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YAGUANG YANG

# SPACECRAFT MODELING, ATTITUDE DETERMINATION, AND CONTROL

Quaternion-Based Approach



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# *Spacecraft Modeling, Attitude Determination, and Control*

## *Quaternion-based Approach*

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**Yaguang Yang**

US Nuclear Regulatory Commission  
Office of Research  
Rockville, Maryland, USA



**CRC Press**  
Taylor & Francis Group  
Boca Raton London New York

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6000 Broken Sound Parkway NW, Suite 300  
Boca Raton, FL 33487-2742

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Printed on acid-free paper  
Version Date: 20181115

International Standard Book Number-13: 978-1-138-33150-1 (Hardback)

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Library of Congress Cataloging-in-Publication Data

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Names: Yang, Yaguang, author.  
Title: Spacecraft modeling, attitude determination, and control : quaternion-based approach / Yaguang Yang (US Nuclear Regulatory Commission, Office of Research, Rockville, Maryland, USA).  
Description: Boca Raton, FL : CRC Press, 2019. | "A science publishers book."  
| Includes bibliographical references and index.  
Identifiers: LCCN 2018051045 | ISBN 9781138331501 (hardback)  
Subjects: LCSH: Space vehicles--Attitude control systems. | Stability of space vehicles. | Rotational motion (Rigid dynamics) | Quaternions. | Vector analysis.  
Classification: LCC TL3260 .Y36 2019 | DDC 629.47/42--dc23  
LC record available at <https://lccn.loc.gov/2018051045>

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*To my parents, my wife,  
my son, and daughter*



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# Preface

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My interest in spacecraft modeling, attitude determination and control started at Orbital Science Corporation. At the end of the summer of 2005, I was looking for a job that would best use my background in controls and optimization. There was an open house for job applicants at Dulles campus of the company. That was the first time I visited Orbital Science Corporation. I was very fortunate to have a chance to talk to Dr. Brian Keller, the deputy director of GNC (guidance, navigation, and controls) at the time. I showed him my publications in controls and explained my work at previous companies, he listened and immediately promised to set up an interview for me. A few weeks later, my future manager at Orbital Science Corporation, Mr. James Bobbett, called me and an interview was scheduled. Both Brian and James knew that I did not have a background in spacecraft and launch vehicles, however they trusted my background in controls and believed that my prior experience to be beneficial in this work. They offered me the job! I joined Orbital Science Corporation in November 2005.

My time at Orbital Science Corporation was delightful. I was deeply involved in the control system designs for two spacecrafts and one launch vehicle. My first assignment was to review and learn the design of ROCSAT III in preparation for designing the next spacecraft. In a few weeks, I realized that the design could be improved and I proposed an alternative method. I was surprised that my manager, Mr. Bobbett, quickly replied to my email with his strong support for my proposal. The proposed changes were implemented and six satellites were launched in April, 2006, all achieving their design requirements.

During my time at Orbital Science Corporation, several textbooks on spacecraft controls, such as M.J. Sidi's book "Spacecraft Dynamics and Control: A Practical Engineering Approach", B. Wie's book "Space Vehicle Dynamics and Control", and J.R. Wertz's book "Spacecraft Attitude Determination and Control", were great source to me in understanding this topic. Although all these books are excellent, I believed that some materials could be improved, especially, the control system design methods. However, my work assignments at Orbital

Science Corporation were very challenging and I did not have time to think about the specific details of these improvements.

I left Orbital Science Corporation to join the US NRC in 2008. At NRC, I have had more free time, after eight hours in office, to think about these problems. I started to publish papers in 2010 on new methods for spacecraft control and algorithms to design spacecraft control systems, trying to address control related problems in different stages of different missions using different sensors and actuators to cover as many design problems as possible. After a few years, my publications covered a few important areas in spacecraft modeling, attitude determination and control.

On May 1, 2015, I received an email from Vijay Primlani from CRC Press, asking if I was interested in publishing a book with this established publisher. My immediate thought was: that is a cool idea. I said “yes, but it might take some time because I want to consider a few more design problems that I have not done yet, besides I had been working and would continue to work only in my spare time for this project.” I did not know that the delay would be a few years but the promise has been the motivation for me to work continuously on this interesting project.

As this project approaches the finish line, I would like to thank a few people, who helped me along the way. First, I would like to thank Dr. Keller and Mr. Bobbett at Orbital Science Corporation for giving me the chance to work in this amazing area. Second, I would like to thank Mr. Primlani at CRC Press for his invitation to write a book with my choice of topic and for his patience with my slow progress. I am also indebted to my former colleague, Dr. Z. Zhou at NASA, who co-authored two papers which are included in this book. Last but not the least, I am grateful to my manager, Mr. Ronaldo Jenkins at the US NRC for his support and approval of writing this book in my spare time.

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# Contents

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<i>Preface</i>	v
<b>1. Introduction</b>	1
1.1 Organization of the Book	3
1.2 Some Basic Notations and Identities	6
<b>2. Orbit Dynamics and Properties</b>	7
2.1 Orbit Dynamics	7
2.2 Conic Section and Different Orbits	11
2.2.1 Circular Orbits	11
2.2.2 Elliptic Orbits	12
2.2.3 Hyperbolic Orbits	14
2.3 Property of Keplerian Orbits	14
2.4 Keplerian Orbits in Three-dimensional Space	16
2.4.1 Celestial Inertial Coordinate System	17
2.4.2 Orbital Parameters	17
<b>3. Rotational Sequences and Quaternion</b>	21
3.1 Some Frequently used Frames	22
3.1.1 Body-fixed Frame	22
3.1.2 The Earth Centered Inertial (ECI) Frame	22
3.1.3 Local Vertical Local Horizontal Frame	23
3.1.4 South-east Zenith (SEZ) Frame	23
3.1.5 North-east Nadir (NED) Frame	23
3.1.6 The Earth-centered Earth-fixed (ECEF) Frame	23
3.1.7 The Orbit (Perifocal PQW) Frame	24
3.1.8 The Spacecraft Coordinate (RSW) Frame	24
3.2 Rotation Sequences and Mathematical Representations	24
3.2.1 Representing a Fixed Point in a Rotational Frame	24
3.2.2 Representing a Rotational Point in a Fixed Frame	26

3.2.3	Rotations in Three-dimensional Space	27
3.2.4	Rotation from One Frame to Another Frame	29
3.2.5	Rate of Change of the Direction Cosine Matrix	30
3.2.6	Rate of Change of Vectors in Rotational Frame	30
3.3	Transformation between Coordinate Systems	31
3.3.1	Transformation from ECI (XYZ) to PQW Coordinate	31
3.3.2	Transformation from ECI (XYZ) to RSW Coordinate	32
3.3.3	Transformation from Six Classical Parameters to ( $v, r$ )	32
3.3.4	Transformation from ( $v, r$ ) to Six Classical Parameters	34
3.4	Quaternion and Its Properties	35
3.4.1	Equality and Addition	36
3.4.2	Multiplication and the Identity	36
3.4.3	Complex Conjugate, Norm, and Inverse	37
3.4.4	Rotation by Quaternion Operator	38
3.4.5	Matrix Form of Quaternion Production	41
3.4.6	Derivative of the Quaternion	41
<b>4.</b>	<b>Spacecraft Dynamics and Modeling</b>	<b>43</b>
4.1	The General Spacecraft System Equations	45
4.1.1	The Dynamics Equation	45
4.1.2	The Kinematics Equation	45
4.2	The Inertial Pointing Spacecraft Model	47
4.2.1	The Nonlinear Inertial Pointing Spacecraft Model	47
4.2.2	The Linearized Inertial Pointing Spacecraft Models	47
4.3	Nadir Pointing Momentum Biased Spacecraft Model	48
4.3.1	The Nonlinear Nadir Pointing Spacecraft Model	48
4.3.2	The Linearized Nadir Pointing Spacecraft Model	49
<b>5.</b>	<b>Space Environment and Disturbance Torques</b>	<b>53</b>
5.1	Gravitational Torques	54
5.2	Atmosphere-induced Torques	56
5.3	Magnetic Field-induced Torques	58
5.4	Solar Radiation Torques	63
5.5	Internal Torques	64
<b>6.</b>	<b>Spacecraft Attitude Determination</b>	<b>65</b>
6.1	Wahba's Problem	66
6.2	Davenport's Formula	67
6.3	Attitude Determination Using QUEST and FOMA	68
6.4	Analytic Solution of Two Vector Measurements	69
6.4.1	The Minimum-angle Rotation Quaternion	69
6.4.2	The General Rotation Quaternion	70
6.4.3	Attitude Determination Using Two Vector Measurements	72

---

6.5	Analytic Formula for General Case	74
6.5.1	Analytic Formula	75
6.5.2	Numerical Test	77
6.6	Riemann-Newton Method	78
6.7	Rotation Rate Determination Using Vector Measurements	80
<b>7.</b>	<b>Astronomical Vector Measurements</b>	<b>83</b>
7.1	Stars' Vectors	83
7.2	Earth's Magnetic Field Vectors	84
7.2.1	Ephemeris Earth's Magnetic Field Vector	84
7.2.2	Measured Earth's Magnetic Field Vector	85
7.3	Sun Vector	85
7.3.1	Ephemeris Sun Vector	85
7.3.2	Sun Vector Measurement	87
<b>8.</b>	<b>Spacecraft Attitude Estimation</b>	<b>89</b>
8.1	Extended Kalman Filter Using Reduced Quaternion Model	90
8.2	Kalman Filter Using Reduced Quaternion Model	94
8.3	A Short Comment	96
<b>9.</b>	<b>Spacecraft Attitude Control</b>	<b>97</b>
9.1	LQR Design for Nadir Pointing Spacecraft	98
9.2	The LQR Design for Inertial Pointing Spacecraft	99
9.2.1	The Analytic Solution	99
9.2.2	The Global Stability of the Design	100
9.2.3	The Closed-loop Poles	102
9.2.4	The Simulation Result	106
9.3	The LQR Design is a Robust Pole Assignment	107
9.3.1	Robustness of the Closed-loop Poles	107
9.3.2	The Robust Pole Assignment	108
9.3.3	Disturbance Rejection of Robust Pole Assignment	113
9.3.4	A Design Example	114
<b>10.</b>	<b>Spacecraft Actuators</b>	<b>119</b>
10.1	Reaction Wheel and Momentum Wheel	119
10.2	Control Moment Gyros	120
10.3	Magnetic Torque Rods	121
10.4	Thrusters	123
<b>11.</b>	<b>Spacecraft Control Using Magnetic Torques</b>	<b>125</b>
11.1	The Linear Time-varying Model	127
11.2	Spacecraft Controllability Using Magnetic Torques	130
11.3	LQR Design Based on Periodic Riccati Equation	137

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11.3.1 Preliminary Results	138
11.3.2 Solution of the Algebraic Riccati Equation	140
11.3.3 Solution of the Periodic Riccati Algebraic Equation	141
11.3.4 Simulation Test	146
11.4 Attitude and Desaturation Combined Control	148
11.4.1 Spacecraft Model for Attitude and Reaction Wheel Desaturation Control	151
11.4.2 Linearized Model for Attitude and Reaction Wheel Desaturation Control	154
11.4.3 The LQR Design	159
11.4.3.1 Case 1: $i_m = 0$	159
11.4.3.2 Case 2: $i_m \neq 0$	160
11.4.4 Simulation Test and Implementation Consideration	161
11.4.4.1 Comparison with the Design without Reaction Wheels	161
11.4.4.2 Control of the Nonlinear System	163
11.4.4.3 Implementation to Real System	165
11.5 LQR Design Based on a Novel Lifting Method	165
11.5.1 Periodic LQR Design Based on Linear Periodic System	166
11.5.2 Periodic LQR Design Based on Linear Time-invariant System	168
11.5.3 Implementation and Numerical Simulation	174
11.5.3.1 Implementation Consideration	174
11.5.3.2 Simulation Test for the Problem in Section 11.3	176
11.5.3.3 Simulation Test for the Problem in Section 11.4	176
<b>12. Attitude Maneuver and Orbit-Raising</b>	<b>179</b>
12.1 Attitude Maneuver	179
12.2 Orbit-raising	181
12.3 Comparing Quaternion and Euler Angle Designs	185
<b>13. Attitude MPC Control</b>	<b>191</b>
13.1 Some Technical Lemmas	193
13.2 Constrained MPC and Convex QP with Box Constraints	194
13.3 Central Path of Convex QP with Box Constraints	198
13.4 An Algorithm for Convex QP with Box Constraints	199
13.5 Convergence Analysis	209
13.6 Implementation Issues	214
13.6.1 Termination Criterion	214
13.6.2 Initial $(x^0, y^0, z^0, \lambda^0, \gamma^0) \in \mathcal{N}_2(\theta)$	214
13.6.3 Step Size	215
13.6.4 The Practical Implementation	218
13.7 A Design Example	219
13.8 Proofs of Technical lemmas	221

---

<b>14. Spacecraft Control Using CMG</b>	<b>235</b>
14.1 Spacecraft Model Using Variable-speed CMG	237
14.2 Spacecraft Attitude Control Using VSCMG	241
14.2.1 Gain Scheduling Control	241
14.2.2 Model Predictive Control	242
14.2.3 Robust Pole Assignment	243
14.3 Simulation Test	244
<b>15. Spacecraft Rendezvous and Docking</b>	<b>249</b>
15.1 Introduction	249
15.2 Spacecraft Model for Rendezvous	251
15.2.1 The Model for Translation Dynamics	251
15.2.2 The Model for Attitude Dynamics	257
15.2.3 A Complete Model for Rendezvous and Docking	259
15.3 Model Predictive Control System Design	261
15.4 Simulation Test	263
<i>Appendices</i>	
<b>Appendix A: First Order Optimality Conditions</b>	<b>267</b>
A.1 Problem Introduction	267
A.2 Karush-Kuhn-Tucker Conditions	268
<b>Appendix B: Optimal Control</b>	<b>271</b>
B.1 General Discrete-time Optimal Control Problem	271
B.2 Solution of Discrete-time LQR Control Problem	272
B.3 LQR Control for Discrete-time LTI System	274
<b>Appendix C: Robust Pole Assignment</b>	<b>279</b>
C.1 Eigenvalue Sensitivity to the Perturbation	279
C.2 Robust Pole Assignment Algorithms	284
C.3 Misrikhanov and Ryabchenko Algorithm	295
<i>References</i>	<b>299</b>
<i>Index</i>	<b>321</b>



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# **Chapter 1**

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## **Introduction**

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Spacecraft attitude determination and control is an important part of a spacecraft to achieve its designed mission. As of today, many spacecrafts have been successfully launched, and most of them have performed well as they were designed. Many research papers have been published to address the attitude determination and control design problems. Several textbooks are available for students to learn the technology and for engineers to use as references.

The most popular spacecraft models for attitude determination algorithms and control design methods are the *Euler angle models* and the *quaternion models*. The Euler angle models have been proved very efficient as the linearized models are controllable, therefore, all standard linear control system design methods are directly applicable. The drawbacks related to the Euler angle methods are: (a) the designs based on linearized models may not globally stabilize the original nonlinear spacecraft, i.e., the design may not work when the attitude of the spacecraft is far away from the point where the linearization is performed; (b) the models depend on the rotational sequences, this can be error prone if several teams work on the same project and they use different rotational sequences; (c) for any rotational sequence, there is a singular point where the model is not applicable; and (d) since most attitude determination methods use quaternion to represent the spacecraft attitude, there is a need to transform quaternion into Euler angles. On the other hand, for quaternion models, people have found controllers that can globally stabilize nonlinear spacecraft systems; the models do not depend on rotational sequences and they have no singular point; and the quaternion is provided by attitude determination system and is ready to use. The main problem with the quaternion model-based control system design is that the linearized quaternion model is not controllable. Therefore, most published design methods heavily rely on Lyapunov functions for the nonlinear spacecraft

system. But there is no systematic way to obtain a desired Lyapunov functions. Moreover, the Lyapunov function-based designs focus on the closed-loop system stability but pay little attention to its performance.

In a series of papers, the author proposed some *reduced quaternion* models which lead to some controllable linearized spacecraft models. Therefore, all standard linear system theory can be directly applied to analyze and design the spacecraft control systems. We showed that, in some cases, the designed control system is not only optimal for the linearized system, but also globally stabilize the original *nonlinear system*. Clearly, the reduced quaternion models do not depend on rotational sequences. Due to the special structure of the linearized spacecraft model, some most important design methods, such as LQR design and robust pole assignment design are very simple, enjoy the analytical solutions for some problems, have direct connection to the performance measures, such as *settling time*, *rising time*, and *percentage of overshoot*. All these features are attractive for high quality control system designs.

The idea mentioned above is then extended to more spacecraft control problems using specific actuators, such as magnetic torque bars and control momentum gyroscopes. These types of actuators may not provide the exactly desired torque. Most existing methods use different conversions to get approximate solutions, meaning that these actuators may generate a torque close to but not equal to the desired one. Using the reduced quaternion models that incorporate the actuators into the system model, the control inputs are not torques but the operational parameters. The main benefit of this idea is that the control actions are not approximate but accurate. As all actuators have their operational limit, design with input constraints have also been considered in this book by using recently developed interior-point optimization techniques.

This book is a result of the author's research for over a decade on the spacecraft attitude determination and control design methods which are focused on using reduced quaternion models because of their merits as stated above. It provides all necessary background materials on orbital dynamics, rotations and quaternion, frequently used reference frames, transformations between reference frames, space environment and disturbance torques, ephemeris astronomical vector calculations and measurement instruments, spacecraft control actuators and their models, so that the readers get a global picture and can apply all this information into the spacecraft system modeling, attitude determination, and spacecraft control system designs, which is the main purpose of this book.

This book is different from existing books as it focuses on the quaternion-based spacecraft control system designs and considers only attitude control system design-related problems, from spacecraft modeling to attitude determination and estimation, to control system design method selection, to control algorithm development, and the simulation of the control system designs. Moreover, this book addresses different attitude control tasks in the spacecraft life cycle, including spacecraft maneuver, orbit raising, attitude control, and rendezvous. Finally,

this book emphasizes the state-space design methods rather than the classical frequency design methods.

## 1.1 Organization of the Book

This book is organized as follows: [Chapter 2](#) is a brief description of orbit dynamics and properties. The treatment is focused on two body systems, which provides necessary background to be used in other chapters, for example, [Chapters 3, 11, and 15](#).

[Chapter 3](#) discusses the frequently used coordinate system, the rotational sequences, and the quaternion mathematics. Similar to [Chapter 2](#), this chapter provides readers the tools and background that will be repeatedly used in the rest of the chapters.

[Chapter 4](#) introduces two spacecraft dynamical systems based on the spacecraft missions, and their representations using the reduced quaternion models. The merit of using reduced quaternion models is that their linearized spacecraft models are controllable while the spacecraft models using full quaternion are not. It is well known that all modern linear control system design methods require the systems to be controllable. This makes the reduced quaternion spacecraft model very attractive. The ultimate goal of this chapter is to establish a few linearized controllable spacecraft models for some mostly desired attitudes for spacecraft, i.e., the inertial pointing attitude and the nadir pointing attitude.

[Chapter 5](#) explains the space environment and the major disturbance torques introduced in the space environment. Most of these torques are difficult to be included in the spacecraft models which are used in spacecraft attitude control system designs. This means that the designed controllers do not consider the effects of these disturbance torques. As a result, the designed controllers may not work in the real space environment because the control torques may not compensate these unmodeled torques. Because of this reason, there is a need to have some simulation test for the designed spacecraft feedback control system to make sure that the designed controller works in the space environment that includes these disturbance torques. This chapter provides the necessary information so that control engineers can build the simulated space environment to test the designed controller.

[Chapter 6](#) discusses the quaternion-based attitude determination methods using vector measurements, including some recently proposed methods. In principle, spacecraft attitude can be determined by a set of observed (measured) astronomical vectors and corresponding ephemeris astronomical vectors at the given time. An important problem is to find some fast, accurate, and robust algorithms to calculate the spacecraft attitude. Though there are other attitude determination methods based on rotational matrix or Euler angle representation, it must be

pointed out that quaternion-based attitude determination methods are the most efficient ones.

[Chapter 7](#) explains how to measure the astronomical vectors and how to calculate the corresponding ephemeris astronomical vectors at any given time. The most widely used astronomical vectors are considered. Given the ephemeris information of the astronomical objects represented in reference frame and measured astronomical vectors represented in body frame, the spacecraft attitude can be obtained using the methods described in [Chapter 6](#).

Since there always exist some random measurement noises, there is a need to have some filtering techniques to reduce the measurement noise effect. Kalman filter was developed in 1960s just for this purpose and this technique was widely used in spacecraft attitude determination. [Chapter 8](#) discusses the attitude estimation problem using extended and traditional Kalman filters.

[Chapter 9](#) talks about attitude control system designs with the desired torques as control variables. This chapter focuses on state-space Linear Quadratic Regulator (LQR) design method. For nadir pointing spacecraft, the solution described in [Appendix B](#) can be applied directly. But for inertial pointing spacecraft, which has an extremely simple linearized model, an analytic solution exists. For this case, the relation between the LQR design and the closed-loop pole positions is established. The analytical solution provides insight for engineers to trade off many conflict requirements. It is shown that the design globally stabilizes the nonlinear spacecraft system even though it is based on the linearized system. As a matter of fact, the LQR design discussed in this chapter is actually a robust pole assignment design. Therefore, the design is insensitive to the modeling error and is good for disturbance rejection.

All designs in [Chapter 9](#) calculate the desired torques that are used to control the spacecraft attitude. These desired torques are supplied by using several different actuators or their combinations. [Chapter 10](#) reviews some widely used spacecraft actuators, including reaction wheel and momentum wheel, control moment gyros, magnetic torque rods, and thrusters. This chapter reveals a fact that several types of actuators are not able to provide the desired torques in all directions. Therefore, the methods discussed in [Chapter 9](#) (when these actuators are used) have a torque realization problem. A better design practice should include the actuators' models in the control system design. This consideration will be the topics in rest of the chapters.

[Chapter 11](#) discusses system designs for spacecraft using magnetic torque rods. Although magnetic torque bars can provide torques only in a plane instead of three-dimensional space at any time, it is shown that the controllability of spacecraft using only magnetic torques is achievable under some mild conditions. Using the fact that the magnetic field is approximately a periodic function of the spacecraft orbit, periodic LQR design is considered in the controller design. Some efficient solutions for the algebraic periodic Riccati equation are proposed.

[Chapter 12](#) discusses the spacecraft control system design using thrusters. A typical operation using thrusters, orbit-raising, is considered in this chapter. The control system models and controller designs depend on the thruster configurations. This chapter describes how to design the controller using the standard linear system theory. Although a particular thruster configuration is considered in this chapter, the idea can easily be used for any other thruster configurations.

[Chapter 13](#) addresses Model Predictive Control (MPC) and its application to the spacecraft attitude control problems. Since MPC needs extensive on-board computation, it was not widely used in spacecraft control, as more powerful computers are installed on spacecraft. MPC is expected to find more applications in aerospace in the future. This chapter establishes the relation between constrained MPC and convex quadratic programming (QP) with box constraints. This formulation is directly applicable to the controller design problem when actuator saturation exists. An efficient interior-point algorithm specifically for this problem is proposed and its convergence is proved. The thruster control problem discussed in [Chapter 12](#) is revisited and it is shown that the problem can be solved by the MPC control method proposed in this chapter.

[Chapter 14](#) is dedicated to the spacecraft attitude control system design using control moment gyros. As it has already been explained in [Chapter 10](#) that for given desired torques obtained in [Chapter 9](#), there are singular points where one cannot find gimbal speeds of the CMGs to achieve the desired torques. This chapter presents a new operational concept for control moment gyros and proposes a MPC design method for this problem. Simulation test is used to demonstrate the feasibility of the proposed method.

[Chapter 15](#) considers coupled orbit and attitude control that is the key technology for spacecraft rendezvous and soft docking. Coupled orbit and attitude control is an extensively studied problem with renewed interest because of installations of powerful on-board computers, availability of advanced theoretical results, and requirements for better performance in future missions. The method considered in this chapter addresses a fundamental requirements for soft docking, i.e., there is no oscillation crossing the horizontal line for relative position and relative attitude between chaser and target spacecraft to avoid collision during the docking process.

Three appendices are included for quick reference for the background used in the control system design methods discussed in this book. [Appendix A](#) is about the first order optimality conditions, which is used in several chapters and in [Appendix B](#). [Appendix B](#) provides LQR problem formulation and numerical solutions. [Appendix C](#) summarizes the background and solutions for robust pole assignment design which has been used in several chapters. For readers who need to know more background information on optimization and control theory, they are referred to some standard textbooks [8, 41, 92, 108, 142, 168, 233] listed in the references.

## 1.2 Some Basic Notations and Identities

In this book, vectors are denoted by small case letters with bold font, for example,  $\mathbf{a}$  is a vector. Vector magnitude is denoted by normal font, for example,  $a$  is the magnitude of  $\mathbf{a}$ . A  $n$ -dimensional linear space is denoted by  $\mathbf{R}^n$ . A collection of all real points is denoted by  $\mathbf{R}$ . Matrices are denoted by capital letters with bold font, for example,  $\mathbf{A}$  is a matrix, its magnitude is denoted by 2-norm  $\|\mathbf{A}\|$  unless it is explicitly indicated that other matrix norm is used. A  $n \times m$  matrix space, or the collection of all  $n \times m$  linear transformation, is denoted by  $\mathbf{R}^{n \times m}$ .

Throughout this book, we will use some common notations. For a column vector  $\mathbf{x} = [x_1, x_2, \dots, x_n]^T$ , we sometimes write it as  $\mathbf{x} = (x_1, x_2, \dots, x_n)$  to save space. For any two vectors  $\mathbf{x}$  and  $\mathbf{y}$ , we will denote by  $\mathbf{x} \cdot \mathbf{y} = \mathbf{x}^T \mathbf{y}$  the dot product of  $\mathbf{x}$  and  $\mathbf{y}$ , by  $\mathbf{x} \times \mathbf{y}$  the cross product of  $\mathbf{x}$  and  $\mathbf{y}$ , by  $\mathbf{x} \circ \mathbf{y}$  the element-wise or Hadamard product of  $\mathbf{x}$  and  $\mathbf{y}$ , by  $\frac{\mathbf{x}}{\mathbf{y}}$  the element-wise division of  $\mathbf{x}$  and  $\mathbf{y}$  if all elements of  $\mathbf{y}$  are not zero, by  $\|\mathbf{x}\|$  the 2-norm of the vector of  $\mathbf{x}$ . For a vector  $\mathbf{x}$ , we use  $\mathbf{X}$  to denote a matrix whose diagonal elements are the vector  $\mathbf{x}$ . Let  $\mathbf{a}$ ,  $\mathbf{b}$ , and  $\mathbf{c}$  be any three dimensional vectors, we will repeatedly use the following identities.

$$\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a} \quad (1.1)$$

$$(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{b} \cdot \mathbf{c})\mathbf{a} \quad (1.2)$$

and

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c} \quad (1.3)$$

and

$$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a} = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = 0 \quad (1.4)$$

We denote

$$\mathbf{i} = (1, 0, 0), \quad \mathbf{j} = (0, 1, 0), \quad \mathbf{k} = (0, 0, 1) \quad (1.5)$$

for the standard basis for  $\mathbf{R}^3$ , and  $\mathbf{S}(\mathbf{x})$  a skew-symmetric matrix function of  $\mathbf{x} = [x_1, x_2, x_3]^T$  defined by

$$\mathbf{S}(\mathbf{x}) = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix} = \mathbf{x}^\times \quad (1.6)$$

The *cross product* of  $\mathbf{x} \times \mathbf{y}$  can then be represented by a matrix multiplication  $\mathbf{S}(\mathbf{x})\mathbf{y}$ , i.e.,  $\mathbf{x} \times \mathbf{y} = \mathbf{S}(\mathbf{x})\mathbf{y} = \mathbf{x}^\times \mathbf{y}$ . We will use  $\bar{\mathbf{p}}$ ,  $\bar{\mathbf{q}}$ , and  $\bar{\mathbf{r}}$  to denote quaternions which will be defined later on.

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