Computational Stats

Group III

Deliveable 1

Exercise 1

1. Consider the continuous random variable X with pdf:

$$f(x) = \begin{cases} \frac{4}{3}(x^3 + x) & 0 < x < 1 \\ 0, & \text{for all others } x \text{ values} \end{cases}$$

Now consider the random variable $Y = g(X)$, where $g(x) = log(x^2 + 4)$. Estimate $P(1.3 < Y < 1.5)$ using the Monte Carlo Method, as well as the estimator standard deviation.

$$P(1.3 < Y < 1.5) \quad = \quad P(1.3 < g(x) < 1.5) \quad = \quad P(1.3 < log(x^2 + 4) < 1.5)$$

Given that x only present values between $0 < x < 1$, that imples:

- the minimun value of $log(x^2 + 4)$ is $log(4)$
- the maximum value of $log(x^2 + 4)$ is $log(5)$

Therefore, we know that:

$$P(1.3 < log(x^2 + 4) < log(4)) = 0$$

With this taken into consideration, the probability that we want to calculate is:

$$P(log(4) < log(x^2 + 4) < 1.5)$$

Which we can know expand into:

$$P(log(4) < log(x^2+4) < 1.5) \quad = \quad P(4 < x^2+4 < e^{1.5}) \quad = \quad P(0 < x^2 < e^{1.5}-4) \quad = \quad P(0 < x < \sqrt{e^{1.5} - 4})$$

So, we now know that the probability we want to calculate can be obtain by the following integral:

$$\int_0^{\sqrt{e^{1.5}-4}} \frac{4}{3}(x^3 + x)dx(\#eq : integralProb) \tag{1}$$

```
mc <- function(t){
  k <- sqrt(exp(1.5)-4)
  return ( (4/3) * ((k*t)^3 + k*t) *k )
}

#t follows an uniform

sample <- runif(100000)
```

```r
pEst <- mean(mc(sample))

varEstimator <- (1/(length(sample)^2))*sum((mc(sample)-pEst)^2)
df <- data.frame(
  probEstimated = pEst,
  varianceMC = varEstimator
)

df
```

```
##   probEstimated   varianceMC
## 1     0.3986619 7.165804e-07
```