Math 10350 – Example Set 06B
Section 3.7 The Chain Rule
Section 3.9 Derivative of the Natural Log

1. Consider the functions $f(x) = e^x$ and $g(x) = \ln x$.

a. Sketch the graph of $f(x) = e^x$ and $g(x) = \ln x$ on the same set of axes. What could you say about their relationship? How are $f(x)$ and $g(x)$ related?

b. Using the fact that $\frac{d}{dx}(e^x) = e^x$ and the chain rule, find a formula for $\frac{d}{dx}(\ln x)$.

c. Using the change of base formula $\log_b x = \frac{\ln x}{\ln b}$, show that $\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$.

2. Find the equation of the tangent line to the graph of $y = \frac{\ln x - 1}{\ln x + 1}$ when $x = 1$.

3. Find the derivatives of the following functions:

a. $f(\theta) = \ln(\sin\theta + 2)$

b. $y = \ln\left(\frac{e^x - 1}{e^x + 1}\right)$

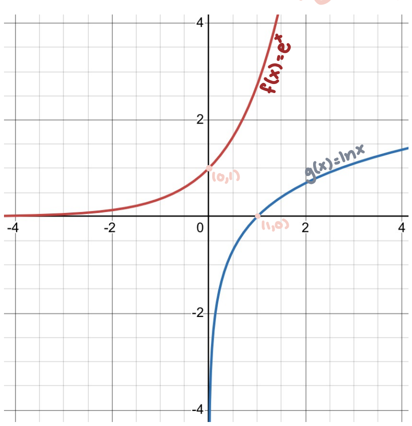
c. $g(z) = \ln(\ln z)$ for $z > 1$.

d. $y = e^{(\ln x)^3}$

e. $x^e + e^x$

e. x^x

1.(a) sketch $f(x)=e^x$, $g(x)=\ln x$



$f(g(x)) = e^{\ln(x)} = x$

$g(f(x)) = \ln(e^x) = x$

(b) Use $\frac{d}{dx}(e^x)=e^x$ & chain rule to find $\frac{d}{dx}(\ln x)$.

$\frac{d}{dx}(e^{\ln(x)}) = e^{\ln(x)} \cdot [\ln x]'$

$\|$

$\frac{d}{dx}(x) = 1$

thus $e^{\ln(x)} \cdot \frac{d}{dx}(\ln(x)) = 1$

$x \cdot \frac{d}{dx}(\ln(x)) = 1$

$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$

similairly $\frac{d}{dx}(a^x) = a^x \ln(a)$

find $\frac{d}{dx}(\log_a(x))$

$\frac{d}{dx}(a^{\log_a(x)}) = a^{\log_a(x)} \cdot \ln(a) \cdot [\log_a(x)]'$

$\|$

$\frac{d}{dx}(x) = 1$

thus $a^{\log_a(x)} \cdot \ln(a) \cdot \frac{d}{dx}[\log_a(x)] = 1$

$x \cdot \ln(a) \cdot \frac{d}{dx}[\log_a(x)] = 1$

$\frac{d}{dx}[\log_a(x)] = \frac{1}{x\ln(a)}$

(c) use $\log_b x = \frac{\ln x}{\ln b}$ to find $\frac{d}{dx}(\log_b x)$

$\frac{d}{dx}[\log_b x] = \frac{d}{dx}\left[\frac{\ln(x)}{\ln(b)}\right] = \frac{1}{\ln(b)} \cdot \frac{d}{dx}[\ln x] = \frac{1}{\ln(b)} \cdot \frac{1}{x} = \frac{1}{x\ln(b)}$

ln(b) is just a constant

2. Find the equation of the tangent line to the graph of $y = \dfrac{\ln x - 1}{\ln x + 1}$ when $x = 1$.

$y = \dfrac{\ln x - 1}{\ln x + 1}\begin{matrix} f \\ g \end{matrix}$ quotient rule: $\dfrac{f'g - g'f}{g^2}$

$y' = \dfrac{(\frac{1}{x})\cdot(\ln x+1) - (\frac{1}{x})(\ln x - 1)}{(\ln x+1)^2} = \dfrac{\frac{1}{x}[(\ln x+1) - (\ln x - 1)]}{(\ln x+1)^2} = \dfrac{\frac{1}{x}[\cancel{\ln x}+1 - \cancel{\ln x}+1]}{(\ln x+1)^2} = \dfrac{\frac{1}{x}\cdot(2)}{(\ln x+1)^2}$

$f'(1) = \dfrac{\frac{1}{1}\cdot 2}{(\ln 1+1)^2} = \dfrac{2}{(0+1)^2} = \dfrac{2}{1} = 2$

$f(1) = \dfrac{\ln(1)-1}{\ln(1)+1} = \dfrac{0-1}{0+1} = -1$

tangent line: $y - f(x_1) = f'(x_1)(x - x_1)$

$y - (-1) = (2)(x-1)$

$y + 1 = 2x - 2$

$y = 2x - 3$

3. Find the derivatives of the following functions:

a. $f(\theta) = \ln(\sin\theta + 2)$ **b.** $y = \ln\left(\dfrac{e^x - 1}{e^x + 1}\right)$ **c.** $g(z) = \ln(\ln z)$ for $z > 1$.

(a) $f(\theta) = \ln(\underbrace{\sin\theta + 2}_{u})$ $f'(u)\cdot u'$

$f'(\theta) = \dfrac{1}{\sin\theta + 2}\cdot\cos\theta$

(b) $y = \ln\left(\underbrace{\dfrac{e^x - 1}{e^x + 1}}_{u}\right)$ $h'(u)\cdot u'$ $u = \dfrac{e^x - 1}{e^x + 1}\begin{matrix} f \\ g \end{matrix}$

$y' = h'(u)\cdot\dfrac{f'g - g'f}{g^2}$

$y' = \dfrac{1}{\frac{e^x - 1}{e^x + 1}}\cdot\left(\dfrac{(e^x)(e^x+1) - (e^x)(e^x - 1)}{(e^x+1)^2}\right)$

$= \dfrac{1}{1}\cdot\dfrac{e^x+1}{e^x-1}\cdot\left(\dfrac{\cancel{e^{2x}}+e^x - \cancel{e^{2x}}+e^x}{(e^x+1)^2}\right)$

$= \dfrac{\cancel{(e^x+1)}(2e^x)}{(e^x+1)^{\cancel{2}}}$

$= \dfrac{2e^x}{(e^x+1)}$

(c) $g(z) = \ln(\underbrace{\ln(z)}_{u})$

$g'(z) = \dfrac{1}{\ln(z)}\cdot\dfrac{1}{z}$

$= \dfrac{1}{z\ln(z)}$