# ECON 3104 Exam 2 Solutions

## Wed. April 12, 2022

1. *(15pts.) Cost Function*: A firm faces a production function: $y = 5\sqrt{k}\sqrt{\ell}$. In the short run, capital is a fixed input, $\bar{k} = 49$. The prices of capital and labor are $p_k = 4$ and $p_\ell = 6$. Derive the firm's short run cost function to produce $y$ units of output: $c(y)$.

   - $c(y) = \frac{y^2}{35^2} \cdot 6 + 49 \cdot 4$

2. *(15pts.) The Edgeworth Box*: An economy consists of two people with utility functions: $u_1 = min(x, y)$ and $u_2 = \sqrt{x}\sqrt{y}$. Their initial endowments are $(x_1^i, y_1^i) = (18, 6)$ and $(x_2^i, y_2^i) = (6, 18)$. Draw the Edgeworth box for this economy, labeling the axes for each person, the initial endowment, and each person's indifference curve at this initial endowment. Shade in the area showing allocations that are Pareto improvements over the initial bundles. You do not need to solve for equilibrium, just sketch the Edgeworth box.
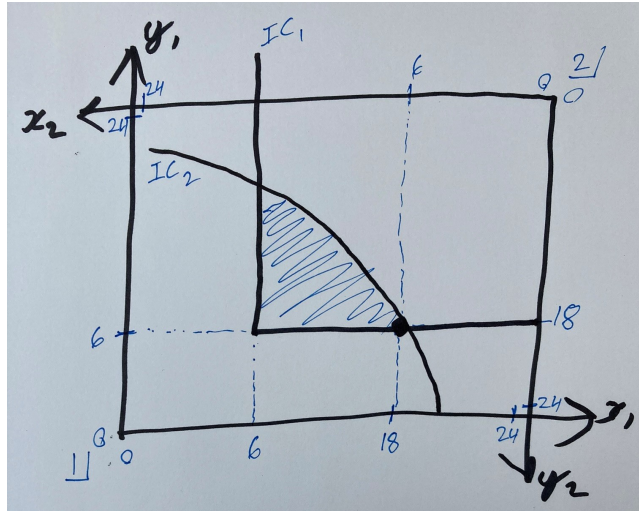


3. *(15pts.) Equilibrium*: An economy consists of two people, both with the utility functions $u_i = x^{\frac{1}{3}}y^{\frac{2}{3}}$. Their initial endowments are $(x_1^i, y_1^i) = (6, 18)$ and $(x_2^i, y_2^i) = (18, 6)$. Using x as the numeraire good, find the equilibrium price and the bundle each person consumes.

   - $p_y = 2$  $x_1 = 14$, $x_2 = 10$, $y_1 = 14$, $y_2 = 10$

4. *(10pts.) Monopoly*: A monopolist sells their output facing the market demand function: $Q = 100 - 2p$ and a cost function: $c(q) = \frac{1}{4}q^2 + 5q + 10$

   (a) Find the firm's profit maximizing output and price choice and calculate their profit at that point.

      - q= 30, p $= 35$, $\pi = 665$

5. *(25pts.) Conceptual questions*

   (a) *(5pts.)* Consider the production function: $y = ln(k + \ell)$. Does this production function exhibit increasing, decreasing, or constant returns to scale? Why?

   - We compare $t \cdot ln(\ell + k)$ to $ln(t \cdot \ell + t \cdot k)$, which simplifies to $ln(t) + ln(\ell + k)$. The returns to scale depend on $\ell$ and $k$, decreasing if they are greater than one and increasing if they are smaller.

   (b) *(10pts.)* A monopolists faces an inverse demand function: $p = 240 - 10q$ and a cost function $20q^2 + 470$.

   - Find their optimal quantity, price, and profit.
     - $q = 4, p = 200, \pi = 10$
   - Compare your results to the cost function. Even though the firm is making positive profits, it's unlikely that a competing firm will enter the market. Why?
     - The monopolist earns very slim proifts relative to their high fixed costs. If another firm enters, they likely earn negative profits

   (c) *(5pts.)* True or false: A firm can have diminishing marginal product for all of their inputs but still have constant returns to scale. Briefly explain.

   - True. Consider $y = \sqrt{\ell}\sqrt{k}$. The marginal prodict of each input is decreasing but the returns to scale are constant.

   (d) *(5pts.)* Capital is a fixed input for a firm, $k = \bar{k}$, in the short run. Their production function $y = f(k, \ell)$ exhibits decreasing returns to scale. If the price of capital, $p_k$ increases, how will they respond in the short run; will their production increase, decrease, or remain the same? Will their profits change?

   - Capital is fixed in the short run so the price change will not affect the amount they use or their output quantity. It will lower their profits though.

6. *Bonus*

    (a) *(5pts.)* A firm sells its output on a perfectly competitive market at a constant price, $p_y = \$20$. They face a production function: $y = 3\sqrt{k}\sqrt{\ell}$. With some mix of capital, K, and labor, $\ell$, they can produce one unit of output at a cost of \$10. What will the firm do to maximize their profit? If there isn't enough information to answer, what else would we need to know?

        • The production function exhibits constant returns to scale, so if they increase their inputs, their outputs increase proportionally. Since there is some mix of inputs where the revenue exceeds the cost, their profits would increase as they increase production. They should make as much as possible to maximize profits.

    (b) *(5pts.)* Using the monopolist firm and market from Question 4, suppose a government regulator mandates that the firm must set their price equal to their marginal cost to produce their last unit of output, $(p_y = \frac{\partial c(y)}{\partial y}.)$ What price and quantity would they choose to produce at? How much profit do they earn?

        • $q = 45, p = 27.5, \pi = 496.25$