MAT4170

Exercises for Spline Methods

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1 Bernstein-Bézier polynomials

Exercise 1.1 It is sometimes necessary to convert a polynomial in BB form to monomial form. Consider a quadratic BB polynomial,

$$p(x) = c_0(1-x)^2 + 2c_1x(1-x) + c_2x^2.$$

Express p in the monomial form

$$p(x) = a_0 + a_1x + a_2x^2.$$

Solution 1.1 Rather than using the explicit formula for conversion, we can just expand the coefficients and collect terms.

$$\begin{aligned} p(x) &= c_0(1-x)^2 + 2c_1x(1-x) + c_2x^2 \\ &= c_0(1-2x+x^2) + 2c_1(x-x^2) + c_2x^2 \\ &= c_0 - 2c_0x + c_0x^2 + 2c_1x - 2c_1x^2 + c_2x^2 \\ &= c_0 + (-2c_0 + 2c_1)x + (c_0 - 2c_1 + c_2)x^2. \end{aligned}$$