Final Exam Practice Problems

1. A consumer has preferences over two goods, x and y, represented by $u(x, y) = x^{\frac{1}{2}} y^{\frac{2}{3}}$. Find the consumer's demand functions (x^*, y^*) (Here the prices are p_x and p_y and income is I). The marginal rate of substitution of x for y ____ as the consumer increases their consumption of x, holding utility constant.

2. A firms uses labor and capital to produce output. Their production function is given by $q = F(K, L) = KL$. The price of labor is w, the price of capital is r. Solve for the firm's demand functions for labor and capital. What is the cost function $c(q)$? What is the marginal cost function?

3. A firms uses labor and capital to produce output. Their production function is given by $q = F(K, L) = ln(K) + L$. The price of labor is w, the price of capital is r. Solve for the firm's demand functions for labor and capital. Find the conditions such that the firm uses a positive amount of both K and L.

4. A consumer has a utility function given by $u(x, y) = (x^{\frac{1}{2}} + y^{\frac{1}{2}})^2$. The marginal rate of substitution of x for y ____ as x increases, holding utility constant. If prices are equal, does the consumer buy more x than y, more y than x, or the same amount of x as they do y?

5. A consumer has a utility function $u(x, y) = xy$, they face prices p_x and p_y and have income I. Graph their income consumption curve. Now, a firm has a production function $F(K, L) = KL$, they face prices w and r. Graph their expansion path (put K on the x axis). What do you notice?

6. Consider a set of N = 500 identical firms, each with the cost function $c(q) = 2 + \frac{1}{2}q^2$. Suppose market demand is given by $Q^d = 200 - 50p$. Solve for the equilibrium price and quantity. What are the profits for each firm? Now, compute the long-run equilibrium price, quantity, and number of firms.

1 Solutions

1. To solve, we set

$$MRS = \frac{p_x}{p_y}$$

The MRS is

$$\frac{\frac{1}{2}x^{-\frac{1}{2}}y^{\frac{2}{3}}}{\frac{2}{3}x^{\frac{1}{2}}y^{-\frac{1}{3}}} = \frac{3y}{4x}$$

Setting equal to the price ratio

$$\frac{3y}{4x} = \frac{p_x}{p_y}$$

$$3y = \frac{p_x}{p_y}4x$$

$$y = \frac{p_x}{p_y}\frac{4}{3}x$$

You then plug this into the budget constraint

$$p_x x + p_x(\frac{p_x}{p_y}\frac{4}{3}x) = I$$

$$p_x x(1 + \frac{4}{3}) = I$$

$$p_x x\frac{7}{3} = I$$

$$p_x x = \frac{3I}{7}$$

$$x^* = \frac{3I}{7p_x}$$

$$y^* = \frac{4I}{7p_y}$$

Now, we are being asked to determine the sign of the partial derivative of the MRS with respect to x. The reason is we are increasing our consumption of x (moving right along the indifference curve), we must keep utility fixed. By inspection, if x is going up, and u is fixed, y must fall. To verify

$$\frac{\partial MRS}{\partial x} = -\frac{3y}{4x^2} < 0$$

2. The optimality condition is where the MRTS = MRT

$$\frac{K}{L} = \frac{w}{r}$$

We can solve this for K in terms of L

$$K = \frac{w}{r}L$$

Plug this into the production function

$$q = KL$$

$$q = (\frac{w}{r}L)L$$

$$q = (\frac{w}{r})L^2$$

$$q\frac{r}{w} = L^2$$

$$L^* = (\frac{qr}{w})^{\frac{1}{2}}$$

$$K^* = (\frac{w}{r})(\frac{qr}{w})^{\frac{1}{2}}$$

$$K^* = (\frac{qw}{r})^{\frac{1}{2}}$$

Total costs are

$$c(q) = wL + rK$$

$$c(q) = w(\frac{qr}{w})^{\frac{1}{2}} + r(\frac{qw}{r})^{\frac{1}{2}}$$

$$c(q) = q^{\frac{1}{2}}(w(\frac{r}{w})^{\frac{1}{2}} + r(\frac{w}{r})^{\frac{1}{2}})$$

$$MC(q) = \frac{\partial c(q)}{\partial q} = \frac{1}{2}q^{-\frac{1}{2}}(w(\frac{r}{w})^{\frac{1}{2}} + r(\frac{w}{r})^{\frac{1}{2}})$$

3. The MRTS is

$$MRTS = \frac{1}{\frac{1}{K}} = K$$

Setting equal to the price ratio

$$K^* = \frac{w}{r}$$

So this firm always uses K. Now,

$$L^* = Q - ln(K^*)$$

$$L^* = Q - ln(\frac{w}{r})$$

The rule for natural logs implies

$$L^* = Q - (w - r)$$

When does the firm use labor? The conditions must be such that $L > 0$. This occurs when

$$Q - (w - r) > 0$$

$$Q > (w - r)$$

Alternatively, if we cared about the ratio of prices,

$$L^* = Q - ln(\frac{w}{r})$$

$$Q - ln(\frac{w}{r}) > 0$$

$$Q > ln(\frac{w}{r})$$

Taking exponents

$$e^Q > \frac{w}{r}$$

4. The MRS for this utility function is

$$MRS = \frac{2(x^{\frac{1}{2}} + y^{\frac{1}{2}})\frac{1}{2}x^{-\frac{1}{2}}}{2(x^{\frac{1}{2}} + y^{\frac{1}{2}})\frac{1}{2}y^{-\frac{1}{2}}} = \frac{y^{\frac{1}{2}}}{x^{\frac{1}{2}}}$$

Taking the partial with respect to x yield

$$\frac{\partial MRS}{\partial x} = -\frac{1}{2}\frac{y^{\frac{1}{2}}}{x^{\frac{3}{2}}} < 0$$

So the MRS decreases as x increases.

Now, set the MRS equal to the price ratio, which is just 1.

$$\frac{y^{\frac{1}{2}}}{x^{\frac{1}{2}}} = 1$$

$$y^{\frac{1}{2}} = x^{\frac{1}{2}}$$

$$y = x$$

So they will consume the same amount of x and y.

5. Set the MRS equal to the price ratio

$$\frac{y}{x} = \frac{p_x}{p_y}$$

$$y = x\frac{p_x}{p_y}$$

Plug this into the budget constraint

$$p_x x + p_y(x\frac{p_x}{p_y}) = I$$

$$p_x x + x p_x = I$$

$$x^* = \frac{I}{2p_x}$$

$$y^* = \frac{I}{2p_y}$$

Now

$$\frac{\partial x^*}{\partial I} = \frac{1}{2p_x}$$

$$\frac{\partial y^*}{\partial I} = \frac{1}{2p_y}$$

So the ICC will be an upward sloping line.

Now, for the firm: Set the MRTS equal to the price ratio

$$\frac{K}{L} = \frac{w}{r}$$

$$K = \frac{w}{r}L$$

Plug into production function

$$q = L\frac{w}{r}L$$

$$q\frac{r}{w} = L^2$$

$$L^* = (\frac{qr}{w})^{\frac{1}{2}}$$

$$K^* = (\frac{w}{r})(\frac{qr}{w})^{\frac{1}{2}}$$

$$K^* = (\frac{qw}{r})^{\frac{1}{2}}$$

$$\frac{\partial L}{\partial q} = \frac{1}{2}q^{-\frac{1}{2}}(\frac{r}{w})^{\frac{1}{2}} > 0$$

$$\frac{\partial K}{\partial q} = \frac{1}{2}q^{-\frac{1}{2}}(\frac{w}{r})^{\frac{1}{2}} > 0$$

So the expansion path is also an upward sloping line.

6. We have 500 identical firms so the total market supply will be 500 times whatever we derive as the individual firm's output decision will be. Each firm's profit function is

$$\pi(q) = pq - (2 + \frac{1}{2}q^2)$$

The first order condition is

$$p - q = 0$$

Or

$$p = q$$

So each firm supplies whatever the price is, market supply is then

$$Q^s = 500p$$

Market demand is

$$Q^d = 200 - 50p$$

Setting equal

$$500p = 200 - 50p$$

$$550p = 200$$

$$p^* = \frac{200}{550} \approx 0.3636$$

Then

$$p^* = q^* = \frac{200}{550} \approx 0.3636$$

Firm profits are then

$$\pi(q) = (\frac{200}{550})^2 - 2 - \frac{1}{2}(\frac{200}{550})^2$$

$$\pi(q) = \frac{1}{2}(\frac{200}{550})^2 - 2 < 0$$

So firms will leave the market.

Now to the long run. The three conditions are

$$p = c'(q)$$

$$p = ATC(q)$$

$$Q^s = Q^d$$

We already know

$$p = q$$

Setting price equal to ATC

$$p = \frac{2}{q} + \frac{1}{2}q$$

Set the terms equal to price equal, use this to solve for each individual firm's output decision.

$$q = \frac{2}{q} + \frac{1}{2}q$$
$$\frac{1}{2}q = \frac{2}{q}$$
$$\frac{1}{2}q^2 = 2$$
$$q^2 = 4$$
$$q^* = 2$$

Since price equals marginal cost (which is just q)

$$p^* = q^* = 2$$

Now we just need the total number of firms.

$$Q^d = 200 - 50p$$
$$Q^d = 200 - 50(2)$$
$$Q^d = 100$$

Total supply is

$$Q^s = Nq = 2N$$

Equating

$$2N = 100$$
$$N = 50$$

So we have verified that there are fewer firms in the market and each individual firm produces more.