Exit Ticket L'Hopital

> **L'Hopital** If both $f(x)$ and $g(x)$ are differentiable functions such that:
>
> - $\lim\limits_{x\to c} f(x) = 0 = \lim\limits_{x\to c} g(x)$ such that $\lim\limits_{x\to c} \frac{f'(x)}{g'(x)}$ exists then $\lim\limits_{x\to c} \frac{f(x)}{g(x)} = \lim\limits_{x\to c} \frac{f'(x)}{g'(x)}$
> - $\lim\limits_{x\to c} f(x) = \infty = \lim\limits_{x\to c} g(x)$ such that $\lim\limits_{x\to c} \frac{f'(x)}{g'(x)}$ exists then $\lim\limits_{x\to c} \frac{f(x)}{g(x)} = \lim\limits_{x\to c} \frac{f'(x)}{g'(x)}$

Identify when you can use L'Hopital. If you can, evaluate the limit:

1. $\lim\limits_{x\to\infty} \dfrac{3x^3+4x^2-3x+5}{5x^4+3x^2-1}$ $\frac{\infty}{\infty}$

L'H x3

$= \lim\limits_{x\to\infty} \frac{18}{120x} = 0$

2. $\lim\limits_{x\to 0^+} \dfrac{ln(x+1)}{\sqrt{x}}$ $\frac{0}{0}$

$= \lim\limits_{x\to 0^+} \frac{\frac{1}{x+1}}{\frac{1}{2}x^{-1/2}} = \lim\limits_{x\to 0^+} \frac{1}{x+1}\cdot\frac{2x^{1/2}}{1}$

$= \lim\limits_{x\to 0^+} \frac{x^{-1/2}}{1} = \lim\limits_{x\to 0^+} \frac{1}{\sqrt{x}} = \infty$

3. $\lim\limits_{x\to\infty} \dfrac{-6e^{2x}+7}{3x^{2x}+5}$ $\frac{\infty}{\infty}$

$= \lim\limits_{x\to\infty} \frac{-12e^{2x}}{6e^{2x}}$

$= \lim\limits_{x\to\infty} -2$

$= -2$

4. $\lim\limits_{x\to\infty} \dfrac{e^{2x}+2e^x+1}{e^x+1}$

$u = e^x$

$= \lim\limits_{u\to\infty} \frac{u^2+2u+1}{u+1} = \lim\limits_{u\to\infty} \frac{(u+1)\cancel{(u+1)}}{\cancel{(u+1)}}$

$= \lim\limits_{x\to\infty} e^x+1$

$= \infty$

5. $\lim\limits_{x\to 0} \dfrac{\sin(x)-\sin(2x)}{\sin(x)+\sin(3x)}$ $\frac{0-0}{0-0} = \frac{0}{0}$

$= \lim\limits_{x\to 0} \frac{\cos(x)-2\cos(2x)}{\cos(x)-3\cos(3x)}$

$= \frac{1-2(1)}{1-3(1)}$

$= \frac{-1}{-2}$

$= \frac{1}{2}$

6. $\lim\limits_{x\to\infty} (1+x)^{\frac{1}{x}}$ ∞^0

$= \lim\limits_{x\to\infty} e^{\ln((1+x)^{1/x})}$

$= \lim\limits_{x\to\infty} e^{\frac{1}{x}\ln(1+x)}$ $e^{\frac{\infty}{\infty}}$

$= \lim\limits_{x\to\infty} e^{\frac{\frac{1}{1+x}}{1}}$

$= \lim\limits_{x\to\infty} e^{\frac{1}{1+x}}$

$= e^0 = 1$

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