Math 10350 – Example Set 05A
Sections 3.1 & 3.2
Product Rule & Quotient Rule

Product and Quotient Rule. Let $f(x)$ and $g(x)$ be differentiable functions. Derive formulas for the derivatives of $p(x) = f(x) \cdot g(x)$ and $q(x) = \dfrac{f(x)}{g(x)}$.

Product Rule: $\dfrac{d}{dx}(f(x)g(x)) = f'(x)g(x) + g'(x)f(x)$

common notation:
$f'g + g'f$
$gdf + fdg$

Quotient Rule: $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$

common notation:
$\dfrac{f'g - g'f}{g^2}$ or $\dfrac{\text{high-d-low minus low-d-high}}{\text{low squared}}$

1. The stationary points in the domain of a function $f(x)$ are the values of x such that $f'(x) = 0$. What can you say about the tangent line at stationary points? Find the stationary points of the following functions:

Since the derivative is the slope of the tangent line, the tangent line at a stationary point is zero i.e. the line is horizontal.

1a. $f(x) = (x^2 - 3)e^x$.

1b. $y = \dfrac{2x - 1}{x^2 + 1}$.

2. Let $p(x) = (x^3 - 5x + 1)g(x)$ and $q(x) = \dfrac{f(x)}{g(x) + 1}$. Given that $f(2) = 2$, $g(2) = 3$, $f'(2) = -1$ and $g'(2) = -4$, find the following values:

a. The instantaneous rate of change of $p(x)$ at $x = 2$.

b. The slope of the tangent line to the graph of $y = q(x)$ when $x = 2$.

1a. $\cancel{f(x)}\ y = \underbrace{(x^2 - 3)}_{f}\underbrace{e^x}_{g}$.

product rule: $f'g + g'f$

$f(x) = x^2 - 3$ $\quad g(x) = e^x$

$f'(x) = 2x$ $\quad g'(x) = e^x$

$y' = (2x)e^x + e^x(x^2 - 3)$

$= 2xe^x + e^x x^2 - 3e^x$

$= e^x(2x + x^2 - 3)$

$= e^x(x^2 + 2x - 3)$

$= e^x(x+3)(x-1)$

when is $y' = 0$?

$e^x = 0$ $\quad x+3=0$ $\quad x-1=0$

never $\quad x=-3$ $\quad x=1$

stationary points: $x = -3, 1$

1b. $y = \dfrac{2x-1}{x^2+1}$.] f] g or high / low

quotient rule: $\dfrac{f'g - g'f}{g^2}$

$f(x) = 2x - 1$ $\quad g(x) = x^2 + 1$

$f'(x) = 2$ $\quad g'(x) = 2x$

$y' = \dfrac{(2)(x^2+1) - (2x)(2x-1)}{(x^2+1)^2}$

$= \dfrac{\cancel{2x^2} + 1 - \cancel{2x^2} + 2x}{(x^2+1)^2}$

$= \dfrac{2x+1}{(x^2+1)^2}$

when is $y' = 0$?

$\dfrac{2x+1}{(x^2+1)^2} = 0$

$2x + 1 = 0$

$2x = -1$

$x = -\frac{1}{2}$

stationary point: $x = -\frac{1}{2}$

2. Let $p(x) = (x^3 - 5x + 1)g(x)$ and $q(x) = \dfrac{f(x)}{g(x)+1}$. Given that $f(2) = 2$, $g(2) = 3$, $f'(2) = -1$ and $g'(2) = -4$, find the following values:

a. The $\overset{\text{derivative}}{\text{instantaneous rate of change}}$ of $p(x)$ at $x = 2$.

$p(x) = \underbrace{(x^3 - 5x + 1)}_{f} \cdot \underbrace{g(x)}_{g}$

power rule: $f'g + g'f$

$f(x) = x^3 - 5x + 1$ $\quad g(x) = g(x)$

$f'(x) = 3x^2 - 5$ $\quad g'(x) = g'(x)$

$p'(x) = (3x^2 - 5) \cdot g(x) + g'(x) \cdot (x^3 - 5x + 1)$

$p'(2) = (3(2)^2 - 5) \cdot g(2) + g'(2) \cdot ((2)^3 - 5(2) + 1)$

$= (3 \cdot 4 - 5) \cdot 3 + (-4) \cdot (4 - 10 + 1)$

$= (12 - 5) \cdot 3 + (-4) \cdot (-5)$

$= 7 \cdot 3 + 20$

$= 21 + 20$

$= 41$

b. The $\overset{\text{derivative}}{\text{slope of the tangent line}}$ to the graph of $y = q(x)$ when $x = 2$.

$q(x) = \dfrac{f(x)}{g(x)+1}$] f] g

quotient rule: $\dfrac{f'g - g'f}{g^2}$

$f = f(x)$ $\quad g = g(x) + 1$

$f' = f'(x)$ $\quad g' = g'(x) + 0$

$q'(x) = \dfrac{f'(x) \cdot (g(x)+1) - g'(x) \cdot f(x)}{(g(x)+1)^2}$

$q'(2) = \dfrac{f'(2) \cdot (g(2)+1) - g'(2) \cdot f(2)}{(g(2)+1)^2}$

$= \dfrac{(-1)(3+1) - (-4)(2)}{(3+1)^2}$

$= \dfrac{(-1)(4) - (-4)(2)}{(4)^2}$

$= \dfrac{-4 + 8}{16}$

$= \dfrac{4}{16} = \dfrac{1}{4}$