Behavioral Finance

Utility of Money

Duke UNIVERSITY

Introduction

- Why are we inclined to **sell** the shares in our portfolio that are performing well, and **hold onto** those that are performing poorly?
- Why should you **always** buy auto insurance and **never** buy electronics insurance?
- Why do we **over-estimate** the probability of plane crashes and **under-estimate** the probability of car crashes?
- Why is it significant that the recent credit crisis, the worst economic recession that the US has seen since the 1930's, took place 75 years after the Great Depression?

Behavioral finance is a relatively new school of thought that addresses and provides insight into questions like these. All of us have innate psychological biases that can lead to predictable "errors" in how we make important financial decisions. Behavioral finance catalogues these errors and helps us to anticipate, and hopefully avoid, these decision-making "traps."

As you work through this course, be prepared to make mistakes…

Rational Expected Utility

In this section, we look at the “traditional” economic model of consumer choice, which seeks to describe how people are expected to behave when making financial decisions.

This traditional model assumes that individuals are rational, and that they make decisions in ways that maximize their expected “utility” (or “happiness”) from money made or lost.

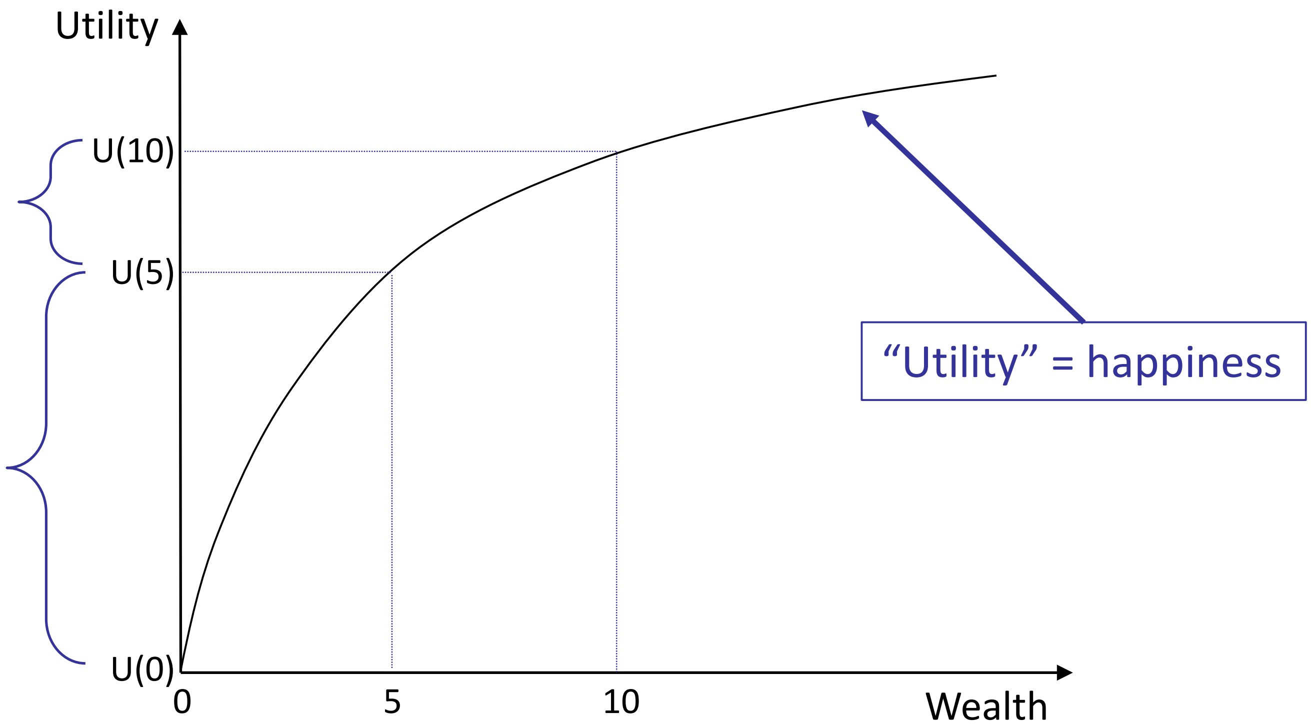
We will specify various axioms (assumptions about behavior that traditional economists view as self-evident), and show under what circumstances we frequently and reliably violate those very assumptions.

When you have completed this section, you will understand:

- What economists mean when they talk about risk aversion
- Budget constraints, indifference curves, and how we utilize both to optimize our “consumption”
- Some rational utility axioms, and under what circumstances these axioms fail to describe the way most people behave.

Utility of Money

See the Lecture: Utility of Money



Implications of the graph:

- More money is undoubtedly better than less: U(10) > U(5), *BUT*
- The incremental (marginal) value of an additional dollar gets smaller as our wealth increases: U(5) – U(0) > U(10) – U(5)

The Utility of a Gamble: Risk Aversion

- Ask yourself what is the most you would pay to play the following game:

  Toss a fair coin.
  If it lands heads, you get $15
  If it lands tails, you get $5

- The fair (or "expected") value of the game is ½ x 15 + ½ x 5 = $10

- Would *you* pay $10 to play? If not, how much *would* you pay?

- What is your "utility" for the game?

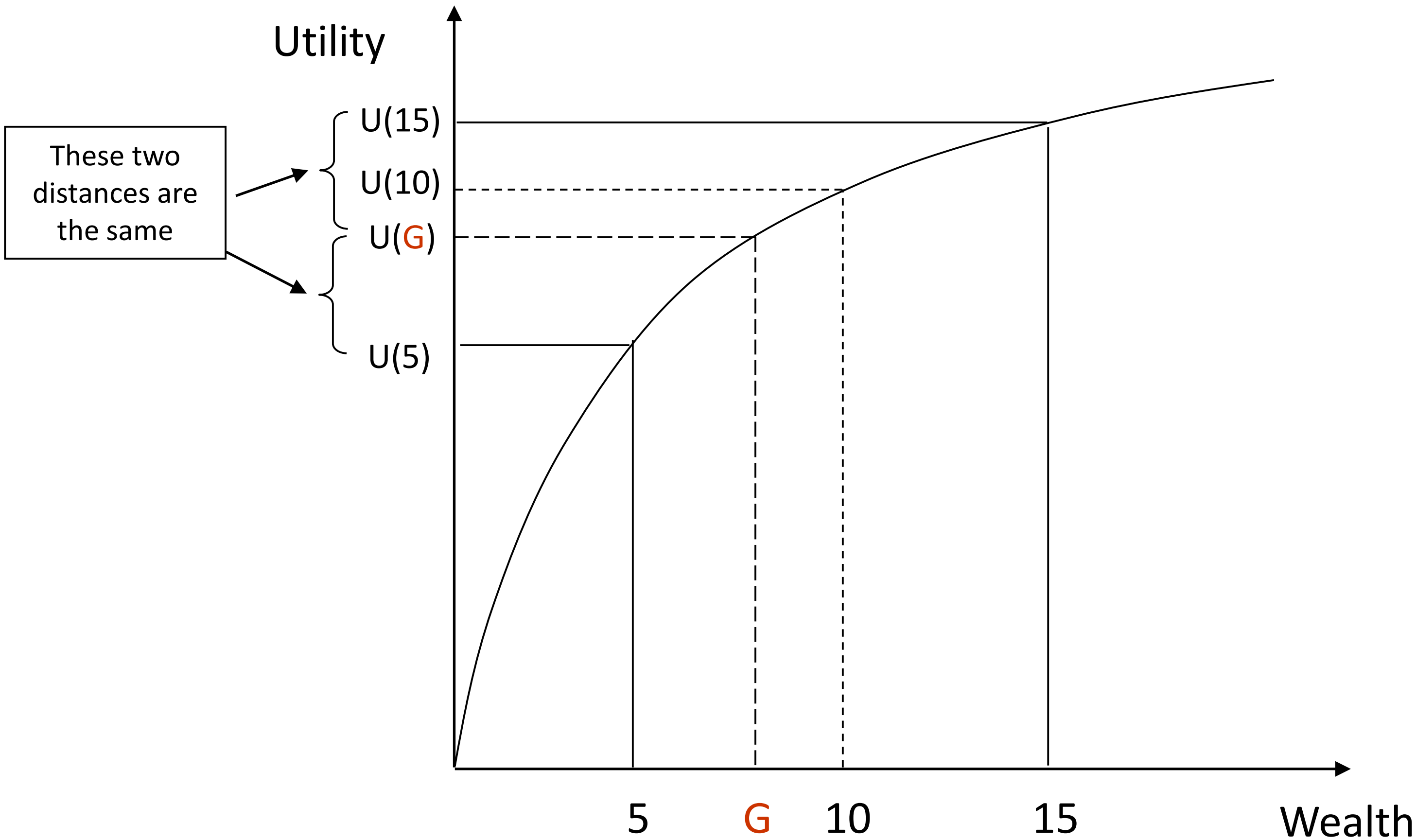
  U(G) = ½ U(15) + ½ U(5)   *where* U(G) = the "utility" of the Game

- How much will you pay to play the game?

  You will pay $G, where:
  U(G) = ½ U(15) + ½ U(5)

*To calculate expected value, multiply each possible outcome with the probability of that outcome, and add all of these probability-weighted outcomes together

Risk Aversion



$$U(G) = \tfrac{1}{2}\,U(5) + \tfrac{1}{2}\,U(15) \quad < \quad U(\tfrac{1}{2} \times 5 + \tfrac{1}{2} \times 15) = U(10)$$

The probability-weighted sum of the utilities for each outcome…

…Is less than…

…the utility of the expected value of the outcomes

The St Petersburg Paradox

A Demonstration of risk aversion

Imagine you are going to play a game with two rules:

(1) An unbiased coin is tossed until it lands on Tails

(2) The player is paid $2 if Tails comes up on the opening toss, $4 if Tails first appears on the second toss, $8 if Tails first appears on the third toss, etc.

Q1: How much would you be willing to play this game?____

Q2: What is the *expected value* of this game (in dollars)?* _____

St. Petersburg Simulation
Harvard professor Dr. Oliver Knill created a simulation available at Mathematik.com. Try the St. Petersburg Paradox for yourself...Play the Simulation! Return here when done.

*To calculate expected value, multiply each possible outcome with the probability of that outcome, and add all of these probability-weighted outcomes together

Make a note of your answers to Q1 and Q2

The St Petersburg Paradox

What's the Expected Value?

There's 50% chance the coin lands on Tails on the first toss. In this case, you earn $2 and the game is over.

There's also the possibility that the coin lands on Heads first. In this case, you'll toss again – if it now lands on Tails, you'll make $4 and the game is over. The probability of Heads, then Tails, is 25%

What about if you get Heads on both the first and second tosses, and then Tails on the third? The probability of *this* outcome is 12.5%, and you'll earn $8. And so on…

Expected Value = (50% * $2) + (25% * $4) + (12.5% * $8) + …

= $1 + $1 + $1 + …

= an infinite amount of money!

So: would you pay a very large amount to play this game?

The St Petersburg Paradox
How much would you be willing to pay?

The amount we are willing to pay can be measured by the Game's "Utility", which, as we've seen, is NOT the same as its Expected Value.

If the coin lands on Tails first (50% probability), you earn $2. Your "Utility" is U[2], with probability 50%.

If the coin lands first Heads then Tails (with probability 25%), you earn $4. Your "utility" is U[4]. As shown on the graph we looked at earlier, your utility for $4 is greater than your utility for $2, but it's not quite twice as great. That is: U[4] < 2 * U[2].

Similarly for each successive outcome. The probability of each successive outcome falls by half each time, but the Utility of each outcome grows by *less than* half. So, unlike the Expected Value, which is infinite, our Utility is significantly less than infinite.

Utility = 50% * U[2] + 25% * U[4] + …

= much less than an infinite amount of $$!

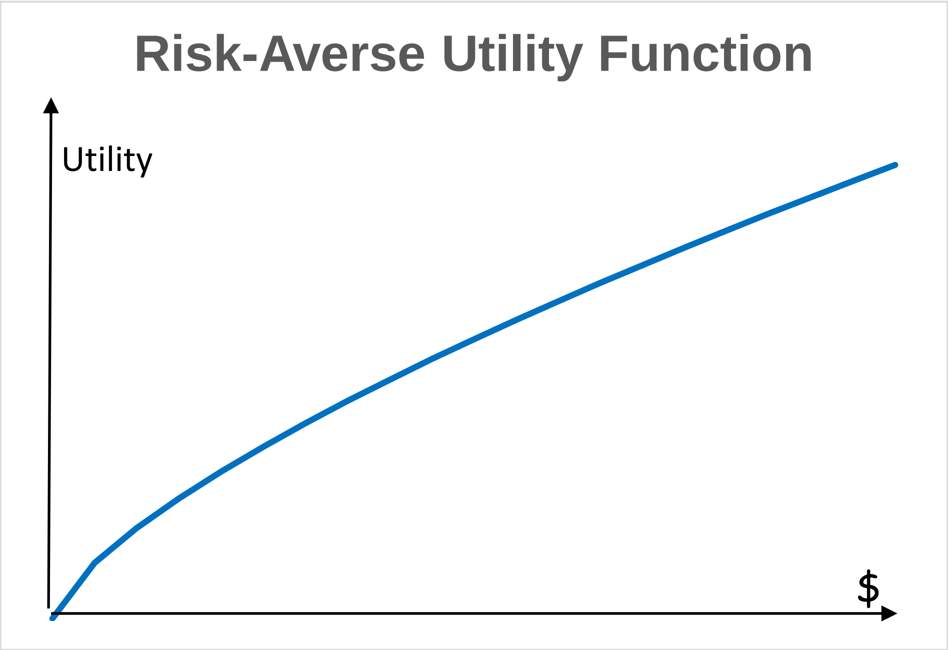
The St Petersburg Paradox

In summary, we see that while the Expected Value of this game is infinite, the Utility of the game – as measured by how much we are willing to pay to play it – is considerably less than infinite.

Most people would only pay a few dollars to play.

Utility = 50% * U[2] + 25% * U[4] + …

= much less than an infinite amount of $$!



This graph shows one example of a risk-averse utility function:

$U[X] = X^{0.7}$

Where X = dollar amount
U[X] = our “utility” (or “happiness”) from those dollars

An individual with this utility function would be willing to pay about $4.35 to play the St Petersburg Paradox game.

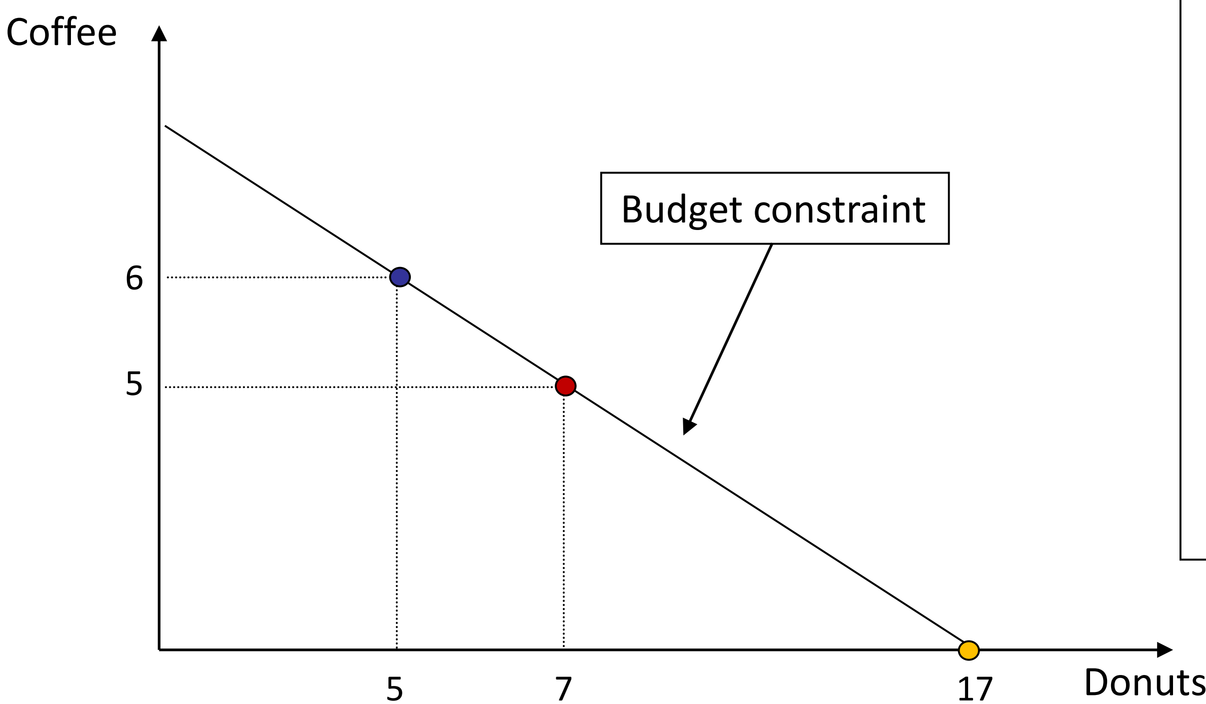
Are we “Rational” Decision-Makers?

Much of behavioral finance involves identifying and categorizing common behavioral patterns that defy the so-called “rational” model of consumer choice.

In the next few slides, we look at the classical economic theory of rational behavior, including three “axioms” of rational consumer choice. We then show some examples of ways in which our day-to-day choices may be considered “irrational,” because they violate the axioms.

Budget Constraint

For simplicity, we will begin with a world in which a consumer must select between just two goods, coffee & donuts, subject to a *budget constraint* that determines the maximum she can afford.



The numbers on the axes reflect units of each good, while the budget constraint line represents all affordable combinations, given consumer’s budget and the costs of each item.

In this example, donuts are $1, while coffee is $2

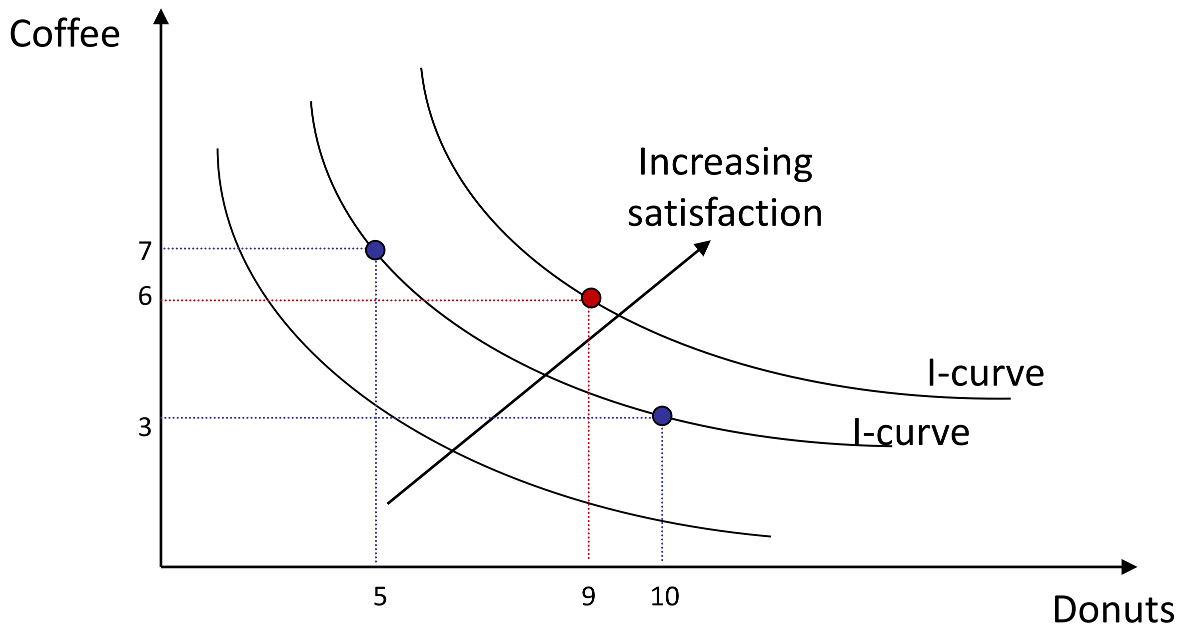
Your budget constraint is $17

- One possible combination is 7 donuts, and 5 cups of coffee (see)
- Another is 5 donuts, and 6 cups of coffee (see)
- An increase in consumption of one good leads to a reduction in consumption of the other, because of the budget constraint
- Does it seem likely that each of the *available* combinations is equally *attractive*? Would the consumer be as happy with 17 donuts but no coffee (see), versus the other two options?

Indifference Curves

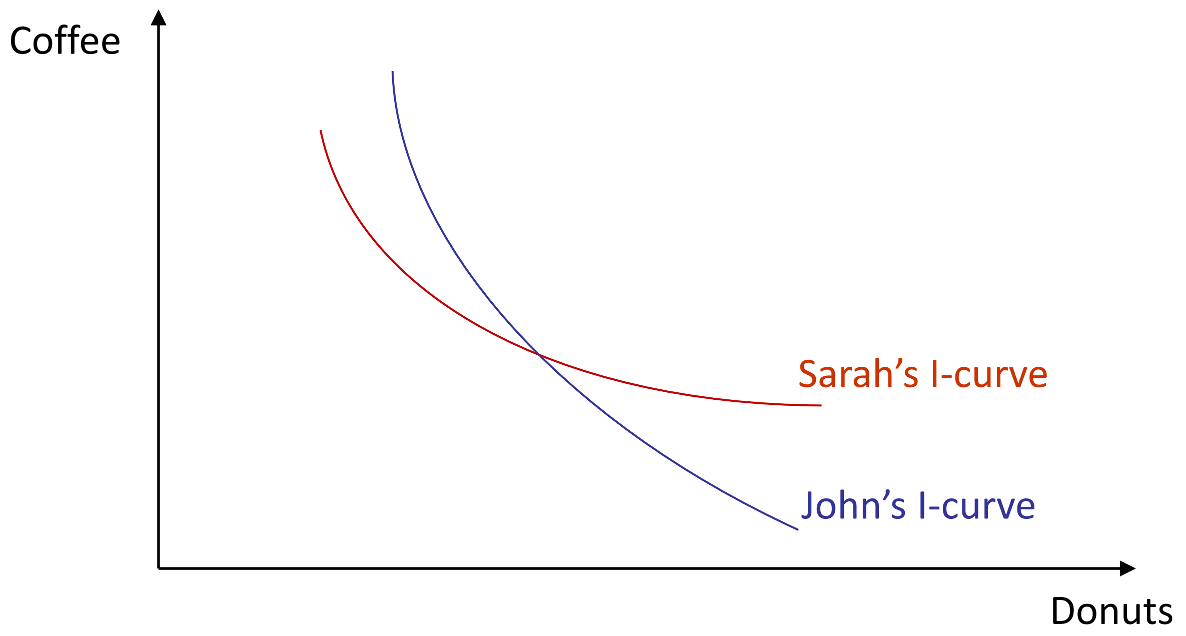
We also assume that each consumer has a set of *indifference curves* that reflect her *preferences* for different combinations of the two goods.

Unlike budget constraints, indifference curves are not typically linear



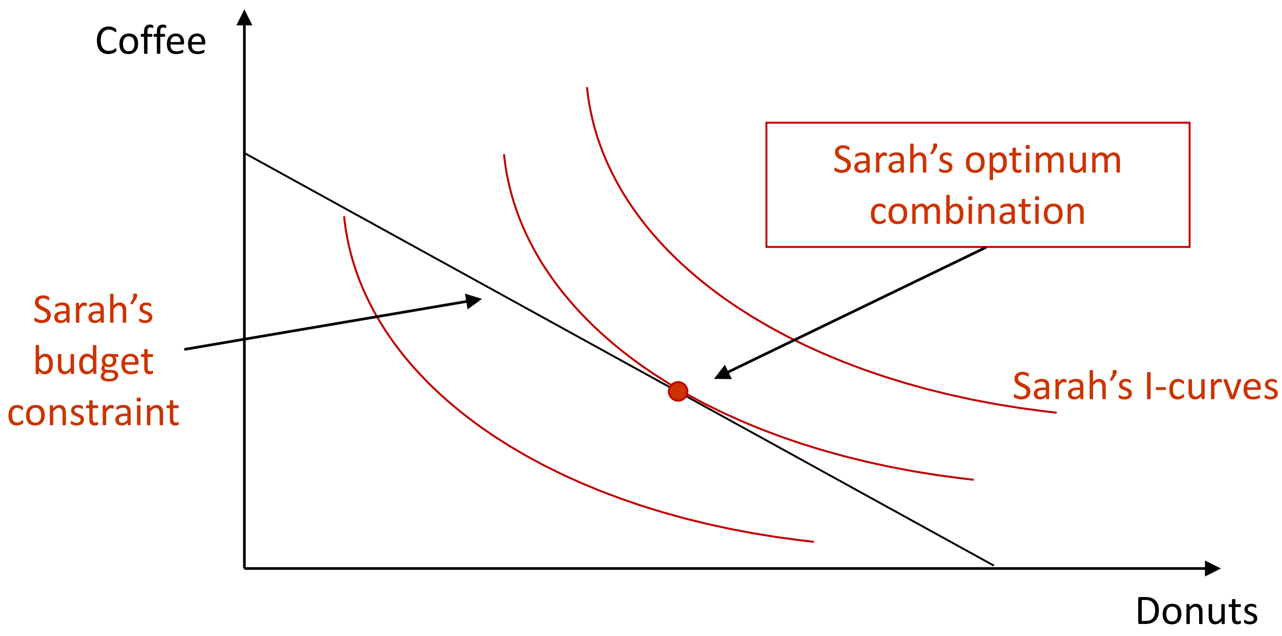
- Each indifference curve reflects a combination of options between which the consumer is indifferent
- She would be equally happy with 7 cups of coffee, and 5 donuts, versus 3 cups of coffee, and 10 donuts
- She would experience greater *overall* happiness if she were able to jump to a higher indifference curve; e.g., one in which she could have 6 coffees, and 9 donuts

Indifference Curves Across Consumers



- Within this theory of consumer choice, it is feasible for different individuals to have different indifference curves:
  - In the above example, John would need a substantial increase in the quantity of his coffee to offset the loss of just one donut (the slope of his indifference curve is steep)
  - Sarah, however, would willingly relinquish several donuts in order to secure an extra coffee

Using Indifference Curves to Optimize Consumption



- Given the shape of Sarah's indifference curves, and her budget constraint, we can identify the optimum combination of coffee and donuts, as indicated.

- With these relatively straightforward concepts in place, we can now identify some *axioms* implied by this model of consumers' indifference curves and budget constraints

Axiom 1: Dominance, or "More Is Better"

In this axiom, we are simply stating that, all else equal, more of a particular good is better than less.

In reality, we need some assumptions about the ability to store, or sell, excess amounts of a good over a given time period. Even John does not want 25 cups of coffee on a single day. If he cannot either profit by selling them, or put them aside to be used tomorrow, then at a certain point, he has a limit to how many he would like to consume.

However, the More is Better axiom fits with the graph of the indifference curves, which clearly represent higher levels of satisfaction given increasing quantities of each good.

Violations of this axiom are common, and many relate to a set of biases known collectively as the "Disposition Effect."

Axiom 2:
Invariance, or “Consistency”

A decision-maker should not be affected by the way alternatives are presented. That is, her preference between option A and option B should not change based on the language used to describe the two.

This axiom sounds universally applicable, but we will see that it is one of the most violated axioms. We are in fact *strongly* influenced by how options are presented to us. It has been shown repeatedly, for example, that meat labeled as “75% lean” will sell better than the same product marked “25% fat.”

We will see numerous examples of violation of this axiom throughout this course. These examples fit within the set of biases generally referred to as “Framing” issues or “Preference Reversals.”*

*In the formal theory of Expected Utility, Preference Reversals are usually considered to defy a slightly different Axiom known as Transitivity. However, distinguishing the two (i.e., separating Framing Issues from Preference Reversals) adds some complexity with minimal additional insight, so for our purposes we have combined the two axioms under the single heading.

Axiom 3:
Independence, or “Cancellation”

This axiom states that choices should be independent of proportional reductions. That is, introducing a third option that has no bearing on the choice between the first two should not affect your earlier choice.

As a more intuitive (if rather flippant) example, consider the following:

> A man comes into a delicatessen and asks what kinds of sandwiches they have. The attendant answers that they have roast beef and chicken. The patron thinks for a minute and then selects the roast beef. The server then says, “Oh, I forgot to mention, we also have tuna.” The patron responds, “Well, in that case I guess I’ll have chicken.”

Unlike Dominance and Invariance, Independence seems somewhat nuanced, although this axiom is as much a requirement of the classical economic theory of rational decision-making as the other two. Violations of this axiom are often related to a bias known as the “Certainty Effect.”

Examples of Irrational Behavior

It is important to keep in mind that rationality is an assumption in economics, not a demonstrated fact*

The following slides invite you to examine your own decision-making, to see whether your choices are consistent with the axioms of rational behavior, or whether you exhibit irrationality in some circumstances.

As you work through these three examples, think about which axiom is violated in each case.

* Professor Richard Thaler, University of Chicago

Example 1:

Two Decisions with Gains and Losses

Here are two decisions, each of which has two alternatives. Please decide which alternative you prefer among the two in Decision 1, and which you prefer in Decision 2. Make a note of your selections before moving on.

Which do you prefer:
A or B?

Decision 1

Alternative A:	A sure gain of $240
Alternative B:	A 25% chance to gain $1,000, and a 75% chance to gain nothing

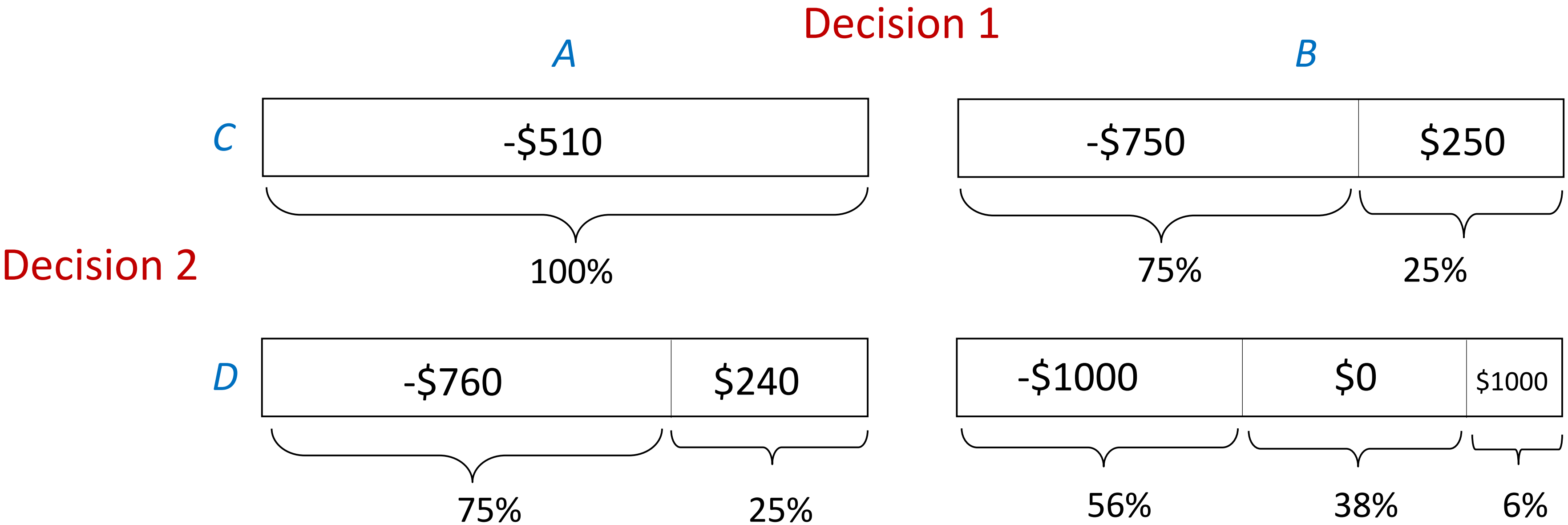
Which do you prefer:
C or D?

Decision 2

Alternative C:	A sure loss of $750
Alternative D:	A 75% chance to lose $1,000, and a 25% chance to lose nothing

Example 1:

Two Decisions with Gains and Losses



Each of the 4 rectangles above shows the range of outcomes and outcome probabilities from the combination of the two decisions, based on the two possible alternatives in each case.

For example, if you picked A & C (the top left outcome), you will lose exactly $510

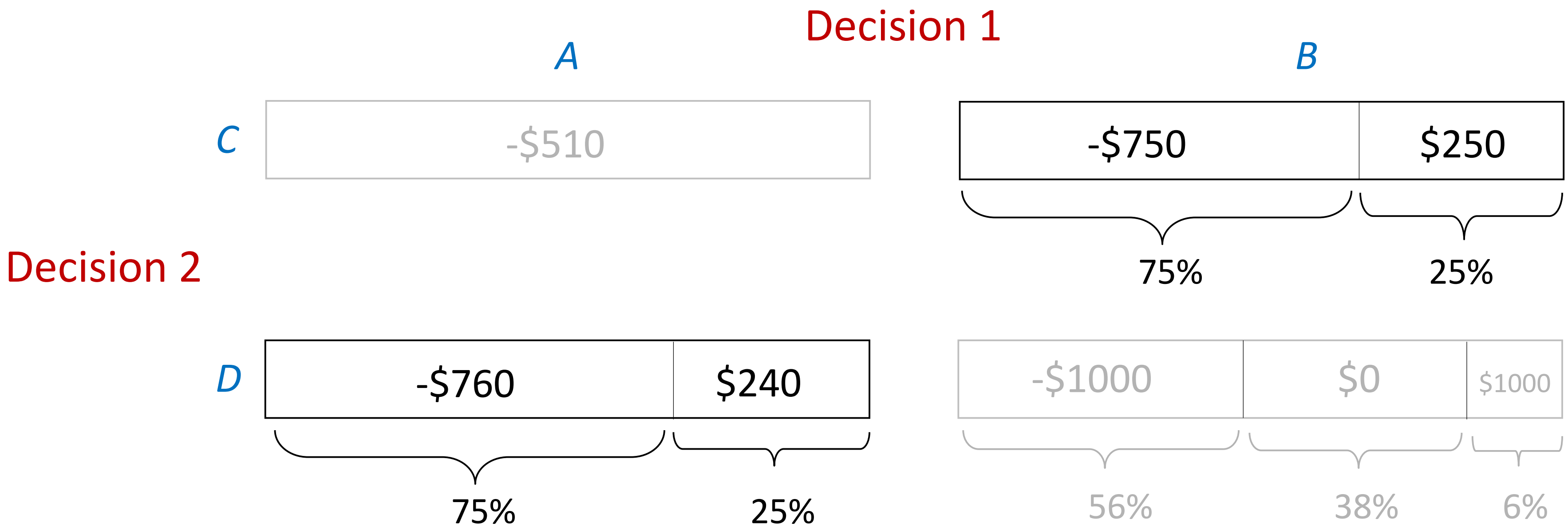
If you picked B & D (bottom right), you will:

- *lose* $1,000 with 56% probability
- neither win nor lose with 38% probability
- *win* $1,000 with 6% probability

But these are not the *interesting* outcomes…keep reading

Example 1:

Two Decisions with Gains and Losses



Did you select A & D (bottom left)? If so, you are like about 50% of people who play this game. You will:

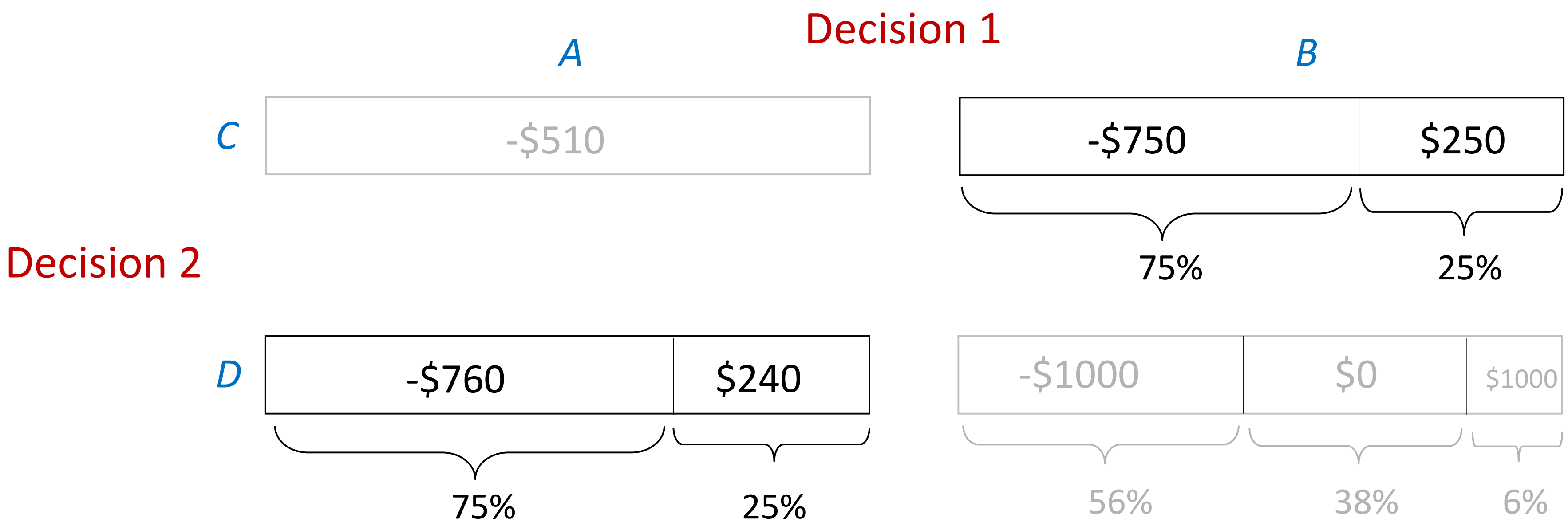
- *Lose* $760 with probability 75%
- *Win* $240 with probability 25%

Now compare A & D (bottom left) with B & C (top right). They look very similar, except that B & C is *better* in both cases! For B & C, you have:

- The same 75% probability of *losing* money, but you *lose* a little *less* (only $750, instead of $760).
- The same 25% probability of *making* money, but you *make* a little *more* ($250 instead of just $240).

Example 1:

Two Decisions with Gains and Losses



Economists would say that B & C *dominates* A & D, because with the former combination, you do a little bit better in either probability scenario. So why do around 50% of respondents typically select the *dominated* A & D outcome?

There are two principal reasons:

(1) We are not good at looking at outcomes over *multiple* games (in this case, Decision 1 followed by Decision 2). We tend to treat each one as a stand-alone decision (we'll see later that this provokes "irrational" decisions about whether to buy product insurance)

(2) We are inclined to be risk–*seeking* in certain predictable scenarios. We will see that this has significant implications across a whole host of financial decisions.

Example 2:
Choosing and Pricing

You are offered the choice between these two Gambles. Make a note of your response before moving on.

Gamble A	*Gamble B*
11/36 chance of winning $80	35/36 chance of winning $20
25/36 chance of losing $7.50	1/36 chance of losing $5

Which Gamble would you PREFER to play: A or B?

Example 2:
Choosing and Pricing

You are now asked a slightly different question relating to these two gambles. Again, make a note of your response before moving on.

Gamble A	*Gamble B*
11/36 chance of winning \$80 25/36 chance of losing \$7.50	35/36 chance of winning \$20 1/36 chance of losing \$5

How much would you be willing to PAY to play Gamble A? How about Gamble B?

Example 2:
Choosing and Pricing

Gamble A	*Gamble B*
11/36 chance of winning $80 25/36 chance of losing $7.50	35/36 chance of winning $20 1/36 chance of losing $5

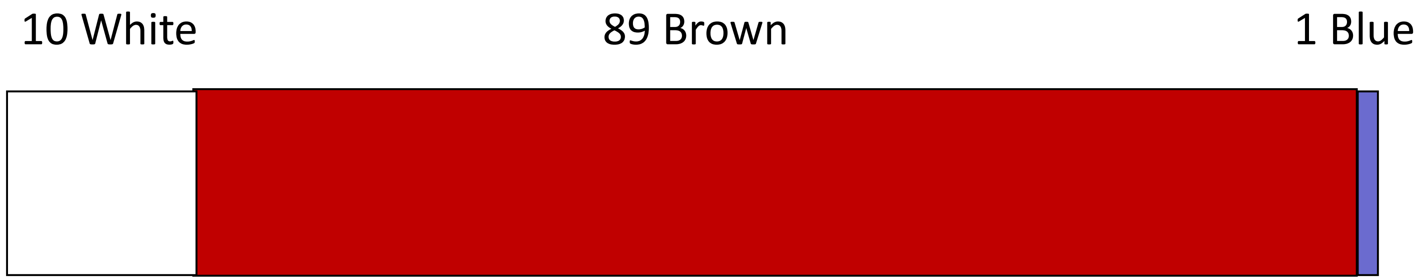
Which gamble would you prefer to play: Gamble A, or Gamble B?

How much would you be willing to pay to play Gamble A vs Gamble B?

On average, people would PREFER to play A… but would PAY MORE to play B

"Choosing" and "pricing" are different psychological processes. Can you think of ways to take advantage of this insight in a business context? Perhaps to "steer" a vote towards your preferred outcome?

Example 3:
The Allais Paradox



Imagine that you are offered a bag containing 100 balls of different colors.

10 of the balls are white 89 of the balls are brown 1 of the balls is blue

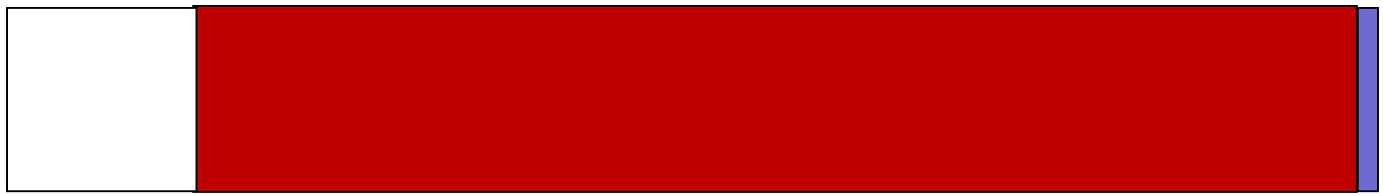
You will reach in and pull out a ball at random, and you will win money depending on the ball's color. Would you prefer to win money on the basis of Option A, or on Option B?

CHOICE 1	White	Brown	Blue
Option A:	$1 million	$1 million	$1 million
Option B:	$2.5 million	$1 million	$0

Example 3:

The Allais Paradox - continued

10 White 89 Brown 1 Blue

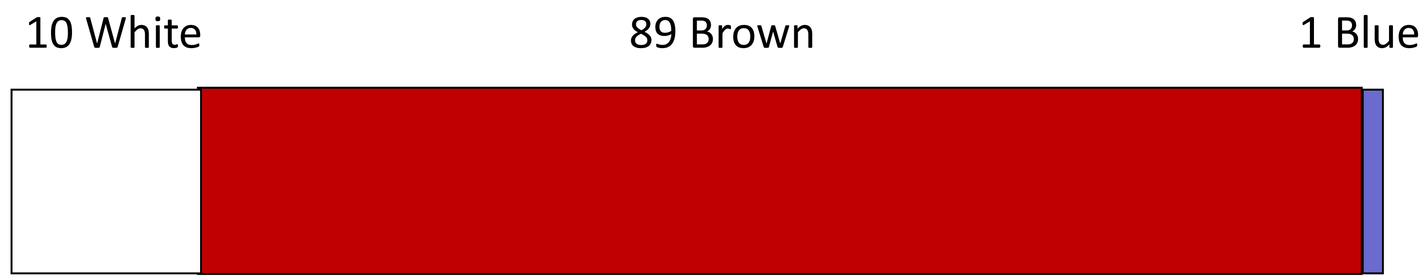


Now imagine that you are offered the same bag, with the same distribution of balls, but a slightly different set of Options. Which option will you pick this time, Option C or Option D?

CHOICE 2	White	Brown	Blue
Option C:	$1 million	$0	$1 million
Option D:	$2.5 million	$0	$0

Example 3:

The Allais Paradox



Most people pick Option A…

CHOICE 1	White	Brown	Blue
Option A:	$1 million	$1 million	$1 million
Option B:	$2.5 million	$1 million	$0

…and Option D

CHOICE 2	White	Brown	Blue
Option C:	$1 million	$0	$1 million
Option D:	$2.5 million	$0	$0

Example 3:

The Allais Paradox

Notice that the two pairs of Options are identical in all respects, except for the outcomes with a Brown ball. In the first pair of Choices (A or B), a brown ball gives you $1 million. In the second pair (C or D), it is worth nothing.

Why do identical outcomes subtracted from each of the two Choices change our view of the desirability of one Option versus the other (i.e., the switch from A in Choice 1 to D in Choice 2)?

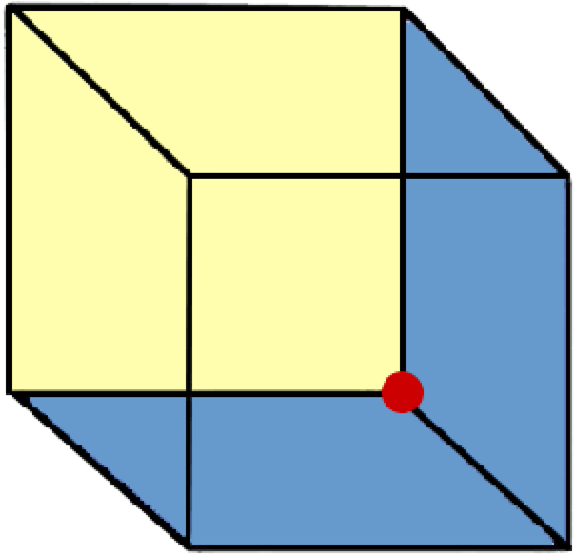
CHOICE 1	White	Brown	Blue
Option A:	$1 million	$1 million	$1 million
Option B:	$2.5 million	$1 million	$0

CHOICE 2	White	Brown	Blue
Option C:	$1 million	$0	$1 million
Option D:	$2.5 million	$0	$0

Visual Illusions

The next few slides contain some so-called “visual illusions”: examples of how our brain interprets the 2-dimensional images in some surprising ways. As you view the illusions, consider whether there may be parallels between how our choices violate the “Rational Decision-Making” axioms, and the way in which our brain perceives and manipulates these images.

The Necker Cube

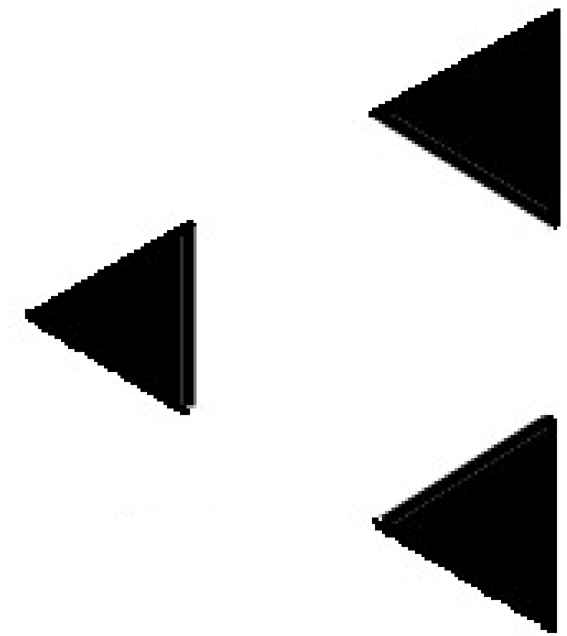


Stare at the red dot on the cube, and keep focusing on the red dot. What happens to the cube as you stare at the dot?

No one fully understands why our brains choose to manipulate the cube in this way. It simply…happens. Can you prevent the change from happening?

Think of this illusion as a metaphor for our violation of the “Invariance” axiom. Quite simply, our preferences are not consistent, and we cannot always control (or even understand) the way in which they may change.

How Many Triangles?

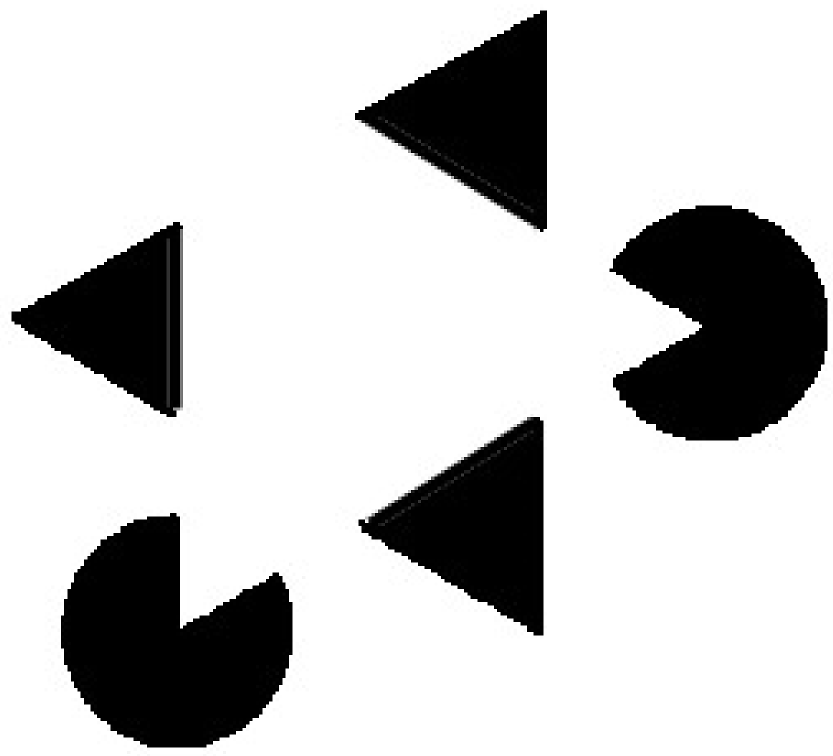


How many triangles do you see on this page?

How Many Triangles?

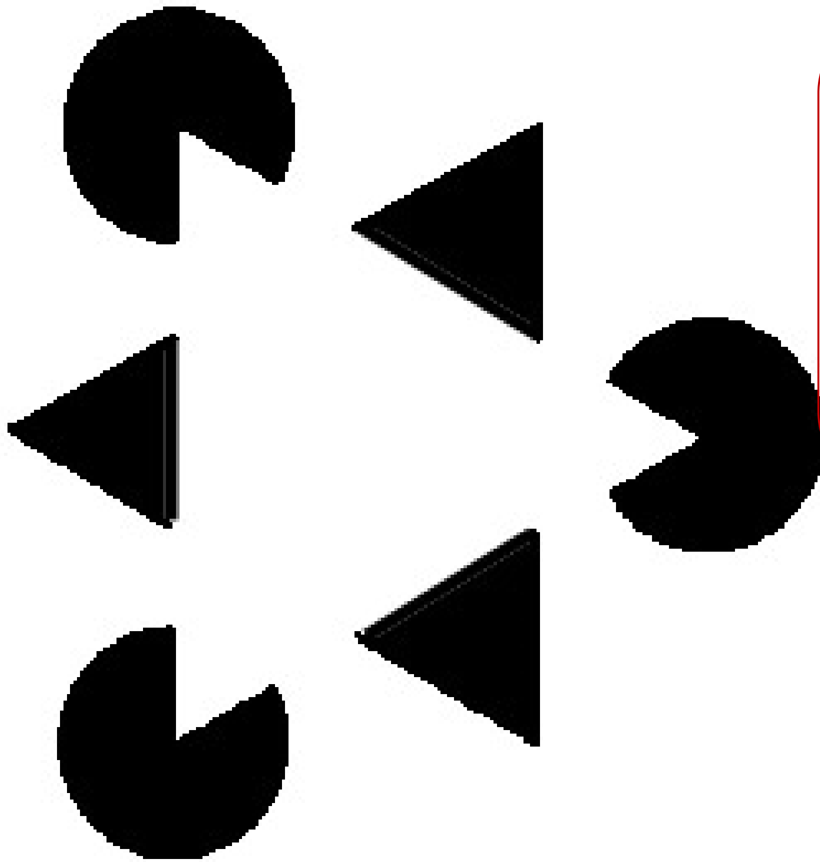
Suppose we add a new shape: an incomplete circle, as shown below. Does it change the number of triangles that you see?

How Many Triangles?



Now let's add a second incomplete circle. Is anything changing in your perception of the number of triangles?

How Many Triangles?



Now we have *three* incomplete circles. We haven't changed the number of *black* triangles, but has the addition of the new shapes altered your perception of the *total number* of triangles on the page?

What have we learned?

In this section, we examined the “traditional” economic model of consumer choice, and examined its assumptions about human behavior:

- that we are rational
- that we make decisions in ways that maximize our expected “utility” (or “happiness”) from money made or lost.

We discussed a number of axioms (assumptions about behavior that traditional economists view as self-evident), and looked at examples in which we violate those very assumptions.

We also defined risk aversion in terms of the rational expected utility model, and examined how we use budget constraints and indifference curves to identify our options as consumers.