

Math 251W: Foundations of Advanced Mathematics, Spring 2020

Portfolio Assignment 1: §1.2-3

Name: August Bergquist

Problem 1.2.6: Let E = “The house is blue,” let F = “The house is 30 years old,” and G = “The house is ugly.” Translate the following statements into symbols.

- (1) If the house is 30 years old, then it is ugly.
- (2) If the house is blue, then it is ugly or it is 30 years old.
- (3) If the house is blue then it is ugly, or it is 30 years old.
- (4) The house is not ugly if and only if it is 30 years old.
- (5) The house is 30 years old if it is blue, and it is not ugly if it is 30 years old.
- (6) For the house to be ugly, it is necessary and sufficient that it be ugly and 30 years old.

Solution

- (1) $F \rightarrow G$
- (2) $E \rightarrow (G \vee F)$
- (3) $(E \rightarrow G) \vee F$
- (4) $(\neg G) \leftrightarrow F$
- (5) $(E \rightarrow F) \wedge (F \rightarrow \neg G)$
- (6) $(G \leftrightarrow G \wedge F)$

Problem 1.2.15: Let P be a statement, let TA be a tautology, and let CO be a contradiction.

- (1) Show that $P \vee TA$ is a tautology.
- (2) Show that $P \wedge CO$ is a contradiction.

Solution

- (1) Since TA is a tautology, by construction it is always true. Similarly, since CO is a contradiction, it is always false. Thus, we can verify that for any statement P , $P \vee TA$ is a tautology using the truth table below.

P	TA	$P \vee TA$
T	T	T
F	T	T

- (2)

P	CO	$P \wedge CO$
T	F	F
F	F	F

Problem 1.3.8: State the *inverse*, *converse*, and *contrapositive* of each of the following statements.

- (1) If it's Tuesday, it must be Belgium.
- (2) I will go home if it is after midnight.
- (3) Good fences make good neighbors.
- (4) Lousy food is sufficient for a quick meal.

Solution

- (1)
 - Inverse: If it's not Tuesday, it must not be Belgium.
 - Converse: If it's Belgium, it must be Tuesday.
 - Contrapositive: If it's not Belgium, it must not be Tuesday.
- (2)
 - Inverse: If it is not after midnight, I will not go home.
 - Converse: If I will go home, it is after midnight.
 - Contrapositive: If I will not go home, it is not after midnight
- (3)
 - Inverse: Not-so-good fences make not-so-good neighbors.
 - Converse: Good neighbors make good fences.
 - Contrapositive: Not-so-good neighbors make not so good fences.
- (4)
 - Inverse: If the food is not lousy, then it is not sufficient for a quick meal.
 - Converse: If the food is sufficient for a quick meal, then it is lousy.
 - Contrapositive: Non-lousy food is not sufficient for a quick meal.

Problem 1.3.10: Negate each of the following statements

- (1) $e^5 > 0$.
- (2) $3 < 5$ or $7 \geq 8$.
- (3) $\sin(\pi/2) < 0$ and $\tan(0) \geq 0$.
- (4) If $y = 3$ then $y^2 = 7$.
- (5) $w - 3 > 0$ implies $w^2 + 9 > 6w$.
- (6) $a - b = c$ iff $a = b + c$

Solution

- (1) $e^5 \leq 0$.
- (2) $3 \geq 5$ and $7 \leq 8$
- (3) $\sin(\pi/2) \geq 0$ or $\tan(0) < 0$.
- (4) $y = 3$ and $y^2 \neq 7$.
- (5) $w - 3 > 0$ and $w^2 + 9 \leq 6w$
- (6) $a - b = c$ and $a \neq b + c$, or, $a - b \neq c$ and $a = b + c$

Problem 1.3.12: Simplify the following statements (making use of any equivalences of statements given so far in the text or exercises).

- (1) $\neg(P \rightarrow \neg Q)$.
- (2) $A \rightarrow (A \wedge B)$.
- (3) $(X \wedge Y) \rightarrow X$.
- (4) $\neg(M \vee L) \wedge L$.
- (5) $(P \rightarrow Q) \vee Q$.
- (6) $\neg(X \rightarrow Y) \vee Y$.

Solution

- (1) $\neg(P \rightarrow \neg Q) \Leftrightarrow P \wedge Q$
- (2) $A \rightarrow (A \wedge B) \Leftrightarrow A \rightarrow B$
- (3) $(X \wedge Y) \rightarrow X \Leftrightarrow X \rightarrow X$
- (4) $\neg(M \vee L) \wedge L \Leftrightarrow M \wedge \neg M$
- (5) $(P \rightarrow Q) \vee Q \Leftrightarrow P \rightarrow Q$
- (6) $\neg(X \rightarrow Y) \vee Y \Leftrightarrow X \wedge Y$