

$$4) I = \int e^{5x} \cdot \sin(2x) \, dx$$

$$u = e^{5x}$$

$$\frac{du}{dx} = 5 \cdot e^{5x}$$

$$I = \int u \cdot dv = u \cdot v - \int v \cdot du$$

$$du = 5 \cdot e^{5x} \, dx$$

$$I = -\frac{e^{5x}}{2} \cdot \cos 2x + \frac{5}{2} \int e^{5x} \cos 2x \, dx$$

$$dv = \sin 2x \, dx$$

$$I = -\frac{e^{5x}}{2} \cos 2x + \frac{5}{2} \int a \, db$$

$$v = -\frac{1}{2} \cdot \cos 2x$$

$$a = e^{5x} \Rightarrow da = 5 \cdot e^{5x} \, dx$$

$$db = \cos 2x \, dx \Rightarrow b = \frac{1}{2} \cdot \sin 2x$$

$$I = -\frac{e^{5x}}{2} \cos 2x + \frac{5}{2} \left[ a \cdot b - \int b \cdot da \right]$$

$$I = -\frac{e^{5x}}{2} \cos 2x + \frac{5}{2} \left[ \frac{e^{5x}}{2} \sin 2x - \frac{5}{2} \int e^{5x} \sin 2x \, dx \right]$$

$$I = -\frac{e^{5x}}{2} \cos 2x + \frac{5}{4} e^{5x} \cdot \sin 2x - \frac{25}{4} I + C$$

$$\frac{29}{4} I = \frac{5}{4} e^{5x} \sin 2x - \frac{e^{5x}}{2} \cos 2x + C$$

$$I = \frac{5 \cdot e^{5x} \sin 2x}{29} - \frac{2 \cdot e^{5x} \cos 2x}{29} + C$$