Behavioral Finance

Probability Weighting

Duke University

Introduction

In the previous section, we talked about some fundamental errors that we tend to make when looking at statistical relationships. We will now discuss a model that predicts how we tent to distort event probabilities. In particular, we tend to *over-weight* low probabilities, and *under-weight* high ones. We will categorize the circumstances in which each of these biases is most prominent; as well as understanding how our brains interpret *changes* in probability, and how sensitive we are to some probability changes but not others.

We will also discuss the many ways in which probabilities are reported inaccurately in the media. These errors creep in partly because of ambiguities in the way we refer to *relative* probabilities versus *absolute* probabilities. The media is always looking for the most eye-catching way in which to report probabilities. Some of the most dramatic examples are taken from the arena of health, rather than wealth – we will examine a number of these and learn how to be a bit more realistic about risk when it is reported in the newspaper or on TV.

Subjective Probability

Imagine that the sadistic kidnapper that we saw earlier is back, and this time he is forcing you to play Russian roulette. However, you are allowed to purchase one bullet from the loaded gun (which has 6 chambers).

- How much would you be willing to pay to reduce the number of bullets in the gun from four to three?
- How much would you pay to reduce the number of bullets from one to zero?
- What about if you are told that the gun is currently fully loaded? How much would you be willing to pay to remove one bullet this time?

Please think about your answers to these questions before moving on

Subjective Probability & the Certainty Effect

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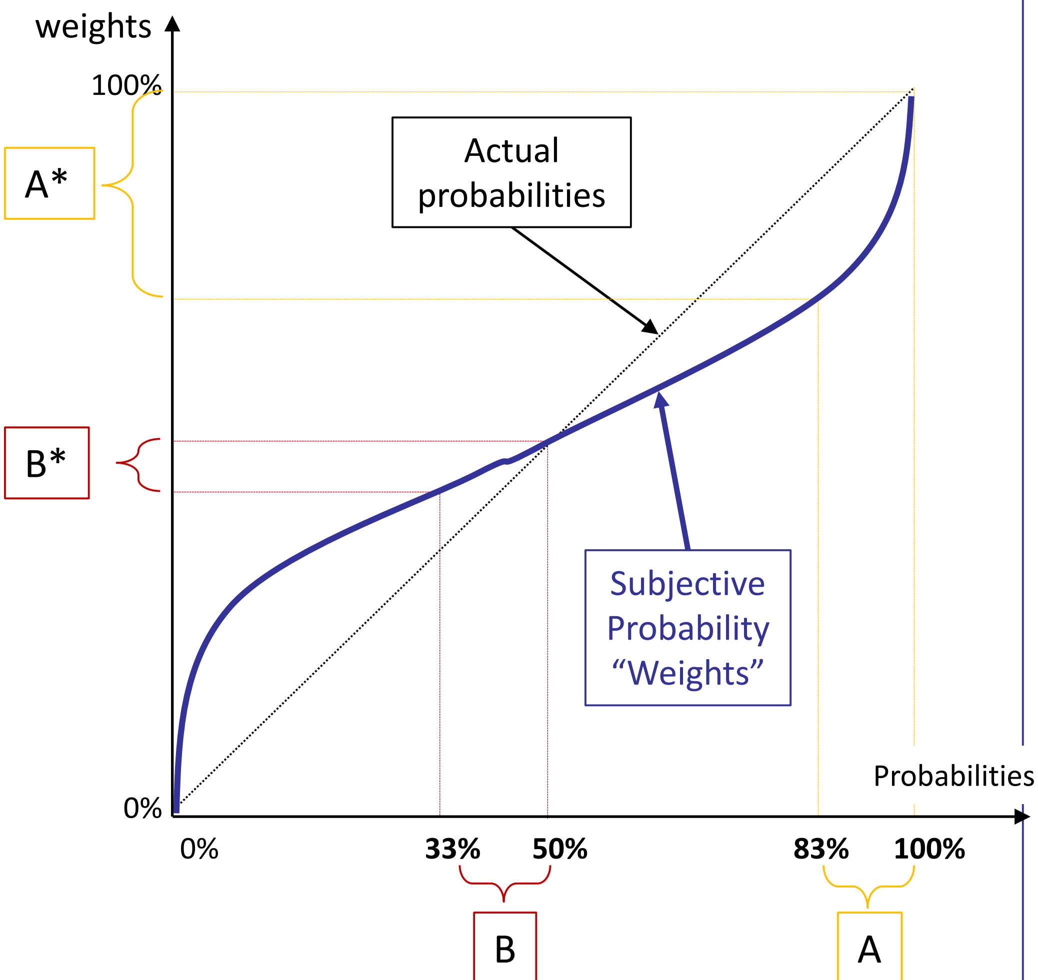
- What about if you are told that the gun is currently fully loaded? How much would you be willing to pay to remove one bullet this time?

When offered this thought experiment, respondents are typically willing to pay significantly more to reduce the number of bullets from one to zero – thus guaranteeing their survival – than to reduce their survival probability from two-thirds to one-half. Even though the objective probability reduction is the same 1/6th in both cases, the move from *probable* to *certain* survival is more emotionally appealing.

Similarly, individuals tend (not surprisingly) to indicate higher willingness to pay for the *possibility* of survival by removing one bullet from a fully loaded gun, than for the equivalent reduction in probability in the middle of the range.

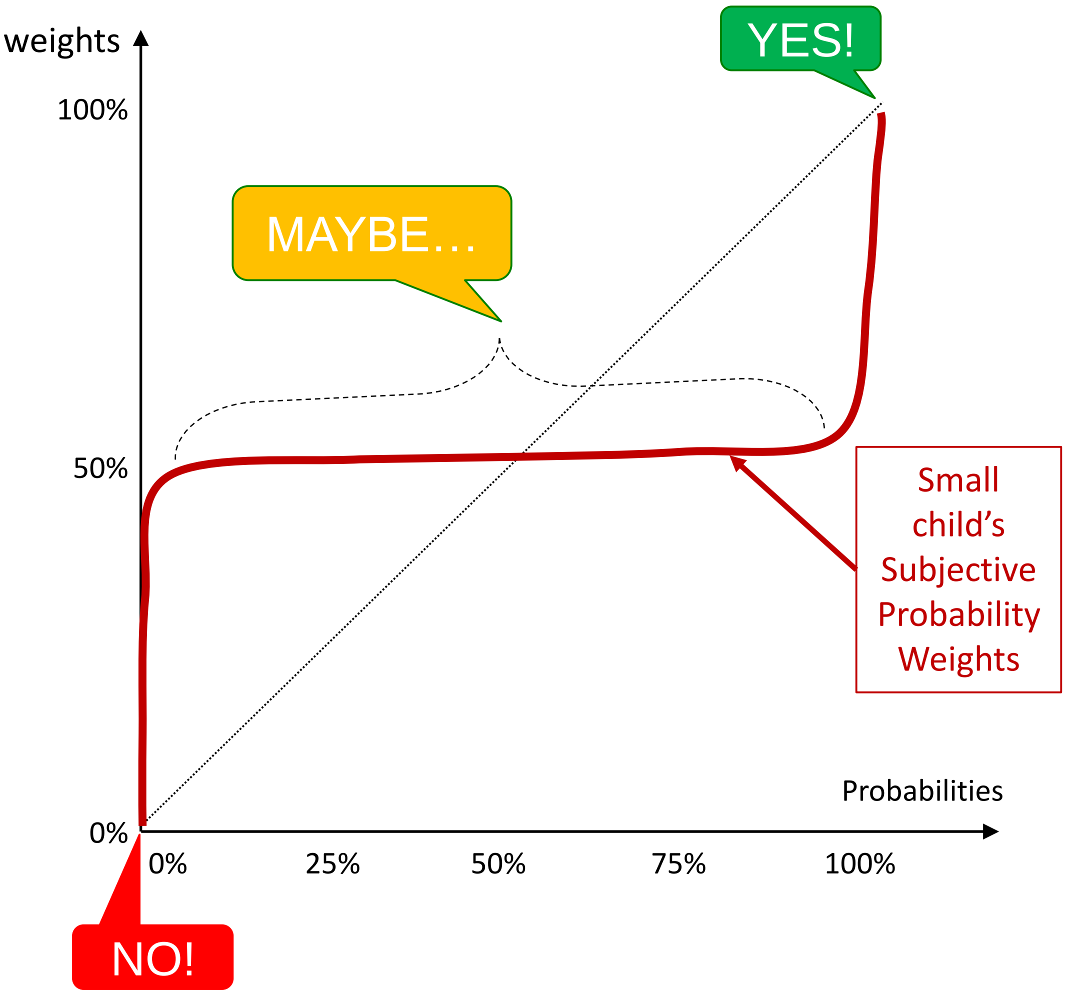
Probability Weighting Function



On this graph, “true” probabilities are on the horizontal axis, while our “subjective weight” interpretation is reflected in the dark blue curve, with values on the vertical axis.

- Look at the *actual* change in probability from 100% to 83% (marked A on the horizontal). This probability decrease of 1/6 (17%) takes us from *certain* death to merely *probable* with the removal of one bullet from the fully loaded gun. Our *subjective* response (A* on the vertical axis) to this change is noticeably larger than the objective probability change.
- Our emotional response to the change from certainty to probability is also significantly greater than our response to the change from 50% to 33% probability (distance B* on the vertical), even though it has the same 17% reduction B in true probability terms.

Subjective Probability & the Certainty Effect



Further examination of the graph helps us to see, more broadly, that we tend to be more sensitive to probability changes that take us from *certainty* to *probability*, than we are to probability changes in the middle of the range.

At the extreme, imagine the small child's weighting function: when asking for a treat (an icecream; an afternoon at the park) she understands the responses "yes" and "no", corresponding to 100% (certainly) and 0% (certainly not). All other probabilities are viewed generically as "maybe."

Probability Weighting: Glossary of characteristics

See the Lecture: Probability Weighting

- We tend to *overweight* low probability events, especially events that are especially "front of mind" or "salient" to us at a particular time (think fear of flying following 9/11)

- We tend to *underweight* high probability events, especially those that are sufficiently common that they tend not to be reported in the media (think automobile accidents)

- We tend to be less sensitive to changes in probability in the middle of the range (e.g., 30% to 40%) than changes that move us from probability to certainty (10% to 0%, or 90% to 100%): the *Certainty Effect*

Probability Weighting Examples

Catastrophic Potential: We tend to overestimate the risk of activities that may injure or kill a large number of people immediately and violently, rather than chronic, but less heavily reported risks

- Americans switched to driving rather than flying after 9/11, with significant uptick in # auto-related deaths per month in Oct – Dec 2001.
- We will return to this concept in the upcoming Section on the *Availability Heuristic*

Familiarity / Controllability: We are more willing to undertake risk when we believe that we are personally in control, or are unusually familiar with a particular situation

- Day-traders selecting individual stocks tend to overestimate the likelihood of making a killing
- Employees investing their pension fund savings in their company's shares are, in fact, insufficiently diversified
- We will see more of this in the Section on *Overconfidence*

Voluntariness: people are less anxious about risks that they voluntarily expose themselves to than those that they are required to engage in

- Smokers…

Probability Weighting – Insurance Example

Probabilistic Insurance

Suppose you have just purchased a house worth $200,000 in a region of NC in which the probability of the house's destruction by flooding is about 1 in 100 (that is, the property may be expected to be destroyed by floods about once every one hundred years).

How much would you be prepared to pay for a flood insurance policy against the value of your home?
$ __________

Suppose the insurance company is offering an alternative form of insurance policy, in which the dollar premium is reduced, but the insurance only applies to certain days of the week. How much would you pay for a flood insurance policy that will pay out only if the flood hits on a Monday, Wednesday, or Friday?
$ __________

Please answer these questions for yourself before moving on.

Probability Weighting – Insurance Example

- People would rather *eliminate* risk than *reduce* it, even if the probability of a catastrophe is diminished by an equal amount in both cases

- From a purely statistical perspective, if you are willing to pay $100 for annual flood insurance on *all* days, you should be willing to pay about $43 (= 3/7 x 100) for the "flood insurance only on MWF" option.

- Most people, however, will pay much less for the partial insurance, since it includes a level of uncertainty that people are not willing to accept. Another way of saying this: we are willing to pay a *premium* to move from uncertainty to certainty, relative to a simple change in probabilities.

- Be aware, however, that there is a *hidden* uncertainty when buying insurance…it doesn't really buy you a *100%* guarantee. *Why not?**

* We'll answer this question later

Relative Probabilities

See the Lecture: Relative Probabilities

As Hugh set out on his habitual morning walk across the park, he pondered the troubling information that he read in the morning's newspaper: that "over-the-counter painkillers such as ibuprofen can double the risk of a heart attack." The article noted that millions "..depend on such drugs to relieve the symptoms of arthritis, headaches, and other common ailments…[but] now sufferers face the dilemma of whether to continue taking some of the most commonly-used painkillers after they were found to carry similar risks to other drugs which have already been withdrawn."

- Suppose that Hugh is 70 years old, has rheumatoid arthritis, and depends on his daily dose of ibuprofen to manage the pain in his joints

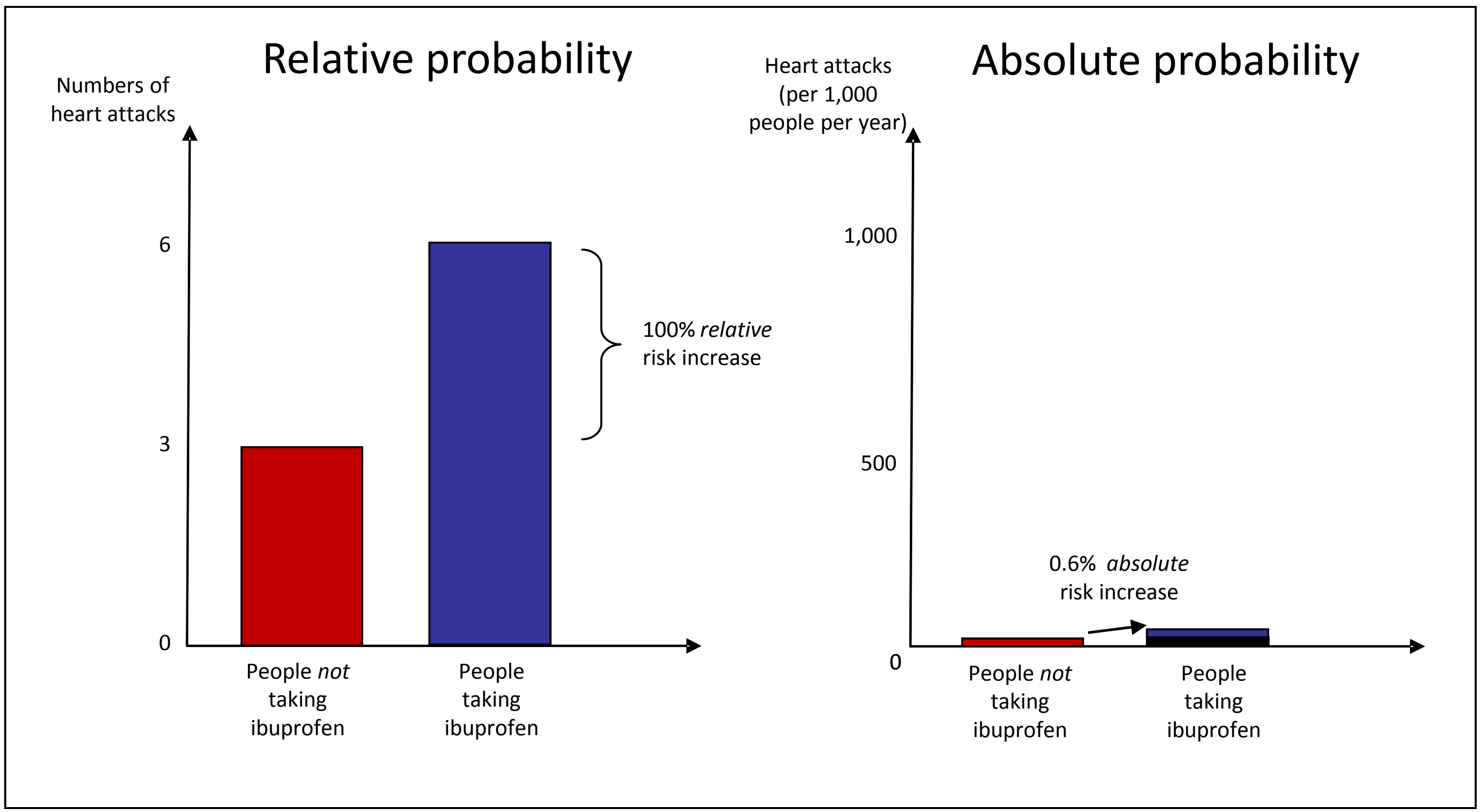
- Suppose that he also has a family history of heart disease, with both his father and older brother dying from heart attacks in their late 70s

- How concerned should he be about the article in his morning newspaper? Is there critical additional information that he should obtain before deciding whether to quit taking ibuprofen for joint pain?

Please answer these questions for yourself before moving on.

Relative vs Absolute Probabilities

- First, we need to know the base rate probability of heart attacks in the population: the chance that any average person (who is not taking daily ibuprofen) might have an attack in a given year
  - The base rate probability of a heart attack is around 0.3% (about 3 people in 1,000)
  - Thus "doubling the risk of a heart attack" translates to a 0.6% chance: 6 people out of 1,000 taking daily ibuprofen are expected to have a heart attack each year
  - When we look at the probability change in *absolute* terms, it becomes a lot less frightening



Relative vs Absolute Probabilities

- Why do newspapers report the *relative* risk change ("twice as many people"), rather than the absolute risk change ("an additional 3 per thousand people"), in stories of this type?
- With so many different risks that we face on a daily basis, why is Hugh suddenly so concerned about heart attacks, just because he read about them in the paper? He has known for many years that heart attacks run in his family…
- In Hugh's case, the base rate should be slightly different than 0.3%. Why?

Please answer these questions for yourself before moving on.

Relative vs Absolute Probabilities

Why do newspapers report the relative risk change (100%), rather than the absolute risk change (0.3%), in stories of this type?

Newspapers are in the business of selling dramatic stories. “Risk doubled” will attract more eyes and sell more papers than “risk increased from 3 in 1,000 to 6 in 1,000”

With so many different risks that we face on a daily basis, why is Hugh suddenly so concerned about heart attacks, just because he read about them in the paper? He has known for many years that heart attacks run in his family…

While we cannot spend all day thinking of every possible risk that we face, we are susceptible to risks that are suddenly made especially *salient* to us – for example because we read about them in the paper and there is an element of the story that we feel applies to us personally

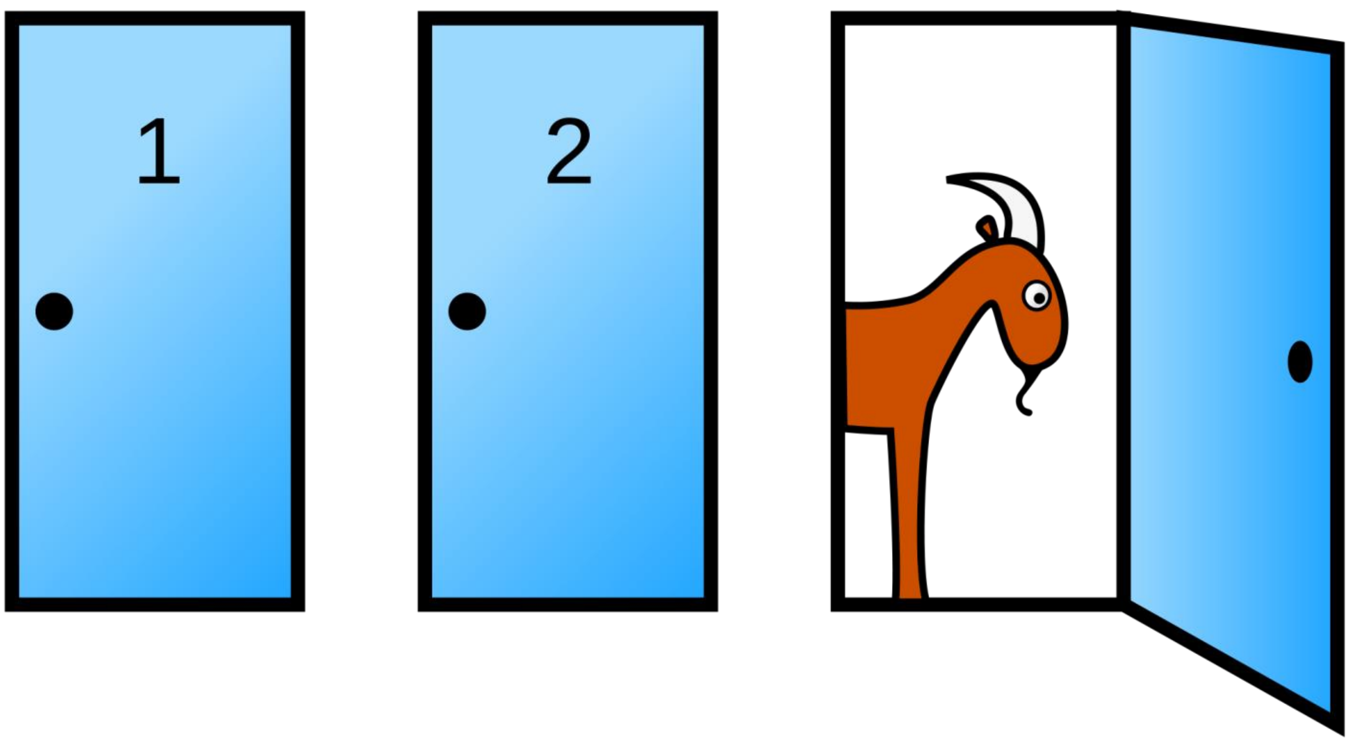
In Hugh’s case, the “base rate” should be slightly different than 0.3%. Why?

Hugh’s family is predisposed to heart attacks. His base rate should therefore be the probability of getting a heart attack in a given year based on his family history, which may be higher than the risk within the overall population*

*Hugh’s age, relative to the overall population, is also a factor

The Monte Hall Problem (or "Where is the goat?")

You are on a game show, and you are given the choice of three doors. Behind one door is a car; behind the others, goats. You pick a door, say #1, and the host (who knows what is behind each door) opens another door, say #3, to show you a goat. He then says to you: "Do you want to pick door #2?" Should you switch?



The Monte Hall Problem Discussion

There are lots of ways to explain this one, and many hours and column inches have been wasted on arguments over the correct answer. The answer is **YES, you should switch.** Here's why:

- In your **initial selection**, suppose you picked a door with a **goat** behind it. The **probability of this scenario is 67%**, since 2 out of 3 doors have goats behind them, and your door selection is completely random.
- The host then opens the other door that has a goat behind it
- If you now **switch** to the remaining door, you will get the **car**.
- The **probability of this scenario is 67%**, since two out of the three doors have goats behind them

- In your **initial selection**, suppose you picked the door with the **car** behind it. The **probability of this scenario is 33%**, since only 1 of the 3 doors has a car behind it, and your door selection is completely random
- The host then opens one of the other two doors, to show a goat.
- If you now **switch** to the remaining door, you will get the other **goat**.
- The **probability of this scenario is 33%**, since only one of the three doors had a car behind it

In summary: if your initial door choice was a goat, you should switch; but if your initial door choice was the car, you should not.
Your initial door choice was random; therefore, 2/3 of the time you would have picked a goat. So your probability of winning the car is 2/3 if you switch, and only 1/3 if you don't. Assuming you want the car, you should switch.

Section Summary

In this section, we have discussed:

- Why we are more sensitive to changes in probability that result in *certain* outcomes, than changes in the middle of the probability range, and how this affects our decisions
- Why we are more likely to take risks in scenarios that involve our areas of interest or expertise – and more likely to *underweight* those risks under such circumstances
- Why the news media is more inclined to report *relative* vs *absolute* probabilities, and why this significantly distorts our interpretation of the importance of the media's message
- How very subjective we are about probabilities, and how this affects our decision-making in many important areas such as health, and wealth.

A Final Thought

Any probabilistic intuition by anyone not specifically trained in probabilistic thinking has a greater than 50% chance of being wrong…

* Here is the answer to my earlier question about flood insurance. What is the *hidden uncertainty* that still remains when we buy *any* type of insurance? Well, our insurance is only as good as the insurance company that sold it to us. If that insurance company fails, valid claims may not be paid in full. (In the United States, the regulations on payment of valid claims from failed insurance companies vary on a state-by-state basis.)