Mandatory Assignment 4

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1 Problem 1: Modelling cross sections of a heart

We are given nine `.dat` files, containing the contours of a heart at different levels. `hj1.dat` is at the bottom, while `hj9.dat` is at the top. Each file contains values of the form $(x_i, y_i, z_i)_{i=1}^n$, where z_i is constant for each file.

We firstly need to order the data, such that we have $(u_i, x_i, y_i, z_i)_{i=1}^n$, where $(u_i)_{i=1}^n$ is an increasing sequence with $u_1 = 0$. We do this through chord length parametrization, setting

$$u_i = u_{i-1} + \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2 + (z_i - z_{i-1})^2} \tag{1}$$

for $i = 2, \ldots, n$. We then approximate the data by a cubic splines, using a least squares approach.

We choose the knots to be uniformly distributed, with 5%, 10% and 20% of the number of data points. As we are seeking cubic splines, we have $d = 3$, and seek the splines

$$g(x) = \sum_{i=1}^{n} c_j B_{j,d,\mathbf{t}}(x) \tag{2}$$

such that

$$\sum_{i=1}^{m} (g(x_i) - y_i)^2 \tag{3}$$

is minimized. With

$$A = [B_{j,d,\mathbf{t}}(x_i)]_{i,j=1}^{m,n} = \begin{bmatrix} B_1(x_1) & \cdots & B_n(x_1) \\ \vdots & \ddots & \vdots \\ B_1(x_m) & \cdots & B_n(x_m) \end{bmatrix} \tag{4}$$

and letting $\mathbf{c} = [c_1, \ldots, c_n]^T \in \mathbb{R}^n$ and $\mathbf{y} = [y_1, \ldots, y_m]^T \in \mathbb{R}^m$, the system we need to solve becomes

$$(A^T A)\mathbf{c} = A^T \mathbf{y}, \tag{5}$$

which we solve by inverting $A^T A$.

Does it then make sense to space the knots with $t_1 = u_1 = 0$ and $t_{n+d+1} = u_n$?