

Kalman
Exos

Exercice 13 : Murs

$$1) d^i = \det \begin{pmatrix} u_1^i & x_1 - a_1^i \\ u_2^i & x_2 - a_2^i \end{pmatrix} + \beta_i$$

$$\Leftrightarrow d = u_1^i (x_2 - a_2^i) - u_2^i (x_1 - a_1^i) + \beta_i$$

$$\Leftrightarrow d^i + u_1^i a_2^i - u_2^i a_1^i = -u_2^i x_1 + u_1^i x_2 + \beta_i$$

$$\Leftrightarrow d^i - \bar{d}^i = \begin{pmatrix} -u_2^i & u_1^i \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \beta_i$$

$$\text{avec } \bar{d}^i = u_1^i a_1^i - u_2^i a_2^i$$

$$Y = \begin{pmatrix} d^1 - \bar{d}^1 \\ d^2 - \bar{d}^2 \\ d^3 - \bar{d}^3 \end{pmatrix} \quad C = \begin{pmatrix} -u_2^1 & u_1^1 \\ -u_2^2 & u_1^2 \\ -u_2^3 & u_1^3 \end{pmatrix} \quad Y = CX + \beta$$

Exercice 18 : Deadreckoning

$$2) \text{ On pose } z = \begin{pmatrix} x \\ y \\ v \end{pmatrix}, \quad \dot{z} = A z + u + a$$

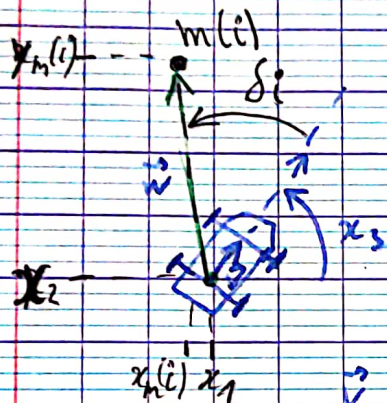
$$\text{avec } A = \begin{pmatrix} 0 & 0 & \cos \delta \cos \theta \\ 0 & 0 & \cos \delta \sin \theta \\ 0 & 0 & 0 \end{pmatrix}, \quad u = \begin{pmatrix} 0 \\ 0 \\ u_1 \end{pmatrix} \text{ et } a = \begin{pmatrix} 0 \\ 0 \\ a_1 \end{pmatrix}$$

$$z_{k+1} = \underbrace{(I + \Delta t A)}_{A_k} z_k + \Delta t^* (u + a) = A_k z_k + u_k + a_k$$

Exercice 10: Géométrie

$$1) \quad \dot{z} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_4 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 & \cos x_3 & \cos x_3 \\ 0 & 0 & \sin x_3 & \sin x_3 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{A_K} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}}_u + \alpha$$

(l'équation d'évolution est linéaire.)



$$\vec{v} = \begin{pmatrix} \cos(x_3 + \delta_i) \\ \sin(x_3 + \delta_i) \end{pmatrix}$$

$$\vec{w} = \begin{pmatrix} x_m(i) - x_1 \\ y_m(i) - x_2 \end{pmatrix}$$

\vec{v} et \vec{w} sont colinéaires donc :

$$\det(\vec{v}, \vec{w}) = 0 \Leftrightarrow \cos(x_3 + \delta_i)(y_m(i) - x_2) = \sin(x_3 + \delta_i)(x_m(i) - x_1)$$

$$\Leftrightarrow \underbrace{-x_m(i) \sin(x_3 + \delta_i) + \cos(x_3 + \delta_i) y_m(i)}_{y_i} = \begin{pmatrix} -\sin(x_3 + \delta_i) & \cos(x_3 + \delta_i) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \beta_i$$

$$y_i = \begin{pmatrix} -\sin(x_3 + \delta_i) & \cos(x_3 + \delta_i) & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_4 \end{pmatrix} + \beta_i$$

$$y(k) = \begin{pmatrix} 0 & 0 & 1 \\ -\sin(x_3 + \delta_1) & \cos(x_3 + \delta_1) & 0 \\ \vdots & \vdots & \vdots \\ -\sin(x_3 + \delta_s) & \cos(x_3 + \delta_s) & 0 \end{pmatrix} z(k) + \beta(k)$$

On a l'équation d'observation associée.

$$2) \quad z(k+1) = z(k) + dt \dot{z}(k) = (I + dt A_K) z(k) + dt u + \alpha(k)$$