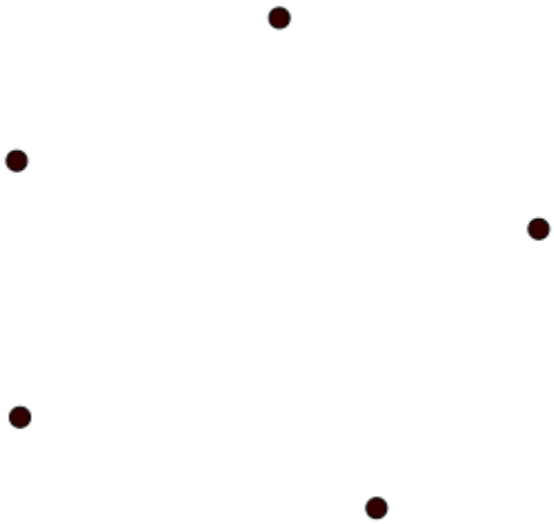
1. Triangulations

***Don't go to the next page yet! It'll spoil the fun!**

Connect the dots with straight line segments! The rules are simple:

a. You can't let two two lines cross
b. When you're done, you shouldn't be able to draw any more lines

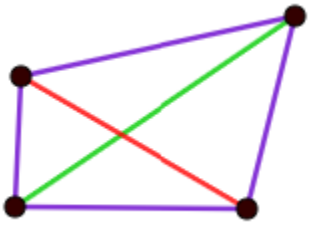


In the space below or on another piece of paper, ask/try to answer whatever questions you want! If you need help getting started, here are some ideas:

- How many edges did you draw?
- What edges did you have to draw?
- Did you notice that you made a choice on any of the edges?
- Can you find a pattern?
- What shape did your edges make with each other?
- How many of these shapes did you get?

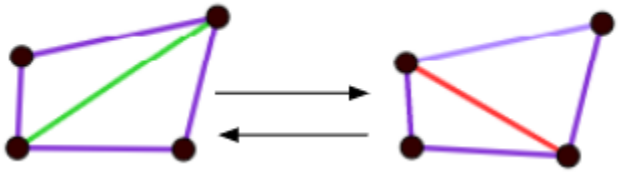
2. Flips

What you just made is called a **triangulation**. Perhaps you noticed that you could have chosen a number of different edges. Let's see if we can find them all! Before we do, let's think about a special type of shape called a convex quadrilateral. Suppose you were triangulating these dots:

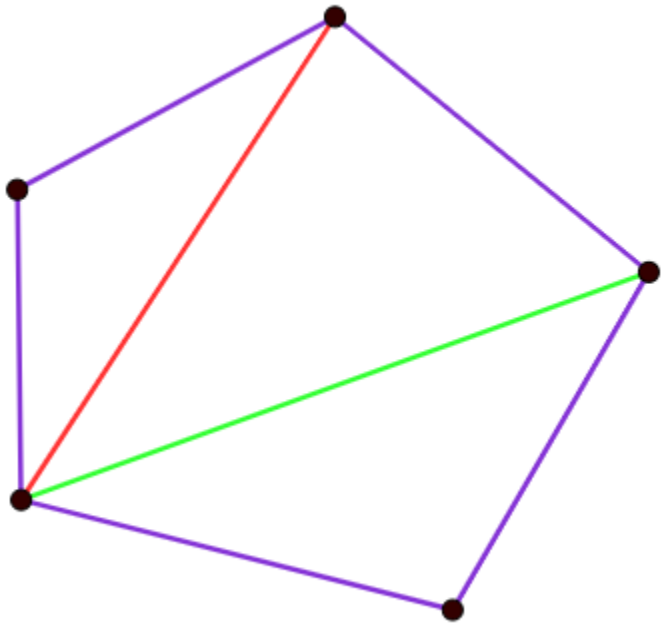


Notice that you'd have to draw the purple lines, but you'd get to chose between the red line and the green one. Which would YOU chose?! Whichever one you chose, you'd end up with one of two possible triangulations.

Suppose you decided to change your mind, and wanted to switch to the other triangulation. Such a maneuver is called a **flip**. Any time you have a convex quadrilateral made out of edges, you can always flip the diagonal edge



Now let's think about the triangulation you made in the last part. Maybe it looked something like this:



If you drew this triangulation, you made a choice. You had to draw the purple edges, but you didn't have to draw the diagonals that I've colored red and green.

Question: How many convex quadrilaterals can you make by drawing line segments between points? Write your answer in the space below.

3. Drawing the flip graph

On the next page, there is an outline!

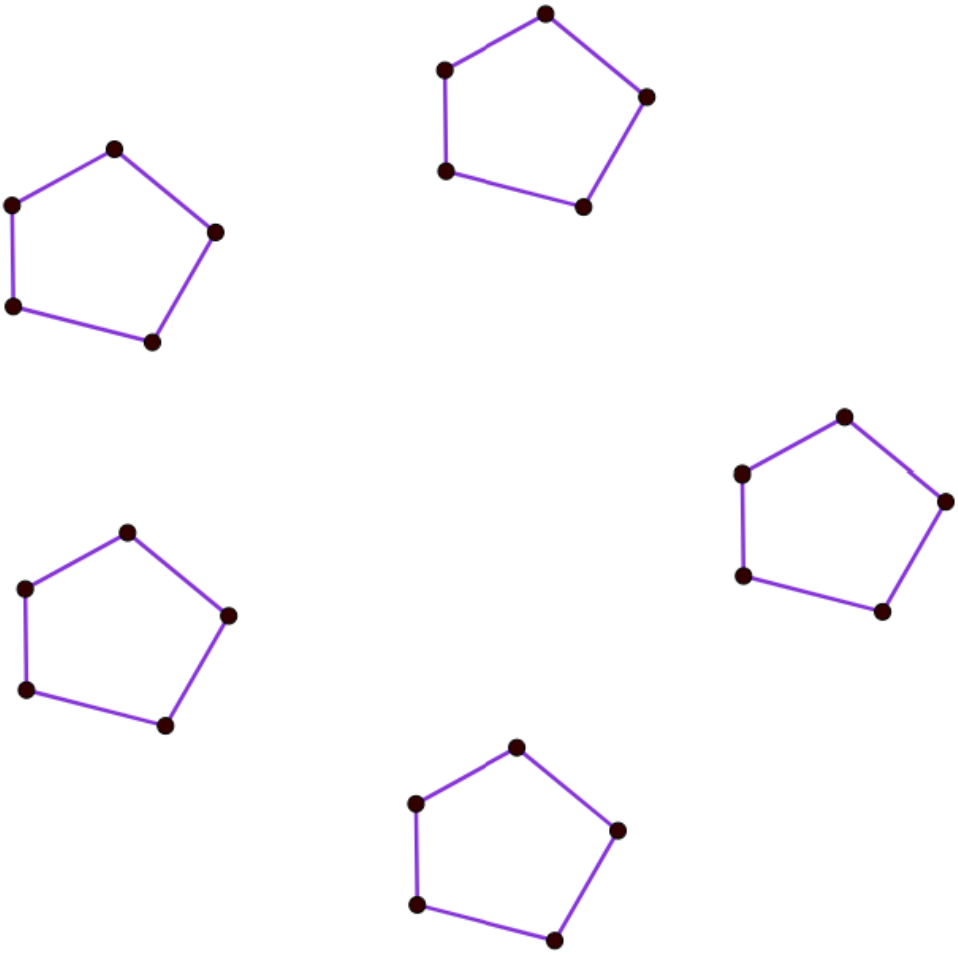
Now copy your original triangulation into the outline on the next page. Chose different colors for both of the diagonals you *chose* (the ones that cut through the pentagon).

Now that you've drawn your single triangulation, flip away! Pick one of your colored edges, and flip it. Now chose a different edge than the one you just flipped. Do it again! Keep repeating this process.

To make it a bit easier to follow: I'll outline these steps more explicitly:

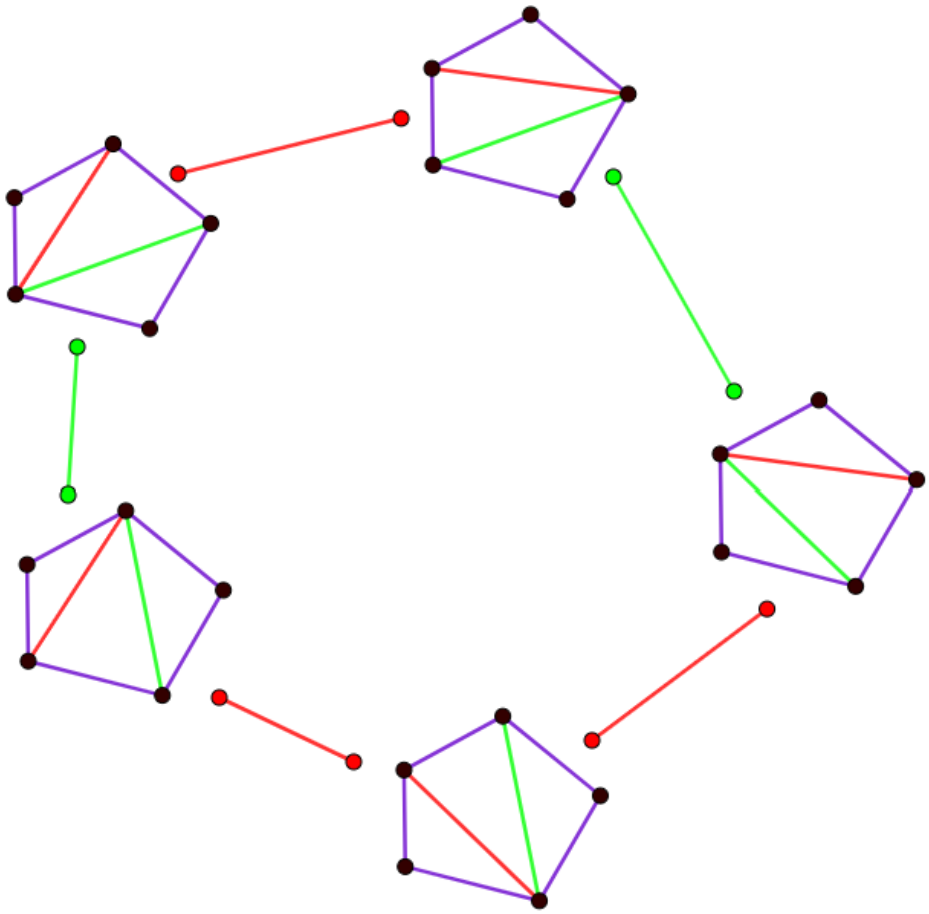
1. Pick an edge, and draw another copy of your triangulation next to your first one (on either side). Color the edge with the same color as the edge you just picked.
2. Now repeat the following steps:
    a. Pick the edge which is colored differently than the one you just chose.
    b. Next to your last triangulation, draw a new triangulation with this one flipped.
    c. If you've arrived back at your triangulation, you're done! If not, go to step a, this time keeping in mind that you have a "new last edge".

If things go as planned, you should find that the red and green edges have been swapped.



4. Answer key

If all went as planned, you should have something like this. Notice how the red and green colors alternate back and forth. Notice also that on the last step, green and red get swapped, even though we are back at the same triangulation!



Hope you had fun!!