Perfect Competition

1 Profit Maximization

Before, we analyzed a firm's cost minimization problem, in these notes, we will studied a firm's profit maximization problem for a perfectly competitive firm. Recall that a perfectly competitive firm is a price taker (their output decisions have no impact on the market price). The firm's profit function has two components, total revenue, and total cost, it is given by

$$\pi(q) = pq - c(q) \tag{1}$$

where p is the price, q is the quantity produced and c(q) is the firm's cost function. The firm simply chooses the quantity produced to maximize their profits. The first order condition is

$$\frac{\partial \pi(q)}{q} = p - c'(q) = 0 \tag{2}$$

where $c'(q)$ is the derivative of the firm's cost function with respect to q. We arrive at the optimality condition

$$p = c'(q) \tag{3}$$

which simply means the firm will produce up to the point that the marginal revenue equals marginal cost. To check, let $c(q) = \frac{1}{2}q^2$. The optimality condition is

$$p = q^*$$

plugging this into the profit function yields

$$\pi(q^*) = p^2 - \frac{1}{2}p^2 = \frac{1}{2}p^2 \tag{4}$$

What is profit if we let $q = \frac{p}{2}$, profit becomes

$$\pi(\frac{p}{2}) = \frac{p^2}{2} - \frac{1}{2}p^2 = \frac{1}{2}p^2 = p^2(\frac{1}{2} - \frac{1}{8}) < \frac{1}{2}p^2 = \pi(q^*) \tag{5}$$

so the firm could earn more profit by producing more. The reverse will be true if we pick a point greater than the optimal quantity, try $q = 2p$ and see what happens to profit, it should be less than the profit earned by producing at q^*.

2 Market Equilibrium

Suppose we have N perfectly competitive firms in a market where the inverse demand curve is

$$Q^D(p) = 200 - 50p$$

and each firm i has a cost function

$$c_i(q_i) = 2 + \frac{1}{2}q^2$$

Our goal is to find the long run equilibrium, which is a tuple

$$(q_i^*, p^*, N^*)$$

where q_i is the amount produced by each firm in the market, p^* is the equilibrium price, and N^* is the number of firms. The perfectly competitive equilibrium is defined as follows

$$p = c'(q)$$

$$p = \frac{c(q)}{q}$$

$$Q^S = Q^D \text{ ,Markets Clear}$$

First, solve the individual firm's problem

$$\max_q pq - c(q)$$

The first order condition is

$$p - q = 0$$

so the solution is

$$q_i^* = p \text{ for each firm i} \tag{6}$$

We also know that in the long run, price will equal average total cost, we can find ATC

$$\frac{c(q)}{q} = \frac{2}{q} + \frac{q}{2}$$

So, we have

$$p = \frac{2}{q} + \frac{q}{2}$$

we also know that p = q, so

$$q = \frac{2}{q} + \frac{q}{2}$$

$$\frac{q}{2} = \frac{2}{q}$$

$$q^2 = 4$$

So

$$q_i^* = 2 \tag{7}$$

is the long run quantity produced by each firm. Since price must equal marginal cost, this implies

$$p^* = 2 \tag{8}$$

Now, we need to find the total number of firms, in equilibrium, markets must clear, so

$$Q^S = \sum_i^N q_i = 100 - 50p = Q^D$$

This becomes

$$Nq_i^* = 100 - 50p^*$$

$$N(2) = 200 - 50(2)$$

$$N(2) = 100$$

$$N^* = 50$$

So the equilibrium is

$$(q_i^*, p^*, N^*) = (2, 2, 50)$$

You can verify that the zero profit condition is satisfied by:

$$\pi(q^*) = (2)(2) - (2 + \frac{1}{2}(2)^2) = 0$$