Kalman Exos

Exercice 14 : Température

1. $$\begin{cases} \hat{x}_{k+1|k} = A_k \hat{x}_{k|k} + u_k \\ \Gamma_{k+1|k} = A_k \cdot \Gamma_{k|k} \cdot A_k^T + \Gamma_{\alpha k} \end{cases}$$ (prédiction)

or ici $$\begin{cases} x_{k+1} = x_k + \alpha_k \\ y_k = x_k + \beta_k \end{cases}$$ avec $\Gamma_\alpha = 4$, $\Gamma_\beta = 3$

donc $A_k = 1$ et $u_k = 0$ soit $\hat{x}_{k+1|k} = \hat{x}_{k|k}$

et $\Gamma_{k+1|k} = \Gamma_{k|k} + \Gamma_\alpha$

$$\begin{cases} \hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \tilde{y}_k \\ \Gamma_{k|k} = (I - K_k C_k) \Gamma_{k|k-1} \\ \tilde{y}_k = y_k - C_k \hat{x}_{k|k-1} \\ S_k = C_k \Gamma_{k|k-1} C_k^T + \Gamma_{\beta k} \\ K_k = \Gamma_{k|k-1} C_k^T S_k^{-1} \end{cases}$$ (correction)

$C_k = 1$ donc $S_k = \Gamma_{k|k-1} + \Gamma_{\beta k} = \Gamma_{k|k-1} + 3$

$K_k = \Gamma_{k|k-1} (\Gamma_{k|k-1} + 3)^{-1}$, $\tilde{y}_k = y_k - \hat{x}_{k|k-1}$

donc $$\begin{cases} \hat{x}_{k+1|k} = \hat{x}_{k|k-1} + \Gamma_{k|k-1} (\Gamma_{k|k-1} + 3)^{-1} (y_k - \hat{x}_{k|k-1}) \\ \Gamma_{k+1|k} = 4 + (I - \Gamma_{k|k-1} (\Gamma_{k|k-1} + 3)^{-1}) \Gamma_{k|k-1} \end{cases}$$

2. $\Gamma_\infty = \Gamma_\alpha + (1 - \Gamma_\infty(\Gamma_\infty + \Gamma_\beta)^{-1})\Gamma_\infty$

$\Rightarrow \quad 0 = -\Gamma_\alpha \Gamma_\infty + \Gamma_\infty^2 - \Gamma_\alpha \Gamma_\beta$

$$\Gamma_\infty = \frac{\Gamma_\alpha + \sqrt{\Gamma_\alpha^2 + 4\Gamma_\alpha \Gamma_\beta}}{2} = 6$$

$$K = \frac{\Gamma_\infty}{\Gamma_\infty + 3} = \frac{2}{3}$$

$$\hat{x}_{k+1} = \hat{x}_k + \frac{2}{3}(y_k - \hat{x}_k)$$

3. $\Gamma_\infty = 0 \Rightarrow$ aucune incertitude

température exacte en ∞.

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