ARTIFICIAL SATELLITES

I. The Motions of Artificial Satellites

1. *Introduction.*—This report is limited, rather narrowly, to the astronomical information that can be obtained from observations of the positions and motions of artificial satellites of the Earth. Studies of artificial satellites on radio wavelengths are the subject of a separate note.

Positional observations may be used in three quite different ways:

(*a*) as a measure of parallactic shift to provide information on the relative positions of observers on the Earth;

(*b*) as a means of determining the instantaneous orbit and thus the secular variations of the elements of the orbit;

(*c*) as a means of determining the position of the satellite in its orbit.

The motion of an artificial satellite is essentially governed by the non-spherical gravitational field of the Earth and by the resistance due to air drag and electrical effects caused by the Earth's atmosphere; perturbations by the Sun, Moon, and other planets are very small. For use (*a*) as geodetic beacons a knowledge of the actual motion is required over only short intervals of time; for satellites close to the Earth simultaneous, or nearly simultaneous, observations may prove useful even if the position and motion are not accurately known. The unknown effects of atmospheric resistance need not therefore be serious. The secular variations of the orbital elements in (*b*) depend almost entirely either on the effects of atmospheric resistance or on the asymmetry of the Earth's gravitational field. Thus, analysis of the rates of decrease of period and eccentricity has already led to improved knowledge of the density of the atmosphere for heights up to several hundred kilometres, and has shown its great variability with position and time; and analysis of the observed rates of precession of the orbital plane (retrogression of the node) has indicated a correction of about 0·3 per cent to the adopted value of the dynamical flattening, or oblateness, of the Earth. The actual position of a satellite as in (*c*) depends on accurate evaluation of both perturbing effects; since accurate prediction of atmospheric resistance is impracticable in the case of low satellites, the observed motion can be used only for detailed studies of atmospheric characteristics. However, for a satellite whose perigee distance is sufficiently large for atmospheric resistance to be small enough to be predicted, the comparison of the observed with the calculated motion will be of the greatest interest to astronomy, and may lead to a new and powerful method for the determination of ephemeris time. The prediction and analysis of such motions presents many difficult problems of both theory and analysis; and accurate observations of such fast-moving objects require special techniques. The existence of observable artificial satellites, with accurately predicted orbits (in the same way as for the minor planets), may also be of significance for problems of geodesy and navigation.

Most of the papers so far published are concerned only with preliminary investigations of the problems of theory, observation and analysis, and it is to be expected that many more detailed discussions will be published in the near future. No attempt has therefore been made to give a complete bibliographical coverage, and only a few selected references to readily accessible journals are given. Further, this report has been restricted to a general survey of the results obtained and of the demands on theory and observational technique that are likely to be made in the future.

No attempt has been made to discuss space probes as it is unlikely that their motions can be observed sufficiently accurately for their orbits to yield new information on the gravitational field of the Earth, or of the solar system.

2. *Orbital data.*—The principal physical characteristics and orbital elements of artificial satellites launched up to February 1959 are given in the accompanying table, which is based on a similar table issued by the Royal Aircraft Establishment. No data are given in respect of the radio transmitters, telemetering systems or other instruments carried.

3. *Parallactic shift.*—There is no record of any results having so far been obtained from observations of artificial satellites treated as geodetic beacons; although, to some extent, the transit observations from the Minitrack and Moonwatch stations form an extreme case of this application. The more straightforward applications, such as the theoretically possible method of determining position at sea by a single observation without reference to the vertical, depend on a knowledge of the position of the satellite in its orbit; predictions to the requisite precision are not possible with existing satellites.

4. *Secular orbital variations.*—For low satellites, that is for satellites with small perigee distance, the most important secular variations of the elements are due to atmospheric resistance. This results in a decrease of the period (the rate of decrease increasing as the period decreases), a corresponding decrease in the semi-major axis with a proportionally larger decrease in distance at apogee than at perigee, and thus in a decrease in the eccentricity of the orbit. Estimates of the life-time of such satellites can be made on the basis of simple theories, in spite of the fact that the day-to-day motions of the satellites are so erratic that precise predictions cannot be made even a few days in advance.

The orbital plane of a satellite precesses, independently of atmospheric resistance, because of the departure of the Earth's gravitational field from spherical symmetry. The longitude of the node of the orbit on the equator retrogrades at a rate proportional to the cosine of the inclination and to the mean distance to the power $-7/2$; the rate therefore increases as the mean distance decreases. The theoretical expression contains terms dependent on the coefficients of the spherical harmonics in the expression of the gravitational potential of the Earth (**19**):

$$U=\frac{fM}{R}\left\{\frac{R}{r}+J\frac{R^3}{r^3}\left(\frac{1}{3}-\sin^2\phi'\right)+D\frac{R^5}{r^5}\left(\sin^4\phi'-\frac{6}{7}\sin^2\phi'+\frac{3}{35}\right)\right\}$$
$$+\sum\sum\frac{A_{ns}S_{ns}}{r^{n+1}}$$

where S_{ns} are the spherical surface harmonics P_{ns} ($\cos s\lambda$, $\sin s\lambda$). The rate of precession can be accurately determined over a long period even from comparatively crude observations; analysis of the differences between the observed and computed values have already shown (**18**) that the currently adopted value of J is too large, corresponding to a change in the coefficient of dynamical flattening from 1/297 to 1/298. This change is nearly three times the standard error of the previous determination from gravity measurements. Further data are rapidly

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Physical Characteristics

Ref. No.	Designation	Name and description	Launching date; life-time; descent date	Mass	Size d = diameter l = length	Maximum visual magnitude
1	1957α2	Sputnik 1 instrumented sphere	1957 Oct. 4 ? 92 days ? 1958 Jan. 4	83 kg	d = 58 cm	+5
2	1957α1	Sputnik 1 rocket	1957 Oct. 4 57 days 1957 Dec. 1	?	d = ? l = ?	−1
3	1957β	Sputnik 2 instrumented rocket	1957 Nov. 3 162 days 1958 Apr. 14	? payload = 508 kg	d = ? l = ? 24 m	−3
4	1958α	Explorer 1 instrumented rocket	1958 Feb. 1 4 years	14 kg	d = 15 cm l = 2 m	+6
5	1958β2	Vanguard 1 instrumented sphere	1958 Mar. 17 ? 200 years	1·5 kg	d = 16 cm	+8
6	1958β1	Vanguard 1 rocket	1958 Mar. 17 ? 100 years	23 kg	d = 51 cm l = 1·2 m	+5
7	1958γ	Explorer 3 instrumented rocket	1958 Mar. 26 93 days 1958 June 28	14 kg	d = 15 cm l = 2 m	+6
8	1958δ2	Sputnik 3 instrumented cone	1958 May 15 600 days	1327 kg	d = 1·7 m l = 3·8 m	+1
9	1958δ1	Sputnik 3 rocket	1958 May 15 202 days 1958 Dec. 3	?	d = ? l = ?	−3
10	1958ε	Explorer 4 instrumented rocket	1958 July 26 450 days	17·5 kg	d = 15 cm l = 2 m	+6
11	1958ζ	Atlas instrumented rocket	1958 Dec. 18 33 days 1959 Jan. 21	3856 kg	d = 3 m l = 24 m	−3
12	1959α1	Vanguard 2 instrumented sphere	1959 Feb. 17 ? 100 years	9 kg	d = 56 cm	+6
13	1959α2	Vanguard 2 rocket	1959 Feb. 17 ?	23 kg	d = 51 cm l = 1·2 m	+5

Notes.—(i) The material in this table has been compiled from a variety of sources, and is not intended to be definitive.

(ii) ? denotes that the value is unknown or is very uncertain.

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Equatorial orbital elements

Ref. No.	Date		Inclination	Nodal period	Semi-major axis	Eccentricity	Daily motion of perigee	Daily motion of node	Height (km) perigee	Height (km) apogee
			°	m			°	°		
1	1957 Oct.	5	65·1	96·2	1·090	0·052	−0·4	−3·1	213	936
	Oct.	26	65·1	95·4	1·084	·047	0·4	3·2	209	860
	Dec.	25	65·0	91·0	1·051	·020	0·5	3·4	191	458
2	1957 Oct.	5	65·1	96·2	1·090	0·052	−0·4	−3·1	213	936
	Nov.	4	65·1	94·0	1·074	·038	0·4	3·2	208	734
	Nov.	24	65·0	91·0	1·051	·020	0·5	3·4	191	458
3	1957 Nov.	4	65·3	103·7	1·147	0·099	−0·4	−2·6	213	1659
	1958 Jan.	4	65·3	100·5	1·123	·080	0·4	2·8	208	1357
	Feb.	21	65·3	97·1	1·097	·060	0·4	3·0	198	1043
	Mar.	25	65·2	93·8	1·072	·040	0·5	3·2	185	732
	Apr.	9	65·2	90·8	1·049	·022	0·5	3·4	167	456
4	1958 Feb.	1	33·2	114·8	1·228	0·139	+6·3	−4·3	369	2541
	1959 Jan.	3	33·1	111·7	1·206	·125	6·8	4·4	348	2276
5	1958 Mar.	17	34·3	134·1	1·362	0·191	+4·4	−3·0	654	3966
	1959 Feb.	12	34·2	134·1	1·361	·190	4·4	3·0	654	3953
6	1958 Mar.	17	34·3	138·5	1·391	0·208	+4·0	−2·8	649	4338
7	1958 Mar.	27	33·3	115·7	1·234	0·166	+6·3	−4·3	187	2800
	Apr.	25	33·3	109·7	1·191	·136	7·0	4·7	185	2251
	June	14	33·3	96·8	1·096	·063	9·2	6·2	172	1047
8	1958 May	15	65·2	106·0	1·165	0·111	−0·3	−2·5	213	1868
	Oct.	1	65·2	104·0	1·151	·101	0·4	2·6	211	1685
	Dec.	9	65·2	102·5	1·140	·092	0·4	2·7	209	1547
9	1958 May	15	65·2	105·8	1·163	0·111	−0·4	−2·5	213	1861
	Aug.	15	65·1	102·0	1·134	·089	0·4	2·7	208	1499
	Oct.	11	65·1	98·0	1·104	·066	0·4	3·0	200	1127
	Nov.	14	65·0	94·0	1·074	·041	0·5	3·2	187	752
	Dec.	1	65·0	90·0	1·043	·017	0·5	3·5	163	389
10	1958 July	27	50·3	110·2	1·194	0·128	+2·9	−3·6	263	2211
	1958 Oct.	25	50·3	107·7	1·176	·115	3·1	3·7	259	1987
	1959 Jan.	3	50·3	105·3	1·159	·103	3·4	3·9	259	1766
11	1958 Dec.	19	32·3	101·3	1·130	0·090	+8·6	−5·7	180	1480
	1959 Jan.	2	32·3	98·1	1·106	·071	9·1	6·0	175	1177
	Jan.	17	32·3	92·7	1·065	·037	10·1	6·7	163	662
12	1959 Feb.	17	32·9	125·7	1·304	0·166	+5·3	−3·5	558	3323
13	1959 Feb.	17	32·9	130·0	1·333	0·184	+4·9	−3·3	558	3691

(iii) The semi-major axis is in units of the Earth's equatorial radius, 6378 km.
(iv) The heights of perigee and apogee are above a sphere of radius 6378 km. For a latitude ϕ these heights should be increased by about $(21{\cdot}4 \sin^2 \phi)$ km.

being accumulated and it is desirable to wait until they have been discussed before a definitive value of J, as derived from satellite observations, is adopted; satellites of differing inclinations are required to separate the corrections to J and D.

Evaluation of the coefficients of the other harmonics, especially those depending on longitude, depends on the comparison of the observed and predicted *positions* of the satellite.

The oblateness of the Earth is also responsible for the motion of the perigee along the orbit. The principal term in the theoretical expression for the secular variation of the argument of perigee (the arc from the ascending node on the equator to perigee) contains the term $(0\cdot2-\cos^2 i)$; for orbits with inclinations i greater than 63° the motion is thus retrograde. For the Sputniks, for which the inclination is near 65°, the motion of the perigee is small. The position of perigee cannot easily be determined accurately from observation, and no detailed comparison of observation with theory has yet been published.

The inclination is, to a first approximation at least, theoretically unaffected by either atmospheric resistance or gravitational asymmetry. But the actual inclinations are observed to decrease gradually. It has been suggested that this is due to a component of air drag, normal to the direction of motion, caused by the rotation of the atmosphere with respect to the orbital plane (**14**). It is essential that possible atmospheric effects should be fully investigated, and allowed for, in the secular variations of the node and perigee before the results derived from the observed values are adopted.

The variations of the magnitude of non-spherical satellites lead to determination of their attitudes and rotational motions. The secular changes in these motions imply changing effective cross-sectional areas and so complicate the interpretation of the air drag effects.

5. *Positional data.*—The motions of the artificial satellites so far launched, as distinct from the positions of their orbital planes, are so disturbed by irregularities of atmospheric resistance that no precise astronomical information has yet been obtained. The applications of such data for satellites not subject to such disturbances are considered in Section 8.

6. *Precision of observations.*—Owing to the high linear and angular speeds of the satellites the determination of the time of observation is as critical as the determination of position—in one millisecond of time a satellite travels about 8 metres and up to 5 seconds of arc. However, observations suitable for maintaining a prediction service and for establishing the principal secular changes in the elements of an orbit require accuracies only of the order of 1 degree in arc and 1 second in time. Such observations can be made by estimating by eye the position of a satellite with respect to bright stars near its track and noting the times from an accurate watch. A more accurate and elaborate method is provided by the Moonwatch programme, organized by the Smithsonian Astrophysical Observatory, and by similar programmes in other countries. A team of observers with small telescopes is used to establish the time of transit of a satellite across the meridian. The method is primarily intended for finding faint satellites.

Observations of great value have been obtained by the use of kinetheodolites. These are special cameras mounted on an altazimuth mounting. The satellite is tracked visually and photographs of the satellite with respect to a grid and of the altitude and azimuth scales defining the centre of the grid are obtained at 1/5 second intervals. The accuracy of a single observation is of the order of $0^{\circ}\cdot01$ and timing accuracy is of the order of $0^{s}\cdot01$. The principal advantage of this type of observation is that a sufficiently long arc of the orbit can be observed so that the elements of the orbit can be obtained directly at a single passage of a bright satellite.

Observations of the highest precision are obtained by the use of Baker-Nunn satellite tracking cameras. As in meteor cameras the trail of a satellite on a star-field is interrupted to establish a time-scale. Each plate gives a position with a probable error of about $4''$ and a timing accuracy of about $0^{s}\cdot002$. Precise observations obtained by these and similar cameras should be of great value in establishing accurate orbits of high satellites and hence for applications to geodesy and time determination.

For low satellites conventional astronomical telescopes are of little value owing to the great angular speed of track and the lack of precise predictions, but are likely to be of value for high satellites especially as such satellites will be faint.

Radio and radar observations have the great advantage that they are not restricted to times of favourable weather conditions during twilight passages of the satellites, but the angular accuracy is low. This is particularly true of positions based on transmissions from the satellites at 20 and 40 Mc/s, as the ionospheric refraction may be so great as to render the observations useless. At 108 Mc/s, however, an accuracy of $0'\cdot3$ in position and $0^{s}\cdot001$ in time is expected from, for example, the American system of Minitrack stations.

7. *Theories of satellite orbits.*—Atmospheric resistance and the oblateness of the Earth give rise to distinct first-order effects and so, up to the present time, have been treated independently. This is only justified for close satellites because the variability of atmospheric resistance (due to varying aspect of the satellite, varying densities and other factors) renders useless any accurate theory. For more distant satellites, for which atmospheric resistance is sufficiently small to be treated theoretically, it will probably be justifiable to include it independently, possibly at the same time as the perturbations by the Moon. The perturbations of the elements of an elliptic orbit due to motion in a resisting medium are treated in many standard text-books on celestial mechanics (e.g. (**17**) and (**15**)); the specific application to artificial satellites of the Earth is considered in references (**14**) and (**16**).

The theory of the motion of a particle in the field of a spheroid has not been fully treated in astronomical literature since in the few cases arising in the solar system the observations are so difficult that they can be adequately represented by a simple theory. The theory of the motion near the equatorial plane of a non-spherical body has, however, been developed by Brouwer (**6**) in connection not only with the orbit of Jupiter V and similar satellites but also as an introduction to the motion of close binary systems, consisting of two spheroids of comparable masses. The effect of the oblateness of the Earth on the motion of the Moon has, of course, been fully discussed by G. W. Hill and E. W. Brown.

The first- and second-order solutions that have so far been published have been adequate for the analyses of the observations of past satellites. However, it may be noted that the work of King-Hele (**10**), who has been responsible for much excellent work in maintaining a prediction service for the British Isles during 1958 and in analysing visual observations made primarily in British Isles, does not follow conventional astronomical methods or usage, though Öpik (**11**) has obtained many of King-Hele's results, or their equivalents, by the application of the method of variation of elements.

Sterne (**13**) and Garfinkel (**8**) have introduced expressions for Hamiltonians, approximating to the actual Hamiltonian of the attraction of a spheroid, which lead to closed solutions. These solutions give much better representations of the actual orbit than does a Keplerian ellipse, and yet retain many of the advantages of the latter. It is not yet clear whether these will be suitable for the development of an accurate theory.

Brouwer (**7**) has shown that his earlier work, based on the Hill-Brown lunar theory, may be extended to non-equatorial orbits. He has also considered Delauney's method and his preliminary investigations have shown that although it offers the perfect analytic solution the complex operations required do not lend themselves to machine calculations. He estimates that an accurate general theory for an individual satellite is of the same order of difficulty as that for a major planet.

Herget and Musen (**8**) are investigating the possible applications of Hansen's method and it is understood that preliminary conclusions will be published in the *Astronomical Journal*. A first-order solution by a similar method has been given by Roberson (**12**).

It is clear from these investigations that the artificial satellites will provide celestial mechanics with a range of problems that will require many years for their satisfactory solution, and that theories that have been superseded for the orbits of natural satellites must be re-examined for their suitability for these new problems.

Satellites whose orbits extend to the distance of the Moon afford a particularly difficult case of the three-body problem, even if only the restricted three-body problem is considered. The possible orbits of such a " lunar probe " have been treated in considerable detail by Yegorov (**3**) who has been forced to adopt numerical integration procedures. The conditions for the establishment of a satellite in a periodic orbit about the Earth and Moon are so critical and the difficulties of observation are so great that it seems likely that numerical integrations will suffice for many years.

In general, numerical integration of the equations of motion will provide an essential check on theoretical methods but will not suffice for long-term investigations, such as those concerned with the determination of time. The periods are so short that the intervals used will have to be very small and building-up errors will soon get out of control.

8. *Applications*.—For distant satellites for which atmospheric perturbations are so small that they can be accurately allowed for in the discussion of the observations (this implies a minimum perigee distance of at least 1000 km above the Earth's surface) the following information of astronomical and geodetic importance should be obtainable, provided that the physical characteristics allow observations of adequate precision to be made.

(*a*) *Ephemeris time*.—Ephemeris time is defined by means of the motion of the Earth in its orbit round the Sun (which moves through $1''$ of longitude in 24^s of time) but determined in practice by the motion of the Moon in its orbit round the Earth ($1''$ of longitude $= 1^s\cdot8$). For an artificial satellite with mean distance $(1+x)$ Earth radii, $1''$ of longitude $= 0^s\cdot004\,(1+x)^{3/2}$, so that for reasonable values of x the precision of time determination from determinations of longitude of comparable accuracy is 300 times that for the Moon. Observations should be possible (if possible at all) with both greater precision and greater frequency than with the Moon; and at closest approach there is a further factor of about 5 between the precisions of observation and of longitude. The precision of the determination of the second is proportional to the precision of the normal places obtained by grouping the observations and to the time-interval between them: a precision of about 3×10^{-10} (of the same order as that at present possible for the frequency of the caesium resonator) should theoretically be obtainable from observations spread over a few months. This assumes precisions of the normal places of about 1″ and a timing accuracy of $0^{s}\cdot001$, corresponding to a precision of about 5 metres in actual position.

The practical difficulties, apart from those of observation, are formidable. Even when an adequate theory of the motion is available, observation over an extended period will be necessary to relate the independent time variable of the theory to ephemeris time. The comparison of the time-scale defined by the satellite itself with, say, that of caesium should, however, provide interesting information.

(*b*) *Fundamental reference systems.*—The possibility of using fast-moving artificial satellites in the determination of the fundamental reference system should not be overlooked. However, apart from the difficulties of observation, very highly accurate theories of motion would be necessary.

(*c*) *Gravitational field of the Earth.*—Observations of position, especially of satellites in orbits at different inclinations to the equator, should provide data from which the coefficients of the higher harmonics in the Earth's gravitational field, especially those depending on longitude, can be determined. According to O'Keefe and Batchlor (**21**) the effect of the term $P_2^2 \cos 2\lambda$, which is the largest that depends on longitude, will give rise to perturbations of the order of 5″ of orbital longitude; the coefficient should be reasonably well determined if observations of the precision assumed are possible. The principal terms, which give rise to secular variations in the elements, will of course be well determined, especially as observations can be combined over long intervals.

(*d*) *Geodetic positions and navigation.*—An observable satellite, whose position can be accurately predicted to a few tens of metres can clearly be used for determination of position on the Earth in many ways. Details are not discussed here.

9. *Desired orbital characteristics.*—Although it would be possible to specify the orbital characteristics, and possibly the physical characteristics as well, desirable for a satellite intended solely for the purely astronomical role considered above, it is certain that other factors will predominate. For a long time, astronomers will have to make use of the satellites as they come; all that can be done is to urge that all feasible steps be taken to make them readily observable by optical methods.

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