 5} dx$

2. $\int \frac{1}{x + 12} dx$

3. $\frac{d}{dx} \left[\ln \left(\frac{1 - x}{1 + x} \right) \right]$

4. $\frac{d}{dx} \left[\ln \left(\frac{2x^2 - 3}{3x^3 - 6} \right) \right]$

5. $\int \frac{2x}{4x^2 + 12} dx$

6. $\int \frac{5x + 7}{5x^2 + 14x + 6} dx$

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Exit Ticket Integral Review

Solve the following integrals and identify the integral rule used:

1. $\int \cot(x) \sin(x) dx$

2. $\int \frac{1 + \cos^2(\theta)}{\cos^2(\theta)} d\theta$

3. $\int \frac{\sin(x)}{1 + \cos^2(x)} dx$

4. $\int 6x(x^2 + 1)^{\frac{1}{2}} dx$

5. $\int \sin(2x) dx$

6. $\int \frac{1}{1 + \sin(\theta)} d\theta$

7. $\int \frac{3x}{(2x^2 + 1)^2} dx$

8. $\int \frac{3x}{(2x^2 + 1)} dx$

Exit Ticket Inverse Trigonometric Functions

Fill in the derivatives and integrals:

1. $\frac{d}{dx} [\arcsin(x)] =$

2. $\int \frac{1}{\sqrt{1-x^2}} dx =$

3. $\frac{d}{dx} [\arctan(x)] =$

4. $\int \frac{1}{1+x^2} dx =$

Use the rules above to find the integrals below:

1. $\int \frac{1}{1+9x^2} dx$

2. $\frac{d}{dx} \left[\arcsin\left(\frac{3}{4}x\right) \right]$

3. $\int \frac{3}{\sqrt{9-4x^2}} dx$

4. $\frac{d}{dx} [\arctan(x^2)]$

5. $\int \frac{5x+1}{4+9x^2} dx$

6. $\frac{d}{dx} [\arcsin(x+1)]$

Exit Ticket Area Between Curves

Area Between curves Assuming that $f(x) \geq g(x)$ for $a \leq x \leq b$, the area between the curves is:

$$\int_a^b [f(x) - g(x)] dx$$

Set up but do NOT solve the integral that finds the areas bounded by the functions below:

1. $y = x^2 + 2$, $y = \sin(x)$, $x = -1$, $x = 2$ 2. $x = y^2 + 1$, $x = 5$, $y = -3$, $y = 3$

3. $y = \frac{1}{x+2}$, $y = (x+2)^2$, $x = -\frac{3}{2}$, $x = 1$ 4. $x = y^2 - y - 6$, $x = 2y + 4$

Exit Ticket Volume of Solids with Uniform Cross-sections

Volume of Solids with Uniform Cross-sections Consider a solid whose base is the region bounded by given function(s) with uniform cross-sections perpendicular to the x -axis.

The volume of the solid is given by:

$$V = \int_a^b [A(x)] dx$$

where $A(x)$ is the area of the cross-section

Set up but do NOT solve the integral that finds the volume of the solid whose base is bounded by $y = x^2 + 2$, $y = \sin(x)$, $x = -1$, $x = 2$ and has uniform cross-sections perpendicular to the x -axis in the shape of:

1. squares
2. triangles of height x^2
3. semicircles
4. rectangles of height \sqrt{x}

Exit Ticket Solids of Revolution

Solids of Revolution Consider a solid formed by rotating a bounded region about a line $y = c$ with cross-sectional area functions $A(x)$, then the volume formula is

$$V = \int_a^b [A(x)] dx.$$

Disk method: $A(x) = \pi r^2$ where r is a function of x

Washer method: $A(x) = \pi [R^2 - r^2]$ where R, r are a functions of x

Shell method: $A(x) = 2\pi rh$ where r, h are a functions of x

Set up but do NOT solve the integral that finds the volume of the solid formed by rotating the region bounded by:

1. $y = \sqrt{x}$, $y = 3$, and the y -axis about the y -axis
2. $y = 10 - 6x + x^2$, $y = -10 + 6x - x^2$, $x = 1$, and $x = 5$ about the line $y = 8$
3. $x = y^2 - 4$, $x = 6 - 3y$ about the line $y = 8$

Exit Ticket Work and Energy

Work and Energy Suppose that the force at any given x is given by $F(x)$, then the work done by the force in moving the object from $x = a$ to $x = b$ is given by

$$W = \int_a^b F(x) dx.$$

Set up but do NOT solve the following integral:

1. A uniform chain 10 m long weighing 30 kg lying completely at the foot of a building 50 m tall.
 - (a) What is the work done against gravity to move one end to the top of the building with the rest of the chain danging free?

 - (b) What is the work done to move one end only 30 m off the ground?

 - (c) What is the work done to move the top end of the chain 5 meters off the ground with the rest of the chain still on the ground?

Exit Ticket Integration by Parts

Integration by Parts Let $u(x)$ and $v(x)$ be two differentiable functions. Integration by parts says

$$\int u dv = uv - \int v du$$

Evaluate the following integrals:

1. $\int 8xe^{6x} dx$

2. $\int 4x \cos(2 - 3x) dx$

3. $\int (2 - x)^2 \ln(4x) dx$

4. $\int \ln(x) dx$

5. $\int e^{-x} \sin(4x) dx$

6. $\int \frac{x^7}{\sqrt{x^4 + 1}} dx$

Exit Ticket Partial Fraction

Partial Fraction Decomposition:

denominator

$$ax + b$$

partial fraction

$$\frac{A}{ax+b}$$

$$(ax + b)^n$$

$$\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \dots + \frac{A_n}{(ax+b)^n}$$

$$ax^2 + bx$$

$$\frac{Ax+B}{ax^2+bx}$$

$$(ax^2 + b)^n$$

$$\frac{A_1x+B_1}{ax^2+b} + \frac{A_2x+B_2}{(ax^2+b)^2} + \dots + \frac{A_nx+B_n}{(ax^2+b)^n}$$

Solve the following integrals using partial fractions:

1. $\int \frac{x^2 + x + 1}{(x + 1)(x + 4)^2} dx$

2. $\int \frac{2x^4 + x^3 + 4x^2 + 1}{x^3 + x} dx$

Exit Ticket Improper Integrals

Improper Integrals

1. If $\int_a^c f(x)dx$ exists for every $t > a$, then $\int_a^\infty f(x)dx = \lim_{c \rightarrow \infty} \int_a^c f(x)dx$ provided that the limit exists and is finite.
2. If $\int_c^a f(x)dx$ exists for every $c < b$, then $\int_{-\infty}^b f(x)dx = \lim_{c \rightarrow -\infty} \int_c^b f(x)dx$ provided that the limit exists and is finite.
3. If $f(x)$ is continuous on the interval $[a, b)$ and not at $x = b$, then $\int_a^b f(x)dx = \lim_{c \rightarrow b^-} \int_a^c f(x)dx$ provided that the limit exists and is finite.
4. If $f(x)$ is continuous on the interval $(a, b]$ and not at $x = a$, then $\int_a^b f(x)dx = \lim_{c \rightarrow a^+} \int_c^b f(x)dx$ provided that the limit exists and is finite.
5. If $f(x)$ is not continuous $x = t$ where $a < t < b$, then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ provided that the limit exists and is finite.

The integral is considered **convergent** if the limit exists and is finite, and **divergent** if the limit doesn't exist or is infinite.

Solve the following integrals using the concept above:

1. $\int_0^\infty \frac{1}{x} dx$

2. $\int_{-5}^1 \frac{1}{10 + 2x} dx$

3. $\int_1^4 \frac{1}{x^2 + x - 6} dx$

4. $\int_{-\infty}^0 \frac{e^{\frac{1}{x}}}{x^2} dx$

Exit Ticket Numerical Integration

Numerical Integration We can estimate the integral $\int_a^b f(x)dx$ using the following formulas,

1. **midpoint:** $\int_a^b f(x)dx \approx \Delta x [f(x_1^*) + f(x_2^*) + \dots + f(x_n^*)]$

2. **trapezoid:** $\int_a^b f(x)dx \approx \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n)]$

3. **simpson's:** $\int_a^b f(x)dx \approx \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) \dots + 2f(x_{n-1}) + 4f(x_{n-1}) + f(x_n)]$

where n is the number of subintervals and $\Delta x = \frac{b-a}{n}$

Estimate the following integrals using each of the rules above:

1. $\int_1^7 \frac{1}{x^3 + 1} dx$

2. $\int_0^4 \cos(1 + \sqrt{x}) dx$

Exit Ticket Double Integrals

Double Integrals The integral over a horizontally simple region $D = \{(x, y) \mid h_1(y) \leq x \leq h_2(y), c \leq y \leq d\}$ is

$$\int \int_D f(x, y) \, dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y) \, dx dy$$

Find the integral $\int \int_D 2x \, dA$ for the regions bounded by the following functions:

1. $y = x^2 + 2$, $y = \sin(x)$, $x = -1$, $x = 2$ 2. $x = y^2 + 1$, $x = 5$, $y = \pm 2$

3. $y = \frac{1}{x+2}$, $y = (x+2)^2$, $x = -1$, $x = 1$ 4. $x = y^2 - y - 6$, $x = 2y + 4$

Exit Ticket Polar Integrals

Polar Integrals The integral over region $D = \{(r, \theta) \mid a \leq r \leq b, c \leq \theta \leq d\}$ is

$$\int \int_D f(r, \theta) \, dA = \int_c^d \int_a^b f(r, \theta) \cdot r \cdot dr d\theta$$

Find the integral $\int \int_D 2x^2 + y^2 \, dA$ for the regions bounded by the following:

1. the circles of radius 1 and 3 centered at the origin
2. the circles of radius 1 and 3 centered at the origin contained in the third quadrant

3. $0 \leq y \leq 1; -\sqrt{1-y^2} \leq x \leq \sqrt{1-y^2}$

4. $-1 \leq y \leq 1; 0 \leq x \leq \sqrt{1-y^2}$

Exit Ticket Separable Differential Equations

Separable Differential Equations The solution to the separable differential equations $\frac{dy}{dx} = p(x)q(y)$ is

$$\begin{aligned}\frac{dy}{dx} &= p(x)q(y) \\ \frac{1}{q(y)} dy &= p(x)dx \\ \int \frac{1}{q(y)} dy &= \int p(x)dx\end{aligned}$$

Find the general solution to the following differential equations:

1. $\frac{dy}{dx} = 6y^2x$

2. $y' = \frac{3x^2 + 4x - 4}{2y - 4}$

3. $y' = 2xe^{-y} - 4e^{-y}$

4. $\frac{dy}{dt} = \frac{\cos^2(y)}{y}$