Review of Calculus A

Basic Properties of Derivatives

Addition: $\frac{d}{dx}[f(x)+g(x)] = [f(x)+g(x)]' = f'(x) + g'(x) = \frac{d}{dx}f(x) + \frac{d}{dx}g$

Subtraction: $\frac{d}{dx}[f(x)-g(x)] = [f(x)-g(x)]' = f'(x) - g'(x) = \frac{d}{dx}f(x) - \frac{d}{dx}g$

Coefficient: $\frac{d}{dx}[c\cdot f(x)] = [c\cdot f(x)]' = c\cdot f'(x) = c\cdot \frac{d}{dx}f(x)$

Product: $\frac{d}{dx}[f(x)\cdot g(x)] = [f(x)\cdot g(x)]' = f'(x)\cdot g(x) + g'(x)\cdot f(x) = \frac{d}{dx}f(x)\cdot g(x) + \frac{d}{dx}g(x)\cdot f(x)$

Quotient: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)\cdot g(x) - g'(x)\, f(x)}{(g(x))^2} = \frac{\frac{d}{dx}f(x)\cdot g(x) - \frac{d}{dx}g(x)\cdot f(x)}{(g(x))^2}$

Chain: $\frac{d}{dx}[f(g(x))] = [f(g(x))]' = f'(g(x))\cdot g'(x) = \frac{d}{dx}f(g(x))\cdot \frac{d}{dx}g(x)$

Basic Properties of Integrals:

Addition: $\int[f(x)+g(x)]dx = \int f(x)\,dx + \int g(x)\,dx$

Subtraction: $\int[f(x)-g(x)]dx = \int f(x)\,dx - \int g(x)\,dx$

Coefficient: $\int c\cdot f(x)\,dx = c\cdot \int f(x)\,dx$

Substitution: $\int f(g(x))\cdot g'(x)\,dx = \int f(u)\,du,\ u=g(x)$

Common Derivatives

Constant: $\frac{d}{dx}(k) = 0$ k is a constant

Power: $\frac{d}{dx}(x^n) = n x^{n-1}$ n is a constant

Trig.: $\frac{d}{dx}(\sin(x)) = \cos(x)$

$\frac{d}{dx}(\cos(x)) = -\sin(x)$

$\frac{d}{dx}(\tan(x)) = \sec^2(x)$

$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$

$\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$

Basic Integrals

Constant: $\int k\,dx = kx + c$

Power: $\int x^n\,dx = \frac{1}{n+1}\cdot x^{n+1} + c$

Trig: $\int \sin(x)\,dx = -\cos(x) + c$

$\int \cos(x)\,dx = \sin(x) + c$

$\int \sec^2(x)\,dx = \tan(x) + c$

$\int \sec(x)\tan(x)\,dx = \sec(x) + c$

$\int \csc(x)\cot(x)\,dx = -\csc(x) + c$

Examples:

1. Find the following derivatives:

a. $\frac{d}{dx}(\underbrace{x^3}\underbrace{\tan(x)}) = (3x^2)(\tan(x)) + \sec^2(x)(x^3)$

product rule $= 3x^2 \cdot \tan(x) + x^3 \sec^2(x)$

$(f \cdot g)' = f'g + g'f$ $= x^3 \sec^2(x) + 3x^2 \tan(x)$

b. $\frac{d}{dx}(\sqrt[3]{2x^2-5x+3}) = \frac{d}{dx}((2x^2-5x+3)^{1/3})$ rewrite as a fraction

power + chain $= \frac{1}{3}(2x^2-5x+3)^{-2/3} \cdot (4x-5)$

$(u^n)' = n u^{n-1} \cdot u'$

2. Find the equation of the tangent line to the curve $x\cos(1+2y) = 2y^2 - 8$ at $(0,2)$.

Equation of the tangent line:

line: $y - y_1 = m(x - x_1)$

slope = derivative: $m = \frac{dy}{dx}\Big|_{x_1} = y'(x_1)$

versions of this question:

given $y = f(x)$ and (x_1, y_1)

given implicit fxn & (x_1, y_1)

given parametric fxns & $t = a$

Implicit Differentiation (if finding $\frac{d}{dx}$ of curve):

- treat x as normal
- take derivative of y as expect then attach $\frac{dy}{dx}$ (or y')
(note this is just chain rule)

Find x_1, y_1, and m.

↳ $(x_1, y_1) = (0,2)$ is given

↳ $\frac{dy}{dx}$ must be found using implicit differentation

↳ attach $\frac{dy}{dx}$

$x\cos(\overbrace{1+2y}^{\text{chain}}) = 2y^2 - 8$ original curve

product rule power rule

$(1)(\cos(1+2y)) + (\sin(1+2y) \cdot 2 \cdot y')(x) = 4y \cdot y' - 0$

$\cos(1+2y) + 2xy'\sin(1+2y) = 4y \cdot y'$

$\cos(1+2y) = 4yy' - 2xy'\sin(1+2y)$ move y' to one side

$\cos(1+2y) = y'(4y - 2x\sin(1+2y))$ factor out y'

$y' = \frac{\cos(1+2y)}{4y - 2x\sin(1+2y)}$ solve for y'

$m = \frac{\cos(1+2(2))}{4(2) - 2(0)\sin(1+2(2))} = \frac{\cos(5)}{8-0} = \frac{1}{8}\cos(5)$ plug in (0,2)

$y - 2 = \frac{1}{8}\cos(5) \cdot (x - 0)$

3. Find the formula for the function f(x) if its slope is given by $x\sin(x^2+1)$ and the graph of f(x) passes through the point (1,2).

Intial value problem:

given slope/derivative and an (initial) point

- take integral of f'(x)
- plug in initial point
- solve for unknown c
- plug back into f(x)

concept: $f(x) = \int f'(x)\,dx$

$f(x) = \int x\sin(x^2+1)\,dx$

u-substitution: function & its derivative

$u = x^2 + 1$ u is normally an "inner" function

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$ you need to recognize this quickly

$f(u) = \int \sin(u) \cdot \frac{1}{2}du$

$= \frac{1}{2}\int \sin(u)\,du$

$= -\frac{1}{2}\cos(u) + c$

$f(x) = -\frac{1}{2}\cos(x^2+1) + c$

f(1) is found by plugging in x=1 to f(x) and was given to us (1,2) ⇒ f(1)=2

$f(1) = -\frac{1}{2}\cos((1)^2+1) + C$

$2 = -\frac{1}{2}\cos(2) + C$

$2 + \frac{1}{2}\cos(2) = C$

remember to plug into c

$f(x) = -\frac{1}{2}\cos(x^2+1) + 2 + \frac{1}{2}\cos(2)$

4. Evaluate $\int_0^1 \frac{x^2+2}{\sqrt{x^3+6x+5}}\,dx$

u-Substitution for $\int_a^b f(u)\cdot u'\,dx$

u= "inner" function

$\frac{d}{dx}(u) = u' \Rightarrow du = u'\,dx$

new bounds:

$u(a) = a'$ (lower)

$u(b) = b'$ (upper)

$u = x^3+6x+5$

$du = (3x^2+6)dx$

$du = 3(x^2+2)dx$

$\frac{1}{3}du = (x^2+2)dx$

$a' = (0)^3 + 6(0) + 5 = 5$

$b' = (1)^3 + 6(1) + 5 = 12$

$$\int_5^{12} \frac{1}{\sqrt{u}} \cdot \frac{1}{3}\,du$$
$$= \frac{1}{3}\int_5^{12} u^{-1/2}\,du$$
$$= \frac{1}{3} \cdot 2\,u^{1/2}\Big|_5^{12}$$
$$= \frac{2}{3}\left[(12)^{1/2} - (5)^{1/2}\right]$$
$$= \frac{2}{3}\left[2\sqrt{3} - \sqrt{5}\right]$$

you are expected to solve using the new bounds