Math 10350 – Example Set 02A
Quadratic Functions: Section 1.6
Basic Exponential Equations: Section 1.6

1. (Completing the Square Review) A particle moving in a straight line has position in meters, measured from a fixed point **O** on the straight line, at time t seconds is given by

$$s(t) = 5 - 4t + 3t^2$$

(i) Sketch the graph of $s(t)$. (ii) Find the time at which the particle is **closest** to the point **O**. (iii) How far can the particle be from the point **O**?

2. (Sect 1.6) Solve the following equations: (a) $4^x = \frac{1}{8}$; (b) $3 \cdot 9^{x+1} = 81^x$.

1. s(t) is the distance from the point O, there is no gaurantee that the particle goes through O i.e. s(t) may never be 0.

(i) Sketch

$s(t) = A(x-h)^2 + k$

$s(t) = 3t^2 - 4t + 5$

$s(t) = 3(t^2 - \frac{4}{3}t) + 5$

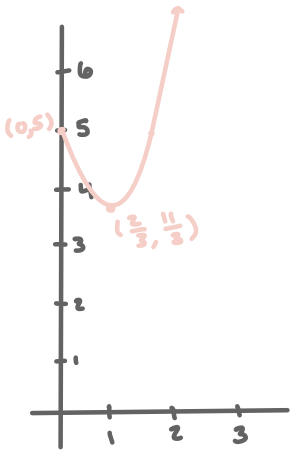
$b = -\frac{4}{3}$; $(\frac{1}{2}b)^2 = (-\frac{4}{6})^2 = (-\frac{2}{3})^2$

$s(t) = 3(t^2 - \frac{4}{3}t + \frac{4}{9} - \frac{4}{9}) + 5$

$= 3(t^2 - \frac{4}{3}t + \frac{4}{9}) - \frac{4}{3} + \frac{15}{3}$

$s(t) = 3(t - \frac{2}{3})^2 + \frac{11}{3}$

$(h,k) = (\frac{2}{3}, \frac{11}{3})$



(ii) minimum distance

s(t) = distance

minimum = vertex

$\frac{11}{3}$ meters away from O at time $\frac{2}{3}$ seconds

(iii) maximum distance

there is no maximum

note: parabolas have a min or a max not both

2. Solve the following equations:

(a) $4^x = \frac{1}{8}$

$\log_4 4^x = \log_4 \frac{1}{8}$

$x \log_4 4 = \log_4 2^{-3}$

$x = -3\log_4 2$

Note: $\log_4 2 = \frac{1}{2}$

$x = -\frac{3}{2}$

alternative:

$\ln(4^x) = \ln(\frac{1}{8})$

$x \ln(4) = -\ln(8)$

$x = -\frac{\ln(8)}{\ln(4)}$

$x = \frac{-\ln(2^3)}{\ln(2^2)}$

$x = \frac{-3\ln(2)}{2\ln(2)}$

$x = -\frac{3}{2}$

(b) $3 \cdot 9^{x+1} = 81^x$

$\log_9(3 \cdot 9^{x+1}) = \log_9(9^{2x})$

$\log_9(3) + \log_9(9^{x+1}) = \log_9(9^{2x})$

$\frac{1}{2} + x + 1 = 2x$

$\frac{3}{2} = x$

alternative:

$\ln(3) + (x+1)\ln 9 = x\ln(81)$

$\ln(3) + (x+1)2\ln(3) = x\ln(3^4)$

$(2(x+1)+1)\ln(3) = 4x\ln(3)$

$2x + 3 = 4x$

$\frac{3}{2} = x$