Math 251W: Foundations of Advanced Mathematics, Spring 2020

Portfolio Assignment 2: §1.4-5

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Problem 1.4.3(2,4,5): Write a derivation for each of the following valid arguments. State whether the premises are consistent or inconsistent.

(2) If warthogs are smart, then they are interesting. Warthogs are not interesting or they are sneaky. It is not the case that warthogs are pleasant or not smart. Therefore warthogs are sneaky.

(4) If music soothes the soul then souls have ears. Music soothes the soul or musicians are calm. It is not the case that souls have ears or musicians are calm. Therefore musicians have souls.

(5) Computers are useful and fun, and computers are time consuming. If computers are hard to use, then they are not fun. If computers are not well designed, then they are hard to use. Therefore computers are well designed.

Solution

(2) Let S be the statement "warthogs are smart," X be the statement "warthogs are interesting," P be the statement "warthogs are pleasant," and Y be the statement "warthogs are sneaky." Thus the given argument can be written symbolically as:

$$\begin{array}{l} S \to Y \\ \neg Y \vee X \\ \neg(P \vee \neg S) \\ \hline X \end{array}$$

We demonstrate this argument is valid with the following derivation.

$$\begin{array}{lll} 1 & S \to Y & \\ 2 & \neg Y \vee X & \\ 3 & \neg(P \vee \neg S) & \\ \hline 4 & \neg P \wedge S & 3, \text{De Morgan's Law} \\ 5 & S & 4, \text{simplification} \\ 6 & Y & 1, \text{modus ponens} \\ 7 & X & 2, \text{modus tollendo ponens} \end{array}$$

The premises are consistent.

(4) Let S be the statement "music soothes the soul," E be the statement "souls have ears," C be the statement "musicians are calm," and M be the statement "musicians have souls." Thus the given argument can be written symbolically as:

$$\begin{array}{l} S \to E \\ S \vee C \\ \neg(S \vee C) \\ \hline M \end{array}$$

We demonstrate the given argument's validity with the following derivation.

1	$S \to E$	
2	$S \vee C$	
3	$\neg(S \vee C)$	
4	$\neg S \wedge \neg C$	3, De Morgan's Law
5	$\neg S$	4, simplification
6	C	2, modus tollendo ponens
7	$C \vee M$	6, addition
8	$\neg C$	4, simplification
9	M	6, 8, De Morgan's Law

The premises are inconsistnent.

(5) Let U be the statement "computers are useful," F be the statement "computers are fun," T be the statement "computers are time consuming," H be the statement "computers are hard to use," and D be the statement "computers are well designed." Thus the given argument can be written symbolically as:

$$\begin{array}{l} (U \wedge F) \wedge T \\ H \to \neg F \\ \neg D \to H \\ \hline D \end{array}$$

We demonstrate this argument is valid with the following derivation.

1	$(U \wedge F) \wedge T$	
2	$H \to \neg F$	
3	$\neg D \to H$	
4	$\neg D \to \neg F$	2, 3, hypothetical syllogism
5	$U \wedge F$	1, simplificaiton
6	F	5, simplification
7	$\neg(\neg D)$	6, 4, modus tollens
8	D	7, double negation

The premises are consistent.

Problem 1.5.11: Write a derivation for each of the following arguments.

(3)

$$\begin{array}{l} (\forall x \in \mathbb{Z})[(A(x) \to R(x)) \vee T(x)] \\ (\exists x \in \mathbb{Z})[T(x) \to P(x)] \\ (\forall x \in \mathbb{Z})[A(x) \wedge \neg P(x)] \\ \hline (\exists x \in \mathbb{Z})[R(x)] \end{array}$$

(4)

$$\begin{array}{l} (\forall x \in W)(\exists y \in W)[E(x) \to (M(x) \vee N(y))] \\ \neg(\forall x \in W)[M(x)] \\ (\forall x \in W)[E(x)] \\ \hline (\exists x \in W)[N(x)] \end{array}$$

Solution

(3)

1	$(\forall x \in \mathbb{Z})[(A(x) \to R(x)) \vee T(x)]$	
2	$(\exists x \in \mathbb{Z})[T(x) \to P(x)]$	
3	$(\forall x \in \mathbb{Z})[A(x) \wedge \neg P(x)]$	
4	$T(a) \to P(a)$	$2,$ Existential Instantiation
5	$A(a) \wedge \neg P(a)$	$3,$ Universal Instantiation
6	$\neg P(a)$	$5,$ Simplification
7	$\neg T(a)$	$4,$ Modus Tollens
8	$A(a) \to R(a) \vee T(a)$	$1,$ Universal Instantiation
9	$A(a)$	$5,$ Simplification
10	$R(a) \vee T(a)$	$8, 9,$ Modus Ponens
11	$R(a)$	$10, 7,$ Modus Tollendo Ponens
12	$(\exists x \in \mathbb{Z})[R(x)]$	$11, 2,$ Existential Generalization

(4)

1	$(\forall x \in W)(\exists y \in W)[E(x) \to (M(x) \vee N(y))]$	
2	$\neg(\forall x \in W)[M(x)]$	
3	$(\forall x \in W)[E(x)]$	
4	$(\forall x \in W)[M(x) \to E(x) \vee N(b)]$	$1,$ Existential Instantiation
5	$(\exists x \in W)[\neg M(x)] \Leftrightarrow \neg M(a)$	$2,$ Universal Instantiation
6	$E(a) \to M(a) \vee N(b)$	$4,$ Universal Instnatiation
7	$E(a)$	$3,$ Universal Instantiation
8	$M(a) \vee N(b)$	$6,$ Modus Ponens
9	$N(b)$	$8, 5,$ Modus Tollendo Ponens
10	$(\exists x \in W)[N(x)]$	$9,$ Existential Generalization

Problem 1.5.N: Negate the following statements by first writing them symbolically as quantified predicates, negating the symbolic expression, and then translating back to English.

(1) Everything was beautiful and nothing hurt.

(2) All happy families are alike; each unhappy family is unhappy in its own way.

(3) A single man in possession of a good fortune, must be in want of a wife.

Solution

(1) Let E be the set of everything, $B(x)$ be the predicate "x was beautiful" and $H(x)$ be the predicate "x hurts". Then the statement is

$$(\forall x \in E)(B(x) \wedge \neg H(x)).$$

Thus, the negation is

$$\neg[(\forall x \in E)(B(x) \wedge \neg H(x))] \Leftrightarrow (\exists x \in E)(\neg B(x) \vee H(x)).$$

In English, we have " there exists a thing which was not beautiful or which hurt."

(2) Let F be the set of all families, M(x,y) be the predicate "x and y" are alike, let H(x,y) be the predicate "x and y are happy," assume that unhappy is the negation of happy, and let O(x) be the predicate "x is unhappy in its own way." Then the statement is

$$(\forall x \in F)(\forall y \in F)[((H(x,y) \to M(x,y)) \wedge (\neg H(x,y) \to \neg M(x,y))]$$

. Thus the negation is

$$\neg\forall x \in F)(\forall y \in F)[((H(x,y) \to M(x,y)) \wedge (\neg H(x,y) \to \neg M(x,y))]$$

$$\Leftrightarrow (\exists x \in F)(\exists y \in F)[(\neg M(x,y) \wedge H(x,y)) \vee (M(x,y) \wedge \neg H(x,y))]$$

. In English, we have "There is a family such that there is another family such that either both families are unalike and happy, or both families are alike and unhappy."

(3) Let M be the set of all men, $S(x)$ be the predicate "x is single," $P(x)$ be the predicate "x is in the possession of a good fortune," and W(x) be the predicate "x is in want of a wife." Then the statement is

$$(\forall x \in M)[(S(x) \wedge P(x)) \to W(x)]$$

. Thus the negation is

$$\neg[(\forall x \in M)[(S(x) \wedge P(x)) \to W(x)]] \Leftrightarrow (\exists x \in M)[S(s) \wedge P(x) \wedge \neg W(x)]$$

. In English we have "There is a man who is single, in possession of a good fortune, and not in want of a wife."