Parenting Decisions and Child Skill Development

August Bruno*

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Abstract

I develop and estimate a dynamic model of child cognitive skill development in a Markov Perfect Equilibrium framework, where both parents and children play an active role in the skill development process. Parents make time and monetary investments in their child, and can augment their study time decisions, and the total factor productivity of the cognitive skill production function, by choosing an authoritarian, authoritative, or permissive parenting style. Counterfactual policy analysis provides evidence that an authoritarian parenting style can hinder child cognitive skill development.

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JEL Classification: J13, D1

*Skidmore College abruno@skidmore.edu

1 Introduction

The relationship between parents and their children is fundamental for the growth and development of children. This importance advocates for a sound understanding of the time and monetary investments that parents make in their children, as well as the style in which they choose to parent, as these different types of investment decisions parents have at their disposal have various consequences on the dynamics of child skill development. Most economic models of child development study the effects of parental time and monetary investments in their children, omitting not only the choice of a parenting style, but the strategic nature of the parent-child relationship[1]. Some economists have recently incorporated parenting style into models of household interaction and child development. These papers typically study parenting style in isolation, ignoring parental time and monetary investments[2]. The absence of a model of child human capital formation that incorporates parental time and monetary investments in their children as well as the strategic nature of the parent-child relationship means that economists are unable to target when certain interventions are most effective for increasing child skills. Should policies be aimed at increasing parental time investment? Or, should policies be focused more on how to get parents to adopt a certain parenting style? Perhaps a combination of policies that target increasing parental time investment in early childhood, with those that nudge parents to adopt to a specific parenting style in later childhood is optimal. Without a model of strategic interaction between parents and children that incorporates all these decisions parents have at their disposal, economists are unable to address these questions.

In this paper, I develop and estimate a dynamic model of parent-child interaction and child cognitive skill development within a Markov Perfect Equilibrium setting. Parents make labor supply decisions, can spend time with their child, and make a discrete choice to adopt an authoritarian, authoritative, or permissive parenting style. Children decide how much of their time they allocate towards studying and leisure. I allow the parents' choice of a parenting style to influence child skill development along two dimensions. First, parenting styles are allowed to affect the marginal value that children place on their leisure time, influencing their time allocation decisions. Second, the choice of parenting style affects the total factor productivity of the cognitive skill production function.

Economic models of child development that only consider parental time and monetary inputs typically constrain the signs of the marginal products in the production function to be positive. This presents parents with the usual trade-offs that spending time with their

[1] See Bernal (2008),Del Boca et al. (2014),Verriest (2022),Brilli (2022), Mullins (2022)

[2] See Cosconati (2009), Lizzeri and Siniscalchi (2008), Hao et al. (2008), Bergman (2021), Seror (2022)

child is beneficial for the development of their skills, but comes at the cost of their leisure time. Similarly, monetary investments, such as schooling or sending a child to a high quality child-care center, are beneficial for child development, but comes at a financial cost to the household. What is less straightforward are the trade-offs parents face when it comes to adopting a parenting style. In this paper, I consider the three major parenting styles defined by Baurmind (1967), permissive, authoritarian, and authoritative.

A permissive parenting style is characterized by allowing children to make their own choices with minimal parental interference. Authoritarian and authoritative parenting styles are more involved in the sense that parents set limits and rules for their children to follow, but, they differ in the ways these rules are monitored and enforced (Baurmind (1967) and Baurmind (1971)). An authoritarian parenting style is characterized by parental demand for obedience and control, and corporal punishment is often used in the event that rules are broken. An authoritative parenting style is an intermediary style that lies between an authoritarian and permissive parenting style. This style is still characterized by the use of rules and limits to nudge children to behave closer to the parents' desired behavior (distinguishing it from a permissive style), but parents give their children an explanation for why the rules are in place instead of demanding obedience, and do not utilize harsh punishment in the event that rules are broken (differentiating it from an authoritarian parenting style). The effects of the different parenting styles and use corporal punishment on child development have been well documented in the sociology, psychology, and medical literatures, but have received limited attention in the field of economics[3].

Outside of Doepke and Zilibotti (2017), economic models that incorporate parenting style typically differentiate between authoritarian and non-authoritarian parenting, ignoring the differences between permissive and authoritative styles, such as their impact on the choices children make, and the consequences for the development of their cognitive skills. Doepke and Zilibotti (2017) present a theoretical motivation for why parents choose certain parenting styles, but do not estimate their model, leaving the magnitude of the trade-offs parents face regarding parenting style choices left unknown. Since permissive parenting is characterized by allowing children to make their choices freely, I assume that a permissive parenting style does not impact the child's time allocation decision, and normalize its impact on total productivity to zero. Authoritarian and authoritative parenting styles are intensive, with parents setting and enforcing rules upon their child. I allow the choice of a parenting style to affect the marginal utility that children place on their leisure time, augmenting their study time decisions. I do not impose ex-ante that an authoritarian or authoritative parenting style increases child study time all else equal. This allows for the potential that a

[3]See Straus (1991), Baumrind (1991), Gershoff (2002), Spera (2005)

stricter parenting style causes children to value their free time more, depressing study time.

Parent preferences over the choice of parenting style are allowed to be a function of their child's age and the parents' level of education. The main driver for parents to adopt a given parenting style is the disagreement in preferences over child skills between parents and children. Parents who place a high value on child skills are more likely to adopt a style that promotes skill development the lower the valuation their children place on their own cognitive skills[4].

I allow the parameters governing the production of child skills to vary with the age of the child, the choice of parenting style, and the level of parent education. Both parent and child preferences are heterogeneous across households, but, the parameters governing how parenting style decisions influence child choices are common across households.

I estimate the model using data from the Panel Study of Income Dynamics (PSID) and its Child Development Supplement (CDS), the parameters governing the model are estimated using Simulated Method of Moments (SMM). The PSID is a nationally representative survey of U.S households, and is well suited for this study because it contains information about parental labor supply as well as time spent with children, child study time, parenting style choices, and child skills. The additional benefit of the PSID is that it is a long panel, therefore, I am able to understand the dynamics of parenting style decisions over a long time horizon.

Identification of the parameters governing child skill production is obtained through variation in child cognitive skill measures, parent and child time investment, parenting style choices, and household income, as well as variation in the age of children when these choices are made. Household preferences are identified through variation in time allocation decisions of parents and their children. Identification of how different parenting styles influence child time allocation decisions is obtained through variation in child study time conditional on the parents' choice of parenting style.

The estimated model fits the data well, I am able to replicate the empirical trends in the data with respect to time allocation and parenting style choices, as well as the evolution of child cognitive skills. The point estimates display that maternal and paternal time investments are more effective at producing both cognitive skills when children are younger, a pattern that is consistent with the existing literature.

The novel result in this paper is that all else equal, an authoritarian parenting style

[4]This disagreement in preferences over child skills is the crux of parenting style decisions in Cosconati (2009) who also models child time allocation decisions. There are other factors that influence parenting style decisions that are omitted from the framework presented in this paper, for a model of parenting style and peer effects, see Agostinelli et al. (2023), for a model of parenting style decisions and risky environments, see Harris (2024).

reduces child time investment, while an authoritative parenting style has the opposite effect. Further, I estimate a negative effect on total productivity of both authoritarian and authoritative parenting (relative to permissive) on the cognitive skill production function.

The model estimates are used to estimate two counterfactual policy experiments. In the first, I force all parents to adopt a given parenting style in each period, and allow them to make all other choices optimally. I find that forcing parents to adopt an authoritarian parenting style each period reduces the average terminal stock of child cognitive skills by about 81% of a standard deviation, while forcing parents to adopt an authoritative or permissive parenting style each period increases the average terminal stock of child cognitive skills by 31% of a standard deviation and 29% of a standard deviation, respectively. The second policy simulates the effect of an after school homework program. In this exercise, I assume that all children 10 and younger spend an hour per week studying in this program, and two hours per week for children older than 10. I find this policy that exogenously raises child study time increases the average terminal stock of child cognitive skills by 16% of a standard deviation, however, consistent with the result in Eren and Henderson (2008), not all children see an improvement in the final stock of their skills as a result of the extra study time.

The remainder of the paper is structured as follows, Section 2 gives a discussion of related literature, Section 3 presents the theoretical model, Section 4 defines and summarizes the data used, Section 5 outlines the econometric assumptions made to estimate the model, Section 7 presents the arguments for identification of the structural parameters, Section 8 discusses the estimation of the structural model, Section 9 presents the model estimates, Section 10 discusses the model fit, Section 11 presents the results of the counterfactual experiments, and Section 12 concludes.

2 Literature Review

2.1 Parent-Child Interaction

Even though the seminal work by Becker and Tomes (1979) was published forty-five years ago, economic research on the dynamics of the parent-child relationship did not garner much attention until the last twenty-five years. This paper attempts to explain the relative importance of parenting style decisions in a larger model of parent-child interaction and child skill development, where the parents' choice of parenting style augments the time allocation decisions of their children and has additional consequences for their cognitive skill development. The closest relative of this work would be Del Boca et al. (2024), who use similar data, to analyze how paying children an allowance conditional on their study time affects their cognitive and non-cognitive skill development. They show that a paying children

an allowance conditional on studying reduces (stochastically) future non-cognitive skills. Their result provides evidence for the theoretical motivation in Benabou and Tirole (2003), who show how short-term incentives can have negative long-term consequences. While I do not have monetary incentives as a lever to increase child study time in my framework, I allow the choice of parenting style to augment child study time, while also allowing for potential negative effects on the development of cognitive skills. Therefore, one could think of this work as an analog to Del Boca et al. (2024), as we both consider dimensions of parenting outside of time allocation choices and their consequences for child development.

There is a growing literature on the use of parental praise and punishment in the economics literature. Weinberg (2001) presents a theoretical model to explain why the use of corporal punishment is more common among low-income households. The thought is that low income households are financially constrained, making it difficult to motivate children to exert effort using an allowance, and they must resort toward punishing the undesirable outcome instead of rewarding the positive outcome. This framework is static, and is unable to state the dynamic effects of corporal punishment on child development. Galiani et al. (2017) propose a theoretical framework to explain the reduction in the use of punishment of children over the course of the 20^{th} century. Their model predicts that a decrease in parental income inequality and a reduction in the number of children in the household shifts the balance of power within homes, which results in a decrease in the use of corporal punishment. Burton et al. (2002) present a model of parental praise of a child's good behavior, and show that children act out more often in poor households. The main takeaway is that child behavior is influenced by a range of socioeconomic factors, but, socioeconomic factors indirectly affect parenting practices through their response to child behavior. Closely related to the result in Burton et al. (2002) is the conclusion Harris (2024) arrives at, which is that disadvantaged parents will implement harsher discipline practices as a rational response to raising their children in a more hazardous environment. Kim (2019) analyzes the parent-child relationship using a signaling model, where the signaling method used by the parent is punishment. He shows that when the parent uses consistent punishment to signal poor behavior, the child will eventually be persuaded to behave optimally even when parents do not supervise their child. Finally, Akabayashi (2006) presents a theoretical model of parental praise and punishment and child effort. Here, effort is unobserved, and Abakayashi presents conditions such that if parents overestimate their child's abilities, maltreatment of children can arise as the household equilibrium.

Even more recent is the explicit consideration of a choice of parenting style as defined by Baurmind (1967) in the economics literature. Doepke and Zilibotti (2017) and Doepke et al. (2019) present a theoretical framework that rationalizes the choice of parenting style

as an optimal decision given the socioeconomic environment of the parents. They provide evidence for why authoritarian parenting styles are more common in unequal societies, while non-authoritarian parenting styles are more common in societies that are more equal. Cobb-Clark et al. (2019) posit the idea that parenting style investments are distinct from time and monetary investments, and provide evidence that the choice of parenting style is important for child cognitive development and that positive parenting styles are positively correlated with socioeconomic status. Cosconati (2009) is perhaps the first to estimate a dynamic model of parenting style and child development, where parents choose how strict to be when setting limits for their child, and the child chooses how much time to spend studying. The model is one of private information where only the child knows their type, and the main result is that a stricter parenting style is beneficial for children who place a low value on their human capital, but harms children who place a high value on their human capital. Agostinelli et al. (2023) allow for authoritarian and non-authoritarian parenting in a dynamic model of child friendship formation and human capital development, but assume that the child is myopic and do not allow for study time decisions by children.

The framework in this paper builds on those mentioned in the previous paragraph by allowing for the differentiation between authoritative and permissive parenting styles, modeling a longer time horizon (Cosconati (2009) on looks at 11 and 12 year old children in a two period model), and allowing for forward looking children (children are not modeled in Cobb-Clark et al. (2019) and are myopic in Agostinelli et al. (2023)).

2.2 Child Skill Development

There is a large literature concerning the economics of child skill development. This paper is closely related to others who have worked on models of time allocation, the timing of investment, and child skill development. Todd and Wolpin (2003) and Todd and Wolpin (2007) present frameworks for estimating the parameters that govern the production of child skills. Bernal (2008) is perhaps the first to estimate a structural model of maternal investment and the development of child cognitive skills, finding that maternal labor supply has a negative effect on the cognitive development of children under five years of age. The channel that is not directly in her model is that when mothers enter the labor market, they are not with their children, and this time is productive for skill development. Del Boca et al. (2014) explicitly consider this channel in their framework and are perhaps the first to include father's time investment into a model of child skill development. Their estimates confirm the results in Carneiro and Heckman (2003) and Cunha et al. (2010) in that the marginal product of parental time is decreasing with the age of the child, stressing the importance of early childhood investment. I extend these models by incorporating the choice of parenting

style into a model of time allocation, by allowing children to have an active role in their skill development, and by allowing for both direct and indirect effects of the choice of parenting style on the cognitive skill development of children.

Brown et al. (2023) considers the role that divorce law plays in a dynamic model of child skill development. They find that children's interests are not necessarily best served by attempting to minimize the divorce rate among parents. Divorce and fertility are absent from my framework, however, parenting style and child time allocation decisions are omitted from their framework.

Bernal and Keane (2011), Brilli (2022) and Verriest (2022) consider the role that non-parental childcare plays in the development of child skills and show that formal childcare is a better substitute for parental time than informal care when it comes to the development of the cognitive skills of children. I do not consider non-parental care decisions, partially because I model households from the age when children are typically already in elementary school, a time where the need for non-parental care is greatly reduced. Further, given that I model child study time, I would need to differentiate between different types of non-parental care, such as tutoring versus athletics, versus music, etc., which would present difficulties in modeling choices and estimation. The three papers mentioned earlier in the paragraph do not model the choices of children, nor do they consider the role parenting style plays in child skill development, and therefore, the framework presented below builds upon their work by considering the role of parenting style in child development.

3 Model

The model consists of a set of households, each including two parents, a mother and a father, and a single child. It is assumed that both the parents and the child are forward looking and each make investment decisions in the child's cognitive human capital development within a Markov Perfect Equilibrium framework. The parents and their child make decisions in a leader-follower structure, with the parents being the first mover. Parents decide how much of their time to allocate toward leisure, labor supply, and child time investment, and decide on a parenting style. Children observe the actions of their parents, and, conditional on parental choices, allocate their remaining time between studying and leisure.

Parenting style enters the model through three distinct channels, parents have preferences over their choice of parenting style, it affects the child's time allocation decision, and, it affects the total factor productivity of the cognitive skill production function.

3.1 Environment

3.1.1 Households

Each household consists of a mother (denoted by superscript m), a father (denoted by superscript f), and a single child (denoted by superscript k). Households face a child cognitive skill production technology that is common to all, but vary in their preferences, initial levels of child skills, initial child age, wage offers and non-labor income. Time is discrete and indexed by the age of the child in years, denoted by t.

Parents operate under a unitary model, their preferences are represented by:

$$u_t^p(l_t^m, l_t^f, c_t, \theta_t, ps_t) = \alpha_l^m ln(l_t^m) + \alpha_l^f ln(l_t^f) + \alpha_c ln(c_t) + \alpha_\theta ln(\theta_t) - \xi(ps_t, X_t) \tag{1}$$

where l_t^m is the mother's leisure time, l_t^f is the father's leisure time, c_t is parent consumption, and θ_t are their child's cognitive skills. Following Del Boca et al. (2014) and many others in the child development literature, I assume that $\alpha_l^m + \alpha_l^f + \alpha_c + \alpha_\theta = 1$. The parameter $\xi(ps_t, X_t)$ captures parental utility from choosing parenting style ps_t when the child is age t, and is allowed to be a function of household demographic characteristics, X_t.

The assumption of a unitary model rules out strategic interaction between parents when making labor supply and child investment decisions. It is also implied that the choice of parenting style is at the household level as assumed by Cosconati (2009) and Agostinelli et al. (2023) (who both consider a child and a representative parent, therefore parenting style is automatically at the household level), ruling out different parenting style decisions by each parent. This decision is driven by data limitations, as I am typically missing information about the father's parenting style, however, when data is available for both parents, the majority of the time, the parents agree on a style. While the unitary model has its limitations, it is a common assumption in the literature (see Weinberg (2001), Akabayashi (2006), Hao et al. (2008), Liu et al. (2010), Del Boca et al. (2014), Berlinski et al. (2024), Bernal (2008), Brilli (2022), etc.)

Each period, parents jointly make a parenting style decision, $ps_t \in \{AR, AT, P\}$, where AR denotes an authoritarian parenting style, AT an authoritative parenting style, and P a permissive parenting style[5]. They also make continuous choices regarding labor supply, monetary investment in their child, and time spent with their child subject to a household budget constraint and individual time constraints. The household budget constraint is given by:

$$w_t^f h_t^f + w_t^m h_t^m + y_t = c_t + g_t \tag{2}$$

[5]For a detailed explanation of the three major parenting styles, see Baurmind (1967). For an introduction to the economics of parenting style decisions, see Doepke and Zilibotti (2017).

where h_t^j represents parent j's hours worked in the labor market at hourly wage w_t^j, y_t is non-labor income and g_t are monetary investments in their child, or child goods. Each parent faces a time constraint

$$112 = \tau_t^j + h_t^j + l_t^j \tag{3}$$

where τ_t^j is the time that parent j spends with their child. It is assumed that each parent has 112 hours each week to allocate toward work, leisure, and time with their child, therefore, all time investment decisions are in hours per week. This implies that total household income, consumption and expenditures on children are also at the weekly level.

The child's utility function is given by

$$u_t^k(l_t^k, \theta_t, ps_t) = \omega(ps_t) ln(l_t^k) + \lambda_\theta ln(\theta_t) \tag{4}$$

where l_t^k is child leisure time and $\lambda_\theta > 0$. The link between the parents' choice of a parenting style and the child's utility function is captured by the function $\omega(ps_t)$, implying that the choice of parenting style is allowed to influence the child's value of leisure time, augmenting their optimal time allocation decision[6]. I assume that a permissive parenting style does not affect child study time decisions, therefore, $\omega(P) = 1$. This is consistent with the definition of a permissive parenting style, which is characterized by letting the child make their own decisions with little interference.

The child only faces a time constraint, which is given by

$$112 = \tau_t^k + l_t^k + s_t + \tau_t^m + \tau_t^f \tag{5}$$

where τ_t^k are the number of hours per week the child spends studying and s_t is exogenous time spent in school. Note that the time parents spend with their child appears in the child's time constraint, effectively setting an upper bound for the maximum amount of time a child can spend studying, and therefore, can crowd out child study time.

The household is subject to a production function for child cognitive skills whose parameters are allowed to vary with the age of the child. It is assumed parental time, child study time, child goods investment, lagged skills, and the choice of parenting style influence the development process. The production function is given by

[6] An alternative formulation would be to allow parenting style choices to set a lower bound for child study time, constraining the child's choice. This would introduce another unobserved state variable, as I do not have data on the specific time limits parents set for their children. Taking this modeling approach would require me to assume homogeneous preferences for children, which conflicts with the observed data, as there is considerable variation in child study time conditional on parenting style. In this paper, parents choose certain parenting styles to increase child study time, the modeling choice presented above accomplishes this goal, while still allowing for heterogeneous preferences.

$$ln(\theta_{t+1}) = ln(R_t(ps_t)) + \delta_t^m ln(\tau_t^m) + \delta_t^f ln(\tau_t^f) + \delta_t^k ln(\tau_t^k) + \delta_t^g ln(g_t) + \delta_t^\theta ln(\theta_t) \tag{6}$$

where $R_t(ps_t)$ is total factor productivity. Note that the choice of parenting style influences child skill production through two channels, indirectly by influencing the time investment choice by children, and directly by affecting the total productivity of the production function[7].

The parents and the child are both forward looking agents. Let the state vector when the child is t years old be denoted

$$\Gamma_t = (w_t^m, w_t^f, y_t, \theta_t)$$

then, the value function for the parents, who anticipate the actions of their child is

$$V_t^p(\Gamma_t) = \max_{a_t^p | \tau_t^k(a_t^p)} u_t^p(l_t^m, l_t^f, c_t, \theta_t, ps_t) + \beta^p E_t V_{t+1}^p(\Gamma_{t+1} | a_t^p, \Gamma_t) \tag{7}$$

where β^p is the parents' discount factor and $a_t^p = (\tau_t^m, \tau_t^f, h_t^m, h_t^f, ps_t, g_t)$ is their choice vector. The child's value function is given by

$$V_t^k(\Gamma_t) = \max_{\tau_t^k | a_t^p} u_t^k(l_t^k, \theta_t, ps_t) + \beta^k E_t V_{t+1}^k(\Gamma_{t+1} | \tau_t^k, a_t^p, \Gamma_t) \tag{8}$$

A driving factor for parents to adopt a certain parenting style is the gap in preferences over child skills between parents and their child. Typically, children place a smaller weight on their cognitive skills than their parents would desire. This source of conflict could cause parents to be more likely to adopt a parenting style that encourages effort that their child would not put forth on their own accord.

The terminal value the parents and the child place on the cognitive skills of the child are assumed to be

$$V_{T+1}^p(\theta_{T+1}) = \psi^p ln(\theta_{T+1}) \tag{9}$$

and for the child

$$V_{T+1}^k(\theta_{T+1}) = \psi^k ln(\theta_{T+1}) \tag{10}$$

where ψ^p and ψ^k are parameters to be estimated.

After the child is T years old, the child development process outlined in this model is no

[7]See Agostinelli et al. (2023) for a similar implementation in their model of child friendship formation, parenting style, and child cognitive skill development.

longer relevant, and the child enters a different stage of their lifecycle. It can be assumed that both the parents' and the child solve different optimization problems than the one described above, and the child's skill production function changes as well. Therefore, these values can be thought of as initial conditions in the next phase of life for both the parents and the child.

As a final note, I do not assume that parents are altruistic in my model. An alternative rationality for parents choosing to be permissive is that they are altruistic and place a high value on their child's utility, in practice, this posed difficulties with theoretical tractability. Del Boca et al. (2024) had the opposite issue, where not allowing for parental altruism presented modeling issues. The difference here lies in our assumptions concerning parenting style. In their framework, parenting style induces child study time with an allowance that is conditional on effort. Children care about their leisure and the amount of allowance they receive, so they trade off leisure for income. This channel is absent in my framework, as a choice of an authoritarian or authoritative parenting can reduce child utility in the current period with no immediate benefit to the child. Incorporating parental altruism thus posed tractability issues when this was attempted in estimation, as there is no manner for parents to compensate children's (potential) utility loss when they adopt an authoritarian or authoritative parenting style[8]. One could posit that a more altruistic parent would face a higher cost to implement an authoritarian or authoritative parenting style, and therefore, altruism may not be completely absent from the current framework even though it is not explicitly modeled.

3.2 Household Equilibrium

Each period, the parents and their child interact in a leader-follower structure where the parents move first and choose $(\tau_t^m, \tau_t^f, h_t^m, h_t^f, ps_t, g_t)$, and, conditional on the parents' actions, the child chooses their study time τ_t^k. Both the utility functions for the parents and their child, as well as the skill production functions, are separable in child skills, implying that the current level of child skills does not alter the solutions to the household's problem[9]. This assumption greatly reduces the computational burden to solve the household's problem.

Consider the parents' problem for the child at any age $t \leq T$, they solve

$$V_t^p(\Gamma_t) = \max_{a_t^p | \tau_t^k(a_t^p)} u_t^p(l_t^m, l_t^f, c_t, \theta_t, ps_t) + \beta^p E_t V_{t+1}^p(\Gamma_{t+1} | a_t^p, \Gamma_t)$$

[8]This is the same issue faced by Cosconati (2009), who also did not assume parents were altruistic.

[9]This is the the assumption utilized in Del Boca et al. (2014),Brilli (2022), Mullins (2022), and Verriest (2022), future work should consider how parenting style decisions depend on the current stock of child skills as done in Agostinelli et al. (2023)

subject to

$$w_t^f h_t^f + w_t^m h_t^m + y_t = c_t + g_t$$

their individual time constraints

$$112 = \tau_t^j + h_t^j + l_t^j$$

for $j \in \{m, f\}$, and must also take into account the child's time constraint, the law of motion for cognitive skills, and the effect of their choice of parenting style on the time allocation decision of their child.

The child being the follower in the model, observes the actions of their parents, and solves

$$V_t^k(\Gamma_t) = \max_{\tau_t^k | a_t^p} u_t^k(l_t^k, \theta_t, ps_t) + \beta^k E_t V_{t+1}^k(\Gamma_{t+1} | \tau_t^k, a_t^p, \Gamma_t)$$

subject to their parents' discrete choice of a parenting style, ps_t, their time constraint

$$112 = \tau_t^m + \tau_t^f + l_t^k + \tau_t^k + s_t$$

and the law of motion for their cognitive skills.

The structure of the model does not allow me to derive analytical solutions for all of the choice variables present in the model. I am able to use a mix of numerical and analytical solutions to solve the household's problem, and the only choice variables that do not have closed form solutions are τ_t^m and τ_t^f.

In the terminal period, the first order condition for the child's maximization problem is

$$\frac{\partial u_T^k(.)}{\partial \tau_T^k} + \beta^k E_T\left(\frac{\partial V_{T+1}^k(\theta_{T+1})}{\partial ln(\theta_{T+1})} \frac{\partial ln(\theta_{T+1})}{\partial \tau_T^k}\right) = 0 \tag{11}$$

A full derivation of household equilibrium can be found in Appendix A, the solution to the child's period T problem is given by

$$\tau_T^k = \frac{\beta^k D_{T+1}^k \delta_T^k}{\omega(ps_T) + \beta^k D_{T+1}^k \delta_T^k}(112 - \tau_T^f - \tau_T^m - s_T) \tag{12}$$

where

$$D_t^k = \frac{\partial V_t^k}{\partial ln(\theta_t)}$$

represents the marginal value the child places on next period log skills. The child's reaction function has the natural properties in that it is increasing in the marginal productivity of their study time and decreasing in parental time investment. Further, a parenting style that

reduces the child's value of leisure activities ($\omega(ps_t) < 1$), increases the number of hours that the child studies each week all else equal, however, note that the effectiveness of the parenting style choice at increasing child study time will depend on the child's preference for their own skills, as they differ in how much they value their future skills. The solution to the child's problem at any age $t < T$ is

$$\tau_t^k = \frac{\beta^k D_{t+1}^k \delta_t^k}{\omega(ps_t) + \beta^k D_{t+1}^k \delta_t^k}(112 - \tau_t^m - \tau_t^f - s_t) \tag{13}$$

and the sequence of D_{t+1}^k terms are derived via backward induction[10]. Equation (13) highlights two key trade-offs parents face when making parenting decisions. The first is that the more time parents spend with their child, the less time the child has available to spend studying, therefore, parental time can crowd out child time investment. The second concerns the relative preferences parents' place on child skill development and parenting styles. An authoritarian or authoritative parenting style could increase or decrease a child's study time all else equal relative to a permissive parenting style. Regardless of whether or not an authoritative or authoritarian parenting style increases or decreases child study time, the parents must consider the trade-offs with respect to future skill development and their preferences for adopting a given parenting style.

The solution to the parents' problem requires a numerical solution for τ_t^m and τ_t^f, however, conditional on these solutions, I am able to derive closed for solutions for labor supply and goods investment. The parents' choose $(\tau_t^m, \tau_t^f, h_t^m, h_t^f, g_t, ps_t)$ to maximize

$$V_t^p(\Gamma_t) = \max_{a_t^p | \tau_t^k(a_t^p)} u_t^p(l_t^m, l_t^f, c_t, \theta_t, ps_t) + \beta^p E_t V_{t+1}^p(\Gamma_{t+1} | a_t^p)$$

subject to the household budget constraint, their time constraints, their child's reaction function, and the law of motion for child skills. The first order conditions with respect to τ_t^m and τ_t^f are

$$\frac{-\alpha_l^m}{112 - h_t^m - \tau_t^m} + \beta^p D_{t+1}^p \left(\frac{\delta_t^m}{\tau_t^m} - \frac{\delta_t^k}{112 - \tau_t^m - \tau_t^f - s_t}\right) = 0 \tag{14}$$

and

$$\frac{-\alpha_l^f}{112 - h_t^f - \tau_t^f} + \beta^p D_{t+1}^p \left(\frac{\delta_t^f}{\tau_t^f} - \frac{\delta_t^k}{112 - \tau_t^m - \tau_t^f - s_t}\right) = 0 \tag{15}$$

I solve for the optimal choices for τ_t^m and τ_t^f numerically, conditional on a pair (τ_t^m, τ_t^f),

[10] See Appendix A for derivation.

parent labor supply is

$$h_t^f(\tau_t^m, \tau_t^f) = \frac{w_t^f(112 - \tau_t^f)(\alpha_l^m + \alpha_c + \beta^p D_{t+1}^p \delta_t^g) - \alpha_l^f(w_t^m(112 - \tau_t^m) + y_t)}{w_t^f(\alpha_l^f + \alpha_l^m + \alpha_c + \beta^p D_{t+1}^p \delta_t^g)} \tag{16}$$

and

$$h_t^m(\tau_t^m, \tau_t^f) = \frac{w_t^m(112 - \tau_t^m)(\alpha_l^f + \alpha_c + \beta^p D_{t+1}^p \delta_t^g +) - \alpha_l^m(w_t^f(112 - \tau_t^f) + y_t)}{w_t^m(\alpha_l^f + \alpha_l^m + \alpha_c + \beta^p D_{t+1}^p \delta_t^g)} \tag{17}$$

With these choices in hand (that are both constrained to be non-negative), the solution for child goods, g_t, is a fraction of total household income

$$g_t(\tau_t^m, \tau_t^f) = \frac{\beta^p D_{t+1}^p \delta_t^g}{\alpha_c + \beta^p D_{t+1}^p \delta_t^g}(w_t^f h_t^f(\tau_t^m, \tau_t^f) + w_t^m h_t^m(\tau_t^m, \tau_t^f) + y_t) \tag{18}$$

The optimal parenting style choice will be the style that generates the highest utility for the parents when taking into account all other decisions they make. Let $\tilde{a}_t^p = (\tau_t^m, \tau_t^f, h_t^m, h_t^f, g_t)$, parents will choose an authoritarian parenting style if and only if:

$$V_t^p(\tilde{a}_t^p, AR) = \max\{V_t^p(\tilde{a}_t^p, AR), V_t^p(\tilde{a}_t^p, AT), V_t^p(\tilde{a}_t^p, P)\} \tag{19}$$

an authoritative parenting style if and only if:

$$V_t^p(\tilde{a}_t^p, AT) = \max\{V_t^p(\tilde{a}_t^p, AR), V_t^p(\tilde{a}_t^p, AT), V_t^p(\tilde{a}_t^p, P)\} \tag{20}$$

and will choose a permissive parenting style otherwise. The optimal parenting style choice considers the parenting style preferences, $\xi(ps_t, X_t)$, the returns to cognitive skills as a function the parenting style choice through increased child effort, δ_t^k, and the quality of the parent-child relationship as a function of the parenting style, δ_t^{ps}.

While I am not able to derive closed for solutions for τ_t^m and τ_t^f, the functional forms for the dynamic problem the household solves still allows me to analytically solve for a majority of the choice variables due to the fact that at any age t, investment is independent of the level of child skills and the wage and non-labor income processes do not depend on any household decisions. While this assumption is common in the literature (see Del Boca et al. (2014), Brilli (2022), Mullins (2022), Verriest (2022)) and reduces the computational burden when estimating the structural parameters, it is not without limitations, as it could be the case that the level of child skills influences parental decisions. Perhaps parents with children who have a lower level of skill will spend more time with them, or perhaps parents with

low skill children are more likely to adopt an authoritarian parenting style to induce more effort. In his framework, Ronda (2017) uses a trans-log cognitive skill production function, and finds a statistically significant evidence of dynamic complementarity of mother's time with the child when the child is younger than 6 years old. This parameter is not statistically different from 0 when the child is between 6 and 16 years of age. Therefore, using similar data, Ronda (2017) does not provide significant evidence for the dynamic complementarity of parental time investment in the age range studied in this paper.

4 Data and Descriptive Statistics

The data used in this paper are taken from the Panel Study of Income Dynamics (PSID) and its Child Development Supplement (CDS). The PSID is a dynastic survey of households in the United States that began in 1968 with a set of approximately 18000 individual from about 5000 families. Between 1968 and 1997, the PSID surveyed households annually, incorporating new members as the original families expanded, since 1997, the PSID has been collected bi-annually. Starting the 1997, the CDS collected information on 3563 children living in 2394 PSID families with children between the age of 3 and 13 years old. The CDS was collected in three waves, the first, CDS-I, was collected in 1997, CDS-II was collected in 2002, and CDS-III in 2007, no new families were added in the second and third waves. The PSID-CDS is well-suited for this study because it contains data on parental labor supply and income and provides information about parent and child time use, parenting style, and child cognitive skills.

The main PSID survey provides information on parent demographic data such as age and education, as well as labor supply, hourly wages, and data on non-labor income. I collect this information for parents from the year their child was 4 years old through the year that their child turns 18, providing a panel from 1986 to 2009. CDS-I, CDS-II, and CDS-III provides data on children's cognitive skills, as well as parent and child time allocation, and information about parenting style decisions. To construct my sample, I follow Del Boca et al. (2024) and require that children have a cognitive skill measure in 1997 and 2002, their biological parents are married when their child is between 4 and 18 years old, and require that the child either have no siblings, or, if there is a household with two children, the younger child is selected. This resulting sample has information on 276 children and their parents[11].

[11]For comparison, Del Boca et al. (2014) have 105 children in their sample, Del Boca et al. (2024) have 248, and Verriest (2022) has 247. All use the PSID-CDS and make similar sample selection decisions over different age ranges.

I measure children's cognitive skills with the Letter-Word (LW) module of the Woodcock-Johnson aptitude test. This assesses children's reading and symbolic identification skills, and has the advantageous property in that it is administered to children in the CDS between the ages of 3 and 18, allowing me to track the development of skills over time.

The CDS does not explicitly ask parents what their parenting style is and I must use alternative questions to infer what style parents choose. I follow Doepke and Zilibotti (2019) who also use the CDS. The CDS asks the primary caregiver (PCG) "If you had to choose, which thing on this list would you pick as most important for a child to learn, to prepare him or her for life?" Potential responses are: "To obey","To be well-liked or popular","To think for himself/herself","To work hard","To help others when they need help". The primary care-giver is then asked this same question, but for their second, third, and fourth most important quality, and the potential answers are the same. I classify parents as choosing an authoritarian parenting style if their first or second most important quality is "To obey". Parents are classified as authoritative if they are not already defined as choosing an authoritarian style, and their first or second most important quality is "To work hard", all other parents are classified as permissive.

The difficulty in classifying parenting styles from survey data lies in differentiating between non-authoritarian parenting styles. Classifying parents who place a high value on obedience as authoritarian is not an unreasonable step, as it follows straight from the work of Baurmind (1967). Permissive parenting is characterized by allowing children to make their own choices with little interference from their parents, while authoritative parenting is characterized by exerting parental control when incentives between parents and children do not align, but, through rational communication and not through a demand for obedience (Baurmind (1971)). Therefore, among parents who are not already classified as authoritarian, a reasonable way to distinguish between authoritative and permissive parenting is to classify the non-authoritarian parents as authoritative if they value hard work, and the remaining as permissive as it is more likely for an authoritative parent to want their child to work hard, and therefore parent in a way to achieve this goal, while a permissive parent would prefer to let their child make their own decisions[12].

Economists working on child development have been drawn to the CDS because of its inclusion of time diary data that provides detailed information about how parents and their children spend their day, including the activities engaged in. The CDS asks children or their PCG to record a detailed, minute by minute timeline of the activities they participated in

[12]See Appendix C for a breakdown of parent rule usage by parenting style. The natural order appears that parents who choose an authoritarian style have rules set with the highest frequency and rules are set with the lowest frequency in households where the parenting style choice is permissive.

and who they participated with for two days of the week, a weekday, and a day on the weekend. I construct parent and child time investment in the following manner. Child study time is classified as time that the child spends in educational activities (studying, reading for pleasure, using the computer for educational purposes, doing homework,etc.) where neither the mother or the father is actively participating in the activity with them. Mother's time investment is considered time that only the mother is actively participating in an activity with her child, and this definition is analogous for the father's time investment in their child. Joint parental time is not a decision in this model, and I avoid double counting by weighting the time both parents are actively participating with their child by the ratio of individual active participation time[13]. Weekly time investments are then calculated by constructing a weighted sum of mother's active time, father's active time, and child study time by $\frac{5}{7}$ times the weekday time diary entries and $\frac{2}{7}$ times the weekend day entry.

4.1 Systemically Missing Data

The PSID has been conducted bi-annually corresponding to the previous year starting in 1997, prior to this, it was conducted annually. The CDS was conducted in three waves, 1997, 2002, and 2007, therefore, a motivation for using simulation methods (described below) is due to systemically missing data in the PSID-CDS. Table 1 shows data availability for a PSID-CDS household with a child that was born in 1993 who would be 16 years old in 2009. The only year where the PSID and CDS surveys overlap is 2002, the typical case is that parental wage and labor supply data is available or, data from the CDS is available.

Table 1. **Structure of Data**

Data	Years Available	Source
Skill Measures	1997, 2002, 2007	CDS
Parent Time	1997, 2002, 2007	CDS
Parenting Style	1997, 2002, 2007	CDS
Child Study Time	1997, 2002, 2007	CDS
Parent Labor Supply	1993-1996, 1998-2009 (bi-annually)	PSID
Parent Wages	1993-1996, 1998-2009 (bi-annually)	PSID
Household Non-Labor Income	1993-1996, 1998-2009 (bi-annually)	PSID

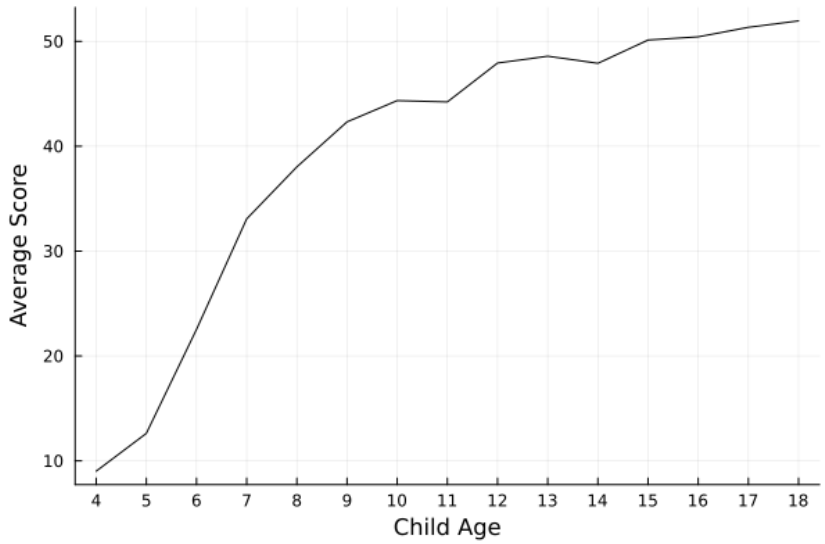
Notes: The PSID-CDS data exhibits non-random missing data due to the survey structure. This table presents a sample of data availability for a PSID-CDS household with a child born in 1993.

[13] I therefore add $\frac{\tau_t^m}{\tau_t^m+\tau_t^f}\tau_t^b$ where τ_t^b is joint parental time, to the mother's child time investment, and $\frac{\tau_t^f}{\tau_t^m+\tau_t^f}\tau_t^b$ to the father's child time investment.

4.2 Summary Statistics

Figure 1 below displays the evolution of the anchoring measures of children's cognitive skills. The upward sloping and concave trend of child performance on the LW Exam is well documented in the literature [14].

Figure 1. **Average Child Skills By Age**



Notes: Average child performance on the LW Exam in the author's sub-sample of the CDS. The exam is out of 57 questions.

Table 2 shows the frequency of parenting style choices as defined above in the CDS. In my sample, authoritative parenting is chosen most often, with a permissive style and authoritarian style being chosen with about the same frequency. In their work Majumder (2016) also report that authoritative parenting is chosen most often in U.S households, albeit they use different data and a different methodology to classify parenting styles.

Table 2. **Frequency of Parenting Style Choices**

Parenting Style	Frequency
Authoritarian	0.28
Authoritative	0.46
Permissive	0.26

Notes: Frequency of parenting style choices in sample are averages of the household-time observations.

One may have concerns that I allow for parenting style to be a choice in my framework and not a type. Table 3 shows that parenting style choices change in the data, providing

[14] Additional summary statistics for the PSID-CDS data can be found in Appendix *D*

evidence that parenting styles should be considered a choice, and not a type. Parenting style is also heavily correlated with demographic characteristics, including education. Table 4 shows that an authoritarian parenting style is adopted more frequently in households where the mother has less than a college degree. This is an empirical fact consistent with the literature that parenting style choices are correlated with socioeconomic status (see Doepke and Zilibotti (2017) and Harris (2024) for example).

Table 3. **Parenting Style Transitions**

	Authoritarian	**Authoritative**	**Permissive**
Authoritarian	0.47	0.30	0.23
Authoritative	0.14	0.64	0.22
Permissive	0.15	0.41	0.44

Notes: Data is taken from the author's subsample of the CDS. Each unit of observation is a household when a child is t years old, and a household when the child is t+5 years old.

Table 4. **Parenting Style By Mother's Education**

Parenting Style	**College**	**No College**
Authoritarian	0.17	0.33
Authoritative	0.53	0.42
Permissive	0.30	0.25

Notes: Data is taken from the author's subsample of the CDS. College represents households where the mother has a college degree (or more), No College represents households where the mother does not have a college degree.

5 Econometric Specifications

To map the model to the data, I make the following econometric specifications with respect to household preferences, the child skill development production function, the wage offer distribution, and the non-labor income process.

5.1 Preferences

The parameters governing parental preferences over their leisure time (α_l^m and α_l^f), consumption (α_c), child skills (α_θ), and parenting style utility $\xi(ps_t, X_t)$ are assumed to be constant across time, and heterogeneous across households. Child preferences over their leisure time (λ_l) and skills (λ_θ) are also constant over time and heterogenous across children. Both the parents' and the child's discount factor are constant across childhood and are exogenously

set to 0.9. Finally, the parameters that allow the choice of parenting style to augment the time allocation decision by children, $\omega(ps_t)$, are assumed to be common to all households and constant across time.

I assume that parental preferences are i.i.d. draws from a multivariate normal distribution that is constructed in the following manner. Define $\boldsymbol{\nu} \sim N(\boldsymbol{\mu_\nu}, \boldsymbol{\Sigma_\nu})$, where

$$\boldsymbol{\mu_\nu} = (\mu_{l^m}, \mu_{lf}, \mu_c)' \tag{21}$$

and

$$\boldsymbol{\Sigma_\nu} = \begin{pmatrix} \sigma^2_{l^m} & 0 & 0 \\ 0 & \sigma^2_{lf} & 0 \\ 0 & 0 & \sigma^2_c \end{pmatrix} \tag{22}$$

The assumption that (22) is a diagonal matrix is used in Berlinski et al. (2024) and is driven by identification and estimation issues when attempting to estimate parameters off the diagonal. Each household draws a vector $\boldsymbol{\nu} = (\nu_{l^m}, \nu_{lf}, \nu_c)$, define $d^p = 1 + \sum_{j \in \{l^m, lf, c\}} exp(\nu_j)$, the utility parameters are then generated by:

$$\alpha^m_l = \frac{exp(\nu_{l^m})}{d^p}, \alpha^f_l = \frac{exp(\nu_{lf})}{d^p}, \alpha_c = \frac{exp(\nu_c)}{d^p}, \alpha_\theta = \frac{1}{d^p} \tag{23}$$

Children's preferences take a slightly different empirical specification. I normalize each child's preference for leisure to be 1, then, each child's preference for their cognitive skills, λ_θ, is drawn from an i.i.d Normal distribution and then transformed according to

$$\nu^k \sim N(\mu^k, \sigma^k)$$

$$\lambda_\theta = exp(\nu^k)$$

where (μ^k, σ^k) are parameters to be estimated.

The parents have preferences over each parenting style and these preferences are allowed to vary with the level of parental education as well as the age of the child. Preferences for a permissive parenting style are normalized to 0 for all households, and thus, one can think of parent preferences for parenting styles being relative to their preference for adopting a permissive parenting style. The empirical specification is given by:

$$\xi(p, X_t) = \xi^p_0 + \xi^p_1 \mathbb{1}\{e^m \geq 16\} + \xi^p_2 t + \upsilon^p \tag{24}$$

where p $\in \{AR, AT\}$ and $\upsilon^p \sim Logistic(0, 1)$. This allows for heterogeneous preferences for

parenting style choices that are correlated with parental education (here represented by the years of education of the mother) and the age of the child.

The parents' choice of a parenting style can augment the child's preference for their leisure time. Permissive parenting is characterized by allowing children to make their own decisions with minimal parental interference, therefore, I assume the parameter governing how a permissive parenting style influences children's preference for leisure is normalized to 1, allowing them to make their own choices according to their own preferences. I allow authoritarian and authoritative parenting to affect the child's preference for leisure. This effect is constant across childhood and common across households. I do not restrict these parameters ex-ante to reduce or increase the child's preference for leisure, it is simply assumed that ω_{AR}, $\omega_{AT} > 0$. This allows for authoritarian and authoritative parenting to increase or decrease (relative to permissive parenting) child study time, all else equal. As an example, suppose the controlling nature of an authoritarian parenting style decreases a child's willingness to read or do other activities that are beneficial for their cognitive development. While they may face a rule that demands them to do a certain number of hours of these activities per week, they do not go above this on their own accord (this could also be true of authoritative parenting ex-ante).

5.2 Skill Production Function Parameters

I allow the parameters governing the production of child skills to vary monotonically with the age of the child. [15] The marginal product of parental time investment is allowed to be a function of the parent's level of education. Finally, the choice of parenting style is allowed to affect the TFP of the production function, and this effect is constant across childhood. The functional forms are as follows:

$$\delta_t^{\iota} = exp(\gamma_0^{\iota} + \gamma_1^{\iota} t) \text{ for } \iota \in \{k, g, \theta\}, \text{ for t} \in \{1, ..., T\} \tag{25}$$

and for the parent j's time investment (j $\in \{m, f\}$)

$$\delta_t^{j} = exp(\gamma_0^{j} + \gamma_1^{j} t + \gamma_2^{j} E^{j}) \text{ for j} \in \{m, f\}, \text{ for t} \in \{1, ..., T\} \tag{26}$$

where E^j is parent j's level of education in years. The empirical specification for TFP is

$$\delta_t^{R} = exp(\gamma_1^{R} + \gamma_2^{R} t + \gamma_p^{R} \mathbb{1}\{ps_t = p\}), \text{for p } \in \{AR, AT\}, \text{ for t} \in \{1, ..., T\} \tag{27}$$

[15] This is a common assumption in the literature, see Del Boca et al. (2014), Brilli (2022), and Del Boca et al. (2024).

where I allow the choice of parenting style to affect the production process directly through its impact of TFP.[16]

5.3 Wages and Non-Labor Income

The log-wage offer distribution for parents is assumed to be only a function of parent characteristics and is not history dependent. This is a common assumption of those who use the PSID-CDS data in the context of child development, and it reduces the computational burden of the solution to the household's problem[17]. Parent wages are assumed to be a function of their age and education, and log-wage offers are drawn each period from the following distribution:

$$\begin{pmatrix} ln(w_t^f) \\ ln(w_t^m) \end{pmatrix} \sim Normal\left(\begin{pmatrix} \eta_t^f \\ \eta_t^m \end{pmatrix}, \begin{pmatrix} \sigma_f^2 & \rho\sigma^w \\ \rho\sigma^w & \sigma_m^2 \end{pmatrix} \right) \tag{28}$$

where

$$\eta_t^j = \eta_0^j + \eta_1^j A_t^j + \eta_3^j E^j \tag{29}$$

for j $\in \{m, f\}$, A_t^j represents parent j's age when the child is t years old, and E^j is parent j's years of schooling.

I follow Verriest (2022) for the non-labor income process, which is assumed to only be a function of the father's demographic characteristics. Non-labor income is drawn using a two-step procedure. First, I draw a Bernoulli random variable to determine whether or not a household has a positive amount of non-labor income in a given period. The probability that a household has any positive non-labor income when their child is t years old is given by

$$Pr(y_t > 0) = \frac{exp(\xi_{y_t})}{1 + exp(\xi_{y_t})} \tag{30}$$

where

$$\xi_{y_t} = \mu_0^y + \mu_1^y A_t^f + \mu_2^y (A_t^f)^2 + \mu_3^y E^f \tag{31}$$

Conditional on a household having positive non-labor income in a given period, the level of non-labor income is drawn according to

$$y_t = exp(\mu_4^y + \mu_5^y A_t^f + \mu_6^y (A_t^f)^2 + \mu_7^y E^f + \epsilon_t^y) \tag{32}$$

where $\epsilon_t^y \sim N(0, \sigma_y^2)$.

[16] This follows from Agostinelli et al. (2023).

[17] This same assumption is made by Del Boca et al. (2014), Brilli (2022), and Del Boca et al. (2024). For an example with a relaxation of this assumption, check out Verriest (2022).

6 Measurement of Child Skills

I assume that child skills, θ_t, are observed by all members of the household, but the econometrician only has access to a noisy measure of each skill, the LW exam. In the model, child skills are a continuous variable, however, in the data, the noisy measures take on discrete values. To derive a mapping from latent child skills to the measures I observe in the data, I follow Lord and Novick (1968). As a matter of notation, I refer to the continuous measures of child skills as θ_t and θ_t^* represent the measures of these skills observed in the CDS.

In the data, cognitive skills are measured by the total number of questions a child answers on the LW exam. There are 57 questions on the LW exam and, as in Del Boca et al. (2014) I make the assumption that each question has an equal level of difficulty and that the probability a child answers a question is a function of their cognitive skills θ_t. Let the probability a child with cognitive ability θ_t answers a question on the LW exam correctly be given by

$$p(\theta_t) = \frac{\theta_t}{1+\theta_t} \tag{33}$$

This assumption has attractive properties such that the probability a question is answered correctly is increasing in child skills, and is bounded between 0 and 1. The general form for the probability is given by

$$p(\theta_t, \lambda) = \frac{exp(\lambda_0 + \lambda_1 ln(\theta_t))}{1 + exp(\lambda_0 + \lambda_1 ln(\theta_t))} \tag{34}$$

however, as with all factor models, the values of λ_0 and λ_1 must be restricted in order to identify the model parameters of interest[18]. Restricting $\lambda_0 = 0$ and $\lambda_1 = 1$ yields (5.12).

Now that the mapping from latent skills to observed data is complete, I can discuss the simulation process for child skills. The initial period for each household takes place in 1997 when their child is some initial age t_0. I assume that the household has an initial prior for $p(\theta_t)$ that is drawn from a Beta(1,1) distribution and I use $\theta_{t_0}^*$ as the initial condition for each household. From this initial measure, I update the household's priors for p according to $\tilde{p} \sim Beta(1+\theta_t^*, 57-\theta_t^*+1)$. The initial latent continuous measures for child skills are then given by

$$\theta_{t_0} = \frac{\tilde{p}}{1-\tilde{p}} \tag{35}$$

From these initial values for θ_t, I can construct a sample path for each household, and simulate decisions and outcomes according to the model structure laid out in Section 3 from

[18] See Agostinelli and Wiswall (2023) for a detailed explanation in the context of a model of child skill development.

the child's initial age, t_0, up through when the child is 18 years old.

7 Identification

7.1 Skill Production Parameters

The parameters governing the production of child skills are allowed to vary with the age of the child and, in the case of the marginal productivity of parental time, the level of education of the parents. Identification of the skill production parameters would be straightforward if I observed all inputs of the production function and child outcomes at both age t and age t+1. Instead, when a child is age t in 1997 (or 2002), I observe parent and child time investment, the parents' choice of a parenting style, and the measure of child skills in that year, while I do not observe parental monetary investments.[19] Additionally, I do not observe child skill measures in consecutive years, instead they are observed every five years.

There is substantial variation in parent and child time investment decisions, parenting style decisions, and child test scores across households and across childhood that allows for identification of the structural parameters governing the skill production function. Given that parental preferences over their leisure time, consumption, and child skills are fixed throughout childhood, and uncorrelated with household observables (outside of preferences over parenting styles), one can use variation in parental time investment across child age and by level of parental education with variation in test score outcomes to identify the parameters governing the marginal product of parental time investment. Similar arguments apply for identifying the parameters that determine the marginal product of children's study time. There is variation in child study time across children and across ages, and this variation coupled with variation in test scores pins down the marginal product of child study time. Variation in child test scores at age t and age t+5 also aids in identification of the marginal product of the stock of child skills.

Total factor productivity is allowed to vary with child age and the parents' discrete choice of a parenting style. There is variation in parenting style decisions and child test score outcomes, identifying the marginal effect of authoritarian and authoritative parenting (relative to permissive parenting) on child skill outcomes. The intercept and slope parameters of TFP show up in a linear and separable form that is independent of parent and child choices, implying that any changes in TFP would only affect simulated child skills, which affect simulated child outcomes, so these parameters are identified by average child test scores at each age.

[19] While the CDS does have some measures of monetary investments, they are quite noisy and typically not used as moment conditions in the literature.

All that remains to identify in the child skill production process is the marginal product of child goods investment. While I do not have explicit data on the monetary expenditures parents make in their children, I reference data from the Consumer Expenditure Survey and match simulated moments from those in the survey.[20] Theoretically, the marginal product of monetary investments in children can be pinned down through remaining variation in child test scores that is not explained by parental or child choices or TFP.[21]

7.2 Wages and Non-Labor Income

Parent log-wage offers are drawn from a Multivariate Normal distribution:

$$\begin{pmatrix} ln(w_t^f) \\ ln(w_t^m) \end{pmatrix} \sim Normal \left(\begin{pmatrix} \eta_t^f \\ \eta_t^m \end{pmatrix}, \begin{pmatrix} \sigma_f^2 & \rho\sigma^w \\ \rho\sigma^w & \sigma_m^2 \end{pmatrix} \right)$$

where

$$\eta_t^j = \eta_0^j + \eta_1^j A_t^j + \eta_3^j E^j$$

for j $\in \{m, f\}$, A_t^j represents parent j's age when the child is t years old, and E^j is parent j's years of schooling. In the main PSID files, I have access to labor supply decisions, accepted wages, as well as information about the age of parents and their years of schooling. The main identification issue is the same that plagues most researchers in labor economics, I do not observe wage offers for individuals who choose not to work. This problem is made more difficult because I assume heterogeneity in the parameters that govern household preferences, therefore, it may be the case that some individuals choose not to work because they receive a low wage offer, because they place a high value on their leisure time, that their time with the child has a high level of productivity so they choose to stay home, or a combination of these reasons. The issue of identification is slightly ameliorated because I do not allow work experience to influence the wage process[22]. Using the data generating process outlined above, I am able to identify the parameters governing the wage offer distribution by simultaneously estimating them with the remaining parameters in the model. I observe variation in accepted wages by parent age and education, as well as variation in accepted wage offers within households over time, simulating both the wage offer distribution as well as parent labor supply decisions allows for identification of said process. The key moments used in identification are the mean and standard deviation of parent (accepted) wages, av-

[20] This follows from Del Boca et al. (2024).

[21] In practice, I match moments concerning the correlation in accepted hourly wages and child observed test scores.

[22] See Del Boca et al. (2014), Del Boca et al. (2024), and Brilli (2022) who make the same assumption with similar samples of the PSID-CDS data.

erage wages within age bins, average wages within parent education bins, correlations across parent wages, and the standard deviation of each parent's wage.

The non-labor income process is simpler to identify as there are no dynamic processes involved an no selection issues to deal with. The non-labor income process is estimated in a first stage before estimating the remaining model parameters. Non-labor income is observed for all households in which it appears in the PSID (biannually after 1997 corresponding to the previous year, that is, an observation in 1999 corresponds to non-labor income in 1998), therefore, estimation of the two stage non-labor income process is straightforward. Given that there is variation in household non-labor income by father's age and education, a logistic regression for positive non-labor income on father's age, education, and age-squared yields consistent estimates of the first stage parameters of the non-labor income process. Then, conditional on a household having positive non-labor income, I again use the variation in the levels of non-labor income by the father's characteristics to identify the parameters that govern the level of non-labor income a household has in a given period.

7.3 Preferences

Parents have preferences over their leisure time (α_l^m and α_l^f), consumption (α_c), their child's skills (α_θ), and derive utility cost from their choice of parenting style ($\xi(ps_t, X_t)$). Children have preferences for leisure time (λ_l which is normalized to 1) and their skills (λ_θ) and face the leisure tax parameters (ω_{AR} and ω_{AT}) if their parents choose an authoritarian or authoritative parenting style.

I begin with identification of the child's preference parameters, along with ω_{AR} and ω_{AT}. Consider a child who always faces a permissive parenting style, then, their choice of study time at any age t is given by

$$\tau_t^k = \frac{\beta^k D_{t+1}^k \delta_t^k}{1 + \beta^k D_{t+1}^k \delta_t^k}(112 - \tau_t^m - \tau_t^f - s_t)$$

where β^k is given, D_{t+1}^k is a function of skill production parameters (which from the previous section are identified) and the child's preferences for their own cognitive skills. Since children's preference for leisure is normalized to 1, any remaining variation in child time investment will be driven by heterogeneity in time investment decisions and I use the first two moments of child study time decisions to identify the parameters governing children's preference for their own skills.

To identify the parameters governing how parenting style affects children's preference for leisure, I first re-arrange the child's solution, when they face a permissive style, it is given

by

$$1 = (\frac{TT - s_t - \tau_t^m - \tau_t^f}{\tau_t^k} - 1)\beta^k D_{t+1}^k \delta_t^k \tag{36}$$

Then, when they face parenting style ps $\in \{AR, AT\}$ the equation becomes

$$\omega_{ps} = (\frac{TT - s_t - \tau_t^m - \tau_t^f}{\tau_t^k} - 1)\beta^k D_{t+1}^k \delta_t^k \tag{37}$$

therefore, ω_{AR} and ω_{AT} are identified by average child choices conditional on the parenting style they face.

The remaining parameters to identify are the ones that govern parent preferences for leisure, consumption, and their child's skills, $\alpha = \{\alpha_l^m, \alpha_l^f, \alpha_c, \alpha_\theta\}$, and the preferences for adopting an authoritarian or an authoritative parenting style $\xi(ps_t, X_t)$. First, consider a household that always adopts the same parenting style in each period. While this does not yield information about $\xi(ps_t, X_t)$, the variation in labor supply and time investment decisions aid in the identification of the parameters governing $\alpha = \{\alpha_l^m, \alpha_l^f, \alpha_c, \alpha_\theta\}$. Conditional on the choice of parenting style, the numerical solution for (τ_t^m, τ_t^f) pins down labor supply. Using average parental time investment, average labor supply, and the fraction of parents supplying any labor allows me to identify the parameters that govern average parent preferences, and variation in these decisions aids in identification of the parameters governing the standard deviation of parent preferences[23].

What remains to be identified are the parameters that govern the parenting style cost distributions, $\xi(ps_t, X_t)$ (recall that the cost for adopting a permissive parenting style is normalized to 0), which are allowed to be a function of whether or not the mother has a college degree, and the age of the child. As pointed out in Del Boca et al. (2024), and explained in great detail in Del Boca and Flinn (2012), each choice of parenting style generates a unique mapping for other household choices by the parents and the child. Unlike in Del Boca and Flinn (2012), I observe parenting style choices, and, by allowing parents to choose a parenting style in each period, and matching simulated moments with those observed in the data (such as parenting style frequency, correlations with child age, i.e the correlation of authoritarian versus non-authoritarian or authoritative versus non-authoritative, correlations with parental education, etc.) allows me to identify parents' preferences for authoritarian and authoritative parenting styles [24]. Additionally, I compared a simulated multinomial

[23]These arguments are straightforward and are similar to those made by Del Boca et al. (2014), Brilli (2022), Verriest (2022), and Del Boca et al. (2024), who all use the PSID-CDS data, and Berlinski et al. (2024), who use different data but impose a similar structure on preferences.

[24]An alternative approach would be to implement a two-stage estimator as done by Flinn and Mullins (2015), and take the average difference in parent value functions when they choose an authoritarian or

logit model of parenting style choices on household characteristics with those observed in the CDS.

8 Estimation

I estimate the structural parameters of the model outlined above using Simulated Method of Moments (SMM). The estimation routine is done in two stages, first, recall that the non-labor income process only depends on the father's demographic characteristics, and is exogenous with respect to household decisions, this allows me to estimate the parameters governing the non-labor income process separately from the remaining model parameters in a first stage, and then use these results in estimation of the full structural model. Let P be the number of parameters to be estimated in the second stage, and $\mathbf{\Omega}$ be a P dimensional vector of structural parameters. My estimator $\hat{\mathbf{\Omega}}_{\mathbf{SMM}}$ is defined as

$$\hat{\mathbf{\Omega}}_{\mathbf{SMM}} = argmin_{\Omega}(m(d) - m(\Omega))'W(m(d) - m(\Omega)) \tag{38}$$

where m(d) is a Z > P dimensional vector of sample moment conditions taken from the data, and m(Ω) is a corresponding set of moment conditions generated by a simulated data set at the parameter vector Ω. The weighting matrix, W, is a diagonal matrix with weights that are bootstrapped standard deviations of the sample moments[25].

I use 30 moments to estimate 9 parameters that govern the non-labor income process. I match moments concerning the probability that a household has non-labor income in a given period as a function of father's characteristics for the logit step, and moments that give information about the mean and standard deviation of the level of non-labor income a household has as a function of the father's characteristics. The second stage of the model has 43 parameters to be estimated and I use 212 moment conditions in my estimation routine. Some of the moments I match are the mean and standard deviation of time allocation decisions for the father, mother, and child, by age, study time by parenting style in each development period, the frequency of parents supplying labor as well as the mean and standard deviation of hours worked, and average skills by child age. Additionally, I match moments about parenting style choices, such as the frequency each style is adopted conditional on

authoritative parenting style, and these moments would converge in probability to the true parameters that govern the utility cost incurred by parents to adopt an authoritarian or authoritative parenting style. This approach would increase the computation time to estimate the model and is unnecessary as I can simply fit moments concerning parenting style choices, such as the overall average take up of each style, as well as the average by child age.

[25] I use 200 bootstrap samples to compute the weight matrix.

child age and the education level of the mother.[26]

Once the parameters governing the non-labor income process have been estimated, estimation of the full structural model is straightforward. I simply take a guess of the parameter vector Ω, and given this guess, simulate the decisions of each household according to the structural model explained in Section 3. I simulate each household R times, where R is set equal to 5, and the only difference for each copy r $\in \{1, ..., R\}$, are the error draws for the wage, non-labor income, and measurement error processes. With this simulated data in hand, I compute the moment conditions and evaluate the weighted distance according to (38). This process then iterates until a convergence criterion is satisfied[27]. To compute the standard errors of my estimates, I re-estimate the model 50 times using non-parametric bootstrapped samples with replacement and compute standard errors following Train (2009) (p. 201).

9 Results

9.1 Skill Productivity

Table 5 below reports the parameter estimates governing the child skill production parameters. Consistent with the literature, I find that the marginal productivity of parental time investment is decreasing with the age of the child and the marginal product of mother's time investment is greater than that of the father. While I estimate that time investment is more productive for higher educated mothers, the opposite is true for fathers. Caucutt et al. (2020) find no evidence that education makes maternal or paternal time investment in children more productive, so the result that the coefficient on father's education being close to zero may just be further evidence of their result.

Past cognitive skills and monetary expenditures on child goods become more productive the older the child becomes. The result that the marginal product of lagged skills is increasing in age supports the result in Cunha et al. (2010) that early investments in skills matter because it will become harder for children to catch up in later years [28].

[26] The full list of the moment conditions used are shown in Appendix E.

[27] I use the NLopt NelderMead algorithm on Julia v1.11.1 to estimate the structural model. The convergence criterion is set to 1e-4.

[28] This also aligns with the result found in Del Boca et al. (2014)

Table 5. **Skill Productivity Parameter Estimates**

Parameter	Estimate	(Std Error)
Mother	-2.515	(0.035)
	-0.079	(0.003)
	0.030	(0.002)
Father	-2.198	(0.043)
	-0.071	(0.003)
	-0.000	(0.000)
Child	-7.473	(0.053)
	0.276	(0.003)
Child Goods	-4.424	(0.041)
	0.054	(0.002)
Past Cognitive Skills	-0.184	(0.005)
	0.003	(0.000)
Authoritarian	-0.081	(0.016)
Authoritative	-0.008	(0.003)
TFP	-1.775	(1.334)
	2.730	(1.549)

Notes: Standard errors are computed by bootstrapping each household with replacement 50 times.

The more novel results in the literature concern the marginal product of children's own time investment and the marginal effects of authoritarian and authoritative parenting on child cognitive development. Consistent with Del Boca et al. (2024), the marginal product of children's time investment is increasing with age, supporting the idea that as children become older, their own time investment becomes relatively more important than the time investment of their parents [29]. The estimates additionally report that authoritarian parenting has a significant negative marginal affect on the TFP process (relative to permissive parenting), in accordance with the result in Agostinelli et al. (2023). The marginal affect of authoritative parenting is also negative and significant relative to permissive parenting, but closer to zero than the marginal affect of authoritarian parenting. This may provide evidence that hands off parenting gives children more freedom to learn on their own, having a beneficial impact on the production of their cognitive skills [30].

[29] A result that is consistent with the finding in Del Boca et al. (2017)

[30] The difference between the results here and those in Agostinelli et al. (2023) are that their model

9.2 Preferences

Table 6 reports the transformed parameter estimates governing household preferences for leisure, consumption, the stock of child skills, as well as the parents' and child's terminal value on the final stock of children's cognitive skills [31].

Table 6. **Preference Parameter Estimates**

Parameter	Estimate	(Std Error)
Mother's Leisure	0.234	(0.003)
Father's Leisure	0.287	(0.004)
Parent Consumption	0.335	(0.005)
Parent Preference Child Skills	0.145	(0.001)
Child Skill Preference	0.688	(0.110)
Parent Terminal Value	30.082	(1.337)
Child Terminal Value	2.052	(0.338)

Notes: Standard errors are computed by bootstrapping each household with replacement 50 times.

The estimates are consistent with those found in the literature, for the parents, the highest weight on average is the weight on consumption, followed by the father's preference for leisure, and the lowest weight is placed on child skills. The result that the value on the mother's leisure time being lower than the father's is common in the literature and driven by the fact that mother's spend more time with children than the father does (and likely also do more other household chores, which is omitted from this framework). The value that children place on their leisure time is normalized to 1, so their preference for skills being less than one implies that children on average care more about their free time than their cognitive skills. Finally, the estimates state the predictable result that parents care more about the final stock of their child's cognitive skills than the children do, supporting the idea that children are more myopic than their parents in this scenario.

Table 7 reports the parameter estimates that govern how authoritarian and authoritative parenting influence the child's marginal utility of leisure as well as parents relative preference for authoritarian and authoritative parenting compared to adopting a permissive parenting style. The first row shows that an authoritarian parenting style increase the child's marginal utility of leisure (decreasing their study time all else equal), while an authoritative parenting

allows for parental decisions to depend on the current stock of skills, an assumption that would increase the computational complexity of the model. In their framework, low cognitive and non-cognitive skills drive the adoption of authoritarian parenting, suggesting reverse causation. The opposite result is presented here, where authoritarian parenting reduces the stock of children's skills.

[31] Reporting of the transformed parameters is the common approach in the literature (see Del Boca et al. (2014)) because the underlying structural parameters themselves are difficult to interpret.

style reduces the child's marginal utility of leisure (increasing study time). The idea that authoritarian parenting reduces child study time might be driven by the fact that children feel too controlled, and while they may meet their parents homework time requirement, they do not feel compelled to go the extra mile and would rather engage in leisure time. The opposite can be said for an authoritative parenting style, which according to the estimates promotes children to spend more time in educational activities than they would have if they faced a permissive parenting style.

The final three rows report the estimates for the parents' relative preference for authoritarian versus authoritative parenting relative to permissive parenting (utility normalized to 0). The main take away is that it is costly for college educated parents to adopt an authoritarian parenting style, and it becomes more difficult to adopt an authoritarian parenting style as children get older. This first result is consistent with the sociology and economic literature that authoritarian parenting is adopted more frequently by lower educated households. [32]

Table 7. **Parenting Style Parameter Estimates**

	Authoritarian		**Authoritative**	
Parameter	**Estimate**	**(Std Error)**	**Estimate**	**(Std Error)**
Leisure Utility	1.084	(0.120)	0.748	(0.073)
Preference Intercept	-0.247	(0.059)	-0.538	(0.067)
Preference Age	0.528	(0.140)	-0.127	(0.146)
Preference College Education	-0.020	(0.004)	0.003	(0.002)

Notes: Standard errors are computed by bootstrapping each household with replacement 50 times.

The second results, that authoritarian parenting becomes less attractive while authoritative parenting becomes more attractive (though, for authoritative parenting this is not significant) is likely driven on two dimensions. Authoritarian parenting is a demanding and controlling parenting style, as children age, they may rebel more against this style, causing friction and strain within the household (Sarwar (2016)). Authoritative parenting may become more attractive when children age because parents may wish to exert a little control over older children rather than letting them make all of their own decisions.

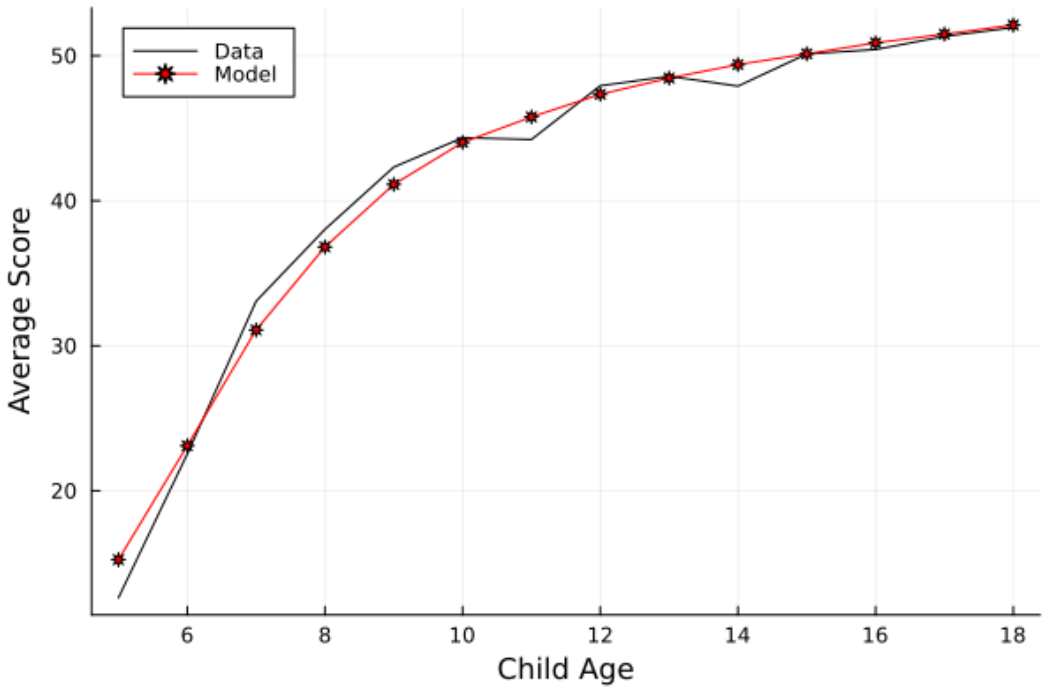
[32] See Harris (2024) and Agostinelli et al. (2023)

10 Model Fit

10.1 Child Skills

Figure 2 below shows that the model is able to accurately capture the upward sloping and concave nature of the evolution of child skills. This suggests that while a simple log-linear production function has been implemented, it is still able to capture the trends present in the raw data.[33]

Figure 2. **Fit of Average Child Skill Evolution**



Notes: Data refers to average child LW scores from the author's CDS sample. Model refers to the average fit of simulated child scores generated by the structural parameters estimates.

10.2 Time Investment

Table 8 displays how the model performs in terms of average time investment in children by parents and their children. The model captures the decreasing time investment of mother's as children age, as well as the increasing self-investment by children. While the model closely predicts average father's time investment by age bin, the model is not always decreasing with child age, increasing between the 6-8 and the 9-11 age, before decreasing again. This is likely driven by the fact that the father's average time investment is relatively flat compared to the

[33]Notice that while the model begins when children are 4 years old, this data point is omitted from the figure. This is because the first test score is used as an initial condition, so they model perfectly matches the data.

mother's and child's. That being said, the model predictions are close to what is observed in the raw data.

Table 8. **Fit of Average Household Time Investment**

	Age	**Data**	**Model**
Mom	4-5	28.65	23.36
	6-8	21.72	22.22
	9-11	19.32	21.03
	13-18	14.40	16.15
Dad	4-5	16.93	14.60
	6-8	14.57	14.43
	9-11	13.56	14.56
	13-18	10.87	12.95
Child	4-5	0.60	0.98
	6-8	1.28	1.24
	9-11	4.03	2.75
	13-18	6.10	6.51

Notes: Data refers to average household time investment from the author's CDS sample. Model refers to the average fit of simulated time investment generated by the structural parameter estimates.

Table 9 shows average child study time conditional on age and parenting style. Across both age bins, the model generates the ordering of study time by parenting style that is observed in the data. Namely, that on average children who face an authoritative parenting style study the most, and children who face an authoritarian parenting style studying the least. The model generates a closer fit for older children, a result that may be driven by the empirical assumption that the effect of the parenting style choice on the child's marginal utility of leisure is constant across childhood. It is likely that how parenting style decisions affect a child's marginal utility of leisure varies over the course of childhood. It might be the case that authoritarian parenting increases child time investment when children are young but reduces it for teenagers. Allowing for this assumption would increase the computational complexity of the model, but is an avenue for future research. That being said, the model still fits the data well across childhood.

Table 9. **Fit of Average Child Time Investment by Parenting Style**

Age	Parenting Style	Data	Model
4-11	Authoritarian	2.02	1.64
	Authoritative	2.55	2.17
	Permissive	2.40	1.66
12-18	Authoritarian	5.05	5.29
	Authoritative	6.64	6.68
	Permissive	6.11	5.87

Notes: Data refers to average household time investment from the author's CDS sample. Model refers to the average fit of simulated time investment generated by the structural parameter estimates.

10.3 Parenting Style Choices

The following tables display how the model performs in terms of fitting the observed data on parenting style choices. Table 10 shows unconditional parenting style choices, the model performs excellently, with the only discrepancy being a rounding error.

Table 10. **Fit of Parenting Style Choices by Child Age**

	Data	Model
Authoritarian	0.28	0.28
Authoritative	0.46	0.46
Permissive	0.27	0.26

Notes: Data refers to the frequency of parenting style choices observed in the author's sample of the CDS. Model refers to the simulated frequency generated by the structural parameter estimates.

Table 11. **Fit of Parenting Style Choices by Child Age**

Age	Parenting Style	Data	Model
4-11	Authoritarian	0.31	0.28
	Authoritative	0.44	0.46
	Permissive	0.24	0.26
12-18	Authoritarian	0.25	0.27
	Authoritative	0.47	0.46
	Permissive	0.28	0.26

Notes: Data refers to the frequency of parenting style choices observed in the author's sample of the CDS. Model refers to the simulated frequency generated by the structural parameter estimates.

The model also performs well when predicting parenting style decisions conditional on the age of the child, as shown above in Table 11. The model captures the trend of decreasing

use of authoritarian parenting when children are older and increasing use of authoritative an permissive parenting.

Finally, and most importantly, the model accurately reflects parenting style choices conditional on the education level of the mother as shown in Table 12. Education is a significant predictor of authoritarian parenting (Harris (2024)), and the model lines up with the data, predicting that households where the mother does not have a college degree adopt an authoritarian parenting style 33% of the time, matching the data, and predicts that households where the mother has a college degree adopt an authoritarian parenting style only 16% of the time (17% in the data).

Table 12. **Fit of Parenting Style Choices by Mother's Education**

Education	Parenting Style	Data	Model
No College	Authoritarian	0.33	0.33
	Authoritative	0.42	0.43
	Permissive	0.25	0.24
College	Authoritarian	0.17	0.16
	Authoritative	0.53	0.53
	Permissive	0.30	0.31

Notes: Data refers to the frequency of parenting style choices observed in the author's sample of the CDS. Model refers to the simulated frequency generated by the structural parameter estimates.

11 Counterfactuals

I perform two counterfactual policy experiments in the sections below. In the first, I force household to choose a given parenting style each period, and allow them to make all other choices optimally. In the second, I examine the effects of a homework program that exogenously increases child study time.

11.1 Forced Parenting Style

In this exercise, I force parents to choose either an authoritarian, authoritative, or permissive parenting style each period and let the household make all other choices. Table 13 below shows how this experiment affects the terminal stock of child skills and child time investment. The results provide evidence that an authoritarian parenting style is the worst style for promoting child development. The average final stock of child skills is reduced by 81% of a standard deviation. This is driven by the results that authoritarian parenting reduces child study time all else equal (also shown in the table below), as well as the negative marginal effect of authoritarian parenting on child skills (relative to permissive parenting). The final

row shows that no children see an improvement in the final stock of their cognitive skills as a result of forcing their parents to be authoritarian each period.

Authoritative and permissive parenting both increase the average terminal stock of child skills relative to baseline. On average, child skills improve by 31% of a standard deviation when parents are forced to be authoritative, and they increase by 29% of a standard deviation when they are forced to be permissive. The channels for the improvements relative to baseline are different for the different parenting styles. While authoritative parenting has a negative marginal effect on next period child skills relative to permissive, it increases the time children spend studying all else equal, therefore, for 50% of households (see final row of Table 13), the returns from child study time outweigh this negative marginal effect. The gains from permissive parenting are driven by avoiding the negative marginal effects associated with authoritarian and authoritative parenting, and 35% of households see an improvement in the final stock of their child's skills.

Table 13. **Forced Parenting Style**

	Baseline	**Authoritarian**	**Authoritative**	**Permissive**
Average Terminal Cognitive Skills	14.47	9.79	16.28	16.15
Average Child Time Investment	4.45	3.83	5.07	4.08
Fraction Cognitive Skills Improved		0.00	0.50	0.35

Notes: This table displays statistics simulated from the model at the structural parameter estimates. The Baseline column refers to statistics generated at the model parameters. The remaining columns reflect outcomes when parents are forced to adopt an authoritarian, authoritative, or permissive parenting style.

11.2 After School Homework Program

The second experiment explores what is called an after school homework program. In this experiment, I force children to spend time in the program. When they are in the program, they are contributing to their study time, effectively setting a higher lower bound on the time they allocate toward developing their cognitive skills. In this exercise, when children are 10 years old and younger, they spend one hour per week in the homework program, and those older than 10 spend two hours per week.

This changes the choice problem that households face. Total child study time is now defined as

$$\tilde{\tau}_t^k = \tau_t^{k,p} + \tau_t^k$$

where $\tau_t^{k,p}$ refers to the time the child spends in the homework program and $\tilde{\tau}_t^k$ is now total

child study time. The production function is now given by

$$ln(\theta_{t+1}) = ln(R_t(ps_t)) + \delta_t^m ln(\tau_t^m) + \delta_t^f ln(\tau_t^f) + \delta_t^k ln(\tilde{\tau}_t^k) + \delta_t^g ln(g_t) + \delta_t^\theta ln(\theta_t)$$

Taken together, the solution to the child's problem for their own study time is

$$\tau_t^k = \frac{\tilde{TT} FV_{t+1}^k - \omega(ps_t)\tau_t^{k,p}}{\omega(ps_t) + FV_{t+1}^k} \tag{39}$$

where $\tilde{TT} = 112 - \tau_t^m - \tau_t^f - s_t - \tau_t^{k,p}$ and $FV_{t+1}^k = \beta^k D_{t+1}^k \delta_t^k$. As a result of the homework policy, a child may choose to spend zero hours per week at home in educational activities if conditions are such that the numerator of (39) is weakly negative.

Table 14 below reports summary statistics for the counterfactual experiment. The average final stock of child skills improves by 16% of a standard deviation, but, consistent with Kalenkoski and Pabilonia (2017) and Eren and Henderson (2008), not all children see an improvement in the final stock of their skills as a result of having them spend this fixed time in the homework program (though an improvement is made in 82% of households). Total child time investment increases on average by about 19 minutes per week. What is notable is that this policy does not influence parenting style decisions. This is because the benefit parents have for certain parenting styles can outweigh potential costs to child skill development.

Table 14. **After School Homework Program**

	Baseline	**Homework Policy**
Average Terminal Cognitive Skills	14.47	15.43
Average Child Time Investment	4.45	4.77
Fraction Cognitive Skills Improved		0.82
Fraction Authoritarian	0.28	0.28
Fraction Authoritative	0.46	0.46
Fraction Permissive	0.26	0.26

Notes: This table displays statistics simulated from the model at the structural parameter estimates. The Baseline column refers to statistics generated at the model parameters. The remaining columns reflect outcomes when children are in the after school homework program.

Table 15 below reports how children fair as a result of the after school homework policy conditional on the parenting style they face the most often. The result of the program is that children whose parents are predominately authoritarian see the largest increase in the average terminal stock of their skills (a gain of 7.4%). Children whose parents are predominately authoritative or permissive see increases in the average terminal stock of their skills by 6.2%

and 6.7% respectively. This is a promising result as this paper presents another angle to analyze the skill gap across children. Parents without a college degree are more likely to adopt an authoritarian parenting style, and this paper shows a negative association between authoritarian parenting and child skills. The counterfactual experiment in this section shows that a program that can increase children's academic investment can improve skills even for children who face an authoritarian parenting style.

Table 15. **After School Homework Program by Parenting Style**

	Baseline	**Homework Policy**
Authoritarian	9.99	10.74
Authoritative	16.17	17.17
Permissive	16.36	17.46

Notes: This table displays statistics simulated from the model at the structural parameter estimates. The Baseline column refers to statistics generated at the model parameters conditional on the parenting style the household uses the most ofter. The policy column shows the average terminal skill after the homework policy is put in place.

12 Conclusion

This paper extends the literature on child skill development by allowing parents to choose one of the three major parenting styles defined by Baurmind (1967), namely, authoritarian, authoritative, and permissive. In this framework, parenting style choices shift the time allocation decisions of children, and have a direct effect on the production of cognitive skills, and indirect effects through the marginal productivity of child study time. Counterfactual experiments present evidence for ranking the three major parenting styles in terms of producing cognitive skills, with authoritative being the best on average, and authoritarian the worst.

The model is not without limitations, however, these present avenues for future research. Relaxing the assumption of log separability of child skills is an important avenue for future work, especially in the context of parenting style decisions, as it may be the case that parenting style decisions are a function of the level of child skills. While the cost of relaxing this assumption is an increase computational burden of the solution of the model, there may be more policy levers available to change parenting style decisions that my current framework is unable to capture.

It is also worth exploring if different parenting style decisions come with different time costs due to differences in monitoring, though, this avenue may not be fruitful due to data limitations. One could posit that it might be more time intensive to adopt an authoritative

parenting style, as parents need to put forth more effort to explain the rational behind their rules and limitations, whereas the demanding nature of authoritarian parenting might generate lower time costs.

It may also be the case that parenting style decisions affect the productivity of parental time investment. Perhaps an authoritarian parenting style reduces the productivity of parental time because children do not enjoy being around their parents due to their demanding nature. The opposite might be true for authoritative parenting with the channel here being the warmth that is typically associated with this parenting style.

Finally, to my knowledge, there is no work being done in the structural economics literature to account for parenting style decisions in multi-child households. Future work should consider this avenue, as well as the strategic interaction between parents when it comes to choosing individual parenting styles. It would be interesting to see a game within game structure where both parents choose an individual parenting style, and the consequences this has on child development.

References

Agostinelli, Francesco and Matthew Wiswall, "Estimating the Technology of Children's Skill Formation," *Accepted at Journal of Political Economy*, 2023.

_ **, Matthias Doepke, Giuseppe Sorrenti, and Fabrizio Zilibotti**, "It Takes a Village: The Economics of Parenting with Neighborhood and Peer Effects," *R and R at Journal of Political Economy*, 2023.

Akabayashi, Hideo, "An equilibrium model of child maltreatment," *Journal of Economic Dynamics and Control*, 2006, *30*, 993–1025.

Baumrind, Diana, "The influence of parenting style on adolescent competence and substance use," *The Journal of Early Adolescence*, 1991, *11* (1), 56–95.

Baurmind, Diana, "Childcare Practices Anteceding Three Patterns of Preschool Behavior," *Genetic Psychology Monographs*, 1967, *75* (1), 43–88.

_ , "Current Patterns of Parental Authority," *Developmental Psychology*, 1971, *4* (1), 1–103.

Becker, Gary S. and Nigel Tomes, "An Equilibrium Theory of the Distribution of Income and Intergener- ational Mobility," *Journal of Political Economy*, 1979, *87* (6), 1153–1189.

Benabou, Roland and Jean Tirole, "Intrinsic and Extrinsic Motivation," *Review of Economic Studies*, 2003, *70* (1), 489–520.

Bergman, Peter, "Parent-child information frictions and human capital investment: Evidence from a field experiment," *Journal of Political Economy*, 2021, *129* (1), 286–322.

Berlinski, Samuel, Maria Marta Ferreyra, Luca Flabbi, and uan David Martin, "Child Care Markets, Parental Labor Supply, and Child Development," *Journal of Political Economy*, 2024.

Bernal, Raquel, "The Effect of Maternal Employment and Child Care on Children's Cognitive Development," *International Economic Review*, 2008, *49* (4), 1173–1209.

_ **and Michael P. Keane**, "Child Care Choices and Children's Cognitive Achievement: The Case of Single Mothers," *Journal of Labor Economics*, 2011, *29* (3), 459–512.

Brilli, Ylenia, "Mother's Time Allocation, Childcare, and Child Cognitive Development," *Journal of Human Capital*, 2022, *16* (2).

Brown, Meta, Christopher J. Flinn, and Joseph Mullins, "Family Law Effects on Divorce, Fertility and Child Investment," *Revise and Resubmit at Journal of Polictical Economy*, 2023.

Burton, Peter, Shelley Phipps, and Lori Curtis, "All in the family: A simultaneous model of parenting style and child conduct," *American Economic Review*, 2002, *92* (2), 368–372.

Carneiro, Pedro and James Heckman, "Human Capital Policy," *NBER Working Paper*, 2003, (9495).

Caucutt, Elizabeth, Lance Lochner, Joseph Mullins, and Youngmin Park, "Child Skill Production: Accounting for Parental Market-based Time and Goods Investment," *NBER Working Paper No. 27838*, 2020.

Cobb-Clark, Deborah A., Nicolas Salamanca, and Anna Zhu, "Parenting Style as an Investment in Human Development," *Journal of Population Economics*, 2019, *32*, 1315–1352.

Cosconati, Marco, "Parenting Style and the Development of Human Capital in Children," *UPenn Dissertation*, 2009.

Cunha, Flavio, James J. Heckman, and Susanne M Schennah, "Estimating the Technology of Cognitive and Noncognitive Skill Formation," *Econometrica*, 2010, *78* (3), 883–931.

Del Boca, Daniela and Christopher Flinn, "Endogenous household interaction," *Journal of Econometrics*, 2012, *166* (1), 49–65.

_ , Chiara Monfardini, and Cheti Nicoletti, "Parental and Child Time Investments and the Cognitive Development of Adolescents," *Journal of Labor Economics*, 2017, *30* (2), 565–608.

_ , Christopher Flinn, and Matthew Wiswall, "Household Choices and Child Development," *Review of Economic Studies*, 2014, *81.1*, 1173–1209.

_ , _ , _ , and Ewout Verriest, "Parenting with Patience: Parental Incentives and Child Development," *Revise and Resubmit Journal of Political Economy*, 2024.

Doepke, Matthias and Fabrizio Zilibotti, "Parenting With Style: Altruism and Paternalism in Intergenerational Preference Transmission," *Econometrica*, 2017, *85* (5), 1331–1371.

_ and _ , *Love Money and Parenting: How Economics Explains the Way We Raise Our Kids*, Princeton University Press, 2019.

_ , Giuseppe Sorrenti, and Fabrizio Zilibotti, "The Economics of Parenting," *Annual Review of Economics*, 2019, *11*, 55–84.

Eren, Ozkan and Daniel J. Henderson, "The Impact of Homework on Student Achievement," *Econometrics Journal*, 2008, *11* (2), 326–348.

Flinn, Christopher and Joseph Mullins, "Labor Market Search and Schooling Investment," *International Economic Review*, 2015, *56* (2), 359–398.

Galiani, Sebastian, Matthew Staiger, and Gustavo Torrens, "When Children Rule: Parenting in Modern Families," *NBER Working Paper 23087*, 2017.

Gershoff, Elizabeth Thompson, "Corporal punishment by parents and associated child behaviors and experiences: A meta-analytic and theoretical review," *Psychological Bulletin*, 2002, *128* (4), 539–579.

Hao, Lingxin, V. Joseph Hotz, and Ginger Z. Jin, "Games Parents and Adolescents Play: Risky Behaviour, Parental Reputation and Strategic Transfers," *The Economic Journal*, 2008, *118* (528), 515–555.

Harris, Jorgen, "The Effect of Children's Safety on Parenting Strategy," *The Review of Black Political Economy*, 2024, *51* (2), 188–227.

Kalenkoski, Charlene Marie and Sabrina Wulff Pabilonia, "Does high school homework increase academic achievement?," *Education Economics*, 2017, *25* (1), 45–59.

Kim, Jun Hyung, "The Economics of Parenting Skill and Child Development," 2019.

Liu, Haiyong, Thomas A. Mroz, and Wilbert van der Klaauwc, "Maternal Employment, Migration, and Child Development," *Journal of Econometrics*, 2010, *156* (1), 212–228.

Lizzeri, Alessandro and Marciano Siniscalchi, "Parental Guidance and Supervised Learning," *Quarterly Journal of Economics*, 2008, *123* (3), 1161–1195.

Lord, F.M and M. Novick, *Statistical Theories of Mental Test Scores*, Addison-Welsley Publishing Company, 1968.

Majumder, Md. Alauddin, "The Impact of Parenting Style on Children's Educational Outcomes in the United States," *Journal of Family and Economic Issues*, 2016, *37* (2), 89–98.

Mullins, Joseph, "Designing Cash Transfers in the Presence of Children's Human Capital Formation," *Working Paper*, 2022.

Ronda, Victor, "The Effect of Maternal Psychological Distress on Children's Cognitive Development," *Unpulbished Manuscript*, 2017.

Sarwar, Samiullah, "Influence of Parenting Style on Children's Behavior," *Journal of Education and Educational Development*, 2016, *3* (2).

Seror, Avner, "Child Development in Parent-Child Interactions," *Journal of Political Economy*, 2022, *130* (9), 2462–2499.

Spera, Christopher, "A Review of the Relationship among Parenting Practices, Parenting Styles, and Adolescent School Achievement," *Educational Psychology Review*, 2005, *17* (2), 125–146.

Straus, Murray A., "Discipline and Deviance: Physical Punishment of Children and Violence and Other Crime in Adulthood," *Social Problems*, 1991, *38* (2).

Todd, Petra E. and Kenneth I. Wolpin, "On the Specification and Estimation of the Production Function for Cognitive Achievement," *The Economic Journal*, 2003, *113* (485), F3–F33.

_ **and** _ , "The Production of Cognitive Achievement in Children: Home, School, and Racial Test Score Gaps," *Journal of Human Capital*, 2007, *1* (1), 91–136.

Train, Kenneth E., *Discrete Choice Methods with Simulation*, Cambridge University Press, 2009.

Verriest, Ewout, "Household Labor Supply, Child Development and Childcare Policies in the United States," *Working Paper*, 2022.

Weinberg, Bruce A., "An Incentive Model of the Effect of Parental Income on Children," *Journal of Political Economy*, 2001, *109* (2), 266–280.

Appendix

A Household Equilibrium

As in Del Boca et al. (2014) and Brilli (2022), I am able to define a recursive structure for the period t marginal (log) utility of child cognitive skills for both the parents and the children. Define $D^p_{t+1} = \frac{\partial V^p_{t+1}}{\partial ln(\theta_{t+1})}$ for the parents and $D^k_{t+1} = \frac{\partial V^p_{t+1}}{\partial ln(\theta_{t+1})}$ for the child. The child's terminal value is given by

$$V^k_{T+1}(\theta_{T+1}) = \psi^k ln(\theta_{T+1}) \tag{40}$$

then,

$$D^k_{T+1} = \frac{\partial V^k_{T+1}}{\partial ln(\theta_{T+1})} = \psi^k \tag{41}$$

Moving to period T, the child's value function is

$$V^k_T(\Gamma_T) = \omega(ps_T) ln(l^k_t) + \lambda_\theta ln(\theta_T) + \beta^k E_T V^k_{T+1}(ln(\theta_{T+1})) \tag{42}$$

Now,

$$D^k_T = \frac{\partial V^k_T}{\partial ln(\theta_T)} = \lambda_\theta + \beta^k E_T(\frac{\partial V^k_{T+1}}{\partial ln(\theta_{T+1})} \frac{\partial ln(\theta_{T+1})}{\partial ln(\theta_T)}) \tag{43}$$

which is

$$D^k_T = \frac{\partial V^k_T}{\partial ln(\theta_T)} = \lambda_\theta + \beta^k E_T D^k_{T+1} \delta^\theta_T \tag{44}$$

and so, for any period t

$$D^k_t = \lambda_\theta + \beta^k E_t D^k_{t+1} \delta^\theta_t \tag{45}$$

It is straightforward to implement this process and compute these continuation values for the parents.

The structure of the model allows me to derive a closed form solution for child study time at any age t, given by

$$\tau^k_t = \frac{\beta^k D^k_{t+1} \delta^k_t}{\omega(ps_t) + \beta^k D^k_{t+1} \delta^k_t} (112 - \tau^m_t - \tau^f_t - s_t) \tag{46}$$

To solve the parents' problem, I use a mix of numerical and analytical solutions to maximize

$$V^p_t(\Gamma_t) = \max_{a^p_t | \tau^k_t(a^p_t)} u^p_t(l^m_t, l^f_t, c_t, \theta_t, ps_t) + \beta^p E_t V^p_{t+1}(\Gamma_{t+1} | a^p_t)$$

where $a^p_t = (\tau^m_t, \tau^f_t, h^m_t, h^f_t, ps_t)$. I need to solve three optimization problems, one for each choice of parenting style. The parents' solution is computed as follows:

1. Choose parenting style $ps_t \in \{AR, AT, P\}$

2. Given ps_t make a numerical guess of (τ_t^m, τ_t^f)

3. Given $(ps_t, \tau_t^m, \tau_t^f)$ labor supply and goods investment are given by

$$h_t^f = \max\{\frac{A_f - B_f}{C_f}, 0\} \tag{47}$$

where

$$A_f = w_t^f(112 - \tau_t^f)(\alpha_l^m + \alpha_c + \beta^p D_{t+1}^p \delta_t^g)$$

$$B_f = \alpha_l^f(w_t^m(112 - \tau_t^m) + y_t)$$

$$C_f = w_t^f(\alpha_l^f + \alpha_l^m + \alpha_c + \beta^p D_{t+1}^p \delta_t^g)$$

and

$$h_t^m = \max\{\frac{A_m - B_m}{C_m}, 0\} \tag{48}$$

where

$$A_m = w_t^m(112 - \tau_t^m)(\alpha_l^f + \alpha_c + \beta^p D_{t+1}^p \delta_t^g)$$

$$B_m = \alpha_l^m(w_t^f(112 - \tau_t^f) + y_t)$$

$$C_m = w_t^m(\alpha_l^f + \alpha_l^m + \alpha_c + \beta^p D_{t+1}^p \delta_t^g)$$

and

$$g_t(\tau_t^m, \tau_t^f) = \frac{\beta^p D_{t+1}^p \delta_t^g}{\alpha_c + \beta^p D_{t+1}^p \delta_t^g}(w_t^f h_t^f(\tau_t^m, \tau_t^f) + w_t^m h_t^m(\tau_t^m, \tau_t^f) + y_t) \tag{49}$$

4. Optimize over (τ_t^m, τ_t^f) to maximize the parents' Bellman given choice of parenting style

5. Do this process for all three parenting style choices

6. The optimal choice is the vector $a_t^p = (\tau_t^m, \tau_t^f, h_t^m, h_t^f, ps_t, g_t)$ that yields the highest value

B Wages and Non-Labor Income

Table 16 shows the point estimates for the parameters governing the wage offer distribution. All else equal, fathers will have a higher wage on average, and the slope with age is greater than that for mothers, although the return to education is larger for mothers. These results

are consistent with Del Boca et al. (2024), who use the same structural setup up and a similar sample of the PSID-CDS data.

Table 16. **Wage Offer Distribution Estimates**

	Mother		Father	
Parameter	**Estimate**	**(Std Error)**	**Estimate**	**(Std Error)**
Intercept	0.460	(0.013)	1.288	(0.009)
Age	0.013	(0.000)	0.008	(0.000)
Educ	0.132	(0.001)	0.117	(0.001)
Sd of Shock	0.509	(0.008)	0.531	(0.005)
Correlation	0.842	(0.022)		

Notes: Bootstrapped standard errors in parentheses.

Table 17 displays the point estimates for the exogenous non-labor income process. The results are approximately those obtained by Verriest (2022), who uses the same structural setup and uses a similar sample of the PSID-CDS data.

Table 17. **Point Estimates for the Non-Labor Income Process**

Parameter	**Estimate**	**(Std Error)**
Logit - Intercept	-5.032	(0.373)
Logit - Dad Age	0.009	(0.019)
Logit- Dad Age Squared	0.000	(0.000)
Logit - Dad Educ	0.463	(0.014)
Conditional - Intercept	3.454	(0.375)
Conditional - Dad Age	0.043	(0.012)
Conditional - Dad Age Squared	-0.000	(0.000)
Conditional - Dad Educ	0.014	(0.015)
Conditional - SD of Shock	0.932	(0.007)

Notes: Bootstrapped standard errors in parentheses.

Table 18 displays the model fit for the wage and non-labor income processes. The model fits the data well, as I am able to fit the average (accepted) wage offers and hours worked quite closely. Further, the model matches the fraction of households having positive non-labor income and the average level of weekly non-labor income well.

Table 18. **Model Fit of Wage and Non-Labor Income Distributions**

	Data	Model
Avg Mom Wage	31.384	32.158
Avg Dad Wage	22.678	21.597
Avg Mom Hours	30.447	31.222
Avg Dad Hours	43.612	44.485
Frac Working Moms	0.834	0.888
Frac Working Dads	0.978	0.995
Avg NLI	162.289	166.466
SD NLI	217.070	225.317
Frac NLI > 0	0.830	0.835

Notes: Data is taken from author's sample of the PSID-CDS data, Simulated corresponds to moments generated by data simulated at the estimated model parameters.

C Parenting Style Analysis

Doepke et al. (2019) provide a methodology to map observed data in the CDS to the parenting styles defined by Baurmind (1967). While it is sensible that authoritarian parents value obedience, distinguishing between authoritative and permissive parenting requires more care. Baurmind (1967) states that authoritarian and permissive parenting are at the extremes, with authoritarian parenting being characterized by the demand for obedience and control, and permissive parenting by allowing children to make their own decisions with minimal interference. Authoritative parenting is the intermediary style, parents will still intervene if their child is not behaving in the manner desired by the parents, separating it from a permissive style, but the corrective measures are not nearly as harsh, separating it from an authoritarian style. Doepke et al. (2019) characterize authoritative parenting as a style that values hard work, which should imply that these parents set rules about homework in order to ensure their children are working hard. Homework does not capture the whole picture of child time investment. Perhaps some children are not assigned homework, so their is no need to have rules. These parents may still want their children to engage in educational activities, and this may imply trying to reduce the amount of time their children spend in unproductive leisure time, like watching television.

Table 19 below reports the proportion of parents who adopt rules about television and homework, conditional on the parenting style adopted in the household. We do see the expected result that this proportion is highest for authoritarian parenting across both rules, supporting that the style characterized by demanding obedience in control is the one enforcing rules with the highest frequency. One would still expect that there would be a higher

proportion of authoritative parents adopting rules about homework than permissive parents, however, they are about the same, with the proportion being slightly higher for permissive parenting. The expected ordering is observed for rules about watching television, with the proportion being highest for authoritarian parenting and lowest for permissive. There are of course, measurement issues. I do not observe any specifics about the rules, which of course are heterogeneous, further, parents might provide answers that conform with social norms, which could be why the proportion reporting there are rules about homework is so high. While it may seem that the result that there is no significant difference in authoritative and permissive parents adopting rules about homework would not support the methodology proposed by Doepke et al. (2019), the result that there is differentiation between authoritative and permissive in terms of television rules supports the methodology, as authoritative parents attempt to push their children away from unproductive leisure time more than permissive parents do.

Table 19. **Parental Rules by Parenting Style**

	Homework Rules	**Television Rules**
Authoritarian	0.75	0.54
Authoritative	0.61	0.41
Permissive	0.63	0.34

Notes: Data is taken from the author's sample of the PSID-CDS. Homework and television rules are averages of data from the 2002 and 2007 waves of the CDS, conditional on the parenting style adopted in that year.

Table 20 below, reports the results from the linear regression of child study time on child age, the education of the mother and the parenting style chosen.

$$\tau_t^k = \beta_0 + \beta_1 t + \beta_2 e^m + \beta_3 \mathbb{1}\{ps_t = AR\} + \beta_4 \mathbb{1}\{ps_t = AT\} + \epsilon_t \tag{50}$$

The reader may note that the only significant regressors are the age of the child and the education level of the mother. These are the expected results that older children will study more, mostly by a function of them being assigned more homework, and that there is a correlation between the education level of the parents and child study time. The more interesting regressors are the coefficients for the marginal effects of authoritarian and authoritative parenting (relative to permissive parenting). One can see that there is a negative marginal effect for authoritarian parenting, and a positive one for authoritative parenting. This suggests that authoritative parenting increases study time while authoritarian parenting decreases study time (relative to permissive parenting). While this is the conclusion reached in this paper, one could not state that authoritarian parenting reduces study time relative to permissive

parenting from an OLS regression. This is because both study time and parenting style are determined jointly, and therefore, the results in Table 20 are biased.

Table 20. Regression of Child Study Time

	Child Study Time
Child Age	0.476***
	(8.52)
Mother's Education	0.649***
	(4.67)
Authoritarian Parenting	-0.365
	(-0.54)
Authoritative Parenting	0.125
	(0.19)
Intercept	-10.31***
	(-4.60)
N	571

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: Regression of child study time on their age, the education of their mother, and the parenting style they face. Data is taken from the author's sample of the PSID-CDS data.

D Additional Tables & Figures

Table 21. **Descriptive Statistics**

	Mean	SD	N
PSID-CDS 1997,2002,2007			
Mother's Age in 1997	38.24	5.50	276
Father's Age in 1997	40.34	6.47	276
Mother's Education	13.83	2.10	276
Father's Education	13.74	2.33	276
Child's Age 1997	8.28	2.59	276
Letter Word Score 1997	30.75	15.63	276
Letter Word Score 2002	48.21	5.07	276
Letter Word Score 2007	50.59	3.50	103
PSID 1988-2010			
Mother's Work Hours per Week	25.41	16.47	2495
Father's Work Hours per Week	42.66	11.60	2423
Mother's Hourly Wage	20.52	14.35	1949
Father's Hourly Wage	31.38	22.68	2370
Non-Labor Income per Week	162.29	217.07	1979

Notes: Parent work hours, wages, and non-labor income statistics are averaged over all years where the child is between 4 and 18 years old, ranging from 1988 to 2010.

Table 22. **Parental Labor Supply by Child Age**

Child Age	**4-6**	**7-10**	**11-18**
Mothers Working	0.79	0.83	0.86
Fathers Working	0.99	0.98	0.97
Mothers Hours Avg	23.15	24.18	27.51
Mothers SD	16.88	16.12	16.04
Fathers Hours Avg	42.83	42.99	42.29
Fathers Hours SD	11.06	10.69	12.56

Notes: Parent work hours, wages, and non-labor income statistics are averaged over all years where the child is between 4 and 18 years old, ranging from 1988 to 2010. In this table, the hours report is conditional on the parents participating in the labor force.

Table 23. **Household Time Investment by Child Age**

Child Age	**4-6**	**7-10**	**11-18**
Mother's Time Investment Avg	26.85	21.49	15.39
Mother's Time Investment SD	12.52	11.86	13.00
Father's Time Investment Avg	15.96	14.03	11.32
Father's Time Investment SD	11.05	9.84	11.85
Child's Time Investment Avg	0.62	2.42	5.77
Child's Time Investment SD	1.18	3.84	7.30

Notes: Data is taken from the author's sample of the PSID-CDS. All data is the average across households by the age of the child.

E Moment Conditions

Table 24. Data versus Simulated Moments

	Data	**Model**
Avg Mother's Time Child Age 4-5	28.654	23.355
Avg Mother's Time Child Age 6-8	21.717	22.220
Avg Mother's Time Child Age 9-11	18.847	21.257
Avg Mother's Time Child Age 12-14	19.495	19.615
Avg Mother's Time Child Age 15-18	13.412	14.634
Standard Deviation Mother's Time Child Age 4-5	12.858	3.957
Standard Deviation Mother's Time Child Age 6-8	10.654	3.843
Standard Deviation Mother's Time Child Age 9-11	11.949	3.432
Standard Deviation Mother's Time Child Age 12-14	12.872	2.897
Standard Deviation Mother's Time Child Age 15-18	12.894	2.452
Avg Father's Time Child Age 4-5	16.927	14.597
Avg Father's Time Child Age 6-8	14.570	14.434
Avg Father's Time Child Age 9-11	13.415	14.553
Avg Father's Time Child Age 12-14	13.854	14.433
Avg Father's Time Child Age 15-18	10.168	12.236
Standard Deviation Father's Time Child Age 4-5	11.040	2.304
Standard Deviation Father's Time Child Age 6-8	9.726	2.339
Standard Deviation Father's Time Child Age 9-11	10.889	2.202
Standard Deviation Father's Time Child Age 12-14	12.144	1.978
Standard Deviation Father's Time Child Age 15-18	11.973	1.604
Avg Child's Time Child Age 4-5	0.602	0.978
Avg Child's Time Child Age 6-8	1.285	1.238
Avg Child's Time Child Age 9-11	3.544	2.432
Avg Child's Time Child Age 12-14	5.332	4.337
Avg Child's Time Child Age 15-18	6.386	7.399
Standard Deviation Child's Time Child Age 4-5	1.141	1.424
Standard Deviation Child's Time Child Age 6-8	2.301	1.626
Standard Deviation Child's Time Child Age 9-11	4.247	3.055
Standard Deviation Child's Time Child Age 12-14	7.358	4.769
Standard Deviation Child's Time Child Age 15-18	7.821	7.110

Table 25. Data versus Simulated Moments (Continued)

	Data	**Model**
Avg Child Study Time Authoritarian Age 4-11	2.015	1.640
Avg Child Study Time Authoritarian Age 12-18	5.048	5.295
Avg Child Study Time Authoritative Age 4-11	2.547	2.169
Avg Child Study Time Authoritative Age 12-18	6.638	6.684
Avg Child Study Time Permissive Age 4-11	2.396	1.665
Avg Child Study Time Permissive Age 12-18	6.107	5.869
Avg Child Study Time Authoritarian	3.493	3.851
Avg Child Study Time Authoritative	4.851	4.931
Avg Child Study Time Permissive	4.569	4.244
Avg Father's Hours Worked Child Age 4-5	42.988	43.013
Avg Father's Hours Worked Child Age 6-8	43.746	42.926
Avg Father's Hours Worked Child Age 9-11	43.713	43.189
Avg Father's Hours Worked Child Age 12-14	43.878	43.786
Avg Father's Hours Worked Child Age 15-18	43.730	46.564
Sd Father's Hours Worked Child Age 4-5	10.348	10.945
Sd Father's Hours Worked Child Age 6-8	9.243	11.270
Sd Father's Hours Worked Child Age 9-11	8.850	10.906
Sd Father's Hours Worked Child Age 12-14	9.655	10.993
Sd Father's Hours Worked Child Age 15-18	10.874	10.520
Avg Mother's Hours Worked Child Age 4-5	28.851	26.791
Avg Mother's Hours Worked Child Age 6-8	29.288	27.398
Avg Mother's Hours Worked Child Age 9-11	29.880	28.694
Avg Mother's Hours Worked Child Age 12-14	30.758	30.288
Avg Mother's Hours Worked Child Age 15-18	33.280	34.998
Sd Mother's Hours Worked Child Age 4-5	13.767	12.964
Sd Mother's Hours Worked Child Age 6-8	13.313	13.833
Sd Mother's Hours Worked Child Age 9-11	12.696	13.543
Sd Mother's Hours Worked Child Age 12-14	13.237	13.874
Sd Mother's Hours Worked Child Age 15-18	12.095	14.189
Pct Father's Who Work Child Age 4-5	0.991	0.994

Table 26. Data versus Simulated Moments (Continued)

	Data	**Model**
Pct Father's Who Work Child Age 6-8	0.983	0.992
Pct Father's Who Work Child Age 9-11	0.985	0.994
Pct Father's Who Work Child Age 12-14	0.980	0.995
Pct Father's Who Work Child Age 15-18	0.952	0.997
Pct Mother's Who Work Child Age 4-5	0.793	0.813
Pct Mother's Who Work Child Age 6-8	0.815	0.823
Pct Mother's Who Work Child Age 9-11	0.853	0.851
Pct Mother's Who Work Child Age 12-14	0.850	0.891
Pct Mother's Who Work Child Age 15-18	0.865	0.943
Cor Father's Time and Education Age 4-5	0.050	-0.256
Cor Father's Time and Education Age 6-8	-0.027	-0.229
Cor Father's Time and Education Age 9-11	-0.043	-0.212
Cor Father's Time and Education Age 12-14	0.226	-0.237
Cor Father's Time and Education Age 15-18	-0.030	-0.178
Cor Mother's Time and Education Age 4-5	0.148	0.158
Cor Mother's Time and Education Age 6-8	0.110	0.086
Cor Mother's Time and Education Age 9-11	0.007	0.076
Cor Mother's Time and Education Age 12-14	0.097	0.092
Cor Mother's Time and Education Age 15-18	-0.081	0.140

Table 27. Data versus Simulated Moments (Continued)

	Data	**Model**
Pct Authoritarian	0.276	0.277
Pct Authoritative	0.458	0.461
Pct Permissive	0.266	0.262
Pct Authoritarian Age 4-10	0.293	0.280
Pct Authoritative Age 4-10	0.437	0.460
Pct Permissive Age 4-10	0.270	0.260
Pct Authoritarian Age 11-18	0.266	0.276
Pct Authoritative Age 11-18	0.470	0.461
Pct Permissive Age 11-18	0.263	0.263
Pct Authoritarian if College	0.168	0.160
Pct Authoritative if College	0.532	0.535
Pct Permissive if College	0.300	0.305
Pct Authoritarian if No College	0.327	0.333
Pct Authoritative if No College	0.423	0.426
Pct Permissive if No College	0.249	0.241
Pct Authoritarian if College Age 4-10	0.200	0.171
Pct Authoritative if College Age 4-10	0.480	0.524
Pct Permissive if College Age 4-10	0.320	0.306
Pct Authoritarian if College Age 11-18	0.148	0.155
Pct Authoritative if College Age 11-18	0.565	0.539
Pct Permissive if College Age 11-18	0.287	0.305
Pct Authoritarian if No College Age 4-10	0.343	0.334
Pct Authoritative if No College Age 4-10	0.414	0.429
Pct Permissive if No College Age 4-10	0.243	0.238
Pct Authoritarian if No College Age 11-18	0.319	0.333
Pct Authoritative if No College Age 11-18	0.428	0.424
Pct Permissive if No College Age 11-18	0.253	0.243
Multinomial Logit Regression Parenting Style: Authoritarian Int	0.768	0.434
Authoritarian: Coefficient on if Mother is has College Degree	-0.866	-0.969
Authoritarian: Coefficient on Child Age	-0.041	-0.009
Multinomial Logit Regression Parenting Style: Authoritative Int	0.424	0.565
Authoritative: Coefficient on if Mother is has College Degree	0.035	-0.007
Authoritative: Coefficient on Child Age	0.009	0.000

Table 28. Data versus Simulated Moments (Continued)

	Data	**Model**
Avg LW Score Age 4	9.021	9.021
Avg LW Score Age 5	12.625	15.253
Avg LW Score Age 6	22.536	23.110
Avg LW Score Age 7	33.074	31.094
Avg LW Score Age 8	38.031	36.817
Avg LW Score Age 9	42.324	41.129
Avg LW Score Age 10	44.351	44.024
Avg LW Score Age 11	44.226	45.782
Avg LW Score Age 12	47.933	47.345
Avg LW Score Age 13	48.583	48.462
Avg LW Score Age 14	47.906	49.394
Avg LW Score Age 15	50.127	50.133
Avg LW Score Age 16	50.431	50.891
Avg LW Score Age 17	51.340	51.493
Avg LW Score Age 18	51.950	52.120
Standard Dev LW Score Age 4	3.959	3.925
Standard Dev LW Score Age 5	4.309	6.665
Standard Dev LW Score Age 6	9.395	8.096
Standard Dev LW Score Age 7	5.470	7.513
Standard Dev LW Score Age 8	5.527	6.383
Standard Dev LW Score Age 9	6.290	5.881
Standard Dev LW Score Age 10	5.328	5.308
Standard Dev LW Score Age 11	4.705	4.824
Standard Dev LW Score Age 12	4.042	4.365
Standard Dev LW Score Age 13	4.032	3.806
Standard Dev LW Score Age 14	4.059	3.430
Standard Dev LW Score Age 15	3.541	3.135
Standard Dev LW Score Age 16	3.766	2.841
Standard Dev LW Score Age 17	4.532	2.743
Standard Dev LW Score Age 18	3.266	2.630

Table 29. Data versus Simulated Moments (Continued)

	Data	**Model**
Cor LW Score and Child Study Time	0.329	0.346
Cor LW Score and Mother's Time	-0.317	-0.451
Cor LW Score and Father's Time	-0.137	-0.204
Cor LW Score t Cognitive Skills t+5	-0.958	-0.956
Cor Father's Wage 1998 LW Score 2002	0.174	0.067
Cor Mother's Wage 1998 LW Score 2002	0.106	0.120
Cor Father's Wage 1998 Δ LW Score	0.049	-0.017
Cor Mother's Wage 1998 Δ LW Score	0.040	-0.028
Cor Δ LW Score and Mother's Time	0.296	0.230
Cor Δ LW Score and Father's Time	0.091	0.039
Cor Δ LW Score and Child Study Time	-0.332	-0.188
Cor Δ LW Score and Mother's Time Age 4-7	0.216	0.177
Cor Δ LW Score and Mother's Time Age 8-11	0.083	-0.056
Cor Δ LW Score and Mother's Time Age 12-18	-0.157	-0.079
Cor Δ LW Score and Father's Time Age 4-7	0.163	0.038
Cor Δ LW Score and Father's Time Age 8-11	-0.023	0.031
Cor Δ LW Score and Father's Time Age 12-18	0.007	0.018
Cor Δ LW Score and Child's Time Age 4-7	-0.210	0.020
Cor Δ LW Score and Child's Time Age 8-11	-0.189	0.115
Cor Δ LW Score and Child's Time Age 12-18	0.110	0.117
Cor Δ LW Score and Lagged LW Score 4-7	-0.907	-0.876
Cor Δ LW Score and Lagged LW Score 8-11	-0.735	-0.823
Cor Δ LW Score and Lagged LW Score 12-18	-0.485	-0.658
Cor Δ LW Score and Father's Wage Age 4-7	0.021	0.054
Cor Δ LW Score and Father's Wage Age 8-11	-0.032	-0.072
Cor Δ LW Score and Father's Wage Age 12-18	-0.036	-0.092
Cor Δ LW Score and Mother's Wage Age 4-7	-0.014	0.089
Cor Δ LW Score and Mother's Wage Age 8-11	-0.072	-0.099
Cor Δ LW Score and Mother's Wage Age 12-18	-0.035	-0.158
Marginal Effect of Authoritarian on Cognitive Skill	-0.996	-1.533
Marginal Effect of Authoritative Parenting on Cognitive Skill	-0.188	0.820

Table 30. Data versus Simulated Moments (Continued)

	Data	**Model**
Avg Father's Wage if Working	31.384	32.158
St Dev Father's Wage if Working	22.678	21.597
Avg Mother's Wage if Working	20.521	21.910
St Dev Mother's Wage if Working	14.347	14.127
Avg Father's Wage if Working Age 0-32	21.636	23.038
Avg Father's Wage if Working Age 33-37	26.456	26.544
Avg Father's Wage if Working Age 38-42	30.234	29.590
Avg Father's Wage if Working Age 43-75	36.136	34.251
Avg Mother's Wage if Working Age 0-32	13.687	14.661
Avg Mother's Wage if Working Age 33-37	17.819	17.516
Avg Mother's Wage if Working Age 38-42	21.046	20.304
Avg Mother's Wage if Working Age 43-75	23.775	24.234
St Dev Father's Wage if Working Age 0-32	18.278	12.813
St Dev Father's Wage if Working Age 33-37	15.876	16.280
St Dev Father's Wage if Working Age 38-42	19.622	19.600
St Dev Father's Wage if Working Age 43-75	26.413	22.797
St Dev Mother's Wage if Working Age 0-32	7.844	8.135
St Dev Mother's Wage if Working Age 33-37	12.875	10.350
St Dev Mother's Wage if Working Age 38-42	14.620	12.982
St Dev Mother's Wage if Working Age 43-75	15.436	15.182
Avg Father's Wage if Working Education 0-12	23.390	23.890
Avg Father's Wage if Working Education 13-15	28.833	30.950
Avg Father's Wage if Working Education 16-50	43.165	42.567
Avg Mother's Wage if Working Education 0-12	15.201	15.675
Avg Mother's Wage if Working Education 13-15	20.253	20.249
Avg Mother's Wage if Working Education 16-50	26.941	29.528
Avg Father's Wage if Less Than HS and Younger Than 40	22.166	22.526
Avg Father's Wage if Less Than HS and Older Than 40	24.500	24.361
Avg Mother's Wage if Less Than HS and Younger Than 40	14.024	13.926
Avg Mother's Wage if Less Than HS and Older Than 40	16.723	16.591
Avg Father's Wage if Less Than College and Younger Than 40	24.278	27.606
Avg Father's Wage if Less Than College and Older Than 40	32.287	32.140
Avg Mother's Wage if Less Than College and Younger Than 40	18.620	17.875
Avg Mother's Wage if Less Than College and Older Than 40	22.004	21.426
Avg Father's Wage if College and Younger Than 40	37.795	38.385
Avg Father's Wage if College and Older Than 40	45.207	43.059
Avg Mother's Wage if College and Younger Than 40	23.879	25.950
Avg Mother's Wage if College and Older Than 40	28.150	30.056
Correlation of Accepted Wages	0.248	0.839

Table 31. Data versus Simulated Moments (Continued)

	Data	**Model**
Avg(NLI if Positive NLI)	195.478	192.490
St Dev(NLI if Positive NLI)	224.209	224.278
Avg(NLI x Father's Age Squared if Positive NLI)	8420.035	8326.036
Avg(NLI x Father's Age Squared if Positive NLI)	372605.209	370618.103
Avg(NLI x Father's Education if Positive NLI)	2701.195	2728.613
Avg(NLI x Father's Education Squared if Positive NLI)	38329.880	39601.288
Avg(NLI x Father's Age x educ if Positive NLI)	116780.601	118754.106
Avg Fraction With Positive NLI	0.830	0.830
St Dev Fraction With Positive NLI	0.376	0.376
Avg(Fraction With NLI x Father's Age)	35.273	35.357
Avg(Fraction With NLI x Father's Age Squared)	1539.461	1551.493
Avg(Fraction With NLI x Father's Education)	11.578	11.698
Avg(Fraction With NLI x Father's Education Squared)	165.705	168.829
Avg(NLI x Father's Age)	6990.458	6909.869
Avg(NLI x Father's Age Squared)	309343.283	307580.048
Avg(NLI x Father's Education)	2242.579	2264.506
Avg(NLI x Father's Education Squared)	31822.129	32865.545
Avg(NLI x Father's Education Squared x Father's Age Squared)	96953.273	98555.341
Avg(NLI if Father's Age Between 25 and 33)	128.130	117.929
Avg(NLI if Father's Age Between 34 and 38)	141.996	142.298
Avg(NLI if Father's Age Between 39 and 43)	155.331	158.973
Avg(NLI if Father's Age Between 44 and 60)	185.732	178.909
Avg(NLI if Positive NLI and Father's Age Between 25 and 33)	182.079	152.299
Avg(NLI if Positive NLI and Father's Age Between 34 and 38)	174.496	180.334
Avg(NLI if Positive NLI and Father's Age Between 39 and 43)	184.326	191.022
Avg(NLI if Positive NLI and Father's Age Between 44 and 60)	215.204	207.386
Fraction Positive NLI if Father's Age Between 25 and 33	0.704	0.774
Fraction Positive NLI if Father's Age Between 34 and 38	0.814	0.789
Fraction Positive NLI if Father's Age Between 39 and 43	0.843	0.832
Fraction Positive NLI if Father's Age Between 44 and 60	0.863	0.863
Average NLI	162.289	159.750
St Dev NLI	217.070	216.737