# Mallow's $C_P$

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- $y_{n\times 1} = X_{n\times m}\beta_{m\times 1} + \epsilon_{n\times 1}$
- model $\mathcal{P}$, $|\mathcal{P}| = p \le m$
- $\hat{\beta}_{\mathcal{P}} = (X_{\mathcal{P}}^T X_{\mathcal{P}})^{-} X_{\mathcal{P}}^T y$, g-inverse
- $\hat{y} = X_{\mathcal{P}}\hat{\beta}_{\mathcal{P}}$
- $y - \hat{y} = (I_n - Q_{\mathcal{P}})y$, where the projection matrix

  $$Q_{\mathcal{P}} = X_{\mathcal{P}}(X_{\mathcal{P}}^T X_{\mathcal{P}})^{-} X_{\mathcal{P}}^T$$

  is idempotent and symmetric
- $RSS_{\mathcal{P}} = \|y - \hat{y}\|_2^2 = (y-\hat{y})^T(y-\hat{y}) = y^T(I_n - Q_{\mathcal{P}})y$
- $Ey = \mu$, $Vary = \Sigma$, then $Ey^TAy = \mu^TA\mu + tr(\Sigma A)$
- $$\begin{aligned} ERSS_{\mathcal{P}} &= (X\beta)^T(I_n - Q_{\mathcal{P}})(X\beta) + tr(\sigma^2(I_n - Q_{\mathcal{P}})) \\ &= (X_{\bar{\mathcal{P}}}\beta_{\bar{\mathcal{P}}})^T(I_n - Q_{\mathcal{P}})(X_{\bar{\mathcal{P}}}\beta_{\bar{\mathcal{P}}}) + \sigma^2(n-p) \end{aligned}$$
- **Define** $C_{\mathcal{P}} = \frac{1}{\hat{\sigma}^2}RSS_{\mathcal{P}} - n + 2p$
- $EC_{\mathcal{P}} \approx n - p - n + 2p = p$ if $X_{\bar{\mathcal{P}}}\beta_{\bar{\mathcal{P}}} = 0$ (the reduced model is true).