Math 10350 – Example Set 11C

1. Find the equations of all vertical and horizontal asymptotes of $y = \dfrac{3x^2 + 2x - 5}{2x^2 + x - 3}$.

2. Sketch the graph of $f(x) = \dfrac{e^x + 1}{e^x - 1}$ by completing the steps below.

$f(x)=0$ $\quad$ $x=0$

a. Find all x-intercepts and y-intercept of the graph of $f(x)$ whenever possible.

$0 = \frac{e^x+1}{e^x-1}$

$0 = e^x + 1$

$-1 = e^x$

never

$y = \frac{e^0+1}{e^0-1}$

$= \frac{1+1}{1-1}$

$= \frac{2}{0}$ undefined

no intercepts

b. Find coordinates of all critical points, vertical asymptotes, and places where $f(x)$ are undefined. $\left(f'(x) = -\dfrac{2e^x}{(e^x-1)^2}\right)$

$f'(x)=0$ or DNE $\quad$ denom. $=0$ $\quad$ domain issues

$f(x) = \frac{e^x+1}{e^x-1}$

$f'(x) = \frac{e^x(e^x-1) - e^x(e^x+1)}{(e^x-1)^2}$

$= \frac{\cancel{e^{2x}} - e^x - \cancel{e^{2x}} - e^x}{(e^x-1)^2}$

$= \frac{-2e^x}{(e^x-1)^2}$

DNE when $x=0$

$0 = \frac{-2e^x}{(e^x-1)^2}$

$0 = -2e^x$

never

vertical asymptote:

$x=0$

$f'(x)>0$ $\quad$ $f'(x)<0$

c. Determine where $f(x)$ is increasing and where it is decreasing.

asymptote

$-$ $\quad$ $-$

test -1 $\quad$ 0 $\quad$ test 1

$f'(-1) = \frac{-2e^{-1}}{(e^{-1}-1)^2}$ $\quad$ $\frac{\text{negative}}{\text{positive}}$ $\leftarrow (\sim)^2$ is always positive

$f'(1) = \frac{-2e^1}{(e^1-1)^2}$ $\quad$ $\frac{\text{negative}}{\text{positive}}$

decrasing: $(-\infty, 0) \cup (0, \infty)$

d. Determine the concavity and coordinates of inflection points of $f(x)$. $\left(f''(x) = \dfrac{2e^x(1+e^x)}{(e^x-1)^3} = \dfrac{2e^x(1+e^x)}{(e^x-1)^2} \cdot \dfrac{1}{e^x-1}\right)$

$f'(x) = \frac{-2e^x}{(e^x-1)^2}$

$f''(x) = \frac{-2e^x(e^x-1)^2 - 2(e^x-1)\cdot(e^x)(-2e^x)}{((e^x-1)^2)^2}$

$= \frac{-2e^x(e^x-1)^2 + 4e^{2x}(e^x-1)}{(e^x-1)^4}$

$= \frac{-2e^x(e^x-1) + 4e^{2x}}{(e^x-1)^3}$

$= \frac{-2e^{2x} + 2e^x + 4e^{2x}}{(e^x-1)^3}$

$= \frac{2e^{2x} + 2e^x}{(e^x-1)^2}$

$= \frac{2e^x(e^x+1)}{(e^x-1)^2}$

$0 = \frac{2e^x(e^x+1)}{(e^x-1)^2}$

$0 = 2e^x(e^x+1)$

$0 = 2e^x$ $\quad$ $0 = e^x + 1$

never $\quad$ $-1 = e^x$

never

DNE when denom. $=0$

$(e^x-1)^4 = 0$

$e^x - 1 = 0$

$e^x = 1$

$x = 0$

$f''(-1) = \frac{(2e^{-1})(e^{-1}+1)}{(e^{-1}-1)^3}$ $\quad$ $\frac{\text{pos.} \times \text{pos.}}{(\text{neg.})^3}$

$f''(1) = \frac{(2e^1)(e^1+1)}{(e^1+1)^3}$ $\quad$ $\frac{\text{pos.} \times \text{pos.}}{(\text{pos.})^3}$

$-$ $\quad$ $+$

test -1 $\quad$ 0 $\quad$ test 1

concave up: $(0, \infty)$ $\quad$ concave down: $(-\infty, 0)$

1. Find the equations of all vertical and horizontal asymptotes of $y = \dfrac{3x^2+2x-5}{2x^2+x-3}$.

denominator $=0$ $\qquad$ $\lim\limits_{x\to\pm\infty} f(x) = L$

vertical:

$2x^2+x-3=0$

$2x^2+3x-2x-3=0$

$x(2x+3)-1(2x+3)=0$

$(x-1)(2x+3)=0$

$x-1=0 \qquad 2x+3=0$

$x=1 \qquad 2x=-3$

$x=-3/2$

horizontal:

$\lim\limits_{x\to\pm\infty} \dfrac{3x^2+2x-5}{2x^2+x-3}$ $\quad \frac{\infty}{\infty}$ L'H

$=\lim\limits_{x\to\pm\infty} \dfrac{6x+2}{4x+1}$ $\quad \frac{\infty}{\infty}$ L'H

$=\lim\limits_{x\to\pm\infty} \dfrac{6}{4}$

$=\dfrac{3}{2}$

$y=\dfrac{3}{2}$

e. Find all asymptotes and limit at infinity whenever applicable.

$f(x) = \frac{e^x+1}{e^x-1}$

denom. $=0$

$\lim_{x \to \pm\infty} f(x)$

vertical: $e^x - 1 = 0$

$e^x = 1$

$x = 0$

$\lim_{x \to +\infty} \frac{e^x+1}{e^x-1}$ $\frac{\infty}{\infty}$

$= \lim_{x \to +\infty} \frac{e^x}{e^x}$

$= \lim_{x \to +\infty} 1$

$= 1$

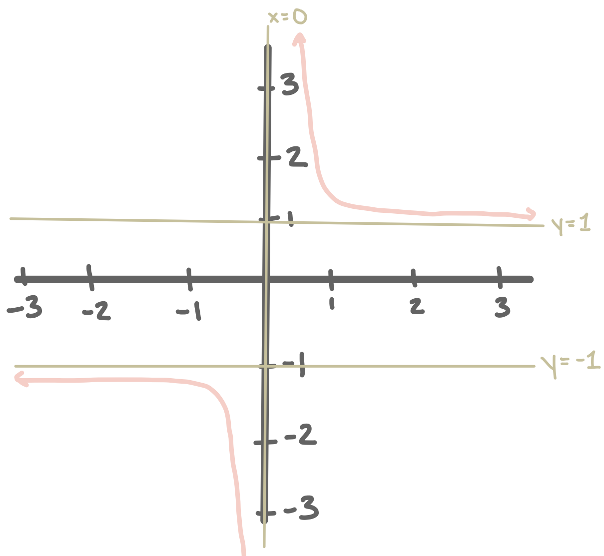
$\lim_{x \to -\infty} \frac{e^x+1}{e^x-1}$ $\frac{\frac{1}{e^\infty}+1}{\frac{1}{e^\infty}-1}$

$= \frac{0+1}{0-1}$

$= \frac{1}{-1}$

$= -1$

f. Sketch the graph below labeling all important features. Your picture should be large and clear.



Recall all points:

- critical points: none
- inflection points: none

Recall intervals:

- increasing: never
- decreasing: $(-\infty, 0) \cup (0, \infty)$
- concave up: $(0, \infty)$
- concave down: $(-\infty, 0)$

Recall asymptotes:

- vertical: $x = 0$
- end behavior:
  - ↳ as $x \to \infty$, $f(x) \to 1$
  - ↳ as $x \to -\infty$, $f(x) \to -1$

It is helpful to know parent functions to inform your sketch, but if you do not know them or any points you can plug in a couple values (like $x = -2, -1, -\frac{1}{2}, \frac{1}{2}, 1, 2$