Math 10350 – Example Set 04C

1. If $f'(a) = \lim_{h\to 0} \dfrac{(3+h)^{10} - 3^{10}}{h}$, what is a possible $f(x)$ and the value of a?

$= \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

$f(x+h) = (3+h)^{10}$

$f(x) = 3^{10}$

guess: $f(x) = x^{10}$, $a=3$

$f'(x) = \lim_{h\to 0} \dfrac{(x+h)^{10} - x^{10}}{h}$

$f'(3) = \lim_{h\to 0} \dfrac{(3+h)^{10} - (3)^{10}}{h}$

my second guess would have been: $f(x)=3^x$, $a=10$

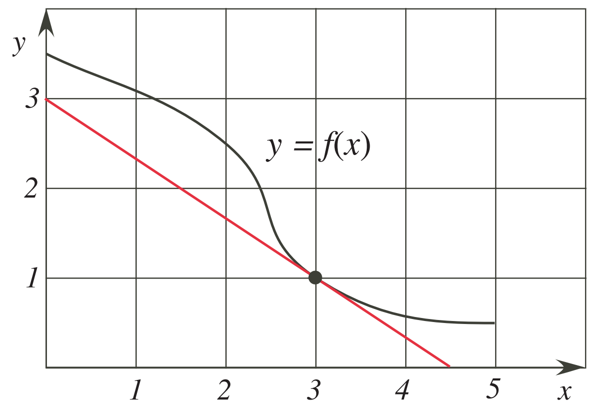
$f'(x) = \lim_{h\to 0} \dfrac{3^{x+h} - 3^x}{h}$

$f'(10) = \lim_{h\to 0} \dfrac{3^{10+h} - 3^{10}}{h}$

not the same

$f(x) \stackrel{?}{=}$ x^{10} and $a \stackrel{?}{=}$ 3

2.



The figure above describes the graph of $y = f(x)$ and its tangent line at $x = 3$. Answer the problems below:

a. Estimate the average rate of change of $f(x)$ over the interval $[0, 5]$.

average rate of change: $\dfrac{f(b)-f(a)}{b-a}$

$f(5) = \frac{1}{2}$, $f(0) = \frac{7}{2}$: $\dfrac{f(5)-f(0)}{5-0} = \dfrac{\frac{7}{2} - \frac{1}{2}}{5-0} = \dfrac{\frac{6}{2}}{5} = \dfrac{3}{5}$

b. $f(3) \stackrel{?}{=}$ 1 and $f'(3) \stackrel{?}{=}$ $-\frac{3}{2}$ slope of red line

c. Find the equation of the tangent line at $x = 3$. Give your answer in slope-intercept form.

$y - 1 = -\frac{3}{2}(x-3) \Rightarrow y - 1 = -\frac{3}{2}x + \frac{9}{2} \Rightarrow y = -\frac{3}{2}x + \frac{11}{2}$

3. The slope of the curve $y = ax^2 + bx$ at the point $(2, 4)$ is -8. Calculate the values of a and b.

4. Find the values of x for which both the graphs of the functions $f(x) = x^3 - 3x^2 + 7x + 8$ and $g(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 5x - 3$ have parallel tangent lines at x. Pick one such location on the graph of $f(x)$ and find the equation of the tangent line there.

3. The slope of the curve $y = ax^2 + bx$ at the point $(2, 4)$ is -8. Calculate the values of a and b.

derivative at $x=2$ is -8: $f'(2) = -8$

$f'(x) = 2ax + b = -8$

$f'(2) = 2a(2) + b = -8$

$4a + b = -8$

this is unsolvable w/o: $f(2) = 4$

$a(2)^2 + b(2) = 4$

$4a + 2b = 4$

solve the system of equations:

(i) $4a + b = -8$

(ii) $4a + 2b = 4$

method of cancellation:

$4a + b = -8$

$-(4a + 2b = 4)$

$0 - b = -12$

$b = 12$

substitute into eq (i):

$4a + 12 = -8$

$4a = -20$

$a = -5$

$a = -5, b = 12$

Check answers:

$f(x) = -5x^2 + 12x$

$f(2) = -5(2)^2 + 12(2)$

$= -5(4) + 24$

$= -20 + 24$

$= 4$

$f'(x) = -10x + 12$

$f'(2) = -10(2) + 12$

$= -20 + 12$

$= -8$

power rule

$\frac{d}{dx}(ax^n) = a \cdot n x^{n-1}$

4. Find the values of x for which both the graphs of the functions $f(x) = x^3 - 3x^2 + 7x + 8$ and $g(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 5x - 3$ have parallel tangent lines at x. Pick one such location on the graph of $f(x)$ and find the equation of the tangent line there.

parallel tangent lines means $f'(x) = g'(x)$

$3x^2 - 6x + 7 + 0 = \frac{1}{3} \cdot 3x^2 - \frac{1}{2} \cdot 2x + 5 - 0$

$3x^2 - 6x + 7 = x^2 - x + 5$

$2x^2 - 5x + 2 = 0$

$2x^2 - 4x - x + 2 = 0$

$2x(x-2) - 1(x-2) = 0$

$(2x-1)(x-2) = 0$

$2x - 1 = 0$ $\quad$ $x - 2 = 0$

$2x = 1$ $\quad$ $x = 2$

$x = \frac{1}{2}$

power rule

$\frac{d}{dx}(ax^n) = anx^{n-1}$

Note: Since $f'(x) = g'(x)$, I can use either to find the slope of the tangent line. I choose $g'(x)$ as it's simpler. But $(x, f(x)) \neq (x, g(x))$ so the tangent line of $f(x)$ at $x = \frac{1}{2}$ is parallel but not equal to the tangent line of $g(x)$ at $x = \frac{1}{2}$.

tangent line at $x = \frac{1}{2}$

$g'(x) = x^2 - x + 5$

$g'(\frac{1}{2}) = (\frac{1}{2})^2 - \frac{1}{2} + 5$

$= \frac{1}{4} - \frac{2}{4} + \frac{20}{4}$

$= \frac{21}{4}$

$f(x) = x^3 - 3x^2 + 7x + 8$

$f(\frac{1}{2}) = (\frac{1}{2})^3 - 3(\frac{1}{2})^2 + 7(\frac{1}{2}) + 8$

$= \frac{1}{8} - 3(\frac{1}{4}) + \frac{7}{2} + 8$

$= \frac{1}{8} - \frac{3}{4} + \frac{7}{2} + 8$

$= \frac{1}{8} - \frac{6}{8} + \frac{28}{8} + \frac{64}{8}$

$= \frac{87}{8}$

$y - (\frac{87}{8}) = \frac{21}{4}(x - 2)$

$y - \frac{87}{8} = \frac{21}{4}x - \frac{21}{2}$

$y - \frac{87}{8} = \frac{21}{4}x - \frac{84}{8}$

$y = \frac{21}{4}x + \frac{3}{8}$

tangent line at $x = 2$

$g'(x) = x^2 - x + 5$

$g'(2) = (2)^2 - (2) + 5$

$= 4 - 2 + 5$

$= 7$

$f(x) = x^3 - 3x^2 + 7x + 8$

$f(2) = (2)^3 - 3(2)^2 + 7(2) + 8$

$= 8 - 12 + 14 + 8$

$= 18$

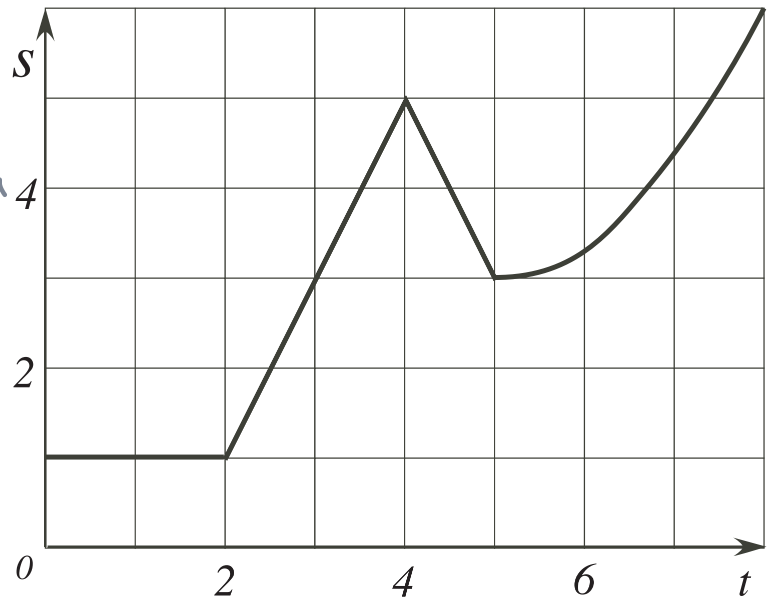
$y - 18 = 7(x - 18)$

$y - 18 = 7x - 126$

$y = 7x - 108$

5. A military craft made with a new technology that could change its velocity on demand in a moment was test driven on a long straight road. The graph of its position $s(t)$ for eight seconds of travel is given below. Sketch in the given axes below the velocity function $v(t)$ indicating clearly places where velocity is undefined.

Unforturnately, a graph does not allow us to get a perfect velocity; however, we can use average rate of change to make a sketch.

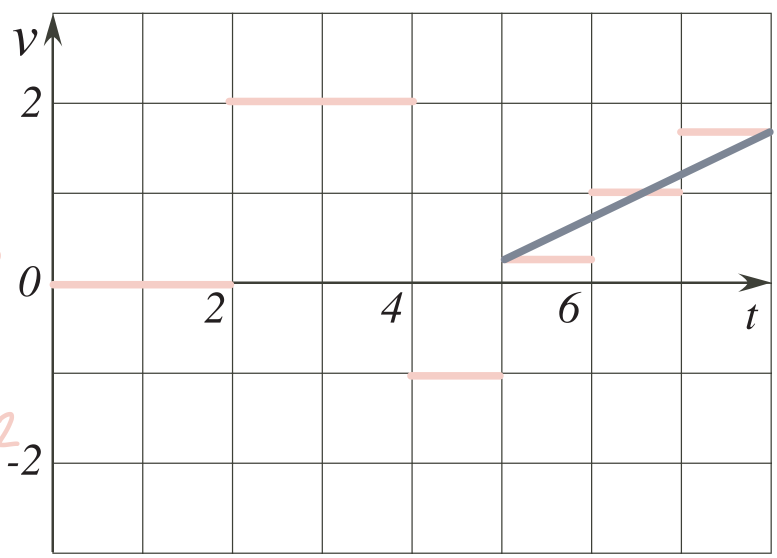


average rate of change:

[0,2]
$\frac{f(2)-f(0)}{2-0}=\frac{1-1}{2}=0$

[2,4]
$\frac{f(4)-f(2)}{4-2}=\frac{5-1}{2}=2$

[4,5]
$\frac{f(5)-f(4)}{5-4}=\frac{3-5}{1}=-2$



the last section is not linear so the slope is not constant we can use smaller ranges to est.:

[5,6]
$\frac{f(6)-f(5)}{6-5}=\frac{3.25-3}{1}=\frac{1}{4}$

[6,7]
$\frac{f(7)-f(6)}{7-6}=\frac{4.25-3.25}{1}=1$

[7,8]
$\frac{f(8)-f(7)}{8-7}=\frac{6-4.25}{1}=1.75=\frac{7}{4}$

the last section resembles a parabola $y=ax^2+b$ which would have a linear slope but it could also be cubic or even $y=ae^x+b$ so the linear slope is merely a guess