Week 13: April 13th, 2023		
Section 14.3: Estimating Partial Derivatives		
Estimating One Varible Two Variables (y) Two Variable (x)		
Estimating One Varible Two Variables (4) Two Variable (x) Forward $f'(x) = \frac{f(x+h)-f(x)}{f(x+h)-f(x)}$ $f_{y} = \frac{f(x,y+h)-f(x,y)}{f(x,y)}$	h= 0 x or h= 0 y	
Difference h		
Backward $f(x) = \frac{f(x) - f(x-h)}{h}$ $f_{y} = \frac{f(x,y) - f(x,y-h)}{h}$ $f_{x} = \frac{f(x,y) - f(x-h,y)}{h}$	h= Dx or h = Dy	
Difference "		
Central $f(x) = \frac{f(x+h) - f(x-h)}{2h}$ $f_1 = \frac{f(x,y+h) - f(x,y-h)}{2h}$ $f_2 = \frac{f(x+h,y) - f(x-h,y)}{2h}$	2n= 0x or 2h= Dy	
example. A two-variable function f(x,y) has selected values given by		
y 2.5 3.0 3.5		
-1.0 6.0 6.5 8.0		
-1.5 6.5 7.0 8.5		
-2.0 5.8 6.9 7.8		
(a) Write down three estimates for the value of $\frac{3f}{3y}$ (3.5,-1.5). State what estimates central difference: $\frac{1}{3y}$ [f(x,y+h)-f(x,y-h)]	tes they are.	
$\frac{\partial f}{\partial y} (3.5, -1.5) = \frac{1}{\Delta y} [f(3.5, -1.0) - f(3.5, -2.0)] ; \Delta y = -1 - (-2) = 1$		
= 1 [8-7.8]		
= 160.2]		
= 0.2		
Forward difference: 54 [f(x,y+h)-f(x,y)]		
$\frac{2f}{2y}(3.5, -1.5) = \frac{1}{0y} [f(3.5, -1.0) - f(3.5, -1.5)]; 0y = -1 - (-1.5) = 0.5$		
= 0.5 [8-8.5]		
= 2[-0.5]		
Backward difference: $\frac{1}{24} [f(x,y) - f(x,y-h)]$ $\frac{2f}{3y} (3.5,-1.5) = \frac{1}{24} [f(3.5,-1.5) - f(3.5,-2.0)]; $		
= 0.5 [8.5-7.8]		
= 2[0.7]		
= 1.4		
(b) Estimate $\frac{\partial f}{\partial x}$ (3.5,-1.5). Which did you use?		
We can only look backwards:		
$\frac{3f}{3x}(3.5,-1.5) = \frac{1}{6x}(f(3.5,-1.5)-f(3.0,-1.5))$; $6x = 3.5-3 = 0.5$		
= 0.5 [8.5-7]		
= 2[1.5]		
Section 14.6: Chain Rule and Implicit Differentation		
Chain Rule		
The chain rule allows us to take a derivative down a tree of functions:		
f(x,y) The derivative down each line is the top over the bottom		
$\frac{df}{x(s,t)} = \frac{df}{ds} = \frac{df}{dx} \cdot \frac{dx}{ds} + \frac{df}{dy} \cdot \frac{dy}{ds} = \frac{df}{dt} \cdot \frac{dx}{dt} + \frac{df}{dy} \cdot \frac{dy}{dt}$		
t's t's This replicates the process of plugging in the parameterizations f(x	((s,t), y(s,t)) and taking the derivative	

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# 3 \$\frac{2}{3} \frac{2}{3} \frac{1}{3} \	(a) Ju at	u=1 and	v=-1				24	2					2	Iax												
**3 (zu) cos(my) - sin(my) - it - vu²) 3x	$\omega(u,v)=3$	x(u,v)·cos(π	4(u,v))				_	_													_			2 v		
= le (1) cos(**(1)) - sin(**(1)) - **(-1)*(1)*(3) = 2) = le (2) cos(**(1)) + le **(sin(**(1)) - 3**(i) x sin(**(1)) - 3**(i) x sin($\frac{3m}{3m} = 3 \frac{3m}{3x}$	COS(TY) + (sinl Ty)). II 34 .	3×		24 =	י של לי	<u>(</u>) = -	1.1	u-2		<u> </u>	3 <u>ax</u>	cast	(yn	+ (-:	Sivia	1))	δΛ ·	π •	3×				<u> </u>
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Consider a function $f(x,y)$ such that its first partial derivatives exist for all points near (a,b) . If (x,y) is a point on the segment found above, find a formula for the rate of change of f with respect to f . $ \frac{2f}{2k} = \frac{2f}{2k} \cdot \frac{2y}{2k} + \frac{2f}{2k} \cdot \frac{2y}{2k} = \frac{2f}{2k} \cdot \frac{2y}{2k} = \frac{2f}{2k} \cdot \frac{2y}{2k} = \frac{2f}{2k} \cdot \frac{2y}{2k} \cdot \frac{2y}{2k} = \frac{2f}{2k} \cdot \frac{2y}{2k} $	(a,b)		(x	w, yw	= latk,	bth)		u	s/ d	t=K	,	ot =	h	_							_				_	
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or a small change in time Δt , let the corresponding change in x be from a be Δx , the corresponding change in y from be Δx and Δf be the corresponding change in f from f(a,b). Then we have $\Delta f \approx \frac{d}{dt} _{s=0}$. We want to show that $f \approx \frac{2f}{2}(a,b) \cdot \Delta x + \frac{2f}{2}(a,b) \cdot \Delta y$ where $\Delta f = f(a t \Delta x, b t \Delta y) - f(a,b)$. This Δf is called the Linear Approximation of change of when (x,y) changes from (a,b) to $(at \Delta x,bt \Delta y)$. $\Delta t = \Delta x \approx \frac{d}{dt} \qquad \Delta f = \frac{f(t+h) - f(t)}{D} \qquad f(t+h) - f(t) \approx \frac{2f}{2}(a,b) \Delta x + \frac{2f}{2}(a,b) \Delta y$ $\Delta x \approx \frac{d}{dt} \qquad \Delta x \approx \frac{d}{dt} \qquad \Delta x = \frac{d}{dt} \qquad \Delta x$					ta	*	25/	T																		
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If $\approx \frac{3x}{2}(a_1b) \cdot \Delta x + \frac{3x}{2}(a_1b) \cdot \Delta y$ where $\Delta f = f(a_1 \Delta x, b_1 \Delta y) \cdot f(a_1b)$. This Δf is called the Linear Approximation of change of when (x,y) changes from (a_1b) to $(a_1 \Delta x, b_1 \Delta y)$. $\Delta f \longrightarrow \Delta x \simeq \frac{dA}{db} \qquad \Delta f = \frac{f(c+h) \cdot f(c)}{\Delta c} \qquad f(c+h) - f(c) \simeq \frac{2f}{2x}(a_1b) \Delta x + \frac{2f}{2y}(a_1b) \Delta y$ **Cample. A two-variable function $f(x,y)$ has selected values given by $y \times [2.5, 3.0, 3.5, -1.5] = [-1, 0, 0, 0.5, 9.0, -1.5, 0.5, 7.0, 9.5, -1.5] = [-1, 0, 0, 0.5, 9.0, -1.5, 0.5, 7.0, 9.5, -1.5] = [-1, 0, 0, 0.5, 9.0, -1.5, 0.5, 7.0, 9.5, -1.5] = [-1, 0, 0, 0.5, 9.0, -1.5, 0.5, 0.5, -1.5] = [-1, 0, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, $																										р
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$\begin{array}{c} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 $		N \	ا دادن	۲۰۰۰ ۲۰۰۰	- C/			1 1					\													
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