Cosmic muons model: Notes

A) Bethe Bloch for protons in water (from source code) (bethebloch.py)

Bethe Bloch formula:

$$\boxed{-\frac{dE}{dx} = \frac{4\pi}{m_e c^2} \cdot \frac{n z^2}{\beta^2} \cdot \left(\frac{e^2}{4\pi \varepsilon_0}\right)^2 \left[\ln\left(\frac{2 m_e c^2 \beta^2}{I \cdot (1-\beta^2)}\right) - \beta^2\right]}$$

Where:

- $\beta = \frac{v}{c}$
- ze : charge of particle
- v : speed of particle
- e : elementary charge
- E : energy of particle
- c : speed of light
- x : length of path
- m_e : mass of electron
- n : numeric density of electrons in material
- I : Mean excitation potential of material

Note:

- Bloch approx: $\boxed{I = 10\,eV \cdot Z}$, Z atomic number of the medium.
- Electron number density: $n = \frac{N_A \cdot Z \cdot \rho}{A \cdot \underbrace{M_u}_{=1}}$

With

- ρ density
- A mass number
- N_A avogadro number

TEST Electron density of water

We know $n_e = 3,34 \cdot 10^{29}\ m^{-3}$

- density of water: $997\ kg/m^3 = 997 \cdot 10^3\ g/m^3$
- avogadro number $6,03\ 10^{23}\ mol^{-1}$
- $Z \simeq 7,42$ (effective)
- $A = 18\ g/mol$

$$\Rightarrow n = \frac{N_A \cdot Z \cdot \rho}{A} = \frac{6,02 \cdot 10^{23}\ mol^{-1} \cdot 7,42 \cdot 997 \cdot 10^3\ g/m^3}{18\ g/mol}$$

$$= \underline{2,47 \cdot 10^{25}}$$

Not exact but good magnitude

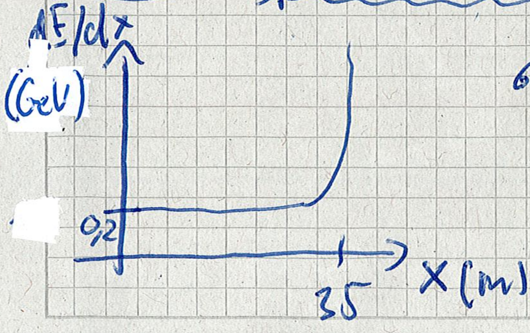
✓ TESTED

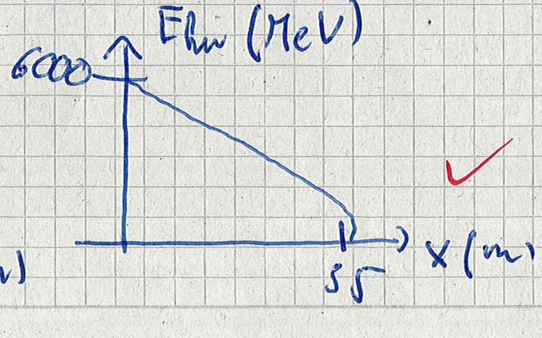
Main Idea of the code

- Start with a particle at a given E_{kin} at $x = 0$
- Compute dE/dx
- $E_{kin} = E_{kin} - dE$, $x = x + dx$

loop until $E_{kin} < 0$

TEST proton in water with $E_{kin} = 6\ GeV$





Coherent!

✓ TESTED

(bethebloch-muons.py)

B) Bethe-Bloch for muons in Air (adiabatic athmosphere)

TEST * Convergence of the previous method for muon in air (sea level)

For air:

- effective Z : 7,6
- A : 28,97 g/mol
- ρ at sea level at 0°C 1,060 kg/m^3

For muon:

- $Z_1 = 1$
- Mass $M_0 = 207 \cdot m_e$

Initial E_{kin} : ~~[illegible]~~ 2 GeV

Results: Distance for $E_{kin} \leq 0$: ~17'000 m

TESTED

TEST ** Muons with E_{kin} 6 GeV after 15'000 m

The energy of muons with initial $E_{kin} = 6$ GeV has a residual energy of 2 GeV after 15'000 m of interaction!

TESTED

Coherent with cosmic.lbl.gov

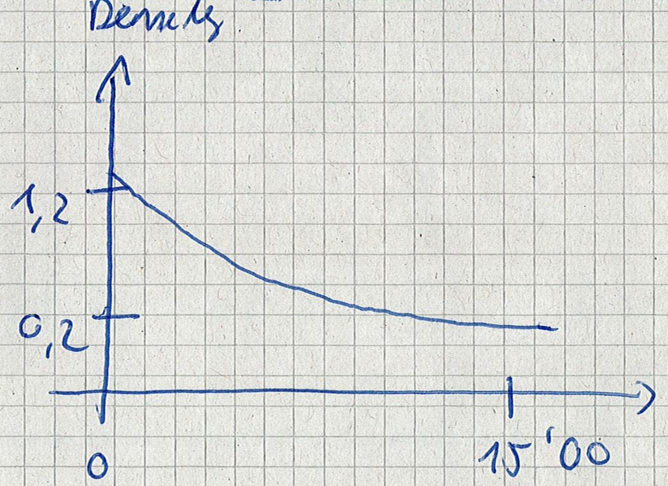
C) Insertion of the adiabatic athmosphere model

Variation of density of air with altitude:

$$\boxed{\rho = \frac{p_0 M}{R T_0}\left(1 - \frac{L h}{T_0}\right)^{gM/RL - 1}}$$

- M: molar mass of air 0,029 kg/mol
- R: ideal gaz constat 8,314 J/(mol·K)
- p_0 sea level pressure: 101325 Pa
- T_0 sea level temperature: 288,15 K
- L: temperature lapse rate: 0,0065 K/m
- h: altitude
- g = 9,81 m/s²

TEST **Variation of density with altitude (0 to 20000 m)**



OK, coherent with cosmic.lbl.gov

✓

TESTED

Intuition in the code we already have in the following script: bethebloch_muons_adia.py

⚠ here, we suppose that the creation altitude of muons is about 15 km. So the "0" distance of the previous code is supposed to be at this creation altitude

TEST Idem as **, but we have an adia. athmosphere

Energy after 15'000 m 4,789 GeV

Ok, but seems very high, but the model is more accurate!

TESTED

TEST Idem as *, but with an adia. athmosphere

muon take approx 20'000 m to stop

Much bigger than the expectations....

TESTED

D) Computation of the max. path length for energies between 0,2 GeV and 100 GeV

Range: 0,2 GeV (minimum ~~to~~ the graph) minimum

100 GeV maximum (after, flux $< 10^{-4}$