Math 10350 – Example Set 08B

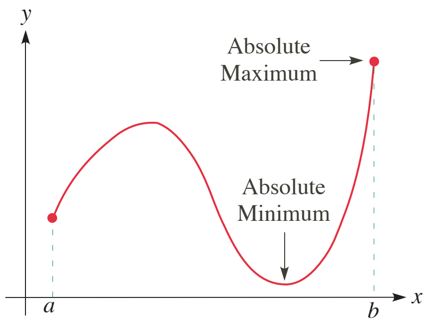


Figure 1

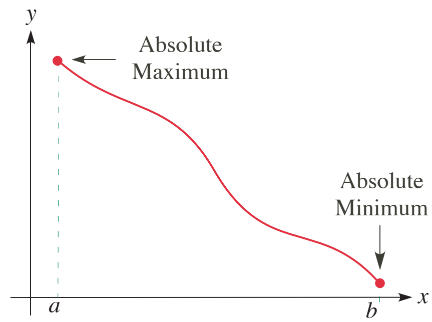


Figure 2

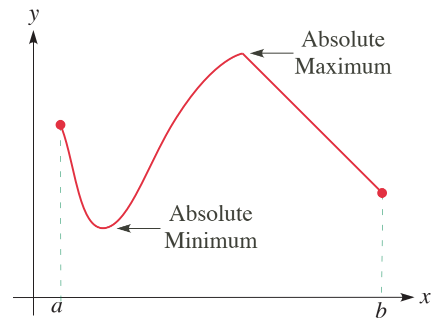


Figure 3

From Figures 1, 2, and 3, we can observe the following fact:

> **The extreme value theorem**
>
> If $f(x)$ is continuous on a closed and bounded interval $[a, b]$ then $f(x)$ takes on a minimum
>
> and takes on a maximum on $[a, b]$.

Q1: If $f(x)$ is continuous, where are the possible places for which absolute maximum and absolute minimum of $f(x)$ occur on $[a, b]$? (See Figures 1, 2, and 3.)

A1: On a closed and bound interval $[a, b]$, a continuous function $f(x)$ attains its absolute maximum and absolute

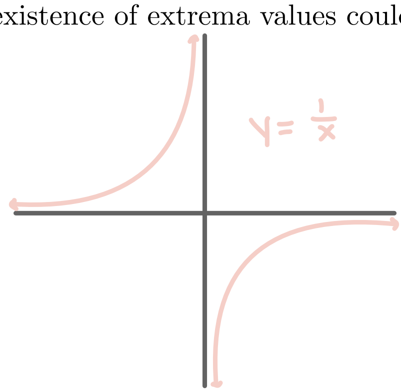
minimum occur at (1) $x=a$, or (2) $x=b$, or (3) anywhere between.

Definition: Let $f(x)$ be defined at c. Then we say that c is a **critical point** of f if (1) $f'(c)=0$,

or (2) $f'(c)$ DNE.

> **Method for finding absolute maxima and minima of f on $[a, b]$**
> 1. Find all critical points in (a, b).
> 2. Evaluate f at all critical points and at endpoints. Then compare the values of f:
>   **highest** = absolute maximum and **lowest** = absolute minimum.

Q2: What about when the interval is no longer closed and bounded? Draw some graphs to illustrate how the existence of extrema values could differ.



Notice that there is no absolute maximum and minimum if we continued this graph forever.
$\Rightarrow$ There does not need to be a max or min on a graph that is not closed or bounded.

1. Find the absolute (global) maximum and minimum of the function $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2 + 1$ on the interval $[0, 3]$.

2. Find all critical points of $f(x) = x - \frac{3}{2}x^{2/3}$ for $-1 \le x \le 1$. Hence determine the maximum and minimum values of $f(x)$ for $-1 \le x \le 1$.

1. Find the absolute (global) maximum and minimum of the function $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2 + 1$ on the interval $[0, 3]$.

Step 1: Find all critical points $f'(x)=0$

$f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2 + 1$

$f'(x) = x^3 - x^2 - 2x$

$= x(x^2 - x - 2)$

$= x(x-2)(x+1)$

$0 = x(x-2)(x+1)$

$x = 0$ $x - 2 = 0$ $x + 1 = 0$

$x = 2$ $x = -1$

Step 2: Compare outputs of critical points inside the interval & end points

note: we throw out $x=-1$ as it is not in the interval $[0,3]$

$f(0) = \frac{1}{4}(0)^4 - \frac{1}{3}(0)^3 - (0)^2 + 1 = 1$

$f(2) = \frac{1}{4}(2)^4 - \frac{1}{3}(2)^3 - (2)^2 + 1$

$= \frac{1}{4}(16) - \frac{1}{3}(8) - 4 + 1$

$= \cancel{4} - \frac{8}{3} - \cancel{4} + 1$

$= -\frac{5}{3}$

$f(3) = \frac{1}{4}(3)^4 - \frac{1}{3}(3)^3 - (3)^2 + 1$

$= \frac{1}{4}(81) - \frac{1}{3}(27) - 9 + 1$

$= \frac{81}{4} - 9 - 9 + 1$

$= \frac{81}{4} - 17$ $\frac{68}{4}$

$= \frac{13}{4}$

absolute max of $\frac{37}{4}$ at $x=3$

absolute min of $-\frac{5}{3}$ at $x=2$

2. Find all critical points of $f(x) = x - \frac{3}{2}x^{2/3}$ for $-1 \le x \le 1$. Hence determine the maximum and minimum values of $f(x)$ for $-1 \le x \le 1$.

critical points: $f'(x)=0$, DNE

$f(x) = x - \frac{3}{2}x^{2/3}$

$f'(x) = 1 - \frac{3}{2}\frac{2}{3}x^{-1/3}$

$= 1 - x^{-1/3}$ $= 1 - \frac{1}{\sqrt{x}}$

$\Rightarrow$ when $x = 0$

$f'(x)$ DNE

$0 = 1 - x^{-1/3}$

$x^{-1/3} = 1$

$\frac{1}{\sqrt[3]{x}} = 1$

$1 = \sqrt[3]{x}$

check critical points are in the interval

$x = 1$ is in $[-1, 1]$

$x = 0$ is in $[-1, 1]$

what is its output:

$f(1) = 1 - \frac{3}{2}(1)^{2/3}$

$= 1 - \frac{3}{2}$

$= -\frac{1}{2}$

$f(-1) = -1 - \frac{3}{2}(-1)^{2/3}$

$= -1 - \frac{3}{2}\sqrt[3]{-1^2}$

$= -1 - \frac{3}{2}$

$= -\frac{5}{2}$

$f(0) = 0 - \frac{3}{2}(0)^{2/3}$

$= 0 - \frac{3}{2} \cdot 0$

$= 0 - 0$

$= 0$

absolute max of 0 at $x=0$

absolute min of $-\frac{5}{2}$ at $x=1$