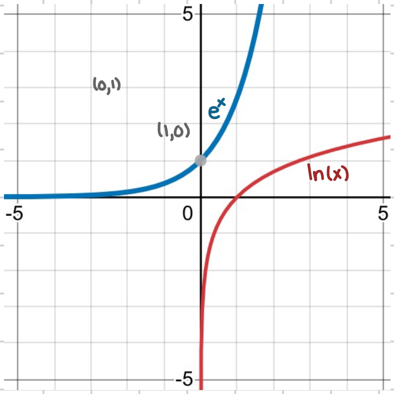
e. Give a sketch of the graph of $y = \ln x$. State clearly the domain and range of $\ln x$. What are the values of $\lim_{x\to 0^+} \ln x$ and $\lim_{x\to\infty} \ln x$?

f. The inverse $g(x)$ of $f(x) = \ln x$ exists. Why? Sketch the graph of $g(x) = \exp(x)$. Infer from (d) that we may write $\exp(x) = e^x$ for all real value x.



$y = \ln(x)$

domain: $0 < x < \infty$
↳ input OR $(0, \infty)$

range: $-\infty < y < \infty$
↳ output OR $(-\infty, \infty)$

$y = e^x$

domain: $(-\infty, \infty)$
OR $-\infty < x < \infty$

range: $(0, \infty)$
OR $0 < y < \infty$

Notice:
$\ln(1) = 0$
$e^0 = 1$

h. Using the fact that $\frac{d}{dx}(e^x) = e^x$, the chain rule and the fact that $e^{\ln b} = b$ $(b > 0)$, show that $\frac{d}{dx}(b^x) = b^x \ln b$.

$$\frac{d}{dx}(b^x) = \frac{d}{dx}\left(e^{\ln(b^x)}\right) = \frac{d}{dx}\left(e^{x\overbrace{\ln(b)}^{\text{constant}}}\right) = e^{x\cdot\ln(b)}\cdot\ln(b)$$

i. Using the change of base formula $\log_b x = \dfrac{\ln x}{\ln b}$, show that $\dfrac{d}{dx}(\log_b x) = \dfrac{1}{x \ln b}$.

$$\frac{d}{dx}(\log_b x) = \frac{d}{dx}\left(\frac{\ln(x)}{\ln(b)}\right) = \frac{d}{dx}\left(\overbrace{\frac{1}{\ln(b)}}^{\text{constant}}\cdot\ln(x)\right) = \frac{1}{\ln(b)}\cdot\frac{1}{x} = \frac{1}{x\ln(b)}$$

Example B. Find the equation of the tangent line to the curve $y = 4 - 2e^x + \ln\left(\dfrac{1-x^2}{1+x^2}\right)$ at $x = 0$.

Equation of the tangent line:
line: $y - y_1 = m(x - x_1)$
slope = derivative: $m = \left.\frac{dy}{dx}\right|_{x_1} = y'(x_1)$

Using log rules in $\frac{d}{dx}$:

$\ln\left(\frac{f(x)}{g(x)}\right) = \ln(f(x)) - \ln(g(x))$

chain + quotient   chain + chain

Given $x_1 = 0$, $y_1 = 4 - 2e^0 + \ln\left(\frac{1+0^2}{1+0^2}\right)$
$= 4 - 2\cdot 1 + \ln(1)$
$= 4 - 2 + 0 = 2$

$y = 4 - 2e^x + \ln\left(\frac{1-x^2}{1+x^2}\right)$
$y = 4 - 2e^x + \ln(1-x^2) - \ln(1+x^2)$
$y' = 0 - 2e^x + \frac{1}{1-x^2}\cdot(-2x) - \frac{1}{1+x^2}(2x)$

$y'(0) = -2e^0 + 0 - 0 = -2\cdot 1 = -2$

$y - 2 = -2(x - 0)$
$y - 2 = -2x$
$y = -2x + 2$