Let $f(x)$ be differentiable at $x = a$. Then the linear approximation for the change Δf in $f(x)$ when x changes from a to $a + \Delta x$:

$$\Delta f = f(a + \Delta x) - f(a) \approx f'(a)\Delta x$$

1. The diameter of a circular disk is given as 10 cm with a maximum error in measurement of 0.2 cm. Use linear approximation to estimate the maximum error (ΔA) and percentage error in the calculated area of the disk. If the disk is made with an expensive titanium sheet that costs \$50 per cm^2, estimate an upper limit for your budget in making a disk of 10 cm diameter (Upper limit for budget is $\$(1250\pi + 50\pi) = \$1300\pi$).

2. A vessel is in the shape of an inverted cone. The radius of the top is 5 cm and the height is 8 cm. Water is poured in to a height of x cm. Find an expression for the volume V of the water in the vessel in terms of x. Use linear approximate to estimate the increased in V when x increases from 4 cm to 4.08 cm. Give units for your answer. (Answer: $\pi/2$ cm^3)

1. $d = 10 \pm 0.2$ find max ΔA & percent error $= 100 \cdot \frac{\text{approx} - \text{exact}}{\text{exact}} = 100 \cdot \frac{\Delta f}{f}$
& find cost to make $C(A) = 50 \cdot A$ for largest disk

$A = \pi r^2$ $\quad a = 10 \quad \Delta r = \pm 0.1$ careful: 0.2 is diameter error

$\Delta A = A'(a) \cdot \Delta r \longrightarrow \Delta A = 10\pi(\pm 0.1)$

$A = \pi r^2$ $\quad = 10\pi \cdot \pm\frac{1}{10}$

$A' = 2\pi r$

$A'(5) = 10\pi$ $\quad \Delta A = \pm \pi$

careful:
diam. = 10cm
radius = 5cm

percent: $100 \cdot \frac{\Delta A}{A} = 100 \cdot \frac{\pm\pi}{25\pi}$
$= \pm 4$

plan to make a disk of area $A + \Delta A$

$C(A + \Delta A) = 50(A + \Delta A) = 50A + 50\Delta A$

$= 50(25\pi) + 50(\pi)$

$= 1250\pi + 50\pi$

$= 1300\pi$

Two ways to compute:
(i) change givens to radius
(ii) change area to diameter

2. Given $\frac{r}{h} = \frac{5}{8}$. Find $V(x)$ and ΔV when $\Delta x = 4.08 - 4$.

$V = \frac{1}{3}\pi r^2 h$
use $r = \frac{5}{8}h$
$= \frac{1}{3}\pi\left(\frac{5}{8}h\right)^2 \cdot h$
$= \frac{1}{3}\pi \cdot \left(\frac{25}{64}h^2\right) \cdot h$
$= \frac{25}{192}\pi h^3$

in terms of x:
$V(x) = \frac{25}{192}\pi x^3$
revert back to $= \frac{1}{3} \cdot \frac{25}{64}\pi x^3$

$V'(x) = \frac{25}{64}\pi x^2$

$V'(4) = \frac{25}{64}\pi(4)^2 = \frac{25}{4}\pi$

$\Delta V = V'(x_1)\Delta x$
$\Delta V = \frac{25}{4}(x - 4)$

For $x = 4.08$:
$\Delta V = \frac{25}{4}(4.08 - 4)$
$= \frac{25}{4}(0.08) = 0.5$

$0.08 = \frac{8}{100}$
$= \frac{4}{50}$