# Multivariate Stats

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## Contents



# 1 Lesson 2

## 1.1 Variance and Corrected Variance

### 1.1.1 Variance

$$S_n^2 = \frac{1}{N}\sum_{i=1}^{N}(X_i - \overline{X})^2$$

$$S_n^2 = \frac{1}{N}\sum_{i=1}^{N}(X_i - \overline{X})(X_i - \overline{X})$$

### 1.1.2 Corrected Variance

$$S_{n-1}^2 = \frac{1}{n}\sum_{i=1}^{n-1}(X_i - \overline{X})^2$$

$$S_{n-1}^2 = \frac{1}{n}\sum_{i=1}^{n-1}(X_i - \overline{X})(X_i - \overline{X})$$

### 1.1.3 Covariance

$$Cov(x,y) = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})$$

$$Cov(x,y) = Cov(y,x)$$

### 1.1.4 Pearson Correlation Coefficient

$$r_{xy} = \frac{Cov(x,y)}{\sqrt{S_x^2 \times S_y^2}}$$

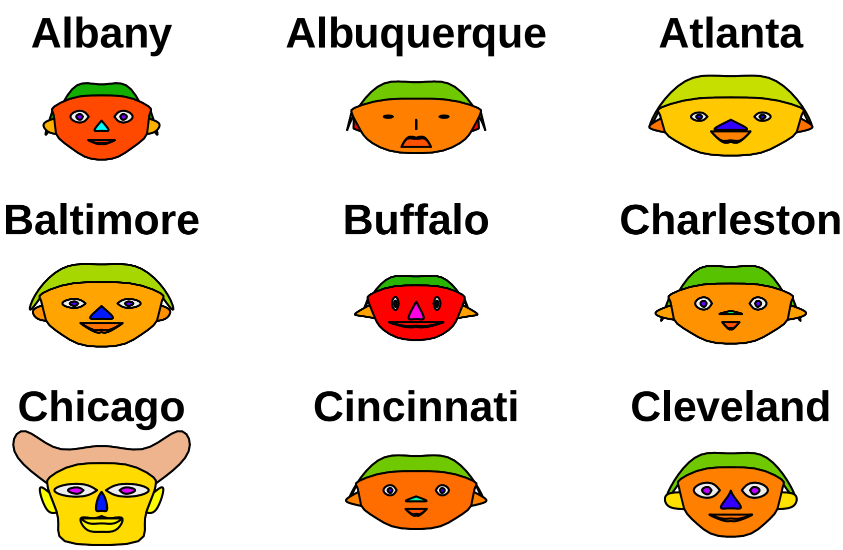
The domain of this coefficient is [-1, 1]

$$\Sigma = V^{\frac{1}{2}}$$

## 1.2 Exercises

```
X = matrix(c(42,52,48,58,4,5,4,3),4)

XMeans <- apply(X, 2, mean)

XVars <- var(X)

Xcor <- cor(X)
```

```
dados <- as.data.frame(readxl::read_xlsx(file.path(datasetsDir,"data1.xlsx")))

aplpack::faces(HSAUR3::USairpollution[1:9,], print.info = F)
```




```
meanVector <- c(5,10)

Sigma <- matrix(c(9,16,16,64),2)
Sigma.eigen <- eigen(Sigma)
```

The eigen values are 68.3158765, 4.6841235

TO obtain the ellipse containing 95% of the population, we must calculate

$$(x-\mu)' \times \Sigma^{-1} \times (x-\mu)' <= \chi^2_{(2)0.95}$$

# 2 Studies & Experiments

Unfortunately, no one can be told what the ***Matrix*** is. You have to see it for yourself

\- Morpheys

## 2.1 Matrixes' Determinants

1. The determinant of a transformation (or matrix) is the area of that transformation
2. The determinant of a transformation (matrix) is the factor by which any other transformation (matrix) will change its area
3. If the determinant of a transformation is 0, it means that transformation squishes all the space onto

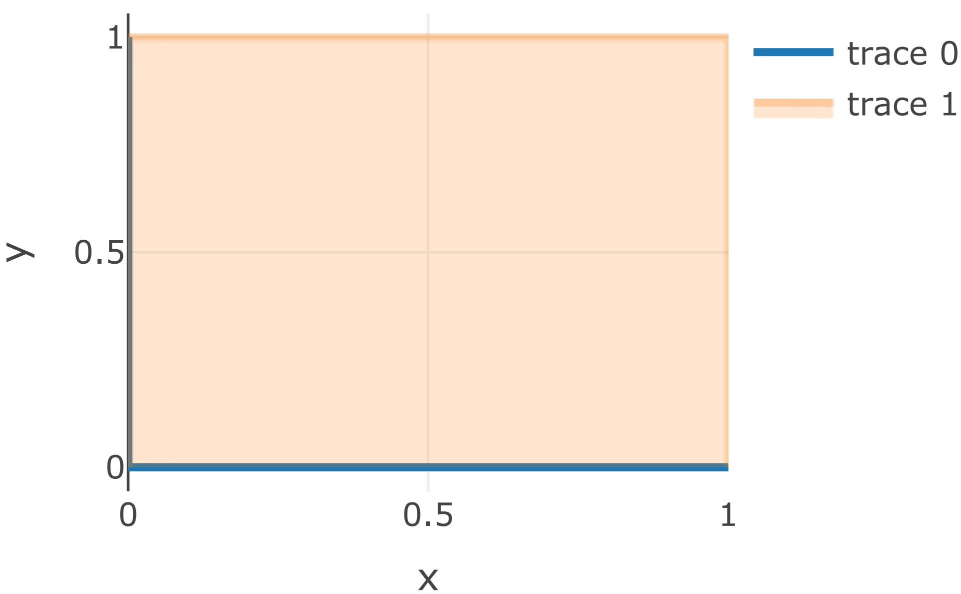
- [2d case] a line, or even into a single point
- [3d case] a plane, a line, or event into a single point

4. The signal of the determinant is related to the orientation: if the determinant of a transformation is negative, this means that the orientation of the plan is changed (like a sheet, from front to back)

```
matrixA2d <- matrix(c(1,0,0,1),2)
matrixA2d
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    0    1
```

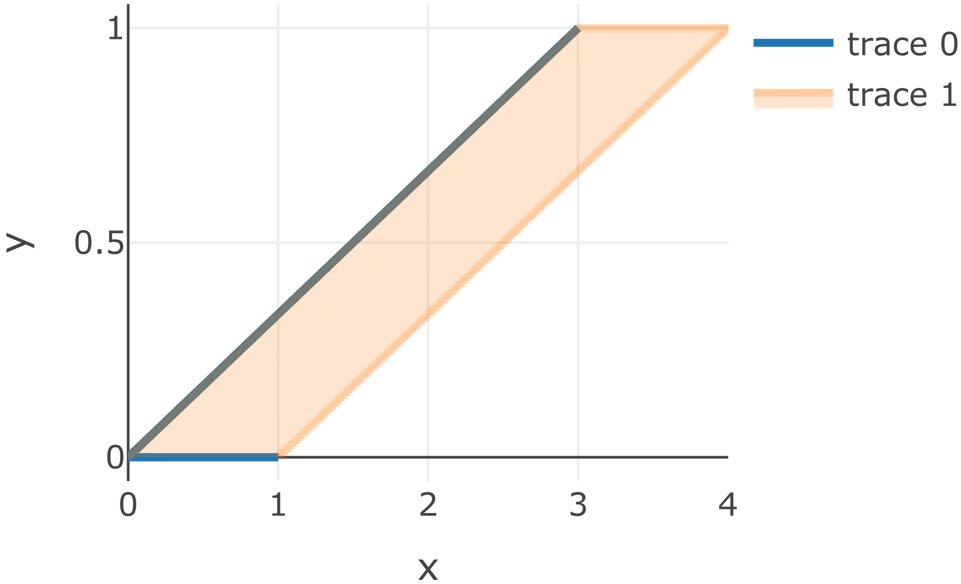
```
drawMatrixWithDet(matrixA2d,dim(matrixA2d)[1])
```




```
matrixB2d <- matrix(c(1,3,0,1),2)
matrixB2d
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    3    1
```

```
drawMatrixWithDet(matrixB2d,dim(matrixB2d)[1])
```




```r
matrixA <- matrix(c(1,0,0,0,1,0,0,0,1),3)
matrixA
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    0    1    0
## [3,]    0    0    1
```

```r
drawMatrixWithDet(matrixA,dim(matrixA)[1])
```

```
## Warning: 'mesh3d' objects don't have these attributes: 'mode', 'line'
## Valid attributes include:
## 'type', 'visible', 'showlegend', 'legendgroup', 'opacity', 'name', 'uid', 'ids', 'customdata', 'sele
```

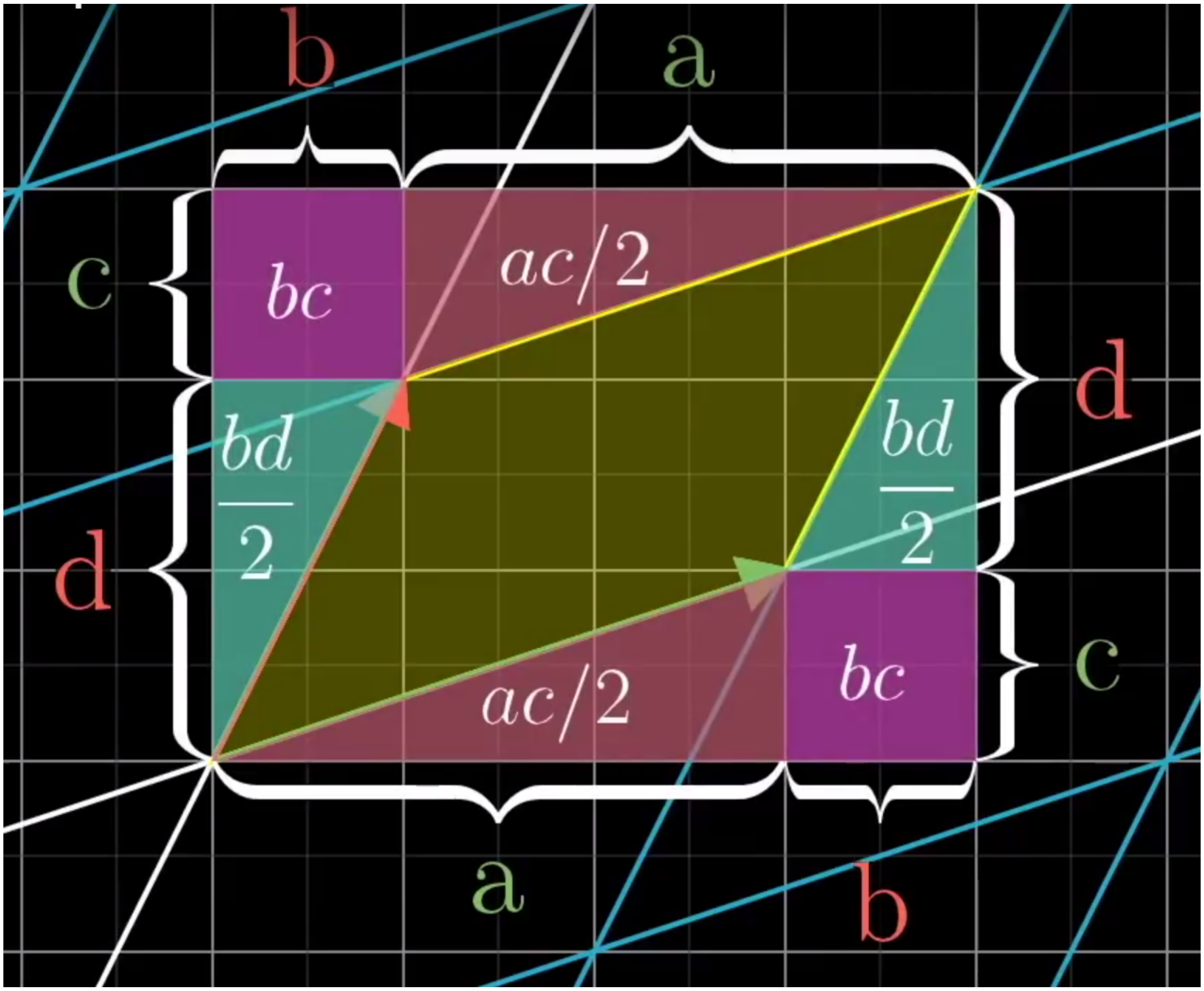
```r
matrixB <- matrix(c(1,2,3,2,2,1,3,2,4),3)
matrixB
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    3
## [2,]    2    2    2
## [3,]    3    1    4
```

```r
drawMatrixWithDet(matrixB,dim(matrixB)[1])
```

```
## Warning: 'mesh3d' objects don't have these attributes: 'mode', 'line'
## Valid attributes include:
## 'type', 'visible', 'showlegend', 'legendgroup', 'opacity', 'name', 'uid', 'ids', 'customdata', 'sele
```

WebGL is not supported by your browser - visit https://get.webgl.org for more info




$$det \left( \begin{matrix} 1 & 2 & 3 \\ [\ 4 & 5 & 6\ ] \\ 7 & 8 & 9 \end{matrix} \right) = (a+b) \times (d+c) - (ac + bd + 2bc)$$

$$= (ad + ac + bd + bc) - ac - bd - 2bc$$

$$= ad - bc$$

#### 2.1.1 More than 3D

> You have to let it all go, Neo. Fear, doubt, and disbelief. Free your mind
>
> - Morpheus

## 3 Exercises

This is an R Markdown Notebook. When you execute code within the notebook, the results appear beneath the code.

Try executing this chunk by clicking the *Run* button within the chunk or by placing your cursor inside it and pressing *Cmd+Shift+Enter*.

# 4 Exercise

## 4.1 Exercise 1 - Linear Algebra

### 4.1.1 Ex 1

```
A = matrix(c(4,7,2,5,3,8),2)

B = matrix(c(3,6,-2,9,4,-5),2)
```

a)

$$A + B =$$

```
A+B
```

```
##      [,1] [,2] [,3]
## [1,]    7    0    7
## [2,]   13   14    3
```

$$A - B =$$

```
A-B
```

```
##      [,1] [,2] [,3]
## [1,]    1    4   -1
## [2,]    1   -4   13
```

b)

$$A' \times A =$$

```
t(A)%*%A
```

```
##      [,1] [,2] [,3]
## [1,]   65   43   68
## [2,]   43   29   46
## [3,]   68   46   73
```

$$A \times A' =$$

```
A%*%t(A)
```

```
##      [,1] [,2]
## [1,]   29   62
## [2,]   62  138
```

### 4.1.2 Ex 2

```
A = matrix(c(1,2,3,-1),2)

B = matrix(c(2,1,0,5),2)
```

a)

$$A \times B =$$

```r
A %*% B
```

```
##      [,1] [,2]
## [1,]    5   15
## [2,]    3   -5
```

$$B \times A =$$

```r
B %*% A
```

```
##      [,1] [,2]
## [1,]    2    6
## [2,]   11   -2
```

b)

$$det(A \times B) =$$

```r
det(A %*% B )
```

```
## [1] -70
```

$$det(A) =$$

```r
det(A)
```

```
## [1] -7
```

$$det(B) =$$

```r
det(B)
```

```
## [1] 10
```

### 4.1.3 Ex 3

```r
A = matrix(c(1,2,5,2,4,10,3,6,15),3)

B = matrix(c(-1,-1,1,1,1,-1,-2,-2,2),3)
```

a)

$$A \times B = 0$$

```r
A %*% B
```

```
##      [,1] [,2] [,3]
## [1,]    0    0    0
## [2,]    0    0    0
## [3,]    0    0    0
```

b)

$$tr(A)$$

```r
#tr(A)
```

$$tr(B)$$

```r
#tr(B)
```

b)

$$det(A)$$

```r
det(A)
```

```
## [1] 0
```

```r
1+1
```